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# 16:332:599:02 – Smart Grid Project Report

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May 9<sup>th</sup>, 2018



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# 1 Case Study/Problem Formulation

For the single-line diagram in Figure 4 convert all positive-sequence impedance, load, and voltage data to per unit using the given system base quantities. Run the power flow program and obtain the bus, line, and transformer input/output voltages

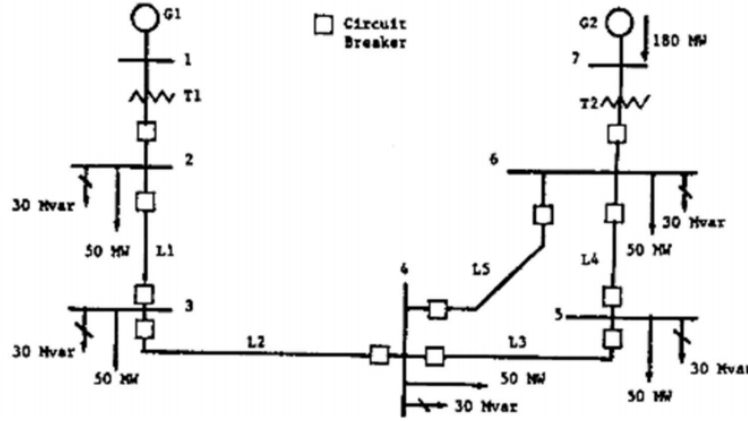


Figure 1: Original Circuit

Generator Ratings	
<b>G1</b>	100MVA, 13.8kV, $x'' = 0.12$
<b>G2</b>	200MVA, 15.0kV, $x'' = 0.12$
<i>The generator neutrals are solidly grounded</i>	

Transformer Ratings	
<b>T1</b>	100MVA, 13.8kV $\Delta$ /230kVY, $x = 0.1$ per unit
<b>T2</b>	200MVA, 15kV $\Delta$ /230kVY, $x = 0.1$ per unit
<i>The transformer neutrals are solidly grounded</i>	

Transmission Line Ratings	
<b>All Lines</b>	230kV, $z_1 = 0.08 + j0.5\Omega/km$ , $y_1 = j3.3E - 6S/km$ , $Max\ MVA = 400$
<b>Line Lengths</b>	$L_1 = 15km$ , $L_2 = 20km$ , $L_3 = 40km$ , $L_4 = 15km$ , $L_5 = 50km$

Power Flow Data	
<b>Bus 1</b>	Swing bus $V_1 = 13.8kV$ , $\partial_1 = 0^\circ$
<b>Bus 2, 3, 4, 5, 6</b>	Load buses
<b>Bus 7</b>	Constant voltage magnitude bus, $V_7 = 15kV$ , $P_{G7} = 180MW$ , $-87MVA_r < Q_{G7} < +87MVA_r$

System Base Quantities	
$S_{base} = 100MVA$ (three-phase)	
$V_{base} = 13.8kV$ (line-to-line) in the zone of $G_1$	

Table 1: Circuit Values

## 2 Detailed Solution

### 2.1 Calculating Per Unit Values

$$\text{per unit (pu) value} = \frac{\text{Actual value}}{\text{Base value}}$$

In zone G1, base values are,

$$S_{base} = 100 \text{ MVA}$$

$$V_{base_{L-L}} = 13.8 \text{ kV}$$

@ G1

$$S_{G1(pu)} = \frac{100 \text{ MVA}}{100 \text{ MVA}} = \boxed{1pu}$$

$$V_{G1(pu)} = \frac{13.8 \text{ kV}}{13.8 \text{ kV}} = \boxed{1pu}$$

$$x''_{(new)} = \boxed{0.12pu}$$

@ T1

$$S_{T1(pu)} = \frac{100 \text{ MVA}}{100 \text{ MVA}} = \boxed{1pu}$$

$$V_{T1(pu)} = \frac{13.8 \text{ kV}}{13.8 \text{ kV}} = \boxed{1pu}$$

$$x_{(pu)} = \boxed{0.1pu}$$

@ Bus 2

$$Q_{Load} = 30 \text{ MVar}$$

$$Q_{Load(pu)} = \frac{30 \text{ MVar}}{100 \text{ MVA}} = \boxed{0.3pu}$$

$$P_{Load} = 50 \text{ MW}$$

$$P_{Load(pu)} = \frac{50 \text{ MW}}{100 \text{ MVA}} = \boxed{0.5pu}$$

@ L<sub>1</sub>

$$L_1 = 15 \text{ km}$$

$$z = 0.08 + j0.5 \Omega/\text{km}$$

$$Z_{total} = (15 \text{ km})(0.08 + j0.5 \Omega/\text{km}) = \boxed{1.2 + j7.5 \Omega}$$

$$\theta = \tan^{-1} \left( \frac{7.5}{1.2} \right) = \boxed{80.91^\circ}$$

$$|Z_{total}| = \sqrt{(1.2)^2 + (7.5)^2} = \boxed{7.59539 \Omega}$$

$$Z_{total} = \boxed{7.59539 \angle 80.91^\circ \Omega}$$

$$Z_{total(pu)} = 7.595 \cdot \left( \frac{100MV A}{(230kV)^2} \right) = \boxed{0.014358 pu}$$

@ Bus 3

$$Q_{Load} = 30MV Ar$$

$$Q_{Load(pu)} = \frac{30MV Ar}{100MV A} = \boxed{0.3 pu}$$

$$P_{Load} = 50MW$$

$$P_{Load(pu)} = \frac{50MW}{100MV A} = \boxed{0.5 pu}$$

@ Bus 4

$$Q_{Load} = 30MV Ar$$

$$Q_{Load(pu)} = \frac{30MV Ar}{100MV A} = \boxed{0.3 pu}$$

$$P_{Load} = 50MW$$

$$P_{Load(pu)} = \frac{50MW}{100MV A} = \boxed{0.5 pu}$$

@ Bus 5

$$Q_{Load} = 30MV Ar$$

$$Q_{Load(pu)} = \frac{30MV Ar}{100MV A} = \boxed{0.3 pu}$$

$$P_{Load} = 50MW$$

$$P_{Load(pu)} = \frac{50MW}{100MV A} = \boxed{0.5 pu}$$

@ Bus 6

$$Q_{Load} = 30MV Ar$$

$$Q_{Load(pu)} = \frac{30MV Ar}{100MV A} = \boxed{0.3 pu}$$

$$P_{Load} = 50MW$$

$$P_{Load(pu)} = \frac{50MW}{100MV A} = \boxed{0.5 pu}$$

@  $L_2$

$$L_2 = 20km$$

$$z = 0.08 + j0.5\Omega/km$$

$$Z_{total} = (20km)(0.08 + j0.5\Omega/km) = \boxed{1.6 + j10\Omega}$$

$$\theta = \tan^{-1} \left( \frac{10}{1.6} \right) = \boxed{80.91^\circ}$$

$$|Z_{total}| = \sqrt{(1.6)^2 + (10)^2} = \boxed{10.127191\Omega}$$

$$Z_{total} = \boxed{10.127191 \angle 80.91^\circ \Omega}$$

$$Z_{total(pu)} = 10.127191 \cdot \left( \frac{100MV A}{(230kV)^2} \right) = \boxed{0.019144pu}$$

@  $L_3$

$$L_3 = 40km$$

$$z = 0.08 + j0.5\Omega/km$$

$$Z_{total} = (40km)(0.08 + j0.5\Omega/km) = \boxed{3.2 + j20\Omega}$$

$$\theta = \tan^{-1} \left( \frac{20}{3.2} \right) = \boxed{80.91^\circ}$$

$$|Z_{total}| = \sqrt{(3.2)^2 + (20)^2} = \boxed{20.254382\Omega}$$

$$Z_{total} = \boxed{20.254382 \angle 80.91^\circ \Omega}$$

$$Z_{total(pu)} = 20.254382 \cdot \left( \frac{100MV A}{(230kV)^2} \right) = \boxed{0.038288pu}$$

@  $L_4$

$$L_4 = 15km$$

$$z = 0.08 + j0.5\Omega/km$$

$$Z_{total} = (15km)(0.08 + j0.5\Omega/km) = \boxed{1.2 + j7.5\Omega}$$

$$\theta = \tan^{-1} \left( \frac{7.5}{1.2} \right) = \boxed{80.91^\circ}$$

$$|Z_{total}| = \sqrt{(1.2)^2 + (7.5)^2} = \boxed{7.59539\Omega}$$

$$Z_{total} = \boxed{7.59539 \angle 80.91^\circ \Omega}$$

$$Z_{total(pu)} = 7.595 \cdot \left( \frac{100MV A}{(230kV)^2} \right) = \boxed{0.014358pu}$$

@  $L_5$

$$L_5 = 50km$$

$$z = 0.08 + j0.5\Omega/km$$

$$Z_{total} = (50km)(0.08 + j0.5\Omega/km) = \boxed{4 + j25\Omega}$$

$$\theta = \tan^{-1} \left( \frac{25}{4} \right) = \boxed{80.91^\circ}$$



$$|Z_{total}| = \sqrt{(4)^2 + (25)^2} = \boxed{25.317978\Omega}$$

$$Z_{total} = \boxed{25.317978\angle 80.91^\circ\Omega}$$

$$Z_{total(pu)} = 25.317978 \cdot \left( \frac{100MV A}{(230kV)^2} \right) = \boxed{0.04786pu}$$

@ Bus 7

$$P_{G7(pu)} = \frac{180MW}{100MV A} = \boxed{1.8pu}$$

$$\frac{-87MV Ar}{100} < Q_{G7(pu)} < \frac{87MV Ar}{100}$$

$$-0.87pu < Q_{G7(pu)} < 0.87pu$$

$$V_{T2(pu)} = \frac{15kV}{13.8kV} = \boxed{1.086956pu}$$

### 2.1.1 Per-Unit One-Line Model

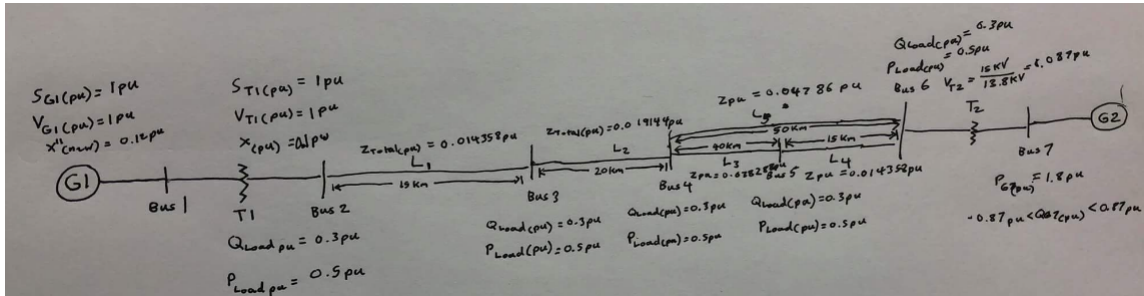


Figure 2: Per-Unit One-line Diagram

## 2.2 Compute All Bus Voltages

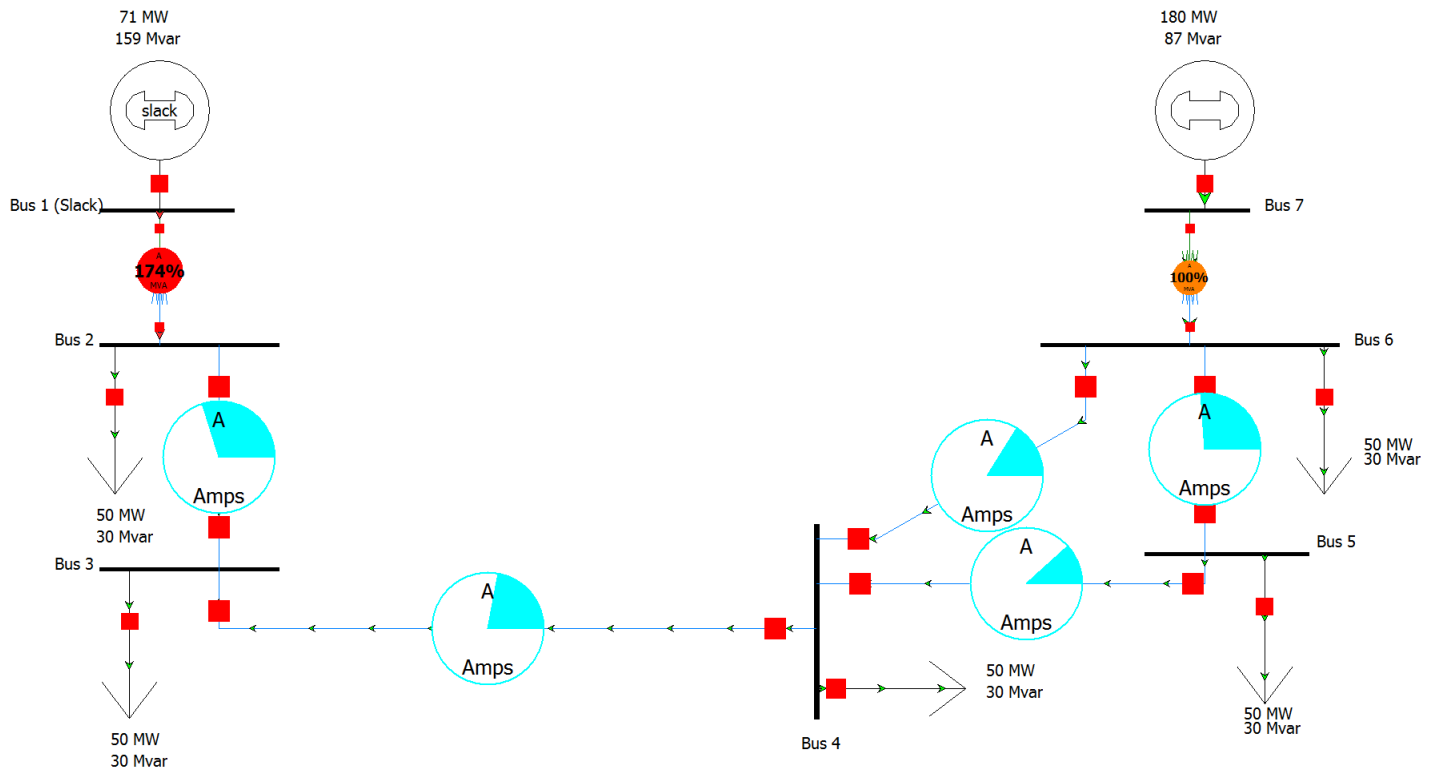


Figure 3: PowerWorld Diagram

## 2.2.1 Newton-Raphson Method

### Bus 1 (Slack)

Bus: Bus 1 (Slack) (1)  
 Nom kV: 13.80  
 Area: 1 (1)  
 Zone: 1 (1)  
 1.0000 pu  
 13.80 KV  
 0.00 Deg  
 Not Valid \$/MWh

System State

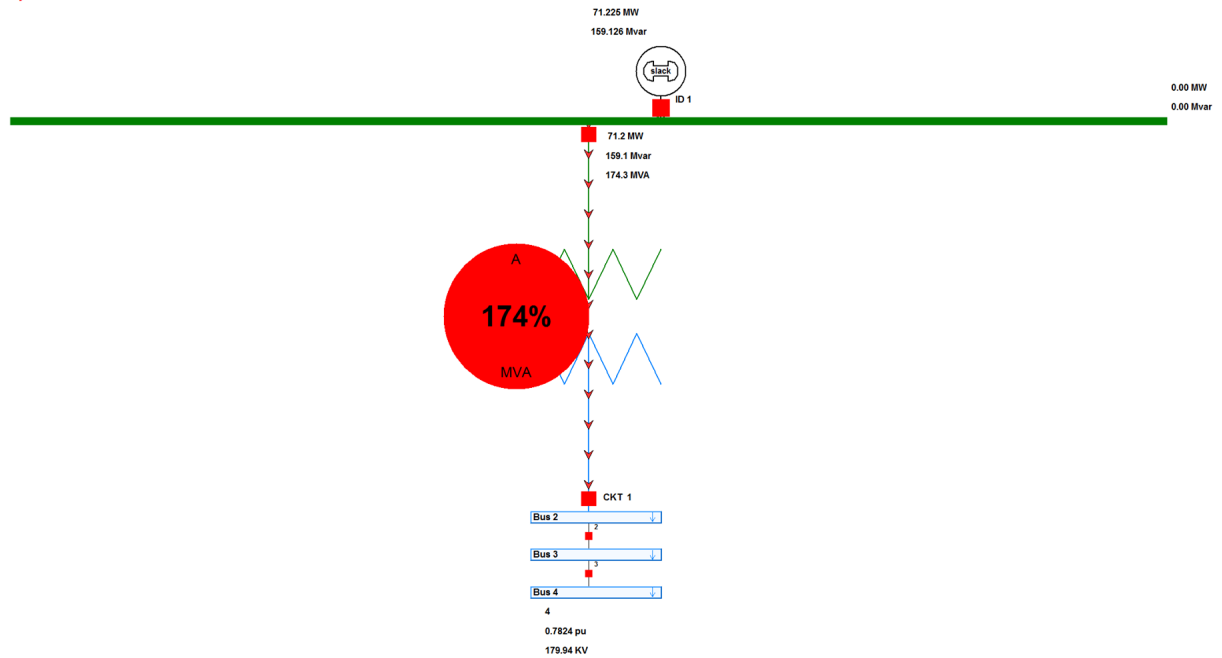


Figure 4: PowerWorld Diagram

### Bus 2

Bus: Bus 2 (2)  
 Nom kV: 230.00  
 Area: 1 (1)  
 Zone: 1 (1)  
 0.8439 pu  
 194.09 KV  
 -4.94 Deg  
 Not Valid \$/MWh

System State

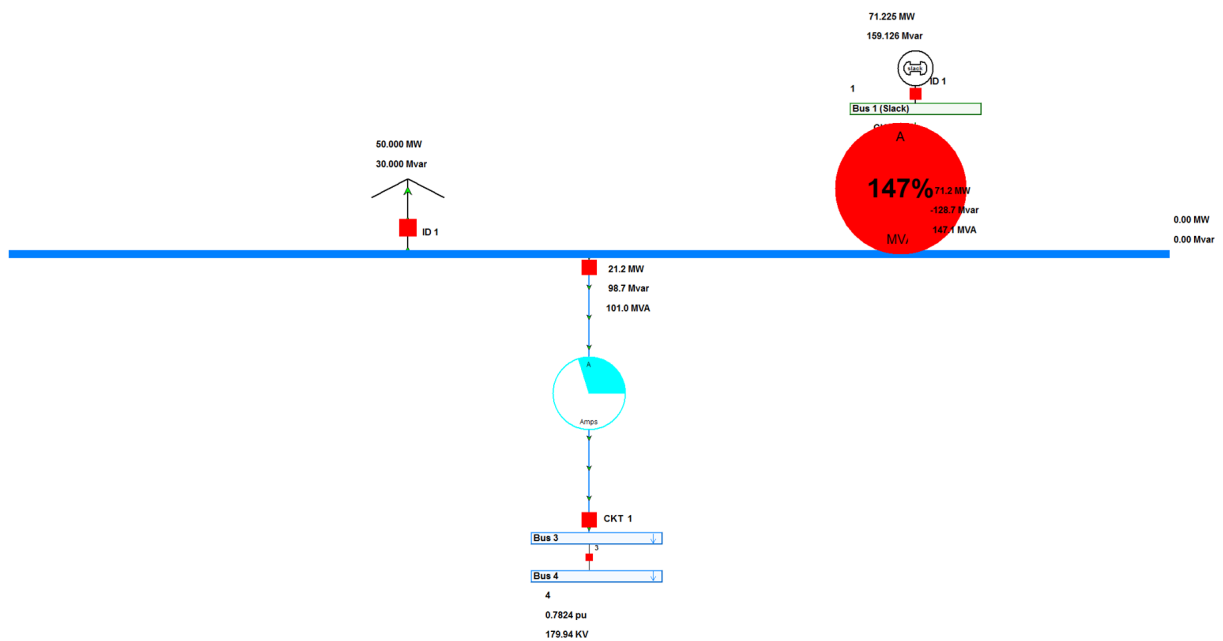


Figure 5: PowerWorld Diagram

**Bus 3**

Bus: Bus 3 (3)  
 Nom kV: 230.00  
 Area: 1 (1)  
 Zone: 1 (1)  
 0.8267 pu  
 190.15 KV  
 -4.90 Deg  
 Not Valid \$/MWh

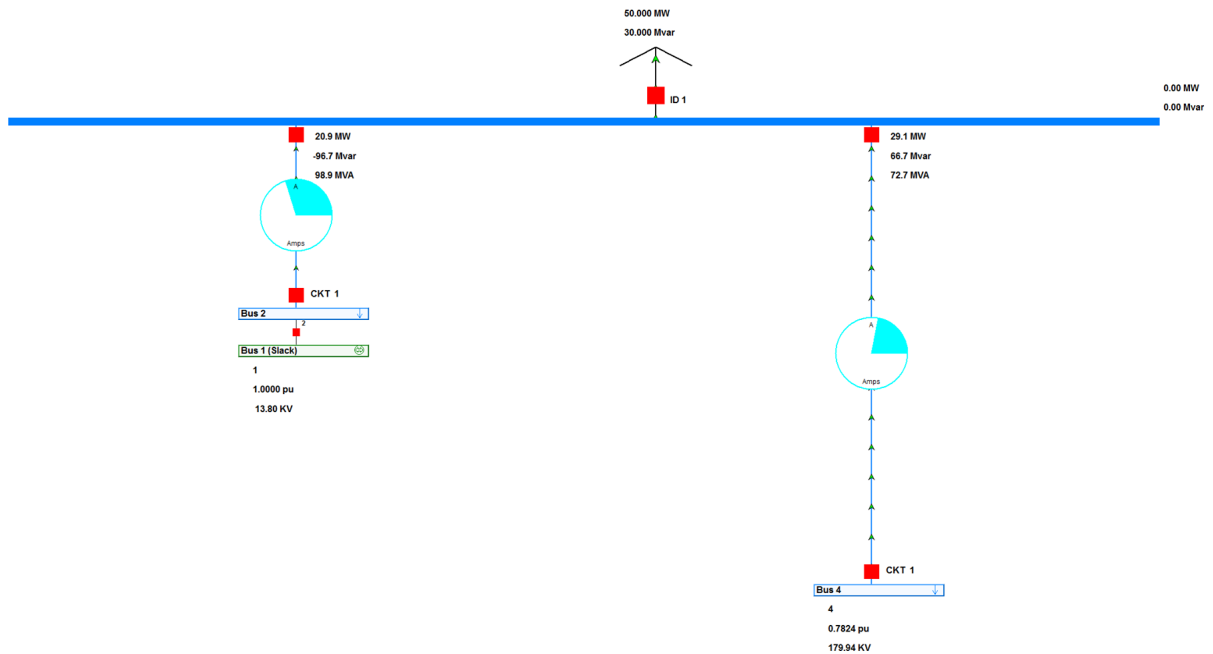


Figure 6: PowerWorld Diagram

**Bus 4**

Bus: Bus 4 (4)  
 Nom kV: 230.00  
 Area: 1 (1)  
 Zone: 1 (1)  
 0.7824 pu  
 179.94 KV  
 -3.26 Deg  
 Not Valid \$/MWh

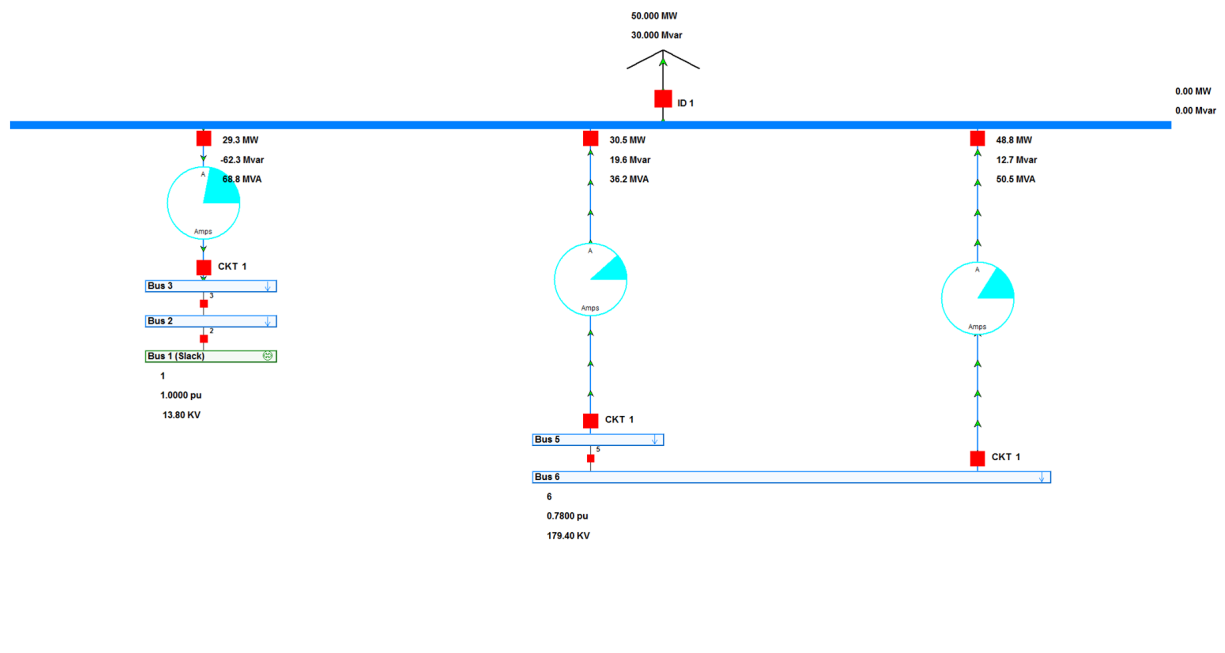


Figure 7: PowerWorld Diagram

**Bus 5**

Bus: Bus 5 (6)  
 Nom kV: 230.00  
 Area: 1 (1)  
 Zone: 1 (1)  
 0.7754 pu  
 178.35 KV  
 -2.06 Deg  
 Not Valid \$/MWh

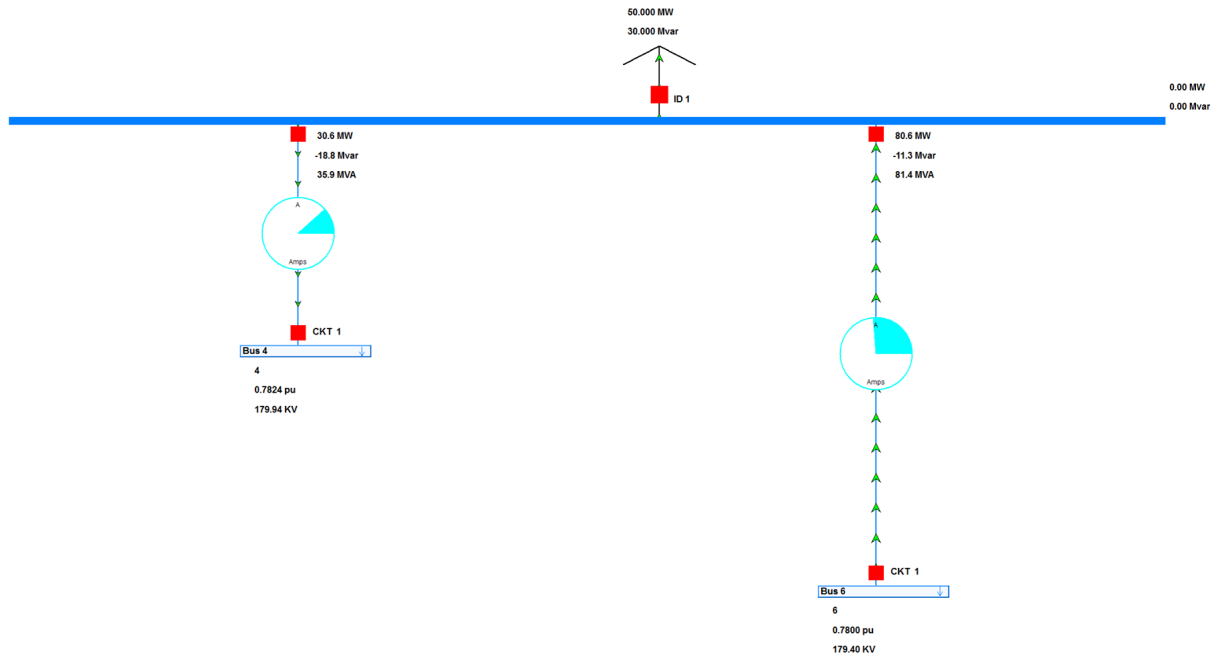


Figure 8: PowerWorld Diagram

**Bus 6**

Bus: Bus 6 (6)  
 Nom kV: 230.00  
 Area: 1 (1)  
 Zone: 1 (1)  
 0.7800 pu  
 179.40 KV  
 -1.01 Deg  
 Not Valid \$/MWh

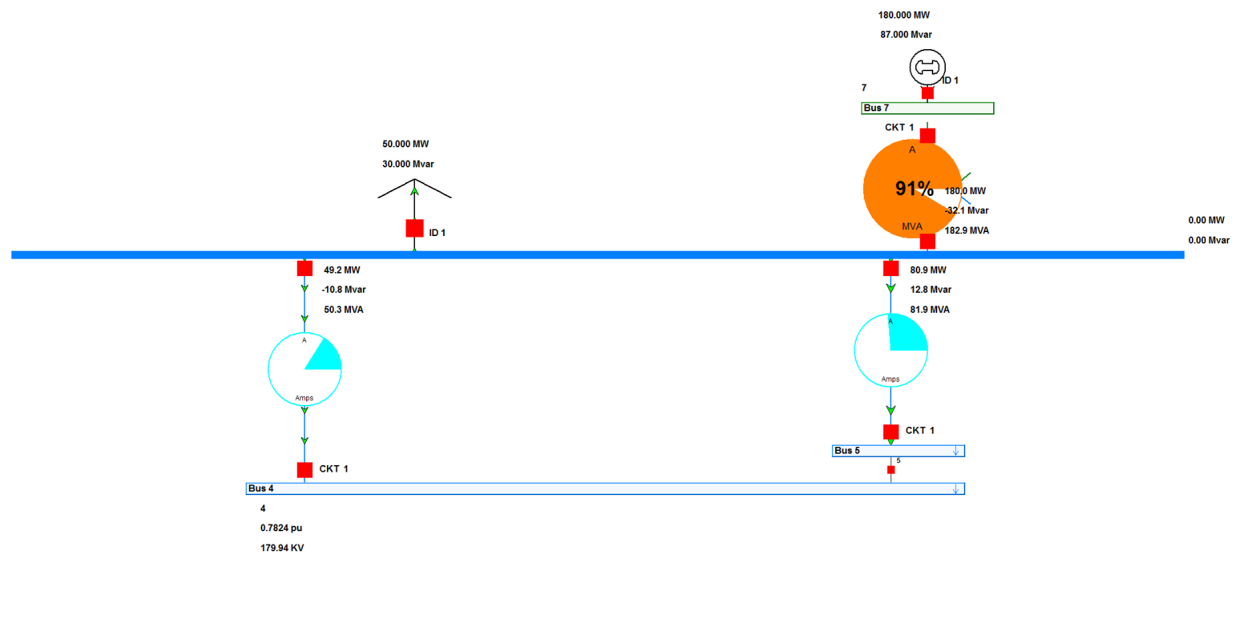


Figure 9: PowerWorld Diagram

**Bus 7**

Bus: Bus 7 (7)  
 Nom kV: 15.00  
 Area: 1 (1)  
 Zone: 1 (1)  
 0.8530 pu  
 12.79 KV  
 14.70 Deg  
 Not Valid \$/MWh

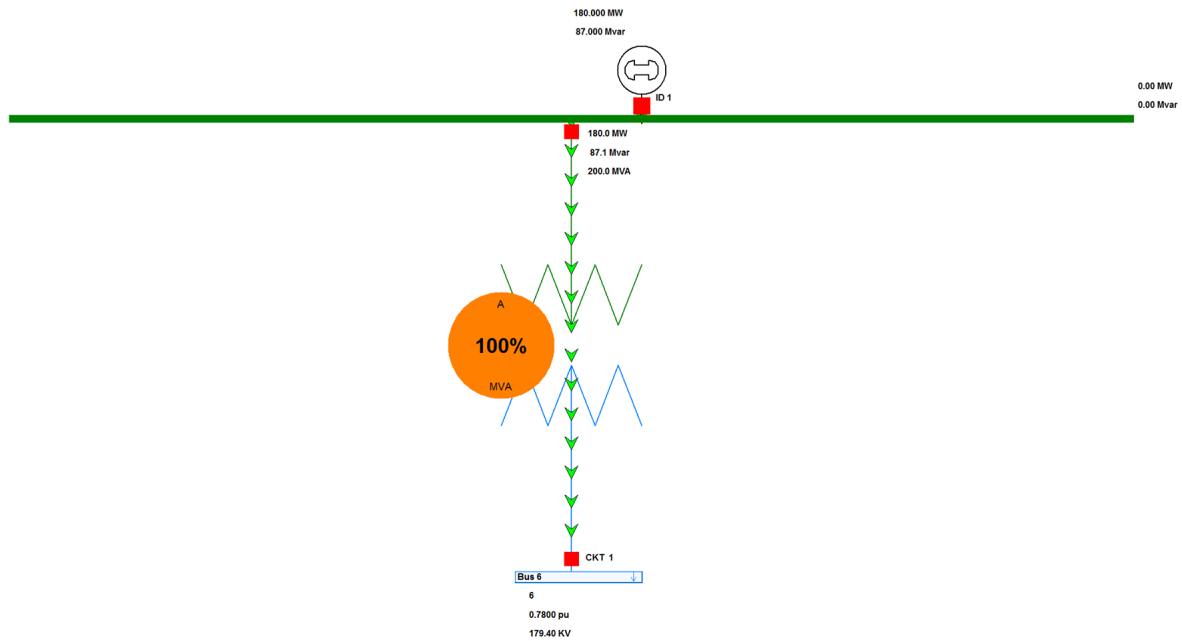


Figure 10: PowerWorld Diagram

Model Explorer: Buses

Explore Fields

Recent

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Branches Input

Branches State

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Number	Name	Area Name	Nom kV	PU Volt	Volt (kV)	Angle (Deg)	Load MW	Load Mvar	Gen MW	Gen Mvar	Switched Shunts Mvar	Act G Shunt MW	Act B Shunt Mvar	Area Num	Zone Num
1	Bus 1 (Slack)	1	13.80	1.00000	13.800	0.00								1	1
2	Bus 2	1	230.00	0.84389	194.094	-4.84	50.00	30.00		71.23	159.13		0.00	0.00	1
3	Bus 3	1	230.00	0.82673	190.147	-4.90	50.00	30.00					0.00	0.00	1
4	Bus 4	1	230.00	0.78236	178.943	-3.26	50.00	30.00					0.00	0.00	1
5	Bus 5	1	230.00	0.77543	178.349	-2.06	50.00	30.00					0.00	0.00	1
6	Bus 6	1	230.00	0.77999	179.397	-1.01	50.00	30.00					0.00	0.00	1
7	Bus 7	1	15.00	0.85297	12.795	14.70			180.00	87.00			0.00	0.00	1

Figure 11: PowerWorld Diagram

Model Explorer: Branches Input

Explore Fields

Recent

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Branches By Type

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	From Number	From Name	To Number	To Name	Circuit	Status	Branch Device Type	Xfmr	R	X	B	Lim MVA A	Lim MVA B	Lim MVA C
1	1	Bus 1 (Slack)	2	Bus 2	1	Closed	Transformer	YES	0.00000	0.10000	0.00000	100.0	0.0	0.0
2	2	Bus 2	3	Bus 3	1	Closed	Line	NO	0.00227	0.01418	0.00000	400.0	0.0	0.0
3	3	Bus 3	4	Bus 4	1	Closed	Line	NO	0.00302	0.05672	0.00000	400.0	0.0	0.0
4	4	Bus 4	5	Bus 5	1	Closed	Line	NO	0.00605	0.03781	0.00000	400.0	0.0	0.0
5	4	Bus 4	6	Bus 6	1	Closed	Line	NO	0.00756	0.04726	0.00000	400.0	0.0	0.0
6	5	Bus 5	6	Bus 6	1	Closed	Line	NO	0.00227	0.01418	0.00000	400.0	0.0	0.0
7	7	Bus 7	6	Bus 6	1	Closed	Transformer	YES	0.00000	0.10000	0.00000	200.0	0.0	0.0

Figure 12: PowerWorld Diagram

Model Explorer: Generators

Explore Fields

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	Number of Bus	Name of Bus	ID	Status	Gen MW	Gen Mvar	Set Volt	AGC	AVR	Min MW	Max MW	Min Mvar	Max Mvar	Cost Model	Part. Factor
1	1	Bus 1 (Slack)	1	Closed	71.23	159.13	1.00000	YES	YES	0.00	100.00	0.00	100.00	None	10.00
2	7	Bus 7	1	Closed	180.00	87.00	1.00000	YES	YES	0.00	180.00	-87.00	87.00	None	10.00

Figure 13: PowerWorld Diagram

Model Explorer: Loads

Explore Fields

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Branches State

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	Number of Bus	Name of Bus	Area Name of Load	Zone Name of Load	ID	Status	MW	Mvar	MVA	S MW	S Mvar	Dist Status	Dist MW Input	Dist Mvar Input
1	2	Bus 2	1	1	1	Closed	50.00	30.00	58.31	50.00	30.00	Closed	0.00	0.00
2	3	Bus 3	1	1	1	Closed	50.00	30.00	58.31	50.00	30.00	Closed	0.00	0.00
3	4	Bus 4	1	1	1	Closed	50.00	30.00	58.31	50.00	30.00	Closed	0.00	0.00
4	5	Bus 5	1	1	1	Closed	50.00	30.00	58.31	50.00	30.00	Closed	0.00	0.00
5	6	Bus 6	1	1	1	Closed	50.00	30.00	58.31	50.00	30.00	Closed	0.00	0.00

Figure 14: PowerWorld Diagram

Model Explorer: Bus Zero-Impedance Branch Group

Explore Fields

Filter Advanced Bus

Number	Name	Area Name	Nom kV	ZBR Bus Primary	ZBR Bus Neighbor List
1	Bus 1 (Slack)	1	13.80	1	
2	2 Bus 2	1	230.00	2	
3	3 Bus 3	1	230.00	3	
4	4 Bus 4	1	230.00	4	
5	5 Bus 5	1	230.00	5	
6	6 Bus 6	1	230.00	6	
7	7 Bus 7	1	15.00	7	

Open New Explorer

Figure 15: PowerWorld Diagram

Model Explorer: Fast Decoupled BP Matrix

Explore Fields

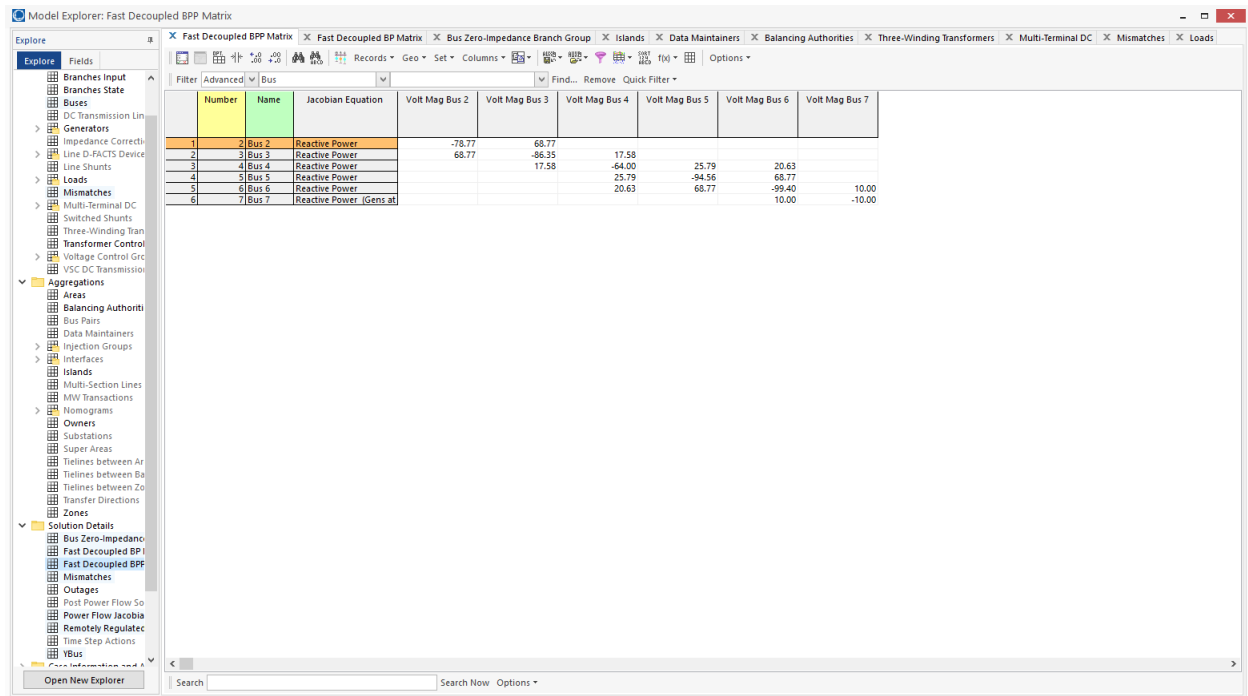
Filter Advanced Bus

Number	Name	Jacobian Equation	Angle Bus 2	Angle Bus 3	Angle Bus 4	Angle Bus 5	Angle Bus 6	Angle Bus 7
1	2 Bus 2	Real Power	-80.53	70.53	17.63			
2	3 Bus 3	Real Power	70.53	-88.16	-65.04	26.45	21.16	
3	4 Bus 4	Real Power		17.63	26.45	-96.98	70.53	
4	5 Bus 5	Real Power			21.16	70.53	-101.69	10.00
5	6 Bus 6	Real Power					10.00	-10.00
6	7 Bus 7	Real Power						

Open New Explorer

Figure 16: PowerWorld Diagram





Model Explorer: Fast Decoupled BPP Matrix

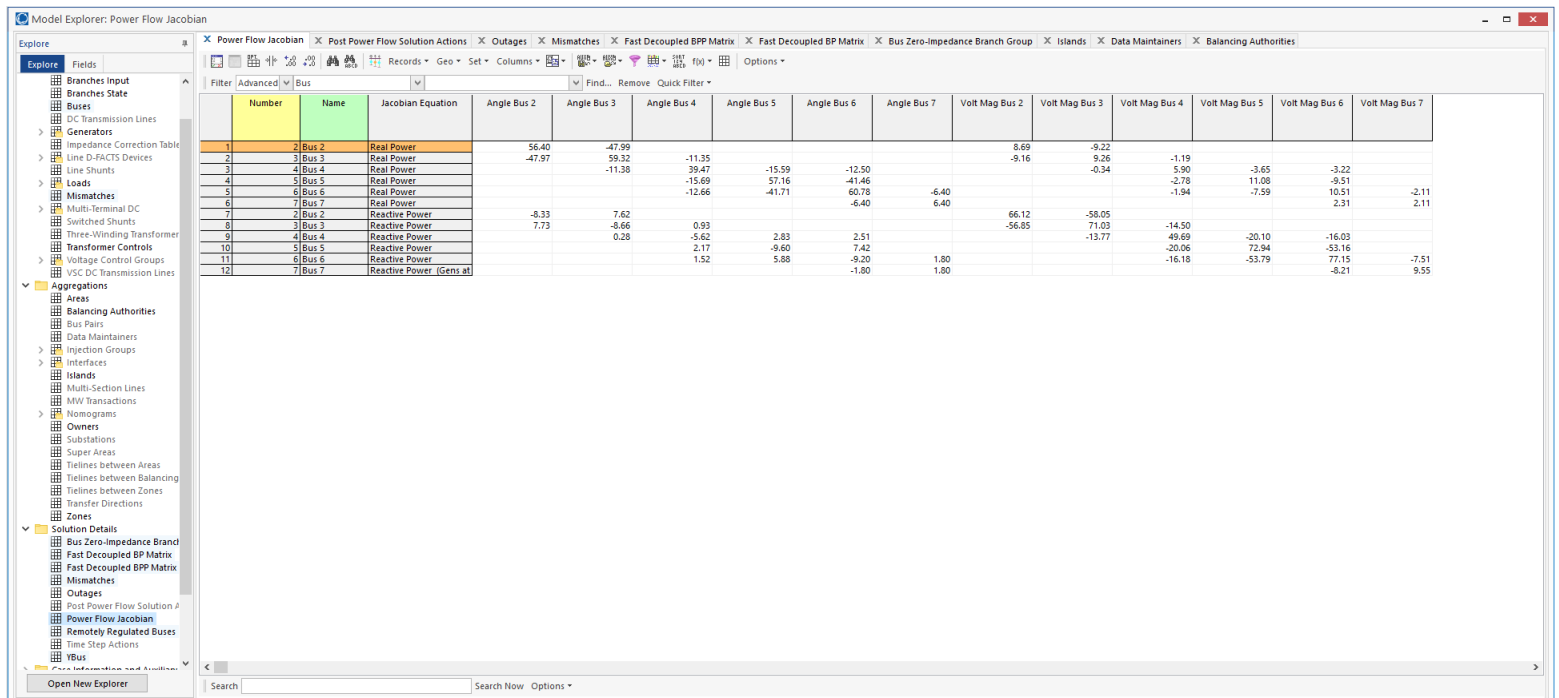
Explore Fields

Filter Advanced Bus

Number	Name	Jacobian Equation	Volt Mag Bus 2	Volt Mag Bus 3	Volt Mag Bus 4	Volt Mag Bus 5	Volt Mag Bus 6	Volt Mag Bus 7
1	2 Bus 2	Reactive Power	-78.77	68.77				
2	3 Bus 3	Reactive Power	68.77	-86.35	17.58			
3	4 Bus 4	Reactive Power		17.58	-64.00	25.79	20.63	
4	5 Bus 5	Reactive Power			25.79	-94.56	68.77	
5	6 Bus 6	Reactive Power			20.63	68.77	-99.40	10.00
6	7 Bus 7	Reactive Power (Gens at					10.00	-10.00

Open New Explorer

Figure 17: PowerWorld Diagram



Model Explorer: Power Flow Jacobian

Explore Fields

Filter Advanced Bus

Number	Name	Jacobian Equation	Angle Bus 2	Angle Bus 3	Angle Bus 4	Angle Bus 5	Angle Bus 6	Angle Bus 7	Volt Mag Bus 2	Volt Mag Bus 3	Volt Mag Bus 4	Volt Mag Bus 5	Volt Mag Bus 6	Volt Mag Bus 7
1	2 Bus 2	Real Power	56.40	47.99					8.69	-9.22				
2	3 Bus 3	Real Power	-47.97	59.32	-11.35				-9.16	9.26				
3	4 Bus 4	Real Power		-11.38	39.47	-15.59	-12.50			-0.34	5.90	-3.65	-3.22	
4	5 Bus 5	Real Power			-15.69	57.16	-41.46				-2.78	11.08	-9.51	
5	6 Bus 6	Real Power			-12.66	-41.71	60.78				-1.94	-7.59	10.51	-2.11
6	7 Bus 7	Real Power				-6.40							2.31	2.11
7	2 Bus 2	Reactive Power	-8.33	7.62					66.12	-58.05				
8	3 Bus 3	Reactive Power	7.73	-8.66	0.93				-56.85	71.03				
9	4 Bus 4	Reactive Power		0.28	-5.62	2.83	2.51			-13.77	-14.50	49.69	-20.10	-16.03
10	5 Bus 5	Reactive Power			2.17	-9.60	7.42				-20.06	72.94	-53.16	
11	6 Bus 6	Reactive Power			1.52	5.88	-9.20	1.80			-16.18	-53.79	77.15	-7.51
12	7 Bus 7	Reactive Power (Gens at					-1.80	1.80					-8.21	9.55

Open New Explorer

Figure 18: PowerWorld Diagram

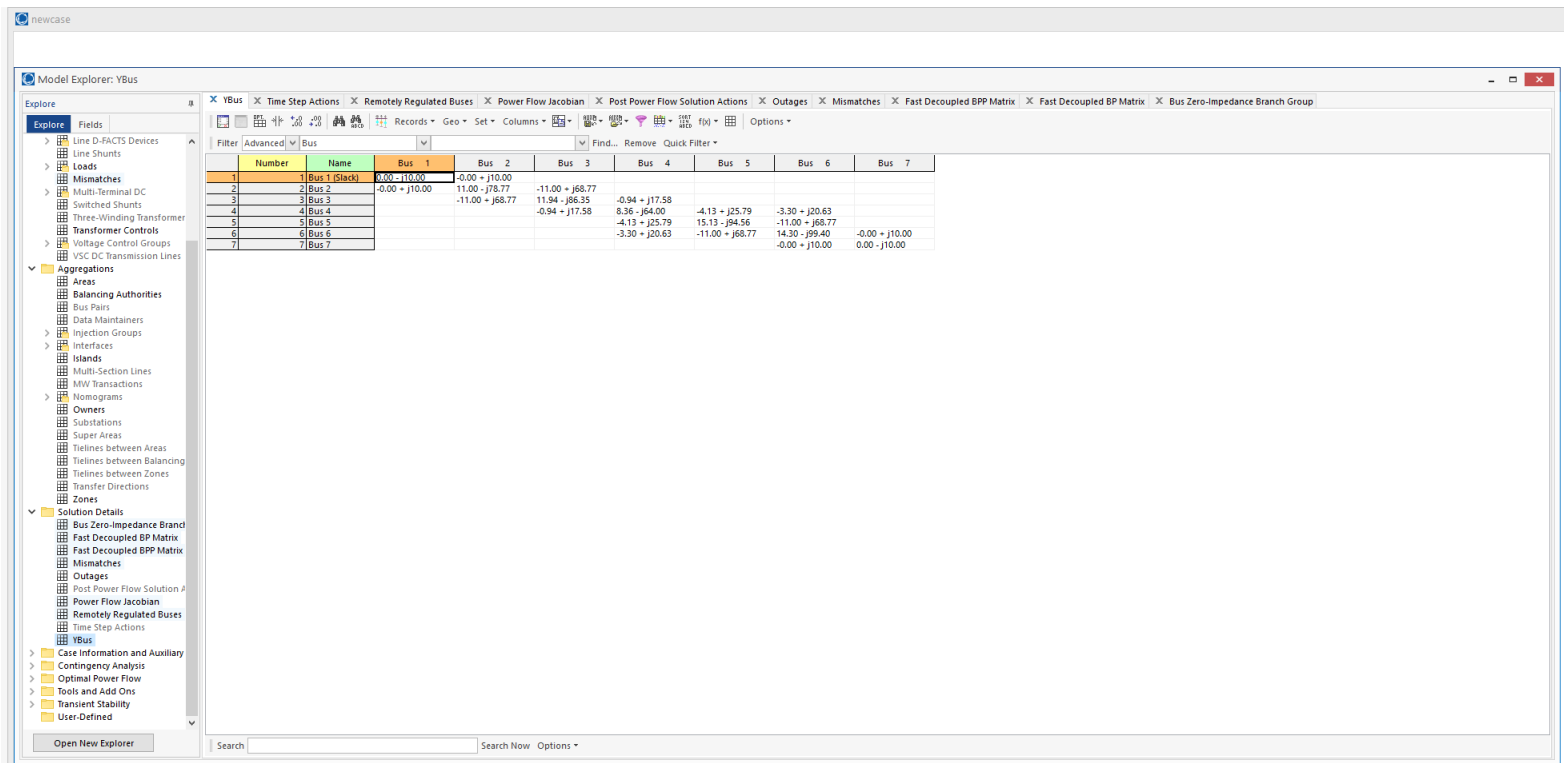


Figure 19: PowerWorld Diagram

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### 3 Conclusions