Homework_2

Math 742

2/2/2026

Casella/Berger

3.1 Find expressions for $\mathbb{E}(X)$ and $\mathbb{V}(X)$ if X is a random variable with the general discrete uniform(N_0, N_1) distribution that puts equal probability on each of the values $N_0, N_{0+1}, , , , N_1$. Where $N_0 \leq N_1$ and N_0 and N_1 are both integers.

3.4 A man with n keys wants to open his door and tries the keys at random. Exactly one key will open the door. Find the mean number of trials if,

(a) unsuccessful keys are not eliminated from further selections.

(b) unsuccesful key are eliminated.

3.23 The *Pareto Distribution*, with parameters α and β, has pdf

$$f(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \text{ where } \alpha < x < \infty, \alpha > 0, \beta > 0$$

(a) Verify that $f(x)$ is a PDF.

(b) Derive the mean and variance of this distribution

(c) Prove that the variance does not exist if $\beta \leq 2$.

6.2 Let $X_1, X_2, ..., X_n$ be independent random variables with densities

$$f_{X_i}(x|\theta) = \begin{cases} e^{i\theta - x} & : x \geq i\theta \\ 0 & : x < i\theta \end{cases}$$

Prove that $T = \min(X_i/i)$ is a sufficient statistic for θ.

Road Map:

(1) Redefine the PDF by using an indicator function:

$$f_{X_i}(x \mid \theta) = e^{\,i\theta - x}\,\mathbf{1}\{x \geq i\theta\}.$$

(2) Find an expression for the joint PDF: $f_{\mathbf{X}}(\mathbf{x} \mid \theta)$

(3) Rewrite the joint PDF using the statistic $T = \min(X_i/i)$.

(4) Use the factorization theorem to show that T is sufficient for θ.

6.8 Let $X_1, X_2, ..., X_n$ be a random sample from a population with location PDF $f(x - \theta)$. Show that the order statistics, $T(X_1, X_2, ..., X_n) = (X_{(1)}, X_{(2)}, ..., X_{(n)})$ are a sufficient statistic for θ and no further reduction is possible. (continued on next page)

More specifically, a population has a location PDF if there exists a fixed PDF f_0 such that for some unknown location parameter θ,

$$f_X(x|\theta) = f_0(x-\theta), \text{ where } -\infty < \theta < \infty$$

Equivalently, if $Y \sim f_0$, then $X = \theta + Y$.

6.9b For the following distribution, let $X_1, ..., X_n$ be a random sample. Find a minimal sufficient statistic for θ.

$$f(x|\theta) = e^{-(x-\theta)}, \text{ where } \theta < x < \infty, -\infty < \theta < \infty$$

More specifically,

$$f(x,y) = \begin{cases} e^{-(x-\theta)} & : x > \theta \\ 0 & : x \leq \theta \end{cases}$$

Extra Problem: (you are required to do this)

Suppose that X_1 and $X_2 \sim$ Bernoulli(p). Let $T(\mathbf{X}) = (X_1 + X_2)$. Show that:

- T is sufficient for p.
- T is also minimal sufficient.