Homework_1

Math 742

1/19/2026

Casella/Berger

1.2d: Prove $A \cup B = A \cup (B \cap A^c)$

After proving the above result, prove: P$(A \cup B)$ =P(A)+P(B)−P$(A \cap B)$

1.3c DeMorgan's Laws. Prove this without Venn diagrams.

1.47b: Prove that $F_X(x) = \frac{1}{1+e^{-x}}, x \in (-\infty, \infty)$ is a CDF.

4.4 A PDF is defined by,

$$f(x,y) = \begin{cases} C(x+2y) & : 0 < y < 1 \text{ and } 0 < x < 2 \\ 0 & : \text{ Otherwise} \end{cases}$$

a) Find the value of C.

b) Find the marginal distribution of X.

c) Find the joint CDF of X and Y.

d) Find the PDF of the random variable $Z = 9(X+1)^2)$

4.10 The random pair (X, Y) has the distribution:

	$X=1$	$X=2$	$X=3$
$Y=2$	1/12	1/6	1/12
$Y=3$	1/6	0	1/6
$Y=4$	0	1/3	0

a) Show that X and Y are dependent.

b) Give a probability table for random variables U and V that have the same marginals as X and Y but are independent.

Wasserman

3.10 $X \sim$ N(0,1), $Y \sim e^X$. Determine $\mathbb{E}(Y)$ and $\mathbb{V}(Y)$

3.19

Extra Problem:

Let X have a Gamma distribution with parameters α, β. Prove that $\mathbb{E}(X) = \alpha\beta$ and $\mathbb{V}(X) = \alpha\beta^2$ The Gamma PDF is defined as:

- $\alpha > 0$
- $\beta > 0$

$f_X(x) = \frac{\beta^\alpha}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$

$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$

Show that: $\mathbb{E}(X) = \alpha\beta$ and $\mathbb{V}(X) = \alpha\beta^2$.