Notes 2: Symmetries in E-mag

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Why we care:

We're trying to solve Master Equation.
It has infinite # of answers, or usually
is not solvable analytically. To help us
succeed in our postulated answers want to
eliminate either impossible solutions or
redundant ones.

In general, symmetries in a system lead to
general understanding of behavior. If we
identify symmetries we can eliminate/reduce
the solution set for the Master Eq.

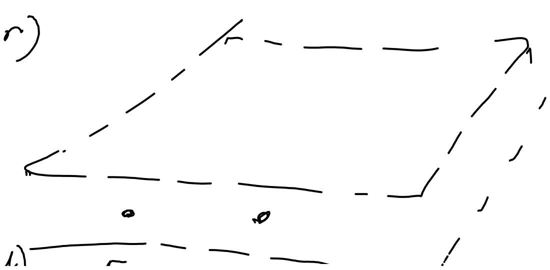
specifically
If we show that some system has symmetry
the eigenfunctions of the symmetry operator
are also eigenfunction of the "system" operator

Symmetries of Interest
Translational : continuous & discrete
Rotational :
Mirror :
Inversion :
Time reversal :

Translational : continuous

definition : a system is unchanged through some
translation d

$G(r-d) = G(r)$



$i\hat{k}z \qquad i\hat{k}(z+d)$

$\hat{T}_{d\hat{z}} e^{j\hat{k}z} = e^{j\hat{k}(z-d)}$ $r = r+d$

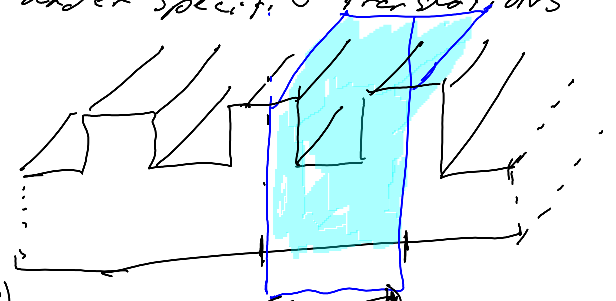
$= \underbrace{e^{-j\hat{k}d}} e^{j\hat{k}z}$

eigenvalue : constant

as move through a medium that is constant the mode stays the same only is changed in phase be e^{-jkd}

Discrete Translational Symmetry

symmetry under specific translations



$\epsilon(r) = \epsilon(r+R)$ $R = la$ a

l = integer a : lattice constant

piece that is repeated over & over is the "unit cell"

eigenmodes again are some plane waves

$$T_R e^{jk_x x} = e^{j\hat{k}_x(x-la)} = \underbrace{e^{-jk_x la}}_{\text{eigenvalues}} e^{-jk_x x}$$

but l = any integer

$$e^{x} = e^{2\pi x} = e^{4\pi x} \ldots$$

$e = e = e \ldots$

$ka = 2\pi$

$k = \frac{2\pi}{a} m$ for any m are the same.

define $b = \frac{2\pi}{a}$ as reciprical lattice vector

since we can build any mode by a linear combination of modes we can find in general

$$\vec{H}_{k_x k_y}(\vec{r}) = e^{jk_x x} \sum_m C_{k_y m}(z) e^{j(k_y + mb)y}$$

$\Uparrow$ for periodicity in $\hat{y}$

$$= e^{jk_x x} e^{jk_y y} \sum_m C_{k_y m}(z) e^{jmby}$$

$$= e^{jk_x x} e^{jk_y y} \vec{u}_{k_y}(y, z)$$

$\vec{u}$ is a periodic function that serves to modulate $\vec{H}$ as you move in (x, y, z)

Bloch's Theorem

because of periodicity, wavevectors separated by a lattice constant $k = k + mb$ are the same.

only need to solve for $\vec{k}$'s that are unique. $-\frac{\pi}{a} \leq k \leq \frac{\pi}{a}$

<u>Brillouin Zone</u>

in 3-D

$\vec{H}_k(r) = e^{jk \cdot r} u_k(r)$ when k lies inside the Brillouin zone

if solve $u_k(r)$ for some one cell of $\vec{r}$, have defined $H(r)$ for that $\vec{k}$

to solve for $u(r)$ plug into the master equation

$$\hat{\Theta} H_k = \left(\frac{\omega(k)}{c}\right)^2 H_k$$

$$\nabla \times \frac{1}{\epsilon(r)} \nabla \times e^{jk\cdot r} u_k(r) = \left(\frac{\omega(k)}{c}\right)^2 e^{jk\cdot r} u_k(r)$$

H.W. Problem

$$\nabla \times e^{jk\cdot r} u_k(r) = (jk + \nabla) \times u_k(r) \quad \checkmark$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{jk_x} u_{k_x}(r) & e^{jk_y} u_{k_y}(r) & e^{jk_z} u_{k_z}(r) \end{vmatrix} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ jk_x + \frac{\partial}{\partial x} & jk_y + \frac{\partial}{\partial y} & jk_z + \frac{\partial}{\partial z} \\ u_{k_x}(r) & u_{k_y}(r) & u_{k_z}(r) \end{vmatrix}$$

$$(j\vec{k} + \nabla) \times \frac{1}{\epsilon(r)} (j\hat{k} + \nabla) \times u_k(r) = \left(\frac{\omega(k)}{c}\right)^2 u_k(r)$$

now solving for $u_k(r)$ will give us the mode profiles we are looking for

new transversality $(jk + \nabla) \cdot u_k = 0$

& $u_k(r) = u_k(r + R)$

Rotation.