Lab 2

Math 742

2026-02-04

Common Parametric Families

1. Simulate 2 samples from a Gamma(α, β) distribution. One should have a sample size of 200 the other 1000. Use: $\alpha = 3, \beta = 2$.

R: x <- rgamma(n, shape = alpha_true, rate = beta_true)

Python: x = np.random.gamma(shape=alpha_true, scale=1/beta_true, size=n)

2. For each sample plot a histogram and overlay the true PDF.
3. Compute the sample mean and variance and compare the the theretical mean and variance.
4. Comment on differences as sample size increases.

Sufficient Statistics

1. For this you will simulate random values from a gamma distribution:

R: x <- rgamma(n, shape = alpha_true, rate = beta_true)

Python: x = np.random.gamma(shape=alpha_true, scale=1/beta_true, size=n)

Use $\alpha = 3, \beta = 2, n = 500$

2. Compute the sufficient statistics:

- $T_1 = \sum(X_i)$
- $T_2 = \sum \log(X_i)$

3. Plot the Gamma log-likelihood surface:

$$\ell(\alpha, \beta) = n(\alpha \log \beta - \log \Gamma(\alpha)) + (\alpha - 1)T_2 - \beta T_1$$

Create a contour plot over a grid of α and β.

R: contour() or ggplot::geom_contour()

Python: plt.contour

4. What does the likelihood surface look like? Is there a ridge? A clear mode?

Method of Moments

This problem also assumes a gamma distribution with $\alpha = 3, \beta = 2, n = 500$.

1. Compute the first 2 theoretical moments based on the values of α and β.

i.e. $\mathbb{E}(X)$ and $\mathbb{V}(X)$

2. Solve for the method of moments estimators for α and β.

3. Calculate the MoM estimators of α and β from your data and compare the the theoretical moments.

How close are your MOM estimates to the true parameters?

Submission

- If using R, Knit your R Markdown file to PDF.
- If you are using Python, Export your notebook to PDF.
- For R users, submit both the .Rmd and the rendered output file.
- For Python users, submit both the .ipynp and the rendered output file.