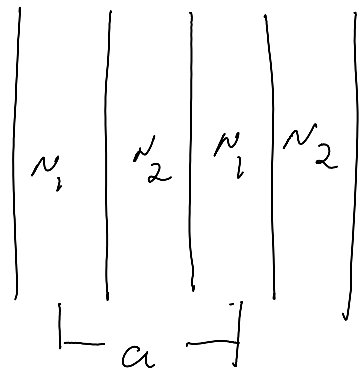
Notes 3

Monday, September 20, 2010
12:20 PM

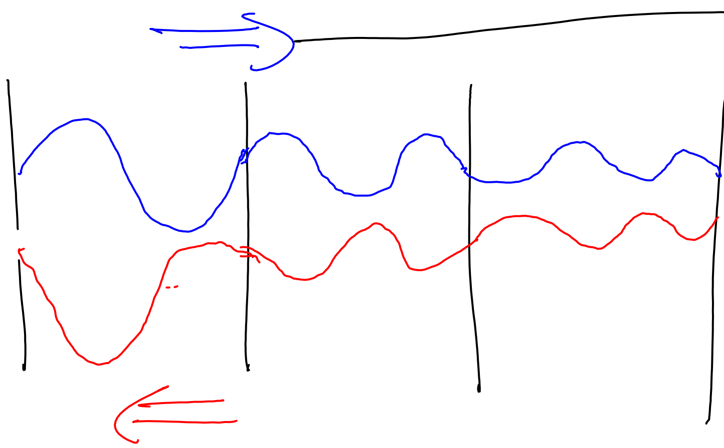
1D crystal

multi-layer film aka Bragg Mirror



Traditional Method

Dielectric Mirror



Transmissions & reflection are out of phase

constructively interfere & can achieve perfect reflection

Let's do it differently

1) Apply symmetries:

1) Continuously translational in x, y
   or has mirror symmetry in x, y plane

2) periodic symmetry in $\hat{z}$
   with lattice vector a

from 1 know we can quantify modes into
$\parallel$ & $\perp$ modes $\Rightarrow k_\parallel$

from 2 know get Bloch modes

$$e^{j k_z z} u_{n, k_z k_\parallel}(z)$$

u: periodic function with
n: band #
k_z: Bloch #
$k_\parallel$: wave # of parallel modes
& z

$$u(z) = u(z+R) \quad R = la$$

Look @ possible values for $k_\parallel$ & k_z

continuous translational symmetry

$k_{\parallel}$ can be anything

k_z : only need Brillouin zone

$a\hat{z}$ lattice vector

reciprical lattice vector

$$\frac{2\pi}{a}\hat{z}$$

$$-\frac{\pi}{a} < k_z \leq \frac{\pi}{a}$$

$$k = \frac{2\pi}{\lambda} \qquad \lambda = \frac{m/s}{1/s} = \frac{c}{f} \qquad f = \frac{\omega}{2\pi}$$

$$= \frac{2\pi c}{\omega}$$

$$= \frac{\cancel{2\pi}}{\frac{\cancel{2\pi} c}{\omega}} = \frac{\omega}{c} = k$$

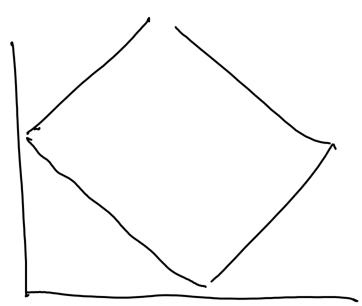
$$c = \sqrt{\frac{\mu}{\epsilon}} \qquad v = c\sqrt{\frac{\cancel{\mu_r}^{\sim 1}}{\epsilon_r}} \qquad v = \frac{c}{\sqrt{\epsilon_r}} = \frac{c}{n}$$

$$\omega = \frac{kc}{\sqrt{\epsilon_r}}$$

Look @ Band Gap figures

freq

$$\frac{\omega a}{2\pi c}$$



$-.5$ wavevector $.5$

$ka/2\pi$

$$\frac{\frac{1}{m}\cdot m}{2\pi}$$ unitless number

that relates $k \to a$

for generalization

$$\frac{\frac{1}{s}\cdot m}{2\pi\ \frac{m}{s}}$$ = unitless ration that relates $f \Rightarrow a$

all charts scale w/ a so

only need to solve for 1 a & have all solutions

fig 1

this is propagation in homogeneous medium

$k = \frac{\omega}{c}$ linear

fig 2

looks similar but

1) "line" is steeper

@ $\frac{ka}{2\pi} \sim \pm .5$ so near edges of

Brillouin zone

the linear relationship breaks down

" $\omega \Rightarrow$

$N=1$ 1^{st} band ends @ ~ 1.48

$N=2$ band starts @ $\stackrel{\omega a}{\sim} 1.49$

for all ω between

$\frac{1.48 \cdot 2\pi c}{a}$ & $\frac{1.49 \cdot 2\pi c}{a}$ No prop.

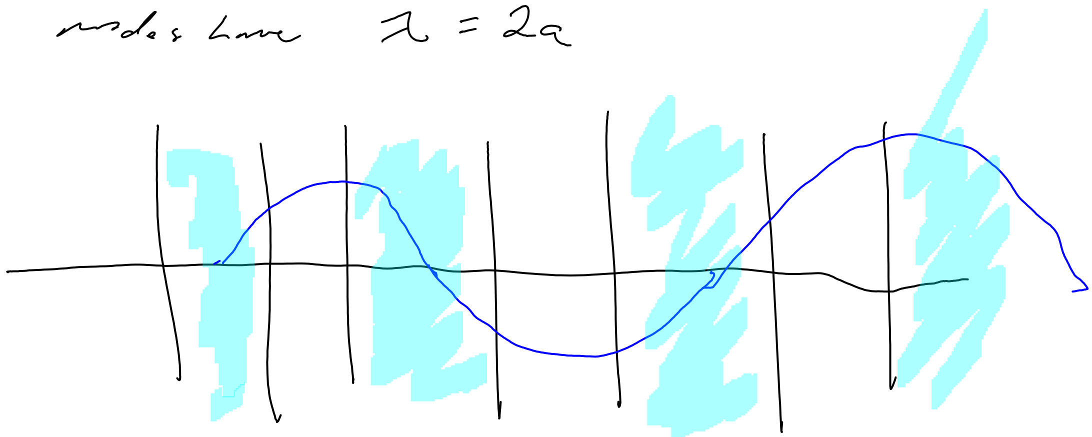
is allowed

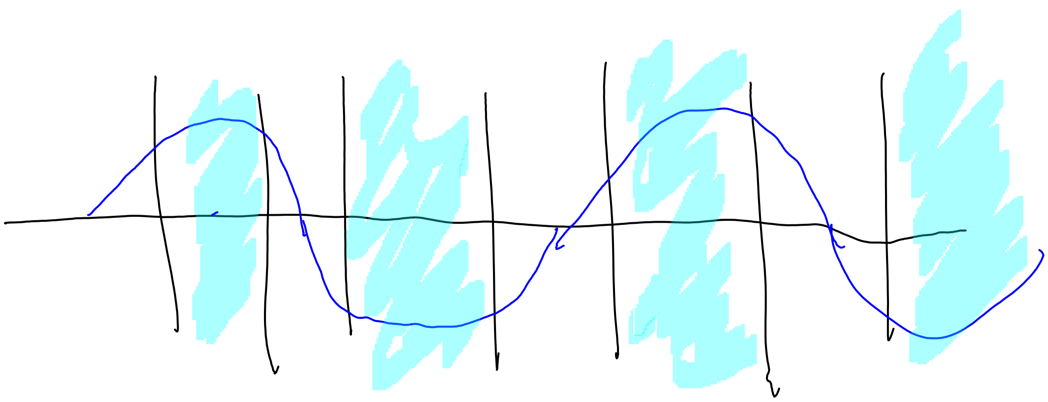
Band Gap!!!

why?

at edge of Brillouin zone $k = \frac{\pi}{a}$

modes have $\lambda = 2a$





two potential ways for E field to

be localized w/ symmetry

<u>fig 3</u>

same effect but much more dramatic

the higher the index radius the larger the gap