Studying nuclear potentials through resonant scattering

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What is resonant scattering?

Scattering phenomena are of immense importance to particle physics. By firing subatomic particles at each other, we gain an insight into their structures and behaviour.

In this project, we are looking at the **scattering cross section** of **nuclei**. When we fire a beam of neutrons at a nucleus, if the beam is larger in diameter than the nucleus, we can't expect all of the neutrons to collide and scatter. The proportion that do can be characterised by the scattering cross section, i.e. the 'cross sectional area' that the target nucleus occupies.

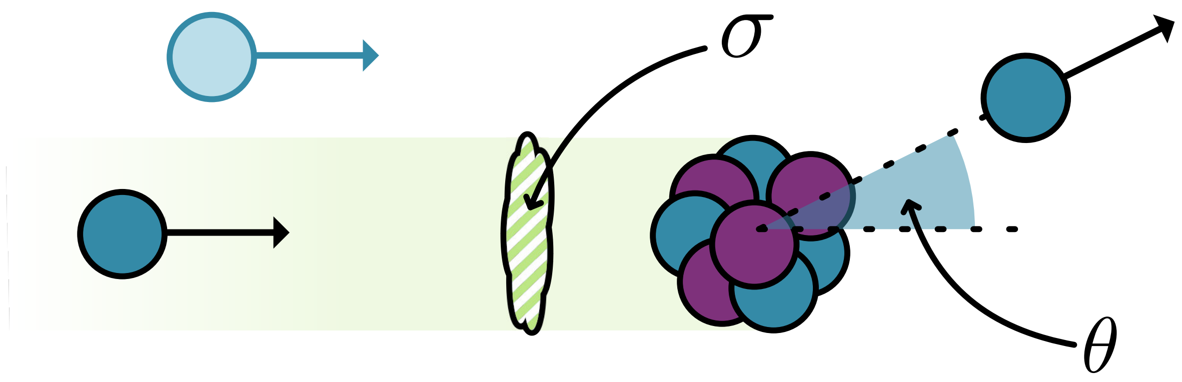

FIGURE 1 Neutrons scattering off a target nucleus. Of the two incoming nucleons, only one will pass through the scattering cross section σ, to collide with the nucleus. This neutron is then scattered at an angle θ.

This being quantum mechanics, we find that the cross section is actually dependant on the energy of the incoming neutrons.

What will we use it for?

The goal of this project is to see if **computational optimisation methods** can be used to determine the potential experienced by a neutron in the presence of a nucleus.

Using the theory of **partial waves**, we can predict how the scattering cross section will vary with the energy of the incoming neutrons, for a given potential. By comparing this with **experimental data**, we can refine our potential to better fit the data. The end goal will be to find a potential which produces the same scattering cross section data as is experimentally measured. This potential could then be considered a good candidate for the nuclear potential.

The starting point for our trial potentials will be the **Saxon-Woods potential**. This has a variety of parameters that can be tweaked. That said, we are by no means limited to the Saxon-Woods potential. It would be possible to vary the coefficients of a say a Taylor series.

Saxon-Woods potential

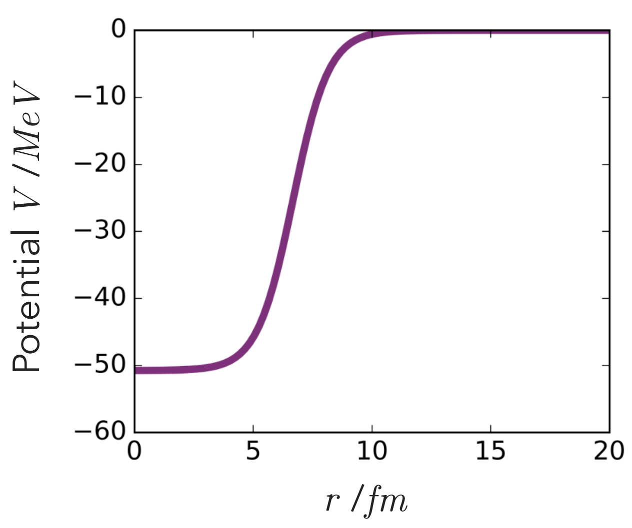

FIGURE 2 The Saxon-Woods potential. This plot shows how the potential experienced varies at various distances from the centre of the target nucleus.

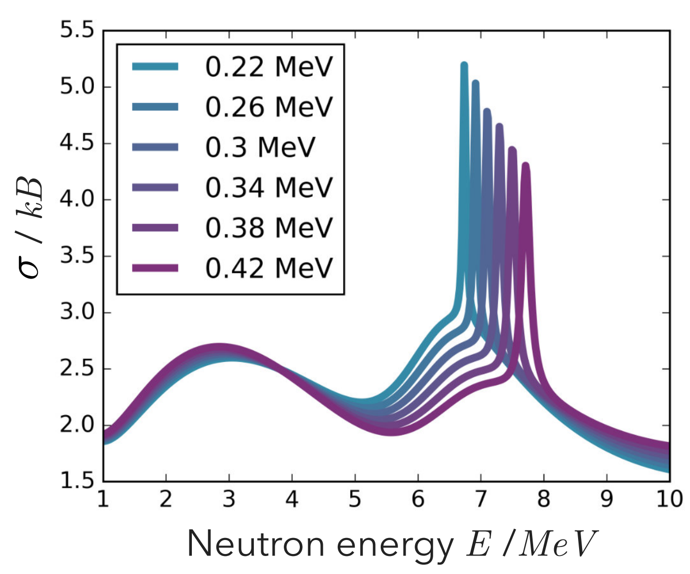

FIGURE 3 Plot of the scattering cross section of against the energy of the incoming neutron E, for six different Saxon-Woods potentials. The key represents the value of V_2 for that potential. Notice how this dramatically shifts the position of the resonance (peak) in the cross section.

The Saxon-Woods potential is a widely used model for the nuclear potential. It is a spherical well potential, with a diffuse boundary (see Fig. 2). The version we shall use to begin with is given by

$$V(r) = -\frac{V_0}{1+e^{\frac{r-R}{a}}}, \quad (1) \qquad V_0 = V_1 - V_2 E - \left(1 - \frac{2Z}{A}\right) V_3. \quad (2)$$

This form we term the 'adaptive' Saxon-Woods potential, due to its dependence on the incoming neutron's energy E. We can manipulate a whole variety of parameters for a given nucleus (given A and Z), such as V_1, V_2, R and a.

The method of partial waves

We use the method of partial waves [1] to predict the scattering cross section of a given spherically symmetric potential $V(\mathbf{r}) = V(r)$.

We describe our incoming neutron by a plane wave

$$\psi \propto e^{i\mathbf{k}\cdot\mathbf{r}}. \quad (3)$$

We describe our outgoing neutron by combination of a plane wave and a modified spherical wave

$$\psi = e^{i\mathbf{k}\cdot\mathbf{r}} + f(\hat{\mathbf{r}})\frac{e^{ikr}}{r}. \quad (4)$$

We can decompose this wavefunction into spherical harmonics, and then by only considering azimuthally symmetric solutions, we can rewrite the wavefunction as

$$\psi = \frac{1}{r}\sum_{l}(2l+1)i^l u_l(r) P_l(\cos(\theta)), \quad (5)$$

where P_l represents the Legendre polynomials. Through this method, we have converted the outgoing wave into a sum of partial waves, each labelled by l. We can solve for numerical solutions of u_l computationally.

By knowing the u_l values, we can solve for $f(\hat{\mathbf{r}})$. We can then use this to find the scattering cross section

$$\sigma = \int |f(\hat{\mathbf{r}})|^2 d\Omega. \quad (6)$$

Experimental data & optimisation method

The EXFOR database

The EXFOR database [3] is a collection of nuclear scattering data, from research facilities all across the globe. The database contains over 100,000 data sets, including scattering cross section data from elastic neutron collision with nuclei.

Results just like that plotted in Fig. 3 can be found in the database, providing empirical data for comparison to the theoretical model.

Genetic algorithms

Genetic algorithms are a class of optimisation algorithm, based on the ideas of natural selection and evolution. They take a set of variable parameters, calculate a metric, and then work to either maximise or minimise that metric. In the case of this problem, that metric would be some value quantifying the difference between the predicted and experimental results (e.g. a mean squared error).

Genetic algorithms can vary in their precise implementation, but they all follow more or less the same structure:

- Begin with a randomised population of potential solutions (configurations of variable parameters)
- Selectively 'mate' pairs of solutions to produce new solutions
- Randomly 'mutate' some of these solutions
- Repeat with a child population of the best new solutions

References

[1] J. J. Sakurai, *Modern Quantum Mechanics*, The Benjamin/Cummings Publishing Company Inc., California, 1985

[2] B. Povh, K. With, C. Scholz, F. Zetsche, W. Rodejohann, *Particles and Nuclei, 7th Ed.*, Springer-Verlag, Berlin & Heidelberg, 1995

[3] *Experimental Nuclear Reaction Data (EXFOR)*, National Nuclear Data Centre, http://www.nndc.bnl.gov/exfor/exfor.htm as of Feb 2017

[4] R. B. Wiringa, V. G. J. Stoks, R. Schiavilla, "Accurate nucleon-nucleon potential with charge-independence breaking", *Physical Review C*, Jan 1995, **Vol.** 51, **No.** 1