Fundamentals of Radio Frequency Control for Particle Accelerators

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Objectives

- Course is aimed at non-electrical engineers with little experience in RF
- Course is designed to illustrate frequency domain system design concepts necessary for the understanding of RF control systems for particle accelerators

References

- Design of a ring RF system D. Boussard(CERN) Jan, 1991
- https://www.scandidavia.com/rfcourses/
- https://github.com/dspsandbox/FPGA-Notes-for-Scientists

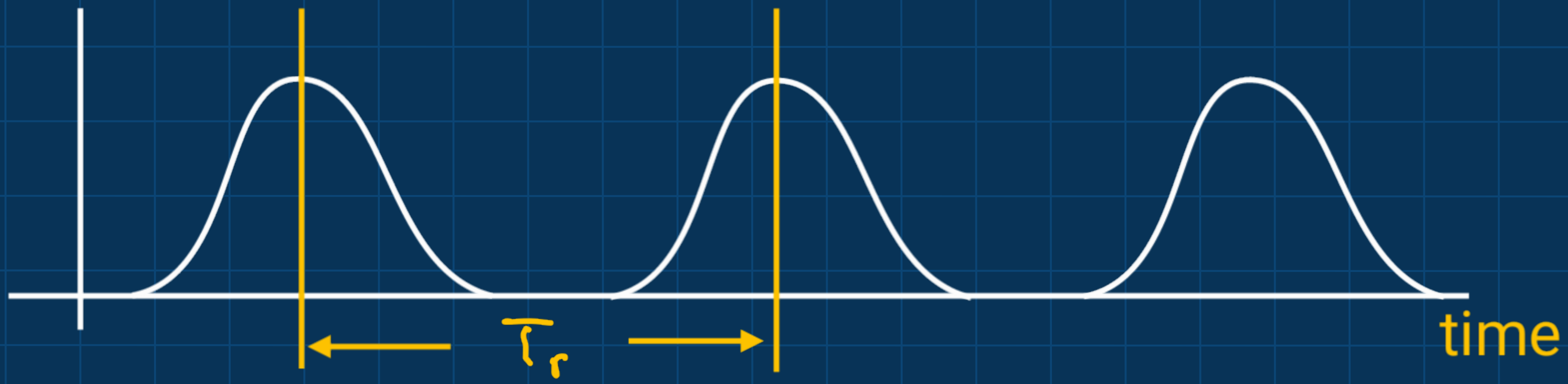
Experience

- Ph.D. from University of Wisconsin- Madison
  - superconducting thin film millimeter amplifiers
- Worked at Fermilab for 23 years
  - Stochastic cooling arrays and systems
  - Beam Stabilization systems
  - 21 cm dark energy FFT Radio telescope
- ESS in Sweden for 7 years
  - RF Group leader
  - Chief engineer responsible for the redesign of the Linac
- MaxIV for 5 years
  - RF Group leader
  - RF Engineer concieved and built Mode 0 system
- BL Monitor and Control AB for 2 years
  - Designed Blinky-Lite control platform

Spectrum Analysis

Periodic Signals

$$v(t) = \sum_{n=-\infty}^{\infty} v_r(t - nT_r)$$



Periodic Signals

Since v(t) is repetitive we can represent v(t) as a sum of orthogonal repetitive functions. We will choose sine waves

$$\boxed{e^{j 2\pi m t/T_r} = \cos\left(2\pi m \frac{t}{T_r}\right) + j \sin\left(2\pi m \frac{t}{T_r}\right)}$$

is periodic at T_r given m is an integer. Let

$$\boxed{\omega_r = \frac{2\pi}{T_r}}$$

Fourier Series

Then we can write the signal as

$$\boxed{v(t) = \sum_{m=-\infty}^{\infty} C_m e^{jm\omega_r t}}$$

where C_m is complex. Multiply both sides by $e^{-jk\omega_r t}$ and integrate over a period.

$$\boxed{\int_{-T_r/2}^{T_r/2} v(t) e^{-jk\omega_r t} dt = \sum_{m=-\infty}^{\infty} C_m \int_{-T_r/2}^{T_r/2} e^{j(m-k)\omega_r t} = C_m T_r \delta_{m,k}}$$

Fourier Series

$$C_m = \frac{1}{T_r} \int_{-T_r/2}^{T_r/2} v(t)\, e^{-jm\omega_r t}\, dt$$

Note that since v (t) is real $C_{-m} = C_m^*$

Let $\omega_m = m\,\omega_r$

$$\tilde{V}(\omega_m) = T_r\, C_m$$ Units of Volts/ Hz

Fourier Transforms

$$v(t) = \sum_{m=-\infty}^{\infty} \frac{1}{T_r} \tilde{V}(\omega_m) e^{j\omega_m t}$$

The spacing between adjacent frequencies is:

$$\Delta \omega_m = m\frac{2\pi}{T_r} - (m-1)\frac{2\pi}{T_r} = \frac{2\pi}{T_r}$$

The Fourier series becomes:

$$v(t) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \tilde{V}(\omega_m) e^{j\omega_m t} \Delta\omega_m$$

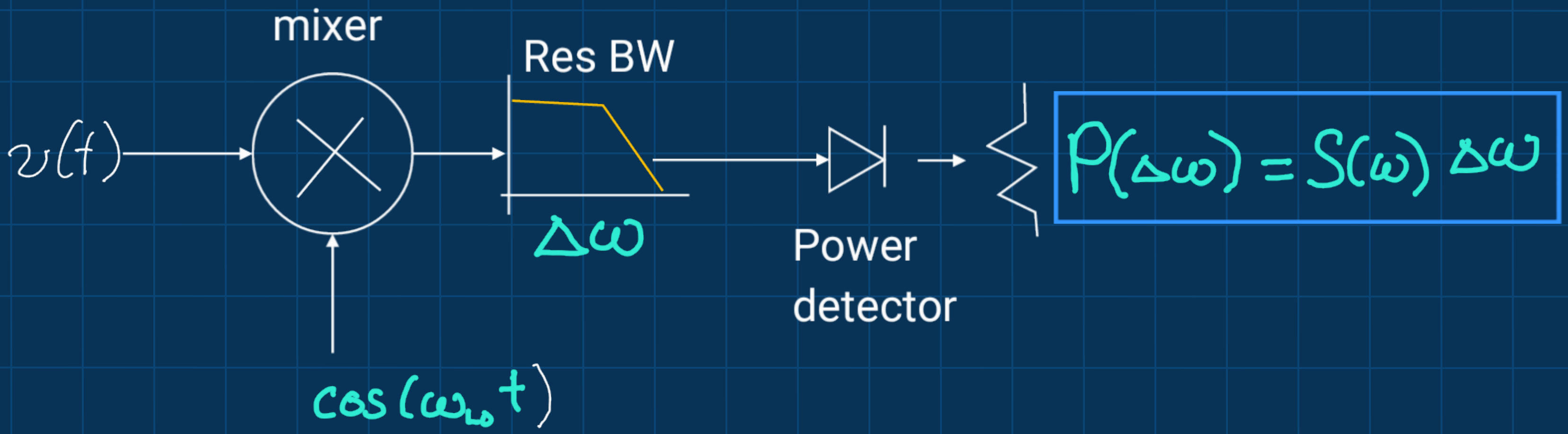
Fourier Transforms

For non-periodic signals, take the limit as $T_1 \to \infty$ $(\Delta\omega_m \to 0)$

$$\boxed{\begin{aligned} v(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{V}(\omega)\, e^{j\omega t}\, d\omega \\ \tilde{V}(\omega) &= \int_{-\infty}^{\infty} v(t)\, e^{-j\omega t}\, dt \end{aligned}}$$

Spectrum Analysis

Swept spectrum analyzers do not measure voltages and currents. They measure power deposited into a filter of width given by the resolution bandwidth.



Power Spectral Density

The power spectral density is defined as:

$$\boxed{\langle p(t) \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega)\, d\omega}$$

But time averaged power is:

$$\boxed{\langle p(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \frac{1}{R} \int_{-T/2}^{T/2} v(t)\, v(t)\, dt}$$

Power Spectral Density

MAX IV RF

Use the fact that because v(t) is real:

$$v(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{V}(\omega_1) e^{j\omega_1 t}\, d\omega_1$$

$$v(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{V}(\omega_2)^{*} e^{-j\omega_2 t}\, d\omega_2$$

and

$$\delta(t-\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega(t-\tau)}\, d\omega$$

$$\delta(\omega-\omega') = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j(\omega-\omega')t}\, dt$$

results

$$\langle p(t)\rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} \frac{|V(\omega)|^2}{R}\, d\omega$$

Power Spectral Density

The power spectral density reduces to:

$$\boxed{S_f(\omega) = \lim_{T \to \infty} \frac{1}{T} \frac{1}{R} |\tilde{V}(\omega)|^2}$$

Since $\tilde{V}(\omega)$ has units of Volts/Hz,

$S_f(\omega)$ has units of Watts/ Hz

Power Spectral Density

Consider the periodic signal again

$$\boxed{V_p(t) = \sum_{n=-\infty}^{\infty} V_r(t - nT_r)}$$

Written as a Fourier Series $\boxed{V_p(t) = \sum_{m=-\infty}^{\infty} C_m e^{jm\omega_r t}}$

Taking the Fourier transform $\boxed{\tilde{V}_p(\omega) = \sum_{m=-\infty}^{\infty} C_m \int_{-\infty}^{\infty} e^{j(m\omega_r - \omega)\tau} d\tau}$

Results with: $\boxed{\tilde{V}_p(\omega) = 2\pi \sum_{m=-\infty}^{\infty} C_m \delta(\omega - m\omega_r)}$

Power Spectral Density

Using the magic of delta functions with the limit as T goes to infinity, the gifted reader can show:

$$\boxed{S_p(\omega) = \frac{2\pi}{R} \sum_{n=-\infty}^{\infty} |C_n|^2 \delta(\omega - n\omega_r)}$$

Since

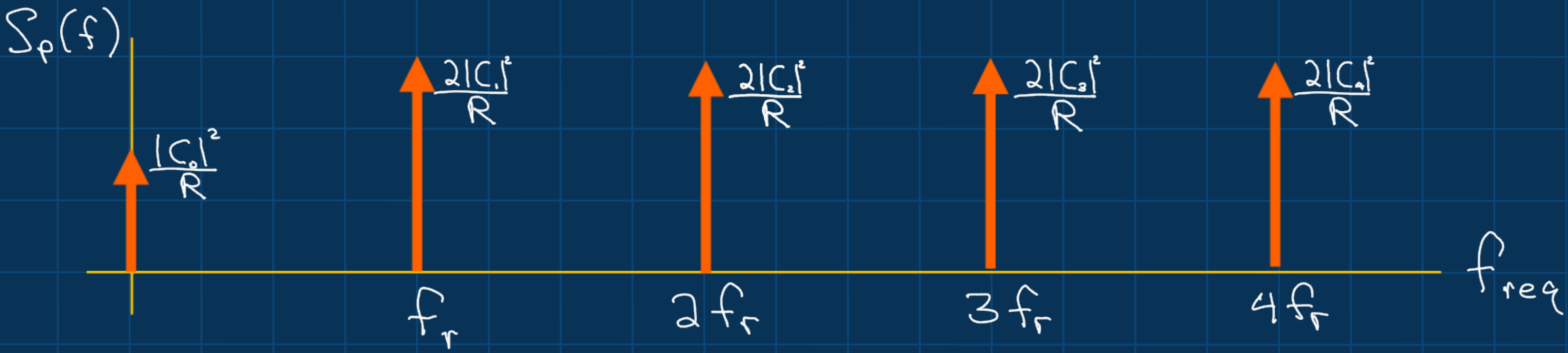
$$\boxed{\omega = 2\pi f}$$

$$\boxed{S_p(f) = \frac{1}{R} \sum_{n=-\infty}^{\infty} |C_n|^2 \delta(f - nf_r)}$$

Spectrum Analysis

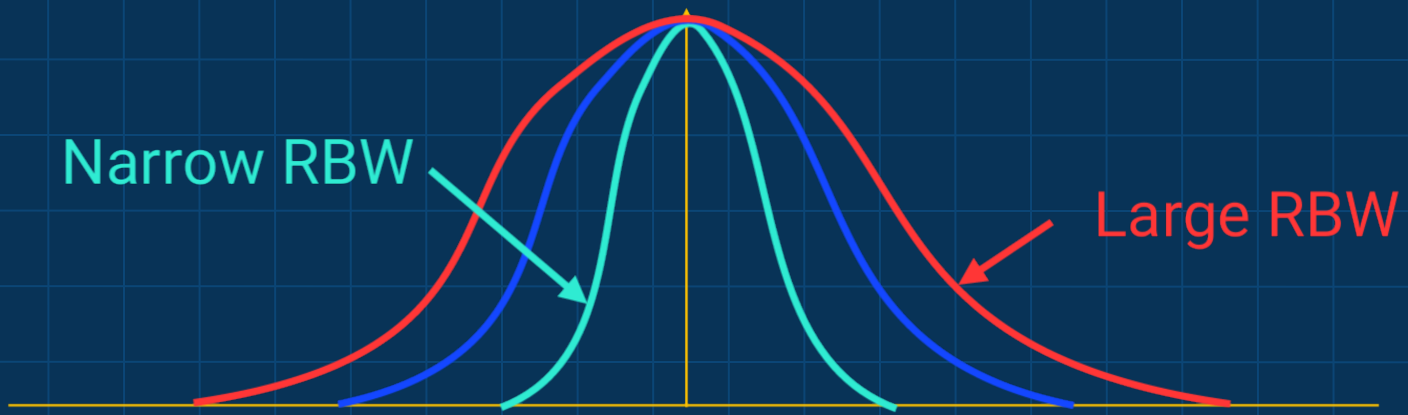
Since spectrum analyzers do not measure phase, they do not display negative frequencies

$$\boxed{S_p(f) = \frac{1}{R}|C_0|^2\delta(0) + \frac{2}{R}\sum_{m=1}^{\infty}|C_m|^2\delta(f-nf_r)}$$



Spectrum Analysis

- Note that a spectrum analyzer does not measure the power spectral density.
- It measures the power deposited in a filter of width called the resolution bandwidth at a frequency f.
- So for a periodic signal, as the resolution bandwidth is changed, the peak signal on a spectrum analyzer does not vary.



Modulation

Amplitude Modulation (AM)

Consider a carrier signal:

$$\boxed{v(t) = A(t)\cos(\omega_c t)}$$

In which the amplitude A(T) varies slowly with respect to the carrier frequency. Let:

$$A(t) = V_0\left(1 + \underbrace{m}_{\text{modulation index}} \cos\left(\underbrace{\omega_m}_{\text{modulation frequency}} t\right)\right)$$

Amplitude Modulation (AM)

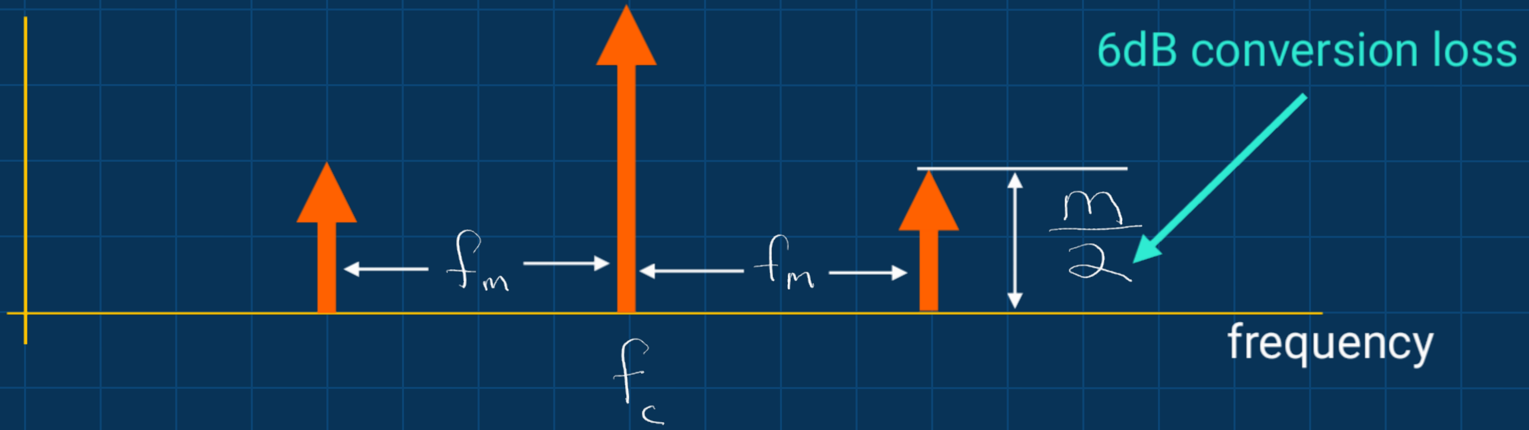
$$\boxed{v(t) = V_0\left(1 + m\cos(\omega_m t)\right)\cos(\omega_c t)}$$

Most important trig identity in RF

$$\boxed{\begin{aligned}\cos(a+b) &= \cos(a)\cos(b) - \sin(a)\sin(b)\\ \cos(a)\cos(b) &= \tfrac{1}{2}\cos(a+b) + \tfrac{1}{2}\cos(a-b)\end{aligned}}$$

Amplitude Modulation (AM)

$$
\begin{aligned}
v(t) = {} & V_0 \cos(\omega_c t) && \text{carrier} \\
& + \frac{mV_0}{2} \cos((\omega_c + \omega_m)t) && \text{upper sideband} \\
& + \frac{mV_0}{2} \cos((\omega_c - \omega_m)t) && \text{lower sideband}
\end{aligned}
$$



Frequency Modulation

$$\boxed{\omega = \omega_c + \Delta\omega_m \cos(\omega_m t)}$$

$\omega_c =$ carrier frequency

ω_m modulation frequency

$\Delta\omega_m$ modulation amplitude

$$\omega = \frac{d\varphi}{dt}$$

$\varphi =$ RF phase

$$\varphi = \int \omega(t)\, dt$$

Frequency Modulation

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$$\phi = \omega_c t + \frac{\Delta\omega_m}{\omega_m}\sin(\omega_m t)$$

$$v(t) = V_0 \cos\left(\omega_c t + \frac{\Delta\omega_m}{\omega_m}\sin(\omega_m t)\right)$$

yikes!

$$e^{j\alpha} = \cos(\alpha) + j\sin(\alpha)$$

$$\cos(\alpha) = \mathrm{Re}\left\{e^{j\alpha}\right\}$$

$$v(t) = V_0\,\mathrm{Re}\left\{e^{j\omega_c t}\, e^{j\frac{\Delta\omega_m}{\omega_m}\sin(\omega_m t)}\right\}$$

Frequency Modulation

MAX IV RF

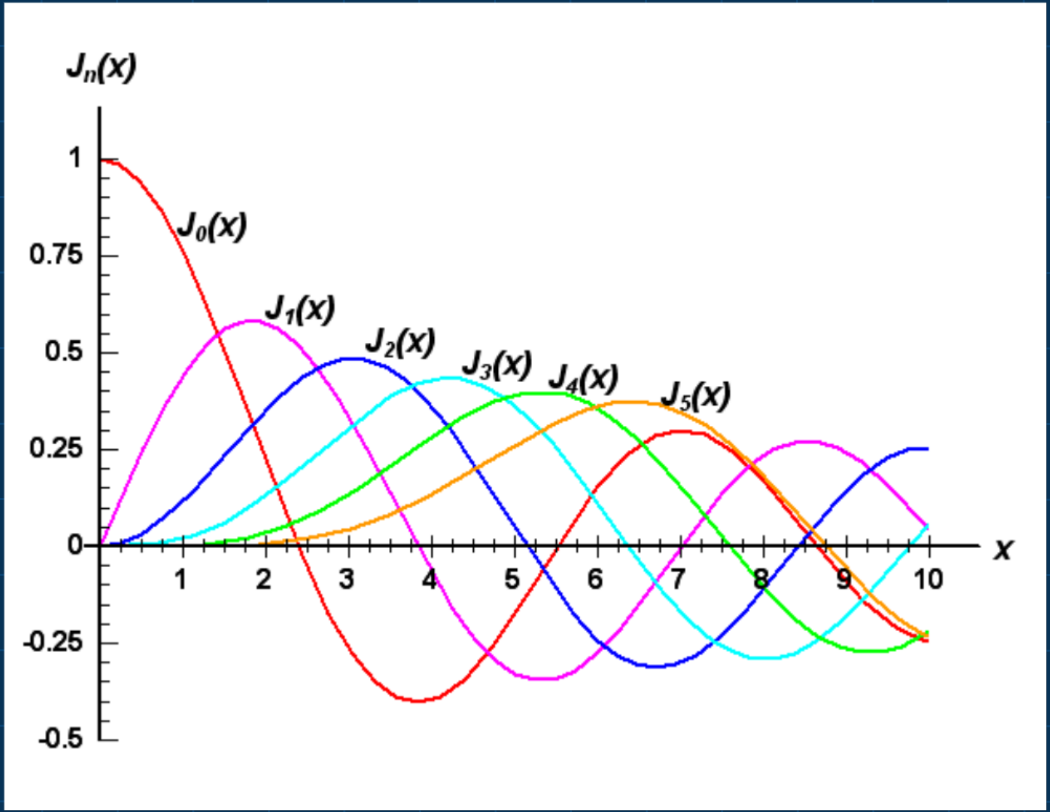
Bessel Identity to the rescue!

$$e^{jz\sin(x)} = \sum_{n=-\infty}^{\infty} J_n(x) e^{jnx}$$

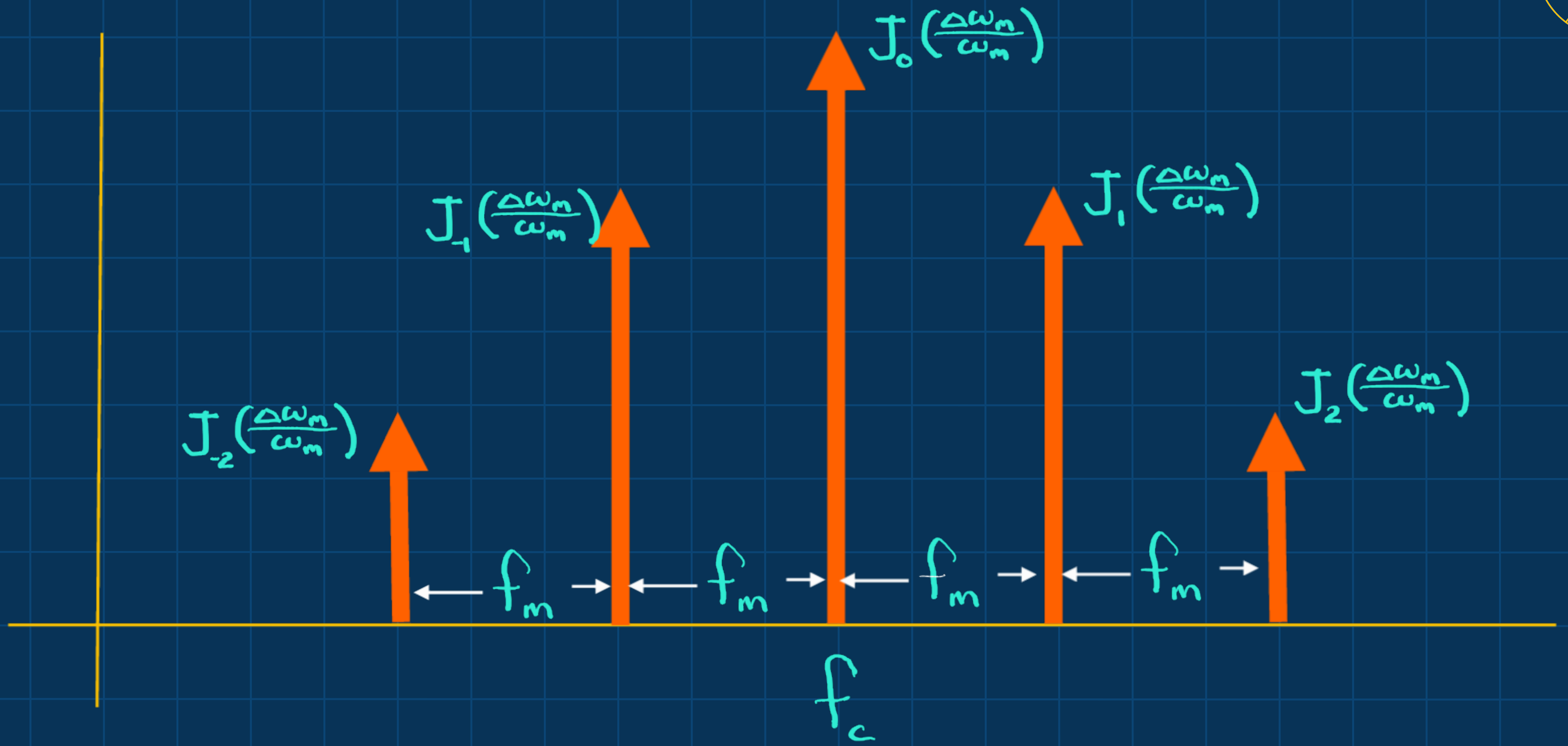
$$v(t) = V_0 \, \mathrm{Re}\left\{ \sum_{n=-\infty}^{\infty} J_n\left(\frac{\Delta\omega_m}{\omega_m}\right) e^{j(\omega_c + n\omega_m)t} \right\}$$

$$v(t) = V_0 \sum_{n=-\infty}^{\infty} J_n\left(\frac{\Delta\omega_m}{\omega_m}\right) \cos((\omega_c + n\omega_m)t)$$

Bessel Functions



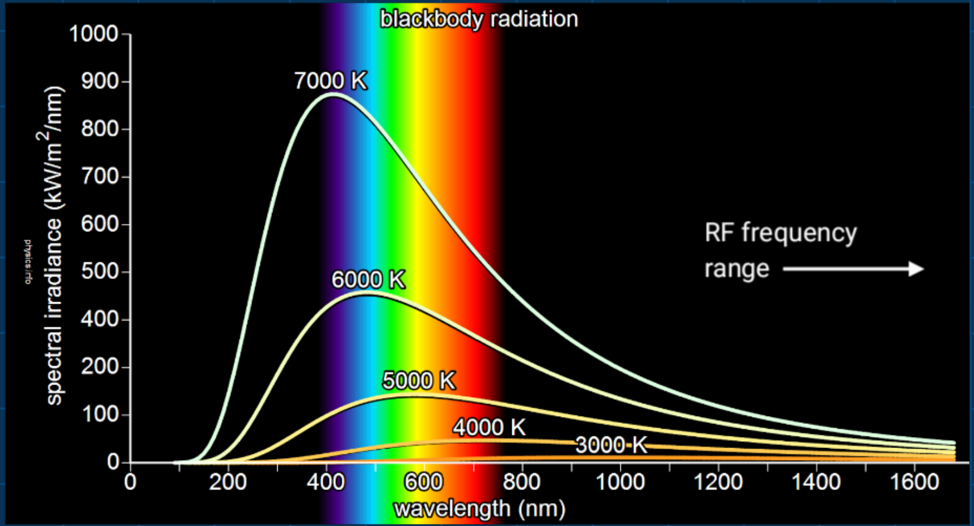
Frequency Modulation Spectrum



Noise

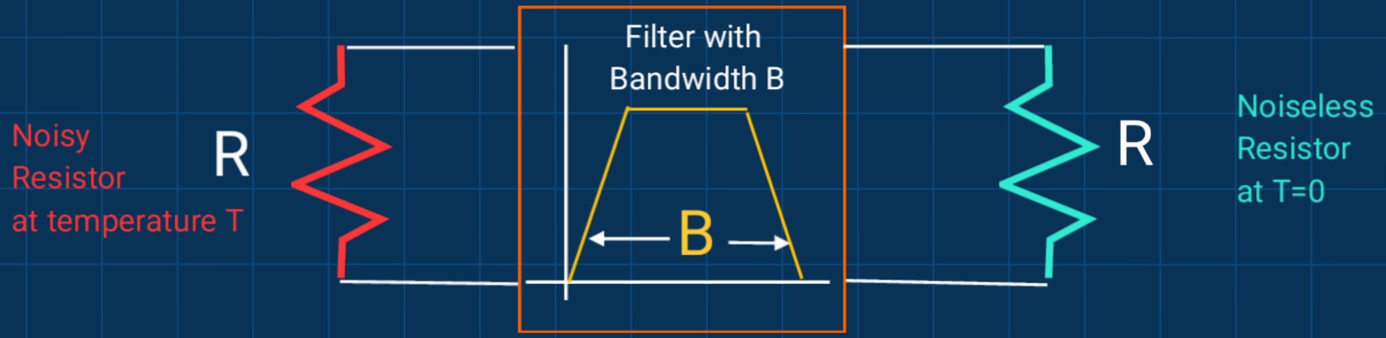
Thermal Noise

Consider a resistor at a temperature T. It will produce photons over a broad spectrum, including the RF frequency range



Band Limited Noise

- Connect a filter with bandwidth B to the hot noisy resistor.
- To collect the noise from the resistor, the filter must be terminated with a matched impedance equal to the source
- Imagine that the matched load is noiseless (ie. at T=0)



Band Limited Noise

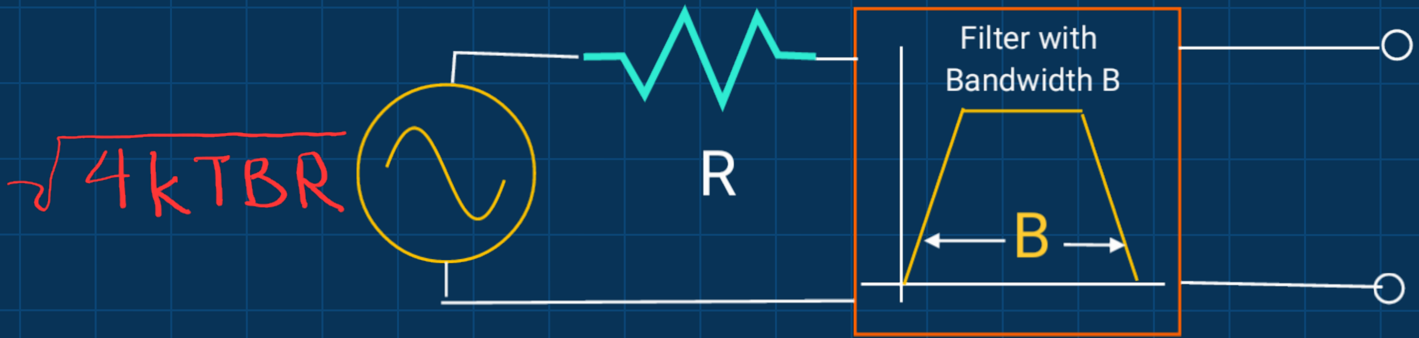
From thermodynamics, energy will be transfered from the hot resistor to the cold resistor at an average rate of:

$$\boxed{\begin{gathered}\langle P \rangle = kTB \\ k = 13.6 \times 10^{-24}\ \mathrm{W/K/Hz}\end{gathered}}$$

Why not just make R small and have no noise power?
Because to match the source to get the maximum power transfer, the load resistor would have to be made small so you cannot win.

Noise Circuit Model

An equivalent circuit model for the noisy resistor is:



The available noise power density from a resistor is.

$$\boxed{S(f) = kT}$$

No matter what the value of the resistance is!

Noise Power Spectral Density

At T=293K (20C)

$$\boxed{S = 4\times 10^{-21}\ W/Hz}$$

$$\boxed{S_{dBm} = 10\log_{10}\left(\frac{4\times 10^{-21}\ W}{0.001\ W}\right) = -174\ dBm}$$

Easy to remember

$$\boxed{S = -174\ dBm \text{ "per Hz" at Room Temperature}}$$

Noise Power Spectral Density

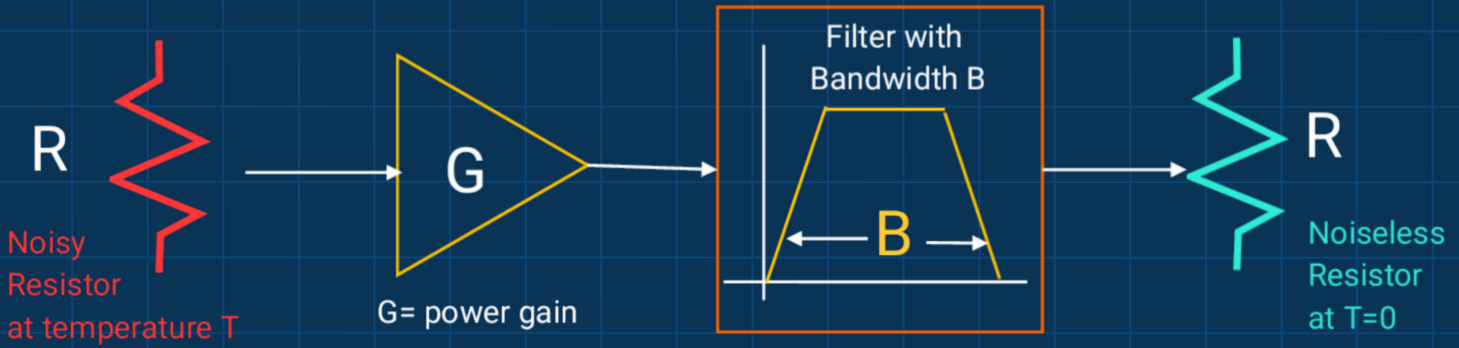
So for a bandwidth of 1 MHz at 293K

$$P_{noise} = -174\,dBm + 10\log_{10}\left(\frac{1\,MHz}{1\,Hz}\right) = -114\,dBm$$

So for a bandwidth of 1 MHz at 77K

$$P_{noise} = -114\,dBm + 10\log_{10}\left(\frac{77K}{293K}\right) \approx -120\,dBm$$

Noise in an Amplifier



A real amplifier will add noise.

For an amplifier, we always define the noise as measured on the output but refered to the input.

$$\boxed{P_o = G(kT_r + S_A)B}$$

Effective Noise Temperature

We can say that the amplifier has an effective noise temperature

$$T_A = S_A / k$$

$$P_o = G(kT_r + kT_A)B$$

We define the noise figure as the ratio:

$$N_f = \frac{T_A + 293K}{293K}$$

Examples

Example 1: An amplifier with a noise figure of 1 dB, what is the noise temperature of the amplifier?

$$1\,dB \Rightarrow 1.26$$

$$T_A = (1.26-1)\,293K = 76\,K$$

Example 2: An passive attenuator of reduces the power by a factor of A. What is the noise figure of the attenuator?

$$P_{out} = \frac{1}{A}\left(kT_A + kT_0\right)B = kT_0B \leftarrow \text{passive}$$

$$N_f = A$$

Examples

Example 3: The noise floor of a spectrum analyzer measured with its input terminated into 50 ohms is -90dBm at a resolution bandwidth of 1MHz.

a). What is the noise figure of the SA

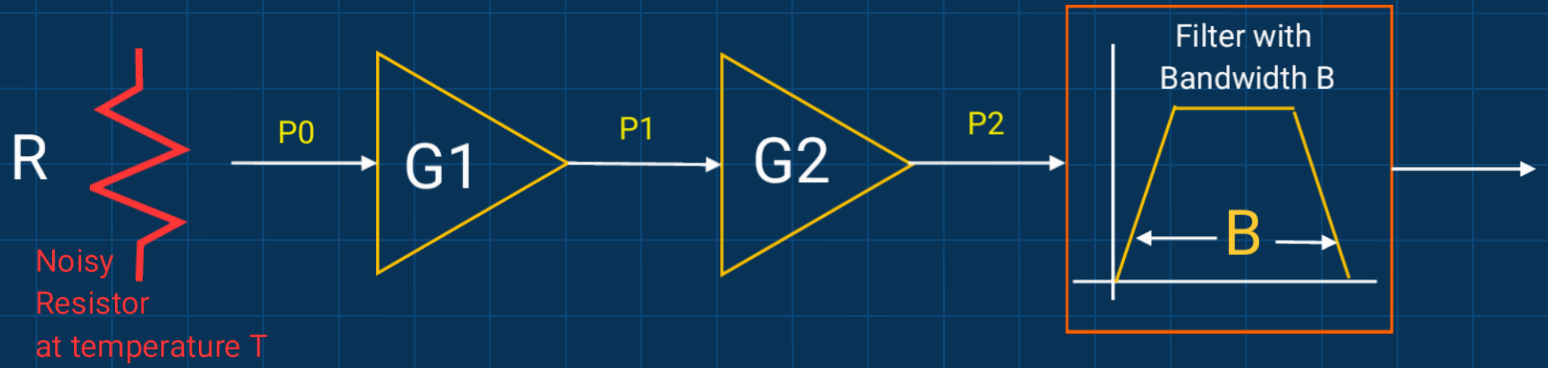
b.) What is the noise temperature?

$$S = -90\,dBm - 60\,dB = -150\,dBm/Hz$$

$$N_f \approx -150\,dB + 174\,dB = 24\,dB$$

$$T \approx 73{,}000\,K$$

Noise Figure of Systems



$$P_1 = G_1 (kT_0 + kT_{A1}) B$$

$$P_2 = G_1 G_2 (kT_0 + kT_{A1}) B + G_2 k T_{A2} B$$

Noise Figure of Systems

$$N_{F_{Total}} = \frac{P_2}{G_1 G_2} \frac{1}{kT_0} = N_{f_1} + \frac{1}{G_1}(N_{f_2} - 1)$$

If there was a third amplifier

$$N_{F_{Total}} = N_{f_1} + \frac{1}{G_1}(N_{f_2} - 1) + \frac{1}{G_1 G_2}(N_{f_3} - 1)$$

Noise figure of a system is dominated by the first element

Control Theory

Laplace Transforms

Fourier Transform Pair

$$\tilde{F}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{F}(\omega)\, e^{j\omega t}\, d\omega$$

The problem with Fourier transforms is that they cannot handle initial conditions due to the infinite bounds on the integral. To handle initial conditions , consider the Laplace transform

$$\mathcal{L}[f(t)] = \tilde{F}(s) = \int_{0}^{\infty} f(t)\, e^{-st}\, dt$$

Laplace Transform Table

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S.no	$f(t)$	$\mathcal{L}\{f(t)\}$	S.no	$f(t)$	$\mathcal{L}\{f(t)\}$
1	1	$\frac{1}{s}$	11	$e^{at}\sinh bt$	$\frac{b}{(s-a)^2-b^2}$
2	e^{at}	$\frac{1}{s-a}$	12	$e^{at}\cosh bt$	$\frac{s-a}{(s-a)^2-b^2}$
3	t^n	$\frac{n!}{s^{n+1}}$	13	$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
4	$\sin at$	$\frac{a}{s^2+a^2}$	14	$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$
5	$\cos at$	$\frac{s}{s^2+a^2}$	15	$f'(t)$	$sF(s)-f(0)$
6	$\sinh at$	$\frac{a}{s^2-a^2}$	16	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
7	$\cosh at$	$\frac{s}{s^2-a^2}$	17	$\int_0^t f(u)du$	$\frac{1}{s}F(s)$
8	$e^{at}t^n$	$\frac{n!}{(s-a)^{n+1}}$	18	$t^n f(t)$ Where $n=1,2,3,..$	$(-1)^n\frac{d^n}{ds^n}\{F(s)\}$
9	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$	19	$\frac{1}{t}\{f(t)\}$	$\int_s^\infty F(s)ds$
10	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$	20	$e^{at}f(t)$	$F(s-a)$

Inverse Laplace Transform

s is a complex number!

$$\boxed{s = \sigma + j\omega}$$

Since **s** is a complex number, the inverse transform is hard to compute directly and using the transform tables in reverse to find the inverse transform is the usual technique.

Initial and Final value Theorems

Initial value theorem

$$\boxed{\lim_{t \to 0^+} f(0^+) = \lim_{s \to \infty} s f(s)}$$

Final value theorem

$$\boxed{\lim_{t \to \infty} f(t) = \lim_{s \to 0} s f(s)}$$

Circuit Elements

Uncharged capacitor

$$\boxed{i(t) = C\frac{dv}{dt}}$$

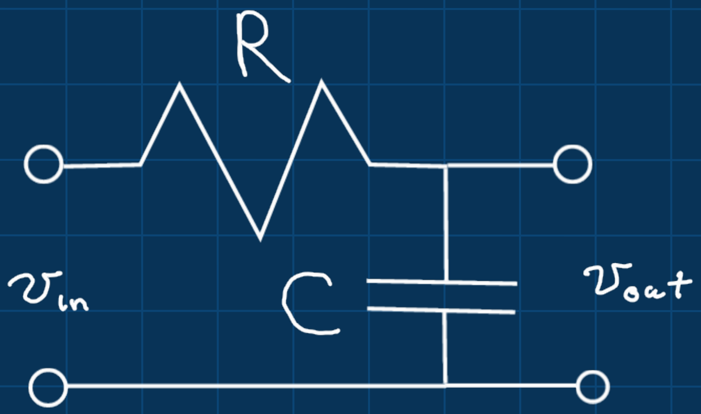
$$\boxed{\tilde{i}(s) = sC\tilde{v}(s) - C\,\cancelto{0}{v(0)}}$$

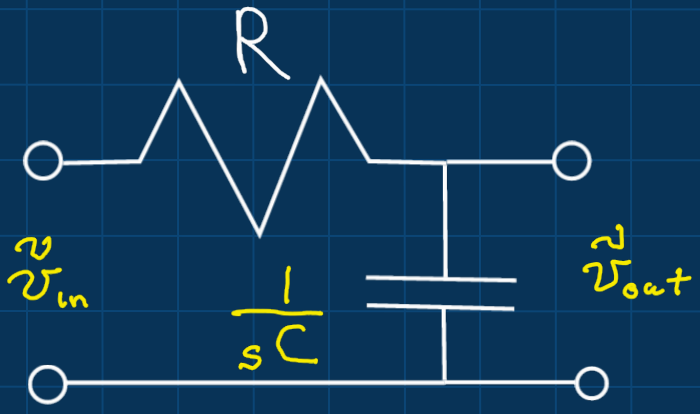
Uncharged inductor

$$\boxed{v(t) = L\frac{di}{dt}}$$

$$\boxed{\tilde{v}(s) = sL\,\tilde{i}(s)}$$

Low Pass Filter





$$\boxed{\frac{\tilde{v}_{out}}{\tilde{v}_{in}} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sRC}}$$

Frequency Response

$$s \Rightarrow j\omega$$

$$\boxed{\frac{\tilde{v}_o}{\tilde{v}_i} = \frac{1}{1 + j\omega RC}}$$

When $\omega = \frac{1}{RC}$

$$\boxed{\frac{V_o}{V_i} = \frac{1}{1+j} = \frac{1-j}{2}}$$

$$\boxed{\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{2}}}$$

$$\boxed{\angle\left(\frac{V_o}{V_i}\right) = -45^\circ}$$

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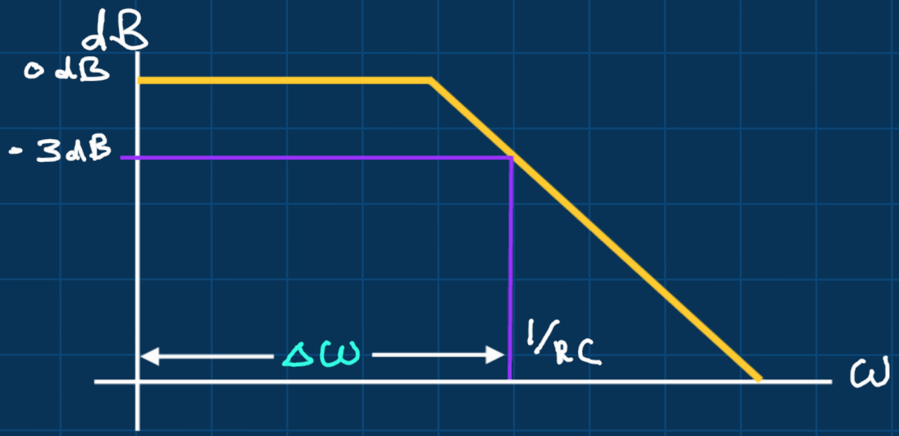
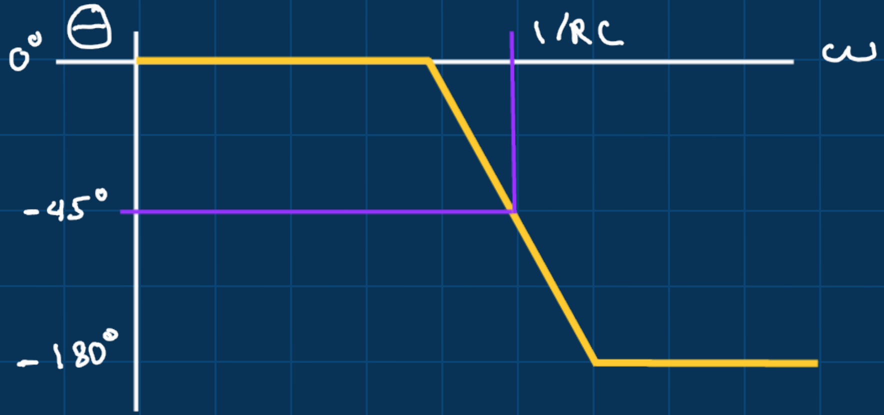
Frequency Response





$$dB = 10\log_{10}\left(\frac{P_o}{P_1}\right)$$

$$dB = 10\log_{10}\left(\frac{|V_o|^2}{|V_i|^2}\right)$$

$$-3dB = 10\log_{10}\left(\frac{1}{2}\right)$$

$$\Theta = \angle \frac{V_o}{V_i}$$

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Impulse Response

$$v_{in}(t) = V_i \, \tau_i \, \delta(t)$$

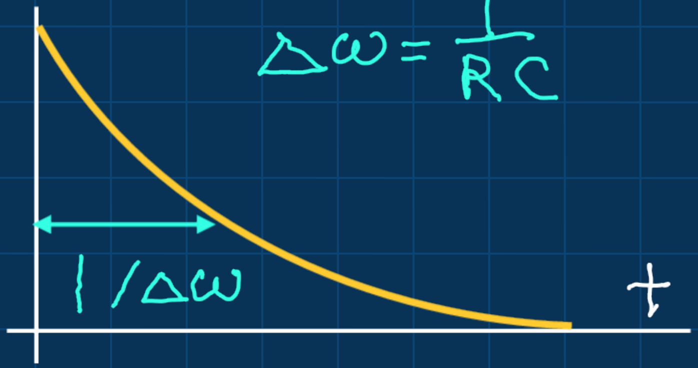
units of 1/time

$$V_{in}(s) = V_i \, \tau_i$$

$$V_o(s) = \frac{v_i \, \tau_i}{1 + sRC}$$

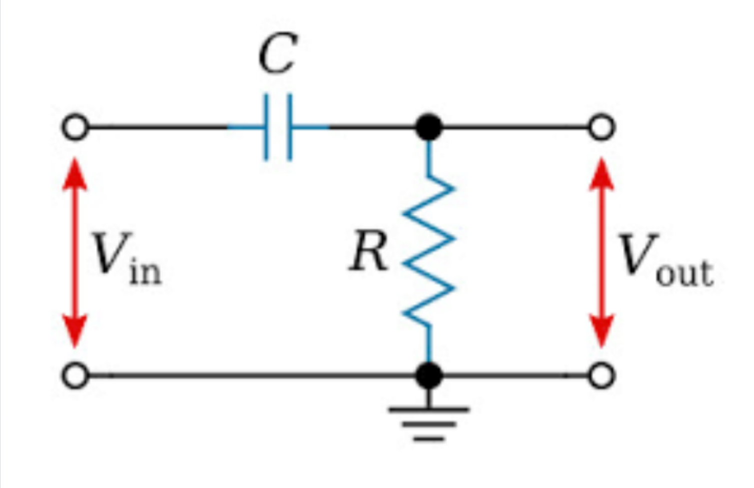
$$v_o(t) = v_i \, \Delta\omega \, \tau_i \, e^{-\Delta\omega t}$$

$$\Delta\omega = \frac{1}{RC}$$



The rms width of this exponential is $1/\Delta\omega$. Thus the bandwidth of a filter is inversely proportional to the time resolution

High Pass Filter



$$\boxed{\frac{V_o}{V_i} = \frac{R}{\frac{1}{sC} + R}}$$

$$\boxed{\frac{v_o}{v_i} = \frac{sRC}{1 + sRC}}$$

The response goes to zero at $s = 0$

The response goes to infinity at $s = -\frac{1}{RC}$

Poles and Zeros

We call $s = 0$ a zero of the filter

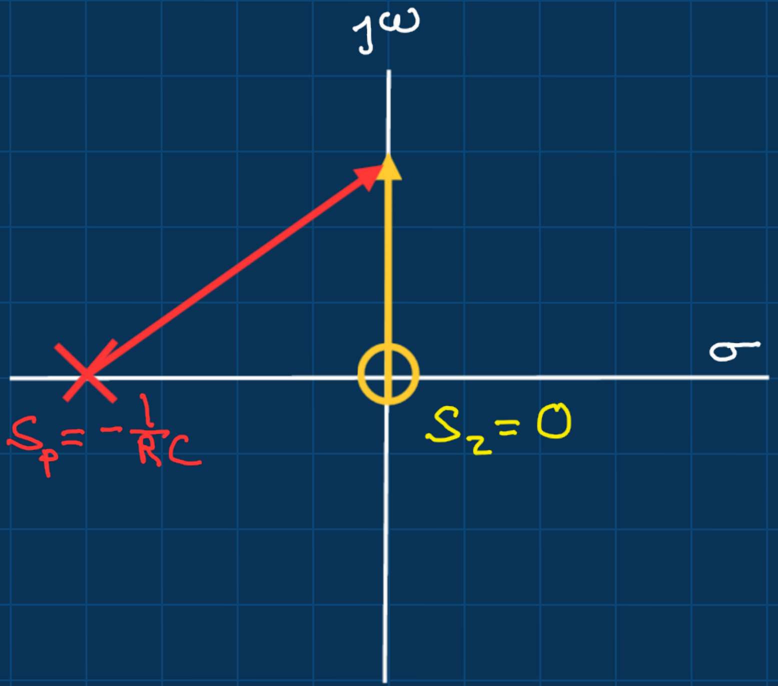
We call $s = -\frac{1}{RC}$ a pole of the filter

In general, any filter can be written as:

$$H(s) = \frac{(s - s_{z1})(s - s_{z2}) \ldots (s - s_{zn})}{(s - s_{p1})(s - s_{p2}) \ldots (s - s_{pn})}$$

Pole- Zero Constellation

We can plot the poles and zeros on the complex s plane



To compute the frequency response, we can draw arrows to any point on the imaginary $j\omega$ axis.

Bode Plots

The magnitude of the response is the product of all the lengths of the zero arrows divided by the product of all the lengths of the pole arrows

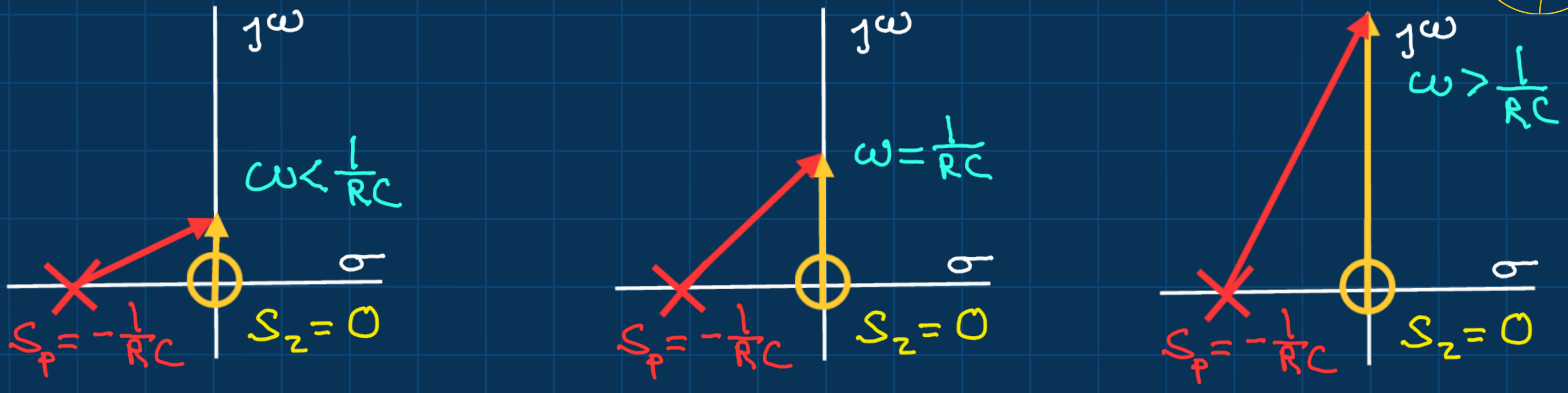
$$|H(j\omega)| = \frac{\prod_i |\vec{l}_{z_i}|}{\prod_i |\vec{l}_{p_i}|}$$

The phase of the response is the sum of all the lengths of the zero arrows subtracte by the sum of all the lengths of the pole arrows

$$\angle H(j\omega) = \sum \angle \vec{l}_{z_i} - \sum \angle \vec{l}_{p_i}$$

Bode Plots

MAX IV RF

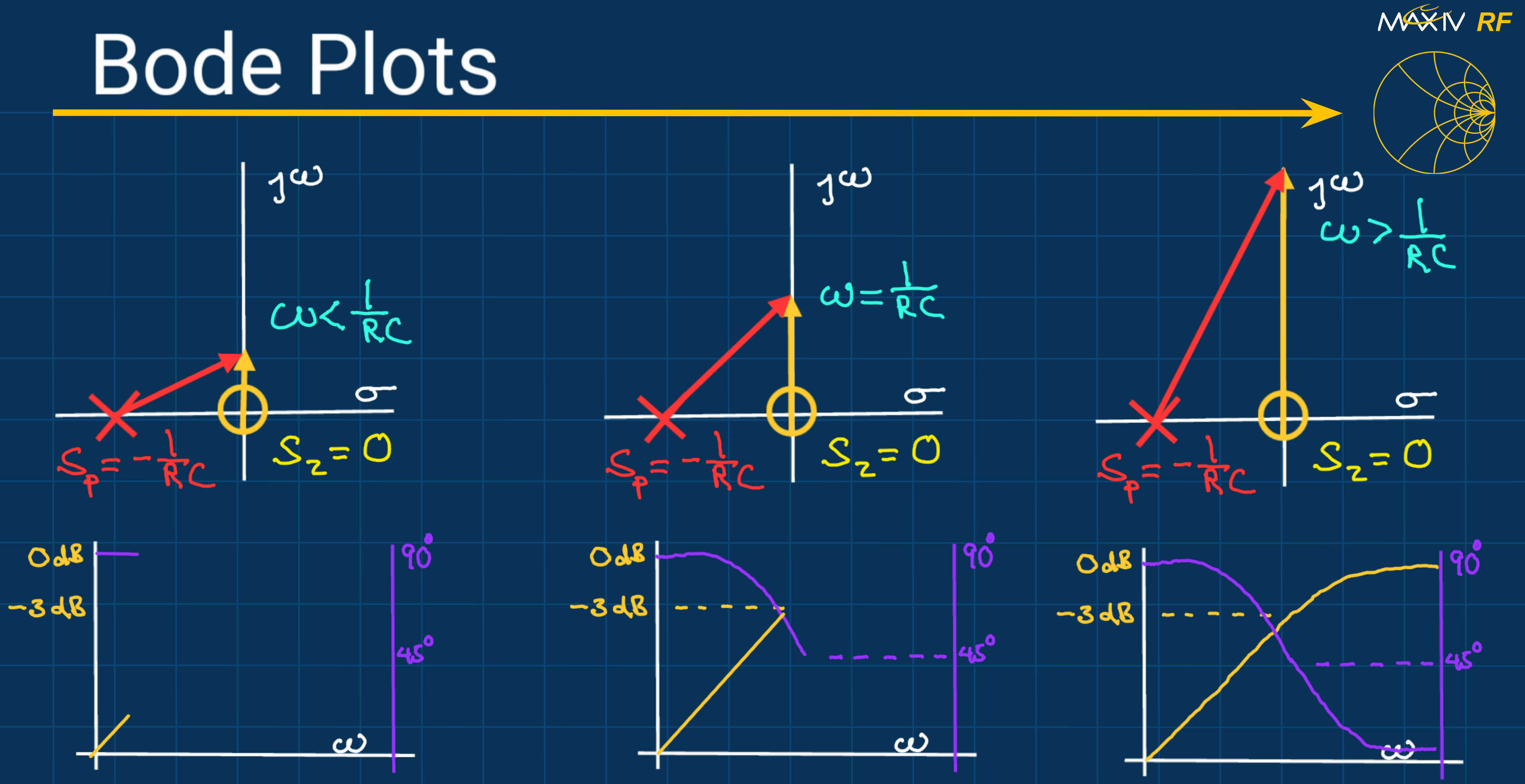


$$|l_z| < |l_p|$$
$$\angle l_z \simeq 90^\circ$$
$$\angle l_p \sim 0^\circ$$

$$\sqrt{2}\,|l_z| = |l_p|$$
$$\angle l_z \simeq 90^\circ$$
$$\angle l_p = 45^\circ$$

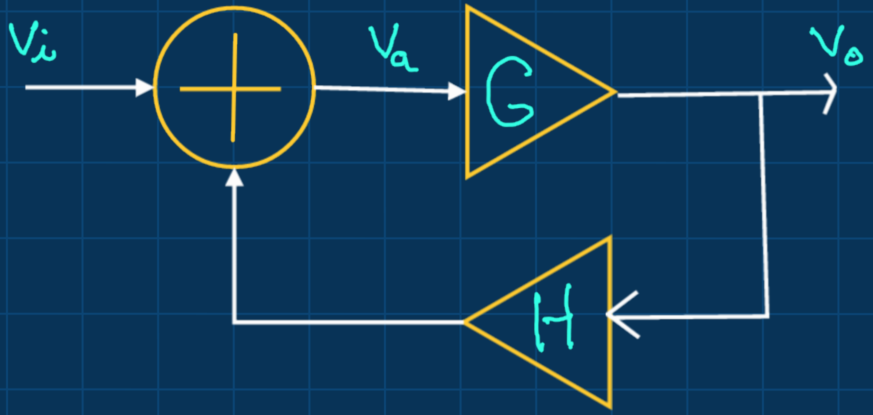
$$|l_z| \sim |l_p|$$
$$\angle l_z \simeq 90^\circ$$
$$\angle l_p \sim 90^\circ$$

Bode Plots



MAX IV RF
jω
ω < 1/RC
σ
S_p = −1/RC
S_z = 0
jω
ω = 1/RC
σ
S_p = −1/RC
S_z = 0
jω
ω > 1/RC
σ
S_p = −1/RC
S_z = 0
0 dB
−3 dB
90°
45°
ω
0 dB
−3 dB
90°
45°
ω
0 dB
−3 dB
90°
45°
ω

Feedback



$$V_a = V_i + H V_o$$

$$V_o = G V_a$$

$$V_o = \frac{G}{1 - GH} V_i$$

Open loop transfer function $= GH$

Closed loop transfer function $= \dfrac{G}{1 - GH}$

Feedback

Let G be a low pass filter

$$G = \frac{G_o}{1 + s/\Delta\omega_o} \qquad H = H_o$$

$$\frac{V_o}{V_i} = \frac{\Delta\omega_o}{\Delta\omega_f}\,\frac{G_o}{1 - s/\Delta\omega_f} \qquad \frac{\Delta\omega_f}{\Delta\omega_o} = 1 - G_o H_o$$

$$v_i(t) = V_i \tau_i \delta(t) \qquad v_o(t) = V_i \Delta\omega_o \tau_i e^{-\Delta\omega_f t}$$

Feedback

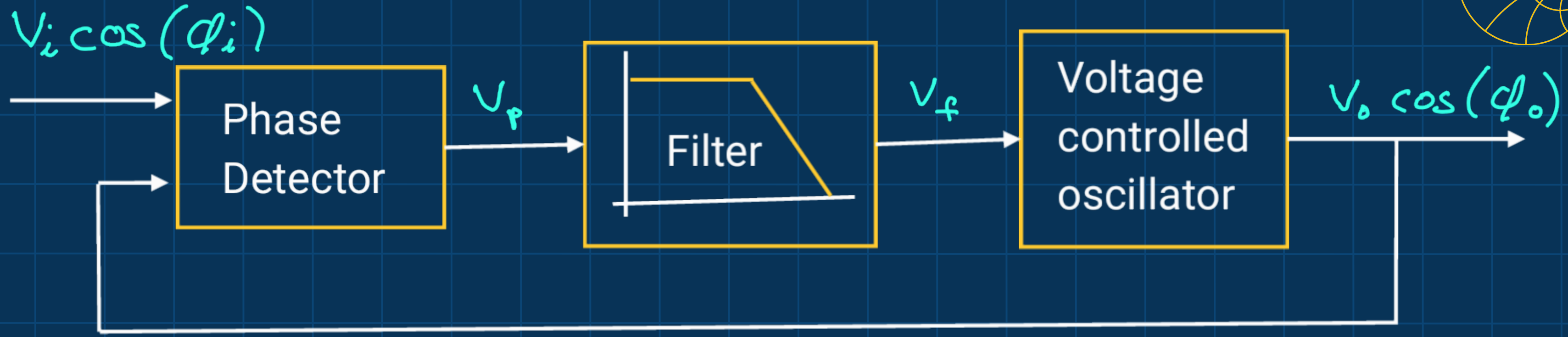
If $G_0 H_0 > 1$ then $\Delta\omega_f < 0$

and the system becomes unstable.

For stability, the poles of a system should be located on the left hand side of the s plane.

If $G_0 H_0 < 0$ the closed loop gain is reduced but the bandwidth is increased.

Phase- Locked Loop



If ω is a constant

$$\phi = \omega t + \psi$$

$$\boxed{V_p(s) = \phi_i(s) - \phi_o(s)}$$

$$\boxed{V_f(s) \quad \frac{1}{1+\frac{s}{\Delta\omega_f}} \quad V_p(s)}$$

Phase- Locked Loop

$$f_o(t) = K\, v_f(t) \qquad \varphi_o(t) = 2\pi \int f_o(t)\, dt$$

$$\varphi_o(s) = \frac{2\pi K}{s} V_f(s)$$

$$\varphi_o(s) = \frac{2\pi K \Delta\omega_f}{s^2 + s\Delta\omega_f + 2\pi K \Delta\omega_f}$$

10	$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}$

When $K = \frac{\Delta\omega_f}{8\pi}$

response is critically damped

$$\varphi_o(s) = \left[\frac{\Delta\omega_f}{2s + \Delta\omega_f}\right]^2$$

Frequency Control of an RF System

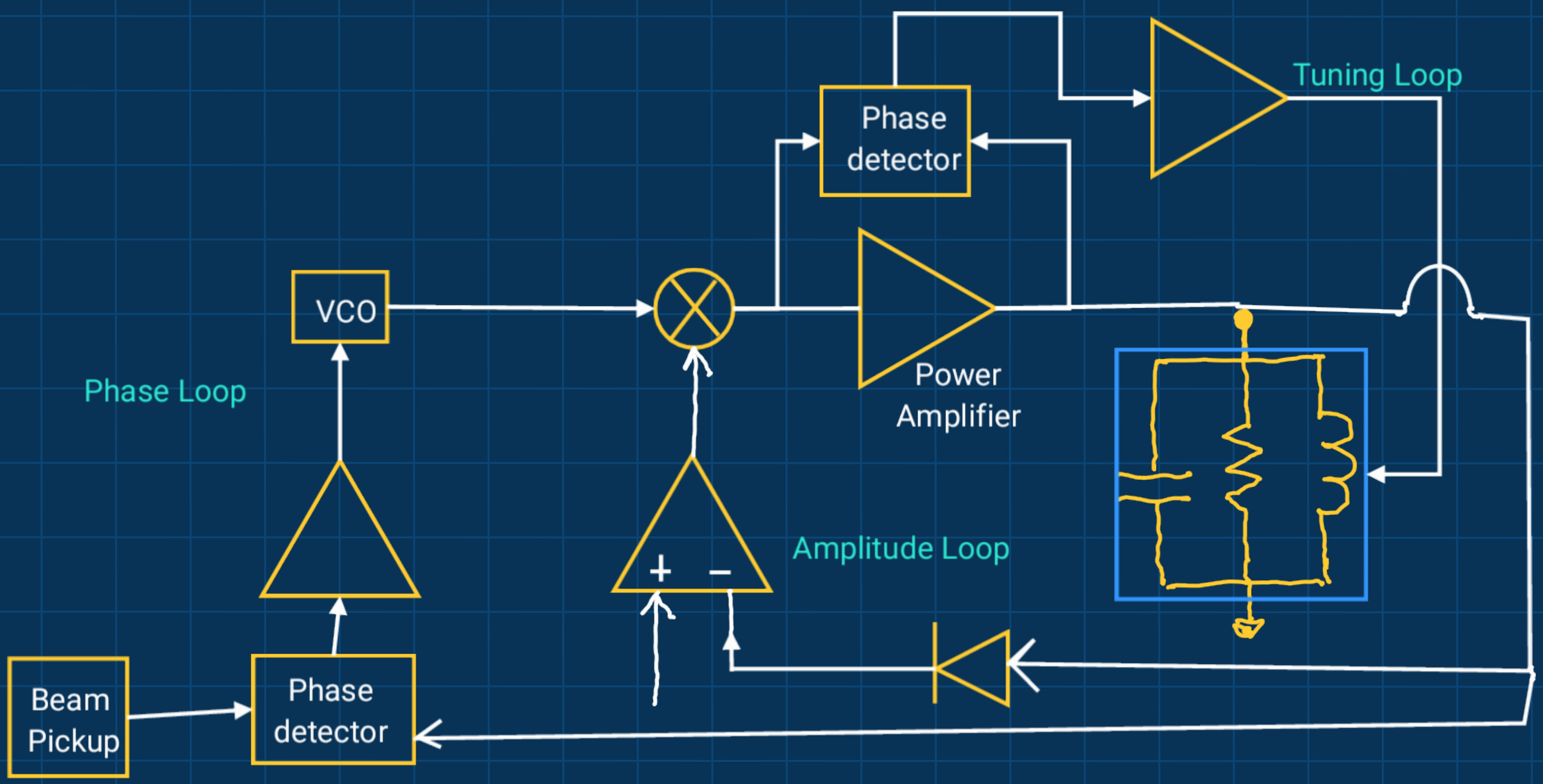
LLRF & HLRF

- Low Level RF (LLRF) is concerned with the frequency control and the amplitude control of the RF waveform that is used to accelerate charge particles.
- Some people restrict LLRF to the realm of frequency control
- and designate High Level RF (HLRF) for amplitude control.
- Before the advent of numerically controlled oscillators (NCO) or direct digital synthesizers (DDS), most RF systems used voltage controlled oscillators(VCO) for phase continuous frequency control

Frequency Sources

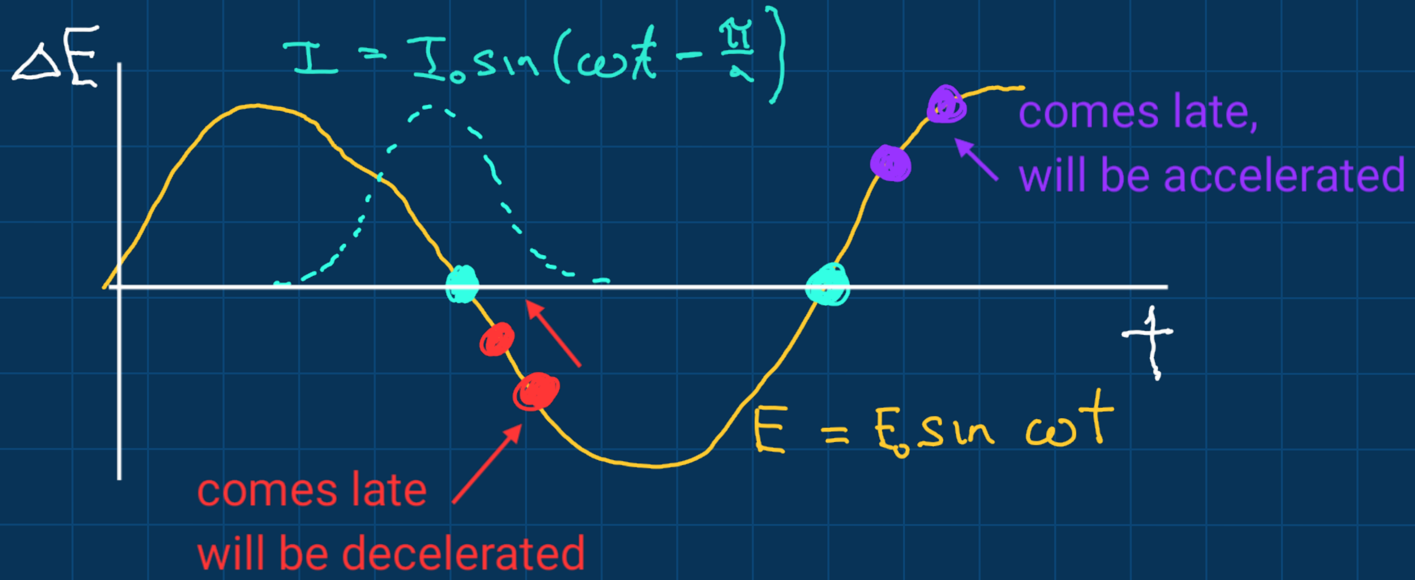
- VCOs can be very tempermental devices requiring great care in component choices, temperature stabilization, and feedback control.
- With NCOs and DDSs, the devices are so stable, the trend for many RF systems is to operate "open loop".
- However, for high intensity accelerators, LLRF systems need to incorporate feed back systems for beam stability.
- A LLRF system using DDS technology can be constructed in an FPGA but the function is still analogous to an analog VCO circuit.

Complete RF System



Phase focusing

Above transition higher energy particles, take a longer time to go around the ring than lower energy particles.



Beam Phase Transfer Function

Change in revolution frequency for a change in beam energy

$$\boxed{\frac{\Delta T_r}{T_r} = \frac{\eta}{\beta^2}\frac{\Delta E}{E_0}}$$

$$\boxed{\Delta\phi = -\frac{2\pi h}{T_r}\Delta T_r}$$

$$\boxed{\frac{\Delta\phi}{\Delta n} = -2\pi h\frac{\eta}{\beta^2}\frac{\Delta E}{E_0}}$$

$$\boxed{\frac{d\phi}{dt} = -\omega_{Rf}\frac{\eta}{\beta^2}\frac{\Delta E}{E_0}}$$

Beam Phase Transfer Function

Change in beam energy for a change in RF phase

$$\boxed{\Delta E = qV\sin\varphi \approx qV\varphi}$$

$$\boxed{\frac{d\,\Delta E}{dt} = q\frac{V\varphi}{T_r} = \frac{\omega_{Rf}}{2\pi h}qV\varphi}$$

$$\boxed{\frac{d^2\varphi}{dt^2} = -\omega_{Rf}\frac{\eta}{\beta^2}\frac{d}{dt}\frac{\Delta E}{E_0}}$$

Beam Phase Transfer Function

$$\frac{d^2\phi}{dt^2} + \omega_s^2 \phi = 0$$

$$\omega_s^2 = \frac{\omega_{rf}^2}{2\pi h} \frac{\eta}{\beta^3} \frac{eV}{E_0}$$

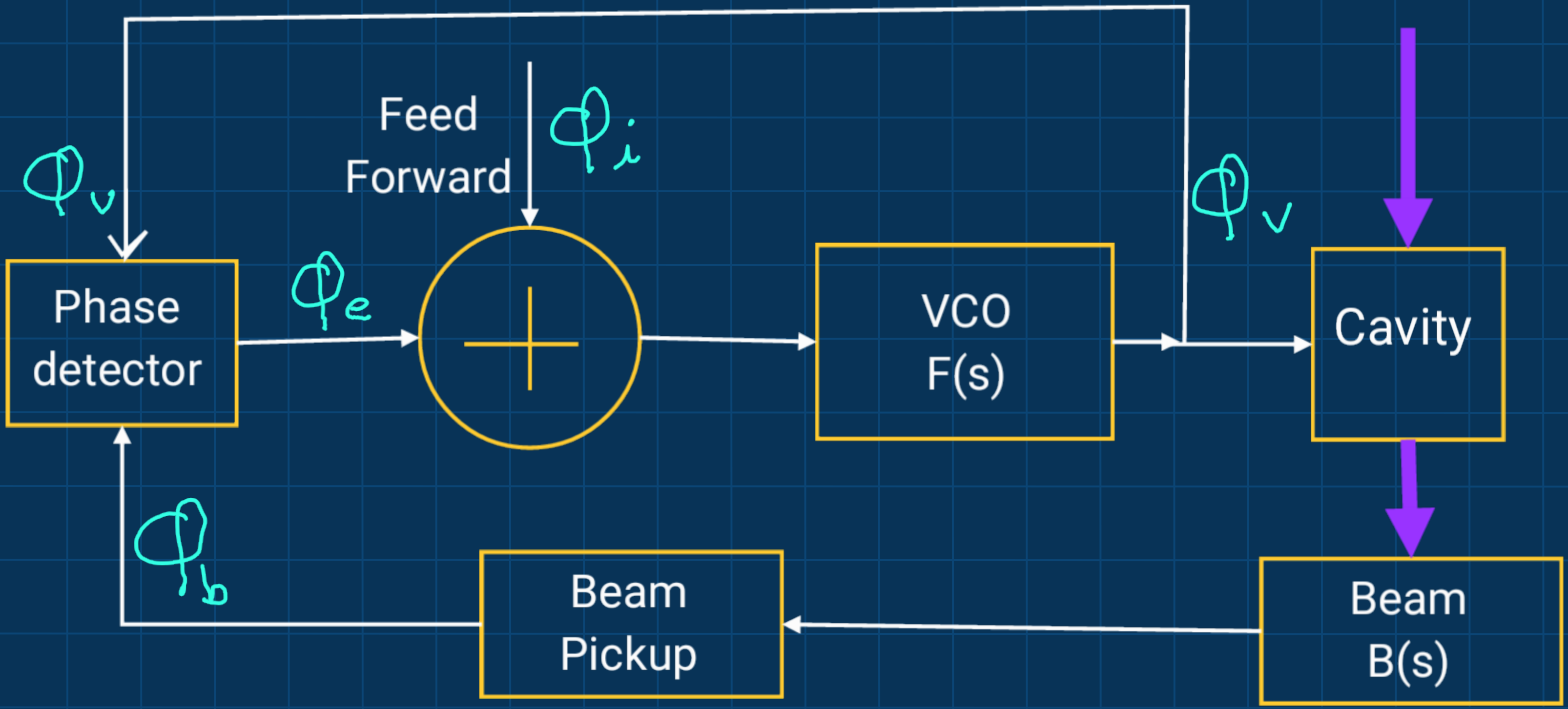
characteristic equation

$$s^2\phi + \omega_s^2\phi = 0$$

$$B(s) = \frac{\omega_s^2}{s^2 + \omega_s^2}$$

undamped oscillator

Phase-locked RF System



Phase-locked RF System Response

$$\phi_e = \phi_v - B(s)\phi_v$$

Phase detector

$$\phi_v = F(s)(\phi_e + \phi_i)$$

VCO

$$\frac{\phi_e}{\phi_i} = \frac{F(s)s^2}{s^2 + \omega_s^2 + F(s)s^2}$$

$$F(s) = \frac{F_0}{s}$$

$$\frac{\phi_e}{\phi_i} = \frac{F_0 s}{s^2 + F_0 s + \omega_s^2}$$

Damped response

Cavity RLC Model

Cavity loss $\Rightarrow R$

Stored magnetic energy $\Rightarrow L$

Stored electric energy $\Rightarrow C$

$$\boxed{\frac{1}{Z} = \frac{1}{R} + \frac{1}{sL} + sC}$$

Cavity RLC Model

Let $\boxed{\omega_o = \frac{1}{LC}}$ $\boxed{\frac{R}{Q} = \sqrt{\frac{L}{C}}}$

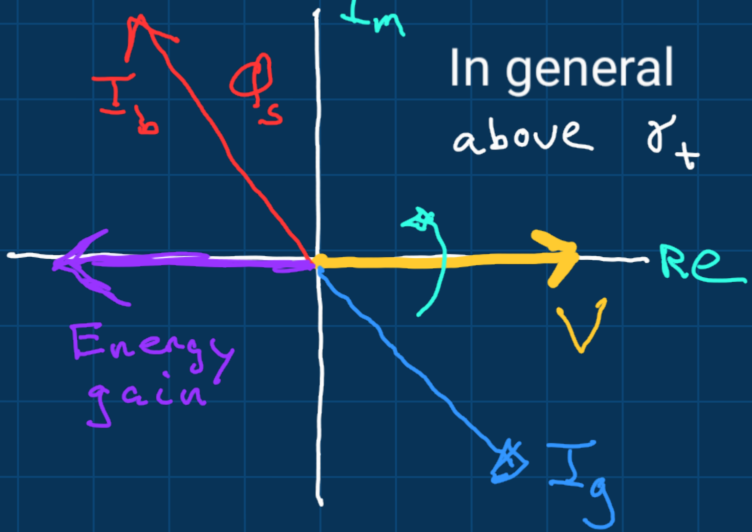
$$\boxed{Z = \frac{s\,\omega_o R/Q}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}}$$

$$\boxed{Z = R\cos\phi\, e^{j\phi}}$$

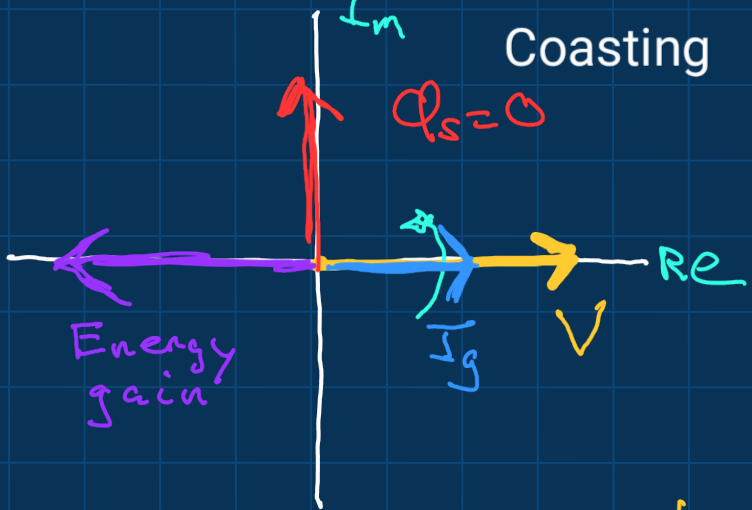
$$\boxed{\tan\phi = \frac{\omega_o^2 - \omega^2}{\frac{\omega\,\omega_o}{Q}}}$$

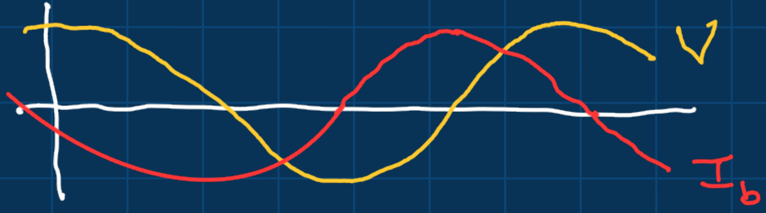
Cavity Phasors

In general
above γ_t



Coasting





$$\overline{V} = V_0 e^{j\omega t}$$

$$\overline{I_g} = I_{g0} e^{j\omega t}$$

$$\overline{I_b} = j I_{b0} e^{j\omega t}$$

Cavity Phasors

$$\bar{V} = \bar{Z}(\bar{I}_g + \bar{I}_b)$$

$$\bar{Z} = R\cos\phi \, e^{j\phi}$$

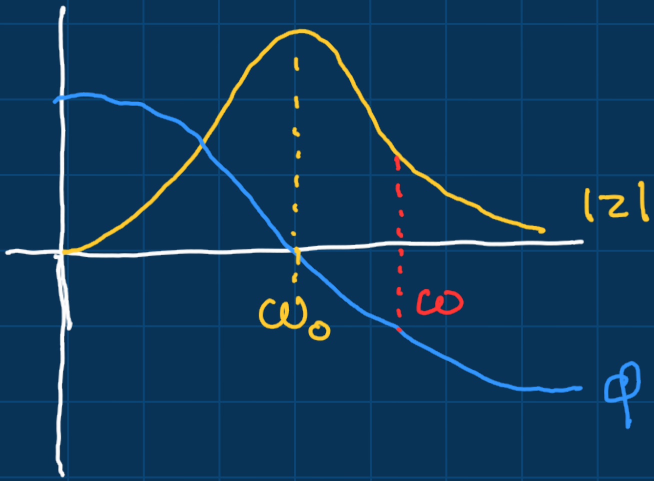
$$V = R\cos\phi\,(I_g\cos\phi - I_b\sin\phi) \quad \text{Real Part}$$

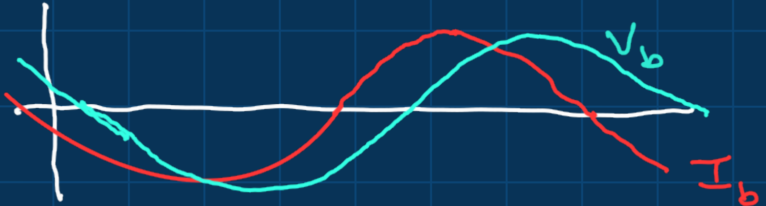
$$0 = I_g\sin\phi + I_b\cos\phi \quad \text{Imag Part}$$

$$\tan\phi = -\frac{I_b}{I_g}$$

$$V = I_g R$$

Cavity Detuning





In the presence of beam current, the cavity must be detuned so as to present a real load to the generator. Above transition the cavity looks capacitive to the beam. i.e the voltage lags the beam current.

This also "assists" phase focusing which keeps the beam Robinson stable

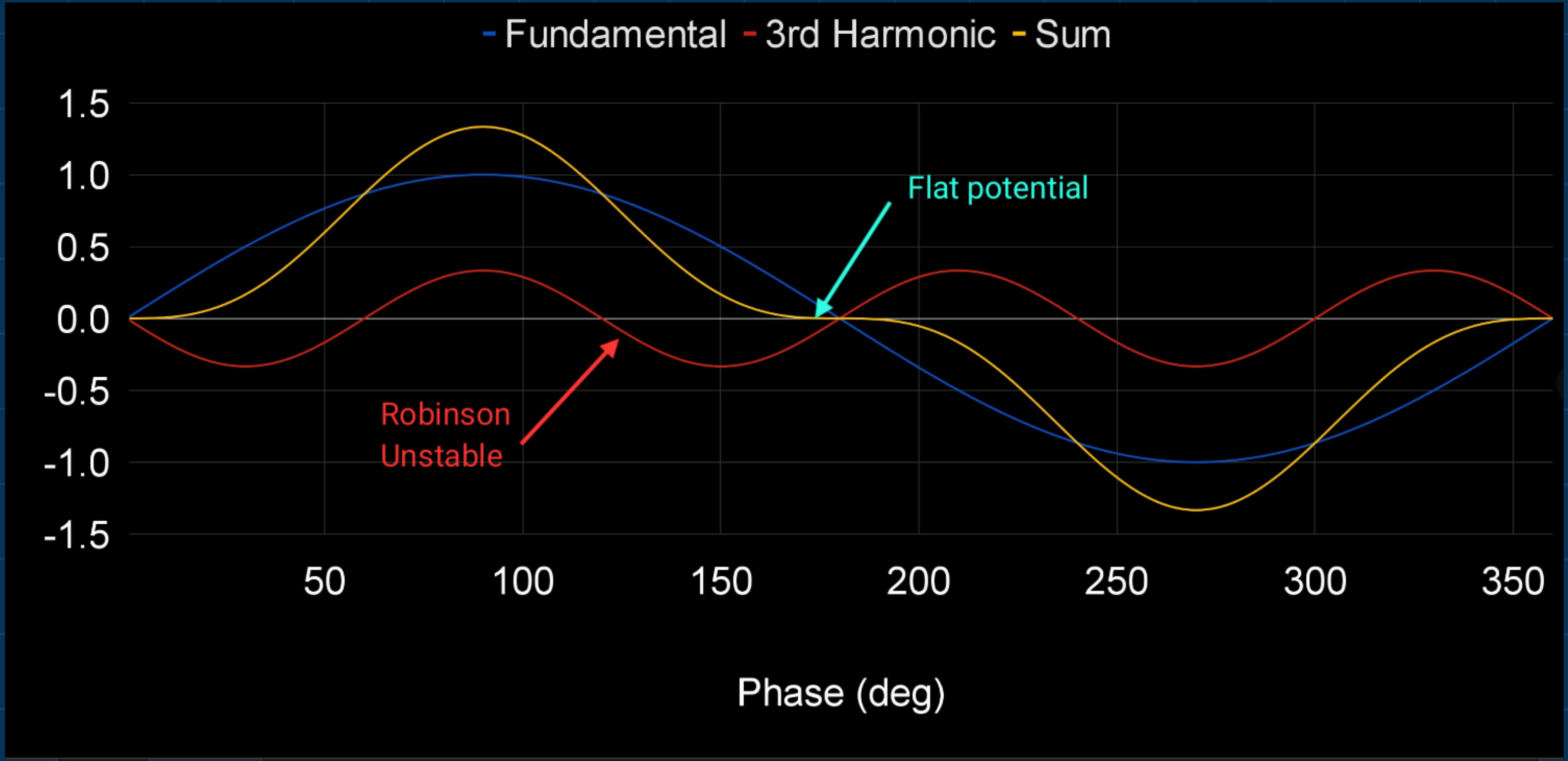
3rd Harmonic Bunch Lengthening

- For light sources, synchrotron radiation will provide a natural damping term.
- If all the cavities are detuned for power match and hence, are Robinson stable, there is no need for a phase loop.
- However, for light sources, low energy spread in the beam is very desireable
- For high frequency single harmonic RF systems, electrons in a bunch will clump together at the synchronous phase resulting in short bunches with large energy spread

3rd Harmonic RF

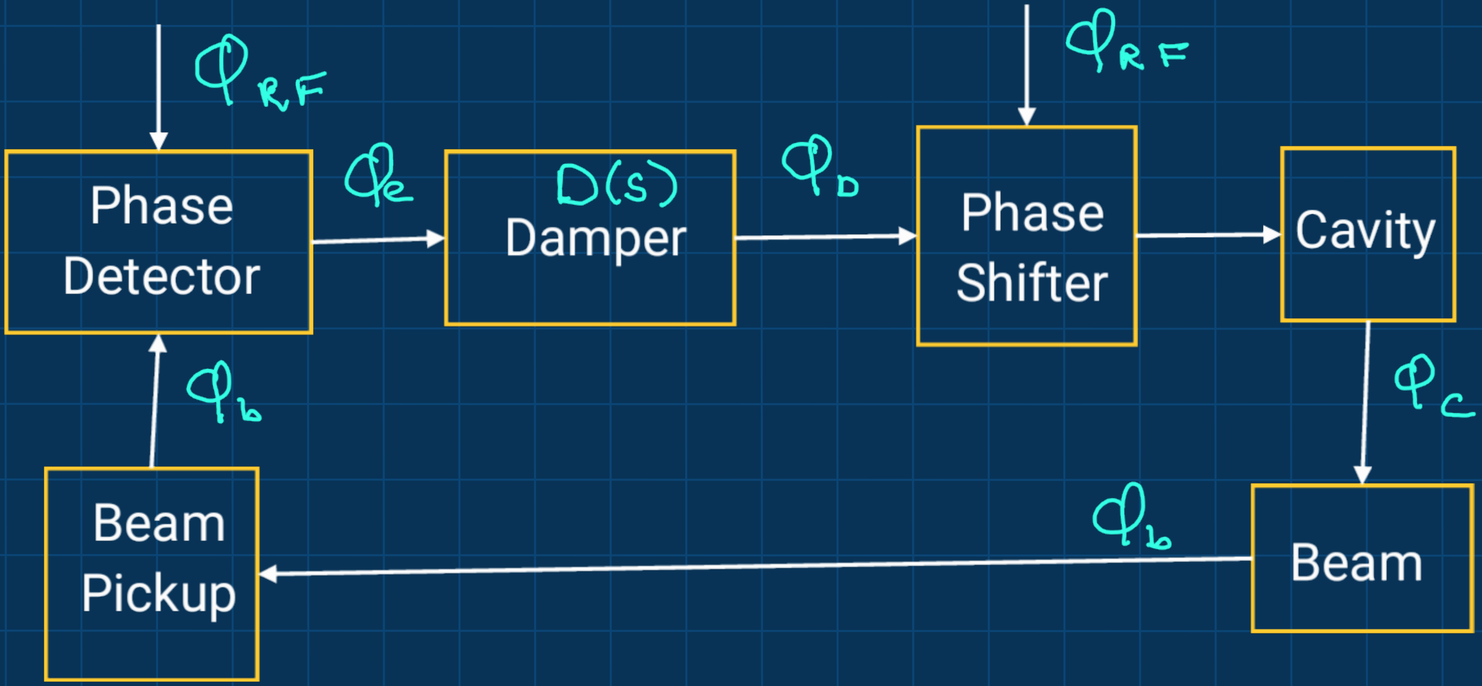
- At Max IV, a relatively low RF frequency (100 MHz) was chosen.
- The bunches are lengthened by the additon of 3rd harmonic cavities (300MHz) providing a flat potential for the electrons.
- The 3rd harmonic cavities are passive and are detuned so that the beam wake provides the field in the cavities. However, this results in the 3rd harmonic cavites being tuned to Robinson unstable!

3rd Harmonic RF



AC Coupled Phase Loop

An AC coupled phase loop can be easily added to an existing RF system



AC Coupled Phase Loop

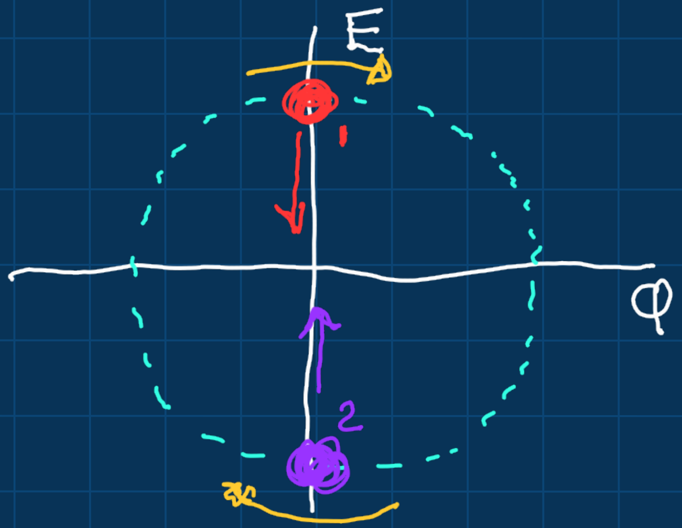
$$\boxed{\varphi_e = \frac{1-B}{1+BD}\varphi_{RF} = \frac{s^2}{s^2+\omega_s^2+D\omega_s^2}\varphi_{RF}}$$

Let $D(s) = D_0 s$ (derivative)

$$\boxed{\varphi_e = \frac{s^2}{s^2 + D_0\omega_s^2 s + \omega_0^2}}$$

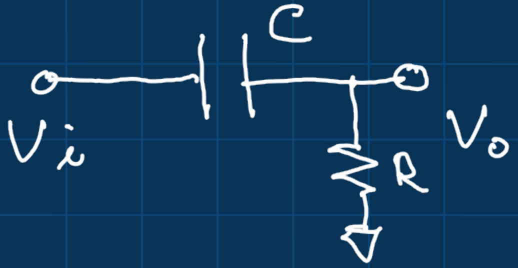
Critically damped for $\boxed{D_0 = 2/\omega_s}$

Practical AC Coupled Phase Loop

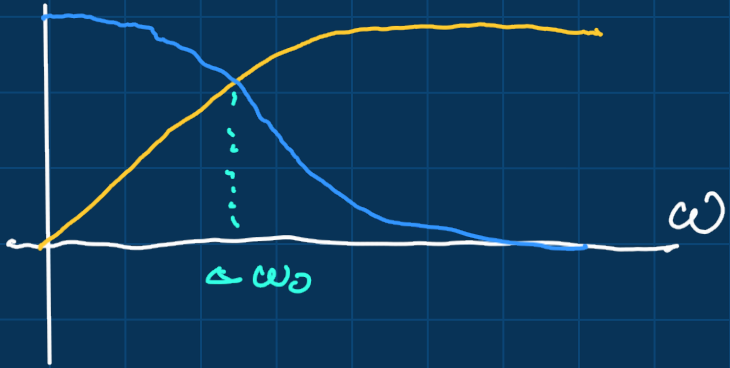


- Situation 1 and situation 2 have the same phase but require different energy corrections.
- However for Situation 1, the phase is increasing. For Situation 2, the phase is decreasing.
- Taking the time derivative of phase distinguishes the different situations.

Analog High Pass Filter Derivative



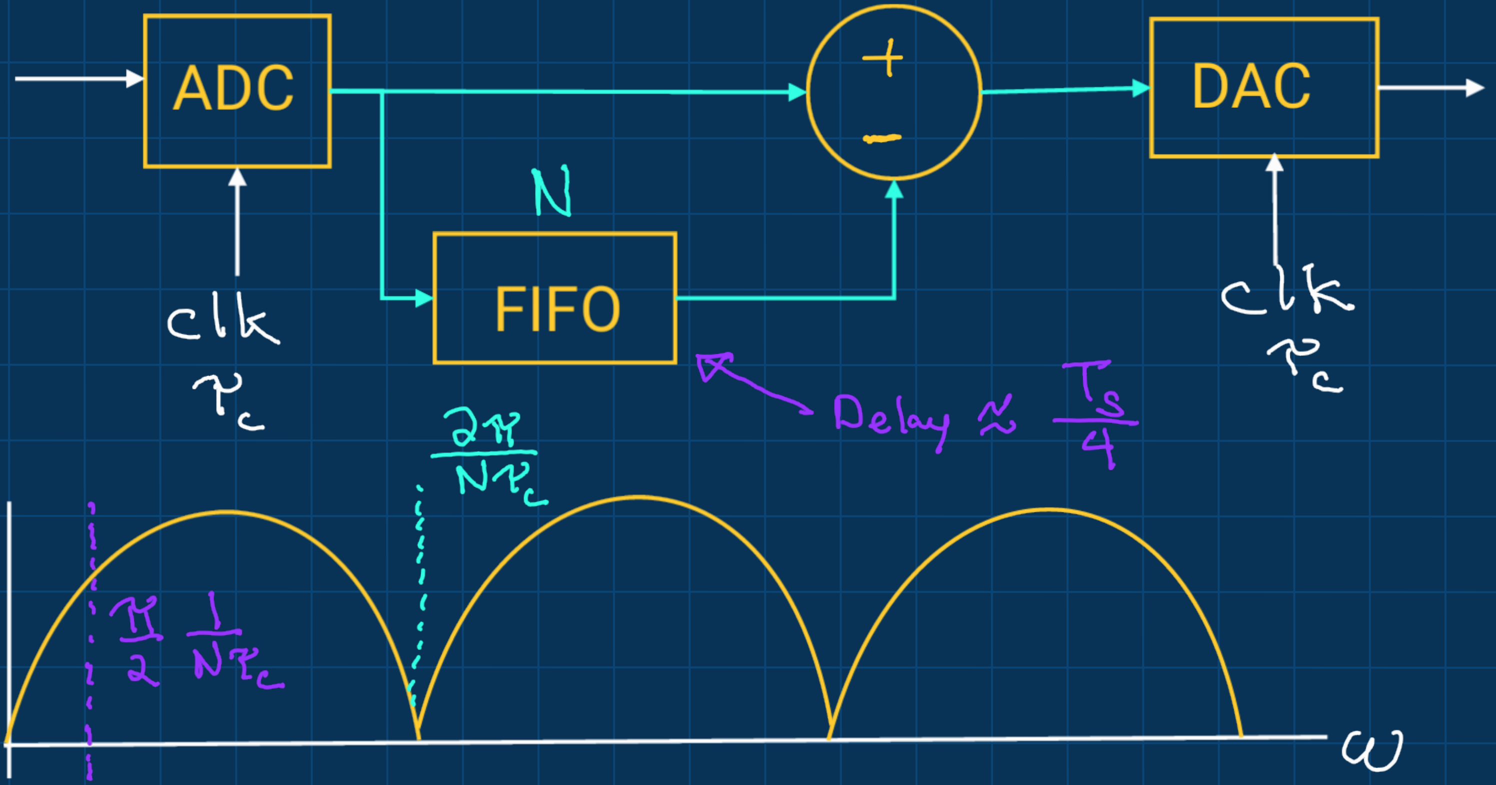
$$\frac{V_o}{V_i} = \frac{s/\Delta\omega_0}{1 + s/\Delta\omega_0}$$



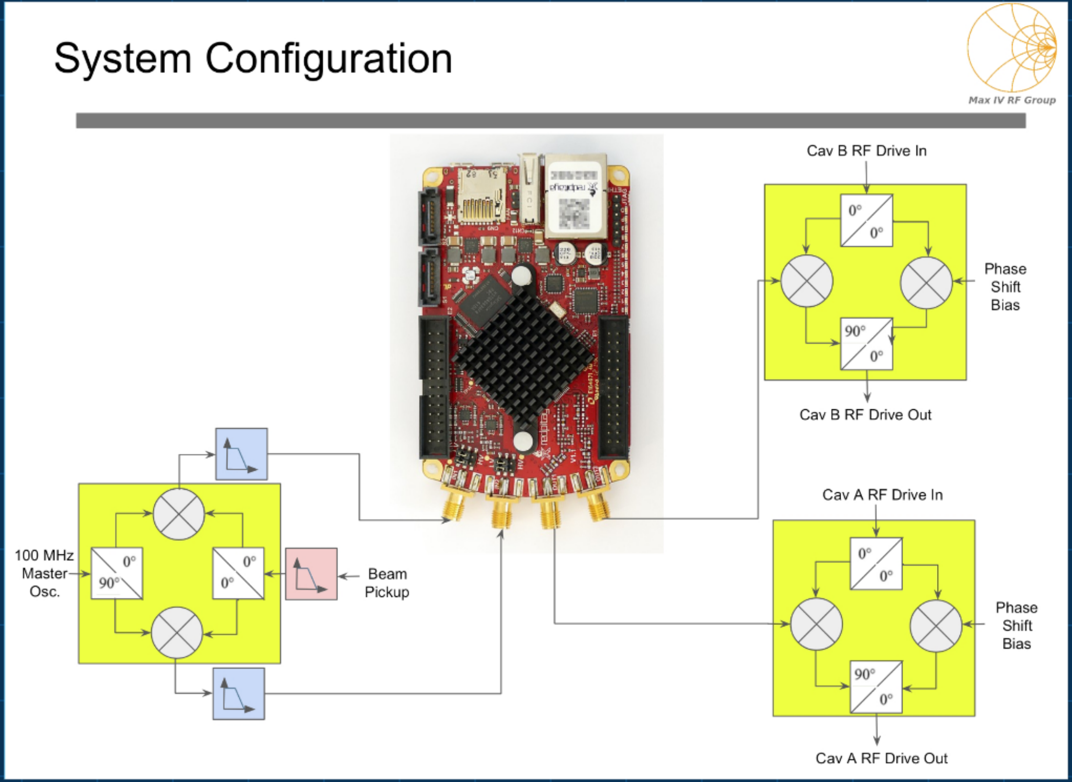
Acts as a derivative for

$$\omega < \Delta\omega_0$$

Digital Notch Filter Derivative



AC Phase Loop Implementation



AC Phase Loop Control