Laplace Transforms

Fourier Transform Pair

$$\tilde{F}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{F}(\omega)\, e^{j\omega t}\, d\omega$$

The problem with Fourier transforms is that they cannot handle initial conditions due to the infinite bounds on the integral. To handle initial conditions , consider the Laplace transform

$$\mathcal{L}[f(t)] = \tilde{F}(s) = \int_{0}^{\infty} f(t)\, e^{-st}\, dt$$

Laplace Transform Table

MAX IV RF

S.no	$f(t)$	$\mathcal{L}\{f(t)\}$	S.no	$f(t)$	$\mathcal{L}\{f(t)\}$
1	1	$\frac{1}{s}$	11	$e^{at}\sinh bt$	$\frac{b}{(s-a)^2-b^2}$
2	e^{at}	$\frac{1}{s-a}$	12	$e^{at}\cosh bt$	$\frac{s-a}{(s-a)^2-b^2}$
3	t^n	$\frac{n!}{s^{n+1}}$	13	$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
4	$\sin at$	$\frac{a}{s^2+a^2}$	14	$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$
5	$\cos at$	$\frac{s}{s^2+a^2}$	15	$f'(t)$	$sF(s)-f(0)$
6	$\sinh at$	$\frac{a}{s^2-a^2}$	16	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
7	$\cosh at$	$\frac{s}{s^2-a^2}$	17	$\int_0^t f(u)du$	$\frac{1}{s}F(s)$
8	$e^{at}t^n$	$\frac{n!}{(s-a)^{n+1}}$	18	$t^nf(t)$ Where $n=1,2,3,..$	$(-1)^n\frac{d^n}{ds^n}\{F(s)\}$
9	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$	19	$\frac{1}{t}\{f(t)\}$	$\int_s^\infty F(s)ds$
10	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$	20	$e^{at}f(t)$	$F(s-a)$

Inverse Laplace Transform

s is a complex number!

$$\boxed{s = \sigma + j\omega}$$

Since **s** is a complex number, the inverse transform is hard to compute directly and using the transform tables in reverse to find the inverse transform is the usual technique.

Initial and Final value Theorems

Initial value theorem

$$\boxed{\lim_{t \to 0^+} f(0^+) = \lim_{s \to \infty} s f(s)}$$

Final value theorem

$$\boxed{\lim_{t \to \infty} f(t) = \lim_{s \to 0} s f(s)}$$

Circuit Elements

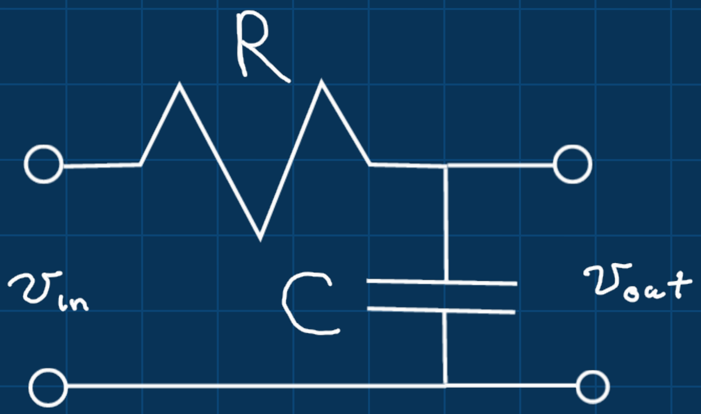
Uncharged capacitor

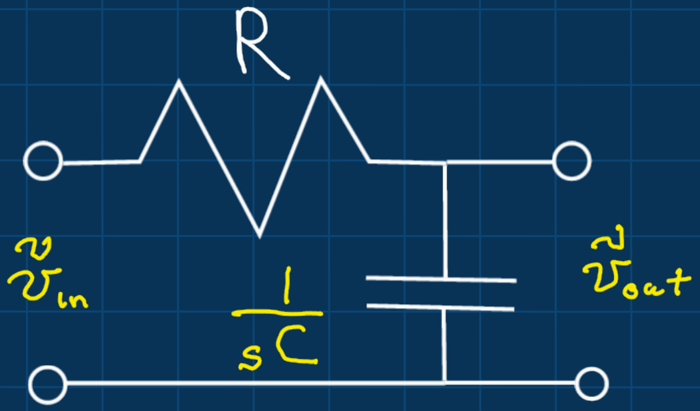
$$\boxed{i(t) = C\frac{dv}{dt}} \qquad \boxed{\tilde{i}(s) = sC\,\tilde{v}(s) - C\,\cancelto{0}{v(0)}}$$

Uncharged inductor

$$\boxed{v(t) = L\frac{di}{dt}} \qquad \boxed{\tilde{v}(s) = sL\,\tilde{i}(s)}$$

Low Pass Filter





$$\frac{\tilde{v}_{out}}{\tilde{v}_{in}} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sRC}$$

Frequency Response

$$s \Longrightarrow j\omega$$

$$\boxed{\frac{\tilde{v}_o}{\tilde{v}_i} = \frac{1}{1 + j\omega RC}}$$

When $\omega = \frac{1}{RC}$

$$\boxed{\frac{V_o}{V_i} = \frac{1}{1+j} = \frac{1-j}{2}}$$

$$\boxed{\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{2}}}$$

$$\boxed{\angle\left(\frac{V_o}{V_i}\right) = -45^\circ}$$