Minimum Width Confidence Intervals

Midterm Exam
STAT 624
Fall 2016

Instructions for this part of the exam:

1. You may use the internet but cannot post to a forum.
2. All of your R code should be written, run, and submitted using `hilbert.byu.edu`.
3. Unless explicitly prohibited in a problem, you may use any documentation, datasets, and functions in base R on `hilbert.byu.edu`. You may *not* use any additional libraries (e.g., no `library()`, `require()`, `::`, or `:::` statements).
4. Submit your R code, complete with clear labels and comments, to address the requested items.

Assuming that $X_1, \ldots, X_n$ are independent and identically distributed from a normal distribution with variance σ^2, a $100(1-\alpha)\%$ confidence interval estimator for σ^2 is $((n-1)S^2/b, (n-1)S^2/a)$, where a and b are such that, for a random variable Y having a chi-squared distribution with degrees of freedom $n-1$, $\Pr(a \le Y \le b) = 1 - \alpha$ and S^2 is the sample variance. There are two popular methods for choosing a and b: 1. *equal-tailed* intervals in which a and b are such that $\Pr(Y \le a) = \Pr(Y \ge b)$, and 2. *minimum-width* intervals in which $g(a) = 1/a - 1/b$ is minimized. Complete the items below. Hint: You might find the following R functions helpful: `rchisq, qchisq, pchisq, rexp, optimize`.

1. Let $\alpha = 0.05$, $n = 10$, and $\sigma^2 = 1$. Perform a simulation study to verify that both of these confidence interval estimators (*equal-tailed* and *minimum-width*) produce intervals that have the stated coverage, i.e., that they both produce intervals that contain the true value of σ^2 95% of the time.
2. Let $\alpha = 0.05$ and $\sigma^2 = 1$, find the expected value of the ratio W_1/W_2, where W_i is the random width of intervals from method i, for $i = 1, 2$ when $n = 10$. Do the same for $n = 25$ and $n = 100$. When is the *minimum-width* interval estimator most advantageous in terms of this ratio?
3. Let $\alpha = 0.05$ and $n = 10$, perform a simulation study to find the coverage of these 95% confidence interval estimators when the data is actually independent and identically distribution from an exponential distribution with rate parameter 1 (and, thus, the variance σ^2 is 1).