November 15, 2017

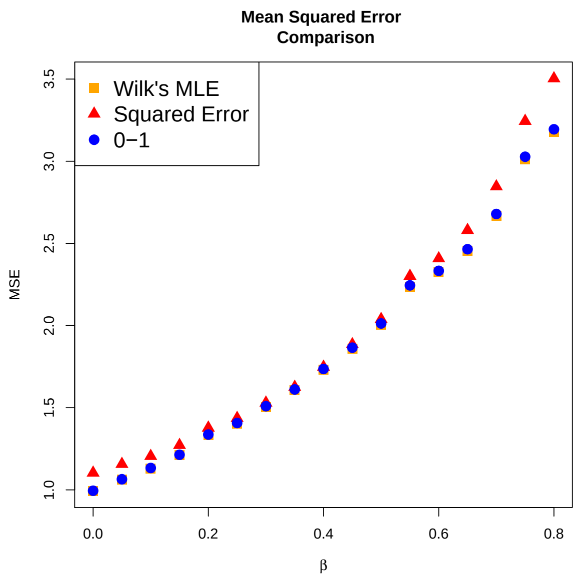
To: Mr. McLapply
From: David Lowe
Subject: Poisson Regression Simulation Study

Mr. McLapply,

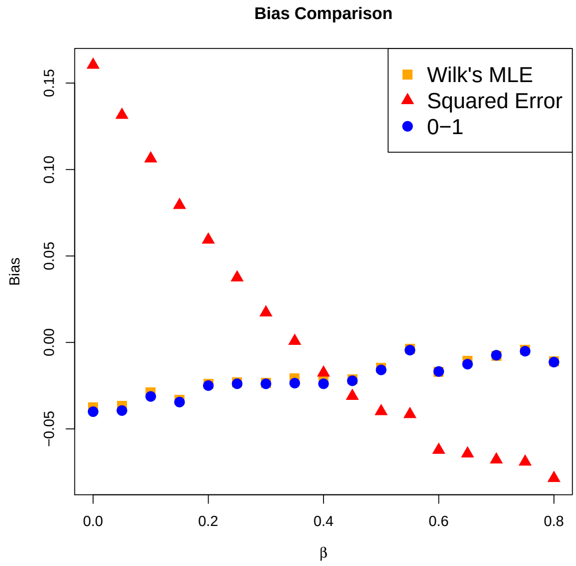
It is difficult to derive a closed-form solution for the parameter, β, when using Poisson regression (see model below). The purpose of this study was to assess the accuracy and precision of various methods for estimating the parameter β. To do this I have developed several different methods for estimating $\hat{\beta}$. The first method employs a frequentist approach to estimate our parameter. I developed an estimator using the Wilk's method, Test-Inversion, and bootstrapping to estimate $\hat{\beta}$. I also approached the problem using Bayesian methods, with loss functions, 0-1, squared error, and absolute error loss. These were tested with both gaussian and uniform priors.

My simulation specifically analyzes the Wilk's method for calculating the MLE, the squared-error loss estimator with a $N(.4, .2)$ prior, and the 1-0 loss estimator with a $Unif(-5, 5)$ prior. Each was estimated for several β values ranging from 0 to 0.8. From these simulations I measured the bias and the mean squared error (MSE).

From the results I observe that the mean squared error is very comparable between the different estimation methods. The trend shows that as β increases, so does the mean squared error. The squared-error loss estimates have slightly higher MSE than Wilk's and 0-1 loss as β nears 0, and again as it approaches 0.8.



The simulations showed that bias did not have the same relationship between tests. Wilk's and squared-error loss methods perform similarly, with a negative bias at β close to 0 and approaches a bias of 0 as they approach 0.8. 0-1 loss, however, shows a strong negative trend, from a positive bias to a negative as β moves from 0 to 1, crossing the zero bias threshold at about $\beta = 0.4$.



It would appear that the Wilk's method for MLE estimation, and squared-error loss gives similar MSE, while keeping a relatively stable bias. This may be helpful as we further use, and estimate the Poisson regression.

Sincerely,

David Lowe
Aspiring Statistician