Lab 1

Math 742

21 Jan 2026

Introduction

In this lab, we will review key concepts from probability and random variables, including:

- Expectations, variances, and covariances
- Transformations of random variables
- Common distributions (Binomial, Poisson, Normal, Multivariate Normal, and Multinomial)
- Simulation and visualization

You can use either R or Python to explore theoretical properties and to simulate data for empirical verification. For each simulation, be sure to set a seed so that I can reproduce your results.

Random Variables and Expectation

1. Generate 10,000 samples from a uniform(0,1) random variable and compute its sample mean and variance.
2. Compare your results to the theoretical mean and variance of a Uniform(0,1) distribution.

Let $Y = X^2$, where $X \sim \text{Uniform}(0, 1)$. Use the sample of x-values from question 1. Compute y=xˆ2 and use R to calculate the mean of y and the variance of y.

Derive analytically $\mathbb{E}(Y)$ and $\mathbb{V}(Y)$. How do your simulation results compare?

Covariance and Correlation

Simulate two correlated normal variables with mean 0 and covariance 0.8. Generate 5000 samples and calculate the correlation. Make a scatter plot.

Interpret the correlation result. How does changing the covariance affect the shape of the scatterplot?

Conditional Distributions

Let X, Y be bivariate normal distributions with $\rho = 0.5$. The conditional distribution of $Y|X = x$ is Normal with mean $\mu_Y + \rho\frac{\sigma_Y}{\sigma_X}$ and variance $(1 - \rho^2)\sigma_Y^2$.

Create 5000 random samples from this bivariate normal distribution.

Fit a linear model to your data. How does the regression slope compare with the theoretical conditional mean relationship?

Common Distributions

- Create 10000 random samples from a binomial distribution with $n = 10, p = 0.4$. Calculate the mean and variance of your sample. Compare to $\mathbb{E}(X)$ and $\mathbb{V}(X)$ where $X \sim$ Binomial(n,p)

- Create 10000 random samples from a Poisson distribution with lambda=2. Calculate the mean and variance of your sample. Compare to $\mathbb{E}(X)$ and $\mathbb{V}(X)$ where $X \sim$ Poisson(rate=lambda). What happens when lambda is large?
- Generate 5000 samples from a bivariate normal distribution with:

$$\mu = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

Make a contour plot of your data. How do the contours of this distribution relate to the covariance structure?

Discussion

Summarize:

How empirical simulations align with theoretical results.

Practical implications for modeling and inference.

Submission

- If using R, Knit your R Markdown file to PDF.
- If you are using Python, Export your notebook to PDF.
- For R users, submit both the .Rmd and the rendered output file.
- For Python users, submit both the .ipynp and the rendered output file.