# IBESTAT SUTs Projections and IOTs Transformation (2015-2018)

Assessment of the SUT-RAS with additional information performance on INE (2016-2019) series and measurement of structural preservation metrics on projected matrices in IBESTAT series.

## 1 INTRODUCTION

The purpose of this document is to compile the whole process applied in IBESTAT regarding the SUT projections from 2015 to 2018 based on 2014 published datasets. Concretely, script on SAS provided by INE is used to apply SUT-RAS model, based on the Umed Temurshoev and Marcel P. Timmer's methodology with additional information (Umed Temurshoev, Marcel P. Timmer, 2010) in order to employ Supply and Use Tables (SUTs).

Specifically, according to this method, Use Table is required to be in purchaser's prices, while Supply Table must be in basic prices with transformation into purchaser's prices, that is, including import, margins and TLS (Taxes Less Subsidies) vertical vectors. Moreover, the methodology is applied on the INE SUTs series 2017-2019 with base year 2016 to obtain an evaluation of the SUT-RAS model on similar years later implemented in IBESTAT.

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# 2 METHODOLOGY

Given the version of SUT-RAS employed by INE, the following data from target years is required: **TPROD** as total output by industries from Supply Table in basic prices, **TCI** as total uses by industries from Use Table in purchaser's prices, **TIMPOR** as total imports by import categories from Supply Table, **TMARG** total margins by margins categories from Supply Table, **TIMPU** as total taxes less subsidies from Supply Table, **TDEMF**[1] as total by final demand categories from Use Table in purchaser's prices. In other words, totals from the Supply Table in basic prices with its transformation to purchaser's prices for products are needed, as well as totals from the Use Table in purchaser's prices.

The data needed from base year is followed: **PROD** as production distribution of products across industries from Supply Table in basic prices, **CI** as intermediate consumption distribution of products across industries from Use Table in purchaser's prices, **IMPOR** as imports by products and import categories from Supply Table, **MARG** as margins by products and margins categories from supply table, **IMPU** as taxes less subsidies by products from supply table, and **DEMF** as final consumption by products and final demand categories from use table in purchaser's prices.

The code implemented in SAS developed by INE, based on Umed Temurshoev and Marcel P. Timmer's methodology of SUT-RAS with additional information, comprises the following steps:

1. Dividing data into ***N*** and ***P*** matrices corresponding to negative and positive values.
2. Creating vectors of: $\boldsymbol{r_u}$, with length equal to the number of products; $\boldsymbol{s_u}$, with length equal to the number of columns in the Use matrix, that is, industries plus final demand categories; and $\boldsymbol{r_v}$, with length equal to the number of columns in the Make matrix, that is, industries plus imports, TLS and margins categories.
3. Vector $\boldsymbol{r_u}$[2] is updated as the proportion between total uses and total production by products, representing the balance that has to be reached. Concretely, the said vector is composed of two components: $\boldsymbol{p_u}$ and $\boldsymbol{n_u}$, where each product is aggregated with the corresponding growth of each column pondered with the $\boldsymbol{s_u}$ vector for the Use Table and $\boldsymbol{r_v}$ for the Supply Table; taking into account in the first component positive uses and negative production values, while in the other case are considered negative uses and positive production values.
4. Vector $\boldsymbol{s_u}$[3] is updated based on the ratio between the total uses by industries from the target year and from the base year or, indirectly, with the yet estimated values due to the accumulation effect along updating vectors.
5. Vector $\boldsymbol{r_v}$ is updated as the proportion between the total production by industries in the target year respected to the estimated values.
6. Steps from 3 to 5 are repeated until convergence is met, that is, when values from proportional vector $\boldsymbol{r_u}$ does not change more than a significant value named as the tolerance.

[1] Given that in IBESTAT case for observed values only aggregated totals are available, then **TIMPOR**, **TMARG** and **TIMPU** are single vectors.
[2] It must be always the case that $\boldsymbol{p_u}$ is not equal to 0. In other words, for each product both weighted sum of positive uses and negative production, must be unequal to 0. Similarly, it applies to $\boldsymbol{n_u}$ as the weighted sum of negative uses and positive production. Thus, some rows in both matrices can sum zero as long as the individual components do not result in zero.
[3] It must be always the case that **x**, as the total output by industries, and **u**, as the total uses by industries, are both strictly positive vectors, otherwise the inverse of diagonal matrices will be undefined.

# 3 SUT-RAS APPLIED IN INE

Below, as an example and with the objective of assessing the performance of the SUT-RAS model with additional information, the same is applied to the 2017-2019 series with the base year of 2016 of the INE SUTs. It should be noted that the years 2020 and 2021, despite corresponding to the same series, are omitted due to the exceptional conditions of Covid-19 and the changes in the production structure not relevant to this case study.

Specifically, this series has been chosen by being the most similar to that estimated for the case of IBESTAT. Thus, as it comes from the national accounts, it will be able to accommodate part of the complexity of the Balearic Islands regional accounts. The main differences that can be seen in the following tables with respect to those of IBESTAT is that they have a greater disaggregation with 81 industries, 110 products and 7 categories of final demand. However, SUT-RAS can be applied on asymmetric tables as we will also see later in the case of IBESTAT.

## 3.1 DATA

The data are extracted from the INE's own website[4] corresponding to the 2019 Statistical Review and are processed using Python in order to obtain the tabular form for each matrix and vector required for the model.

## 3.2 SPECIAL CONSIDERATION

Given that the industry ***81. Actividades de organizaciones y organismos extraterritoriales*** does not produces any product/service and at the same time the product/service ***110. Servicios de organizaciones y organismos extraterritoriales*** is not produced nor consumed, then, both are not considered in the analysis and modelling. On the other hand, given that product ***109. Servicios de los hogares como empleadores de personal doméstico*** is only produced by one industry and, in turn, is consumed by one final demand category, it is decided to eliminate it from the analysis and reduce noise by its certainty in value. Similarly, the industry ***80. Actividades de los hogares como empleadores de personal doméstico o como productores de bienes y servicios para uso propio*** is omitted by not consuming any product/service. Overall, 108 products, 79 industries, 2 margin categories and 7 final demand categories are treated.

## 3.3 RESULTS

In this section the performance of the SUT-RAS model will be assessed with additional information on the exposed INE data. Specifically, it will be analysed how well the projected data of the model fits the data observed in the corresponding publications, by means of goodness-of-fit measures. As well as it will be assessed how well the structures of each projected matrix and vector approximate those of the base year. With respect to this last point, the aim is to see whether the model has kept the original structure as close as possible to that of the base year regardless the experimented growth.

Firstly, to evaluate the performance of the SUT-RAS projections using official Spanish SUTs data from INE for the years 2017, 2018, and 2019, we rely on five core goodness-of-fit metrics, as defined and applied in Temurshoev and Timmer (2011) — specifically Table 6 of their paper:

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[4] https://www.ine.es/dyngs/INEbase/operacion.htm?c=Estadistica_C&cid=1254736177059&menu=enlaces&idp=1254735576581

- **MAPE (Mean Absolute Percentage Error)**: reflects the average relative deviation between each projected and observed element. It treats all deviations equally, regardless of their size, and therefore may disproportionately penalize small entries.
- **WAPE (Weighted Absolute Percentage Error)**: improves upon MAPE by weighting each deviation according to the size of the corresponding observed value. This makes it more representative of the economic relevance of the errors.
- **SWAD (Standardized Weighted Absolute Difference)**: also weights errors by the observed values but normalizes them over the entire squared sum of true entries. It serves as an overall intensity measure of deviation, favouring projections that closely replicate the large components of the matrix.
- **PSI (Psi Statistic)**: originates from information theory and captures the divergence between the distributions of true and projected values. It penalizes structural differences without being sensitive to the absolute scale, making it useful for comparing matrix patterns.
- **RSQ (Coefficient of Determination)**: measures the degree of linear association between the true and projected values. A value close to one indicates that the structural shape of the matrix is well preserved.

**Table 1. Metrics of Goodness of fit for 2017-2019 projections**

| | MAPE | WAPE | SWAD | PSI | RSQ |
|---|---|---|---|---|---|
| **2017 PROD** | 48.554 | 4.542 | 0.016 | 0.004 | 0.999 |
| **2017 IMPOR** | 26.972 | 9.959 | 0.066 | 0.008 | 0.986 |
| **2017 MARG** | 16.977 | 5.963 | 0.017 | 0.003 | 0.999 |
| **2017 IMPU** | 13.184 | 6.115 | 0.039 | 0.002 | 0.997 |
| **2017 CI** | 65.599 | 13.369 | 0.079 | 0.016 | 0.985 |
| **2017 DEMF** | 99.541 | 5.513 | 0.028 | 0.004 | 0.997 |
| **2018 PROD** | 61.925 | 6.535 | 0.025 | 0.007 | 0.997 |
| **2018 IMPOR** | 32.859 | 12.938 | 0.098 | 0.010 | 0.977 |
| **2018 MARG** | 24.557 | 7.710 | 0.031 | 0.004 | 0.998 |
| **2018 IMPU** | 13.964 | 7.850 | 0.062 | 0.003 | 0.994 |
| **2018 CI** | 107.243 | 18.375 | 0.152 | 0.022 | 0.963 |
| **2018 DEMF** | 74.467 | 7.300 | 0.033 | 0.005 | 0.996 |
| **2019 PROD** | 93.522 | 6.824 | 0.026 | 0.008 | 0.997 |
| **2019 IMPOR** | 31.394 | 12.597 | 0.090 | 0.012 | 0.980 |
| **2019 MARG** | 39.448 | 9.474 | 0.037 | 0.008 | 0.998 |
| **2019 IMPU** | 20.170 | 9.146 | 0.058 | 0.004 | 0.992 |
| **2019 CI** | 68.031 | 20.305 | 0.123 | 0.026 | 0.970 |
| **2019 DEMF** | 105.408 | 8.988 | 0.041 | 0.007 | 0.994 |

Across all years, the results presented in **Table 1** reveal consistently high **RSQ** values (typically above 0.99), indicating that the overall structure of the matrices is very well preserved. **WAPE** values are also relatively low, particularly for matrices such as PROD, MARG, IMPU, and DEMF. In contrast, **MAPE** shows greater variability, especially in CI and DEMF, reflecting substantial relative errors in cells with small values. Meanwhile, the **SWAD** and **PSI** statistics fall within the expected range (typically below 0.1), confirming a structurally coherent fit.

Looking more closely at year-by-year trends, the projections for 2017 show strong structural accuracy, although DEMF and CI present high **MAPE** values due to errors concentrated in low-value cells. Nevertheless, their low **WAPE** and **PSI** values suggest the overall structure remains sound. In 2018, a slight deterioration is observed in **MAPE** and **SWAD**, particularly for CI, while the main matrices—PROD, MARG, and IMPU—remain relatively stable. In 2019, **MAPE** continues to rise in CI and DEMF, though **RSQ** and **WAPE** remain robust. A moderate increase in **PSI** values suggests some growing structural divergence in certain matrices.

Overall, the values of **WAPE**, **SWAD**, and **PSI** are of the same order of magnitude as those reported in Table 6 of Temurshoev and Timmer (2011), confirming the validity of the calculations. Although **MAPE** is somewhat higher for DEMF and CI in the current results—likely due to their structural dispersion—**RSQ** remains similar or even higher.

**Table 2. Metrics of Structural Preservation for 2017-2019 projections respect to the base year**

| | Theil | Cosine Industries | Cosine Products | Frobenius |
|---|---|---|---|---|
| **2017 PROD** | 0.00137 | 0.99999 | 0.99986 | 0.00559 |
| **2017 CI** | 0.00206 | 0.99985 | 0.97160 | 0.00390 |
| **2018 PROD** | 0.00279 | 0.99997 | 0.99982 | 0.00850 |
| **2018 CI** | 0.00538 | 0.99971 | 0.97039 | 0.00694 |
| **2019 PROD** | 0.00397 | 0.99996 | 0.99972 | 0.01051 |
| **2019 CI** | 0.00705 | 0.99944 | 0.96939 | 0.00624 |

The structural metrics obtained for the matrices PROD and CI across the years 2017 to 2019 reveal consistent and interpretable patterns regarding the stability of projected economic structures under the SUT-RAS framework. In the case of the production matrix (PROD), the results indicate an exceptionally high level of structural preservation throughout the period. The **cosine similarity** values for both industries and products remain extremely close to one. The **Theil index** and **Frobenius distance** for PROD show a slight upward trend, particularly by 2019, but their magnitudes remain very low, confirming that any divergence from the observed structure is within an acceptable range.

By contrast, the intermediate consumption matrix (CI) displays a somewhat more complex dynamic. While the cosine similarity for industries remains stable, consistently above 0.999, the **similarity across products** decreases slightly over time, dropping to approximately 0.969 by 2019. This gradual decline suggests that the projection model maintains the general column-wise distribution of demand relatively well, but exhibits increasing divergence in the distribution across products. This is further supported by the growth in the **Theil index**, which more than triples from 2017 to 2019. Despite these changes, the **Frobenius distance** for CI remains low in absolute terms, which confirms that the overall deviation is still moderate.

In summary, the structural metrics validate that the projected production matrices retain a highly faithful reproduction of the observed structure, while the intermediate consumption matrices exhibit more variation over time, especially in terms of product-level distribution. These results are entirely consistent with the economic complexity of the matrices involved and reflect a realistic behaviour of the SUT-RAS model when applied to multi-year projections.

**Table 3. Cosine Similarity by industries for 2017-2019 projections**

| | 2017 PROD | 2017 CI | 2018 PROD | 2018 CI | 2019 PROD | 2019 CI |
|---|---|---|---|---|---|---|
| **count** | 79 | 79 | 79 | 79 | 79 | 79 |
| **mean** | 0.99999 | 0.99985 | 0.99997 | 0.99971 | 0.99996 | 0.99944 |
| **std** | 0.00002 | 0.00020 | 0.00008 | 0.00039 | 0.00011 | 0.00059 |
| **min** | 0.99985 | 0.99882 | 0.99948 | 0.99736 | 0.99920 | 0.99674 |
| **25%** | 0.99999 | 0.99982 | 0.99999 | 0.99963 | 0.99997 | 0.99924 |
| **50%** | 1.00000 | 0.99991 | 1.00000 | 0.99985 | 0.99999 | 0.99959 |
| **75%** | 1.00000 | 0.99996 | 1.00000 | 0.99994 | 1.00000 | 0.99988 |
| **max** | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 0.99997 |

Moving on, the cosine similarity by industry measures the similarity of the production or consumption structure by industry of the projected PROD or CI matrix with respect to that of the base year. Thus, a value or proportion is obtained for each industry in this case which, subsequently, the statistics set out in **Table 3** are calculated. Thus, it can be seen that all the values are above 99% similarity for all the years and matrices.

**Table 4. Cosine Similarity by products for 2017-2019 projections**

| | 2017 PROD | 2017 CI | 2018 PROD | 2018 CI | 2019 PROD | 2019 CI |
|---|---|---|---|---|---|---|
| **count** | 108 | 108 | 108 | 108 | 108 | 108 |
| **mean** | 0.99986 | 0.97160 | 0.99982 | 0.97039 | 0.99972 | 0.96939 |
| **std** | 0.00061 | 0.16500 | 0.00049 | 0.16481 | 0.00088 | 0.16466 |
| **min** | 0.99517 | 0.00000 | 0.99706 | 0.00000 | 0.99399 | 0.00000 |
| **25%** | 0.99996 | 0.99901 | 0.99994 | 0.99686 | 0.99992 | 0.99521 |
| **50%** | 0.99999 | 0.99960 | 0.99999 | 0.99925 | 0.99999 | 0.99863 |
| **75%** | 1.00000 | 0.99995 | 1.00000 | 0.99991 | 1.00000 | 0.99984 |
| **max** | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |

Similarly, the cosine similarity by products and their respective statistics shown in **Table 4** are applied. In this case, null values can be seen that correspond to certain products that, despite having produced values, no consumption in both base year and projected year is recorded. These are: ***65. Servicios de comercio al por menor***, ***84. Rentas inmobiliarias imputadas*** y ***104. Servicios de juegos de azar y apuestas***. In other words, these are purely services. Apart from this, the rest of the values also show a similarity of over 99%.

# 4 SUT-RAS APPLIED IN IBESTAT

This section applies the SUT-RAS model with additional information seen throughout the paper on the 2015-2018 series of the IBESTAT SUTs, with base year 2014, as well as the structural preservation metrics previously applied in the INE data. Specifically, each projection is made with respect to the base year 2014 and not sequentially.

## 4.1 DATA

IBESTAT datasets related to SUTs published in 2014 are in tabular or extended format in the internal database as well as the corresponding estimations from 2015 to 2018 years. However, transformations are needed for being prepared with the proper format required in the script, that is, in tabular format but for each matrix or numerical vector.

The datasets for all years are structured by three levels of aggregation: **Matriz**, **Agregado** and **Agregado2**. **Matriz** indicates whether refers to production, imports or auxiliar matrices (including margins and taxes less subsidies), while **Agregado** and **Agregado2** disaggregate in a wider way these categories. Entering more into detail, values are expressed in euros, in basic prices, calculated both together public administrations and others as definitive amounts, including illegal activities with no distinction and given for Balearic Island with no regional disaggregation.

Moreover, classifications comprehend until 66 industries or economic activities and 70 products, that is, 2 digits classification. Then, two columns of products and industries present the corresponding classification tag, and for those that represent totals or subtotals have this entry with a NULL assignation.

**Table 5. Levels of aggregation for Supply Table (2015-2018)**

| Tabla | Matriz | | Agregado | | Agregado2 | |
|---|---|---|---|---|---|---|
| TO | P_1 | Producción | P_1 | Producción | P_1 | Producción Total |
| | | | | | P_11 | Producción de mercado |
| | | | | | P_12 | Producción para uso final propio |
| | | | | | P_13 | Producción no de mercado |
| | Mbs | Importación bienes y servicios | P_7 | Importación | P_7 | Total |
| | | | | | P_7a | Resto de España |
| | | | | | P_7b | Unión Europea |
| | | | | | P_7c | Resto del Mundo |
| | | | O_PB | Oferta a pb | O_PB | Oferta a Precios básicos |
| | Aux | Auxiliar | Mc | Margen comercio | Mc_ma | Margen de comercio mayorista |
| | | | | | Mc_mi | Margen de comercio minorista |
| | | | | | Mcvh | Margen venta de vehículos |
| | | | Mt | Margen transporte | Mt | Margen de transporte |
| | | | D_21 | Impuestos a los productos | D_211 | Impuesto IVA |
| | | | | | D_212 | Impuesto Importaciones |
| | | | | | D_214 | Impuestos Especiales |
| | | | D_31 | Subvenciones a los productos | D_319 | Subvenciones a los productos |
| | | | O_PA | Oferta a padq | O_PA | |
| | Ajustes | Ajustes | CIFFOB | Ajuste CIF/FOB | CIFFOB | |
| | | | OP_P6 | Compras de no residentes en el territorio económico | OP_P6 | |
| | | | OP_P7 | Compras de residentes fuera del territorio económico | OP_P7 | |

## Table 6. Levels of aggregation for Use Table (2015-2018)

| Tabla | Matriz | Agregado | Agregado 2 | Tabla | Matriz | Agregado |
|---|---|---|---|---|---|---|
| TD | P_2 | CI | P_2 | CI | P_2 | Consumos Intermedios |
| | DF | DF | P_3 | GCF (Hogares residentes, no residentes, AAPP e ISFLSH) | P_3 | Gasto en consumo final TOTAL |
| | | | | | P_3_14R | GCFH Residentes |
| | | | | | P_3_14NR | GCFH No Residentes |
| | | | | | P_3_13 | GCF AAPP |
| | | | | | P_3_15 | GCF ISFLH |
| | | | P_5 | FBK (FBKF, VE) | P_5 | TOTAL |
| | | | | | P_51 | Formación bruta de capital fijo |
| | | | | | P_52 | Variación de existencias |
| | | | P_6 | Exportaciones | P_6 | Total |
| | | | | | P_6a | Resto de España |
| | | | | | P_6b | Unión Europea |
| | | | | | P_6c | Resto del Mundo |
| | | | U_PB | | U_PB | Total Usos a precios básicos |
| | | | U_PA | | U_PA | Total Usos a precios de adquisición |
| | Ajustes | Ajustes | CIFFOB | Ajuste CIF/FOB | CIFFOB | |
| | | | OP_P6 | Compras de no residentes en el territorio económico | OP_P6 | |
| | | | OP_P7 | Compras de residentes fuera del territorio económico | OP_P7 | |
| | Aux | Auxiliar | D_21X31 | Impuestos netos de subvenciones sobre los productos | D_21X31_Macromagnitud | Impuestos netos de subvenciones sobre los productos (con macromagnitud a la que están ligados) |
| | IP | Input Primarios | DI_PA | | DI_PA | Marginales de Consumos Intermedios a Precios de adquisición |
| | | | D_1 | Remuneraciones de Asalariados | D_1 | Total |
| | | | | | D_11 | Sueldos y salarios |
| | | | | | D_12 | Cotizaciones sociales a cargo de los empleadores |
| | | | D_29X39 | Otros impuestos sobre producción | D_29X39 | Otros impuestos sobre producción |
| | | | B_2g | EBE/ Renta mixta | B_2 | EBE/ Renta mixta |
| | | | B_1g | VAB | B_1 | VAB |
| | | | P_1 | Producción | P_1 | Producción |
| | EM | Empleo | PT | Puestos de trabajo total | PT_1 | Total |
| | | | | Puestos de trabajo asalariado | PT_11 | Asalariado |
| | | | | Puestos de trabajo resto | PT_12 | Resto |
| | | | EJC | Puestos de trabajo EJC total | EJC_1 | Total |
| | | | | Puestos de trabajo EJC asalariado | EJC_11 | Asalariado |
| | | | | Puestos de trabajo EJC resto | EJC_12 | Resto |
| | | | HRS | Horas de trabajo EJC total | Hrs_1 | Total |
| | | | | Horas de trabajo EJC asalariado | Hrs_11 | Asalariado |
| | | | | Horas de trabajo EJC resto | Hrs_12 | Resto |

## 4.2 SPECIAL CONSIDERATION

Given that the industry ***66. Activitats de les llars***[5] only produces the product/service ***70. Serveis de les llars*** and at the same time the final demand category ***Despesa en consum final llars residents*** is the only one which consumes it, then, the vector of total intermediate uses by industries presents one zero value for the mentioned industry. For this reason, values for this industry are aggregated together with the industry ***65. Altres serveis personals*** to avoid singularity case in the inverse matrix when calculating vector $\boldsymbol{s_u}$. Overall, 70 products, 65 industries, 1 margin category and 7 final demand categories are considered.

Another common problem when applying the SUT-RAS model is the rounding of data. Specifically, the totals for each row or product in each of the SUTs do not match to the decimal point. To address this problem, the difference between the totals in the Supply Table and the Use Table has been added across Margin, TLS and Import categories, in this order of priority whenever there are not zero values. So, at no point does this result in increases greater than a few euros. Additionally, in order to avoid problems with exact decimal calculations given the optimization of binary operations that programming languages such as Python tend to perform, the data has been converted to pence units beforehand.

Finally, some other cases become relevant to understand before applying the SUT-RAS model. Specifically, product ***27. Serveis de comerç al detall*** besides having a zero total supply and use, its weighted sum of negative values from Supply Table and positive values from Use Table is not equal to zero. Similarly, it applies to the weighted sum of positive values from Supply Table and negative values from Use Table[6]. This case also is related to the products ***10. Coc i refinació de petroli***, ***59. Serveis d'educació de no mercat***, ***61. Serveis d'atenció sanitària de no mercat***, ***63. Serveis socials de no mercat*** and ***70. Serveis de les llars***, where their total production or use is equal to zero, however, their total supply and use remain positive.

## 4.3 RESULTS

In this section, unlike the previous section of the INE, we will look at the structural preservation metrics, that is, the goodness-of-fit metrics will not be analysed, given that the observed data are not available. On the other hand, structural preservation metrics compare the matrix structure of the projected year with that of the observed year. Therefore, part of the value of the following indices is affected purely by a changing growth and not always by the performance of the model.

**Table 7. Metrics of Structural Preservation for 2015-2018 projections respect to the base year**

| | Theil | Cosine Industries | Cosine Products | Frobenius |
|---|---|---|---|---|
| **2015 PROD** | 0.01562 | 0.99954 | 0.98163 | 0.01175 |
| **2015 CI** | 0.02041 | 0.99898 | 0.91420 | 0.00841 |
| **2016 PROD** | 0.01867 | 0.99953 | 0.98108 | 0.01625 |
| **2016 CI** | 0.02305 | 0.99830 | 0.91308 | 0.00861 |
| **2017 PROD** | 0.02205 | 0.99950 | 0.98224 | 0.02127 |
| **2017 CI** | 0.02624 | 0.99802 | 0.91035 | 0.01090 |
| **2018 PROD** | 0.02403 | 0.99947 | 0.98288 | 0.02462 |
| **2018 CI** | 0.03070 | 0.99687 | 0.90906 | 0.01558 |

As seen in the INE section, structural preservation metrics include: **Theil Index**, as a measure of how much the distribution of flows differs, with 0 being identical structures; **Cosine Similarity**, as the cosine similarity between columns or rows of the matrices, with 1 being identical structures; and **Frobenius Norm**, as the Euclidean distance between two matrices, which grows with absolute differences in values, not just in proportions. As a detail, cosine similarity in products is generally

[5] The classification number for the activity or product/service corresponds to that set out in the SUTs for the base year 2014.

[6] These are conditions for calculating vector $\boldsymbol{r_u}$.

more sensitive with respect to industries, because more complex patterns are added, such as imports, margins and taxes less subsidies, which tend to vary more over time.

With all this, the estimate data show little change in the productive structure by industry. Meanwhile, for the rest of the metrics, greater differences can be seen than those observed in the case of the INE. Where the INE adjustment is much closer to the observed base distribution, in IBESTAT the structure differs more, especially in CI. On the other hand, with regard to the **cosine similarity by product**, the IBESTAT projection changes the patterns of use/supply by product more significantly. Finally, IBESTAT presents up to double or triple the Euclidean distance in some projections.

**Table 8. Cosine Similarity by industries for 2015-2018 projections**

| | 2015 PROD | 2015 CI | 2016 PROD | 2016 CI | 2017 PROD | 2017 CI | 2018 PROD | 2018 CI |
|---|---|---|---|---|---|---|---|---|
| **count** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| **mean** | 0.99954 | 0.99898 | 0.99953 | 0.99830 | 0.99950 | 0.99802 | 0.99947 | 0.99687 |
| **std** | 0.00357 | 0.00147 | 0.00355 | 0.00230 | 0.00363 | 0.00266 | 0.00380 | 0.00434 |
| **min** | 0.97121 | 0.99258 | 0.97131 | 0.98844 | 0.97067 | 0.98794 | 0.96932 | 0.97622 |
| **25%** | 0.99999 | 0.99901 | 0.99996 | 0.99798 | 0.99993 | 0.99760 | 0.99994 | 0.99659 |
| **50%** | 1.00000 | 0.99947 | 0.99999 | 0.99918 | 0.99999 | 0.99910 | 0.99999 | 0.99850 |
| **75%** | 1.00000 | 0.99980 | 1.00000 | 0.99964 | 1.00000 | 0.99969 | 1.00000 | 0.99946 |
| **max** | 1.00000 | 0.99997 | 1.00000 | 1.00000 | 1.00000 | 0.99997 | 1.00000 | 0.99991 |

**Table 9. Cosine Similarity by products for 2015-2018 projections**

| | 2015 PROD | 2015 CI | 2016 PROD | 2016 CI | 2017 PROD | 2017 CI | 2018 PROD | 2018 CI |
|---|---|---|---|---|---|---|---|---|
| **count** | 70 | 70 | 70 | 70 | 70 | 70 | 70 | 70 |
| **mean** | 0.98163 | 0.91420 | 0.98108 | 0.91308 | 0.98224 | 0.91035 | 0.98288 | 0.90906 |
| **std** | 0.12004 | 0.26006 | 0.12046 | 0.25899 | 0.11969 | 0.25878 | 0.11970 | 0.25906 |
| **min** | - | - | - | - | - | - | - | - |
| **25%** | 0.99976 | 0.98715 | 0.99939 | 0.98167 | 0.99917 | 0.97596 | 0.99954 | 0.98043 |
| **50%** | 0.99995 | 0.99676 | 0.99991 | 0.99529 | 0.99987 | 0.99054 | 0.99991 | 0.99089 |
| **75%** | 1.00000 | 0.99890 | 0.99999 | 0.99804 | 0.99999 | 0.99783 | 0.99999 | 0.99750 |
| **max** | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |

It can therefore be seen that all the cosine similarity values by industry and by product are reasonable. However, as mentioned above, a lower cosine similarity is shown when looking at the distribution by products globally. Similar to what was mentioned in the case of the INE, there are specific zero values as totals per row in both the PROD and CI matrices that have a decisive influence on the aggregate calculation and which, nevertheless, are not purely linked to the performance of the model.

In other words, there are rows that represent purely services and to which, therefore, no intermediate consumption is applied, such as cases ***27. Serveis de comerç al detall***, ***59. Serveis d'educació de no mercat***, ***61. Serveis d'atenció sanitària de no mercat***, ***63. Serveis socials de no mercat*** and ***70. Serveis de les llars***. In these cases, as they have no value, they mathematically obtain a zero value as a result of their corresponding index. This is also the case for product ***10. Coc i refinació de petroli***, which is not produced in the Balearic Islands but is consumed there.

Thus, taking into account all the casuistry of the productive structure and uses by specific services and oil production in the Balearic Islands, the following table shows that the rest of the values maintain a high cosine similarity of around 98-99%. It is worth noting the case of product/service ***56. Serveis administratius d'oficina i altres serveis d'ajuda a les empreses***, which has a significantly lower cosine similarity of around 70%.

## Table 10. Detailed Cosine Similarity by products for 2015-2018 projections

| PRODUCT | 2015 PROD | 2015 CI | 2016 PROD | 2016 CI | 2017 PROD | 2017 CI | 2018 PROD | 2018 CI |
|---|---|---|---|---|---|---|---|---|
| **1** | 0.99990 | 0.99893 | 0.99987 | 0.99666 | 0.99979 | 0.99643 | 0.99995 | 0.99505 |
| **2** | 0.99999 | 0.99978 | 0.99998 | 0.99973 | 0.99987 | 0.99973 | 0.99992 | 0.99885 |
| **3** | 0.99995 | 0.99998 | 0.99998 | 1.00000 | 1.00000 | 0.99999 | 0.99998 | 0.99999 |
| **4** | 0.99929 | 0.99871 | 0.99852 | 0.99735 | 0.99583 | 0.99698 | 0.99620 | 0.99438 |
| **5** | 0.99998 | 0.99741 | 0.99985 | 0.99114 | 0.99987 | 0.98369 | 0.99999 | 0.98961 |
| **6** | 0.99985 | 0.98839 | 0.99967 | 0.99529 | 0.99916 | 0.99678 | 0.99927 | 0.98720 |
| **7** | 1.00000 | 0.99991 | 0.99999 | 0.99995 | 0.99999 | 0.99994 | 0.99999 | 0.99989 |
| **8** | 0.99991 | 0.98419 | 0.99991 | 0.98184 | 0.99991 | 0.97461 | 0.99998 | 0.99788 |
| **9** | 0.99995 | 0.98845 | 0.99851 | 0.97643 | 0.99882 | 0.98193 | 0.99743 | 0.98040 |
| **10** | - | 0.99886 | - | 0.99950 | - | 0.99899 | - | 0.99875 |
| **11** | 0.99996 | 0.99882 | 0.99991 | 0.99860 | 0.99979 | 0.99794 | 0.99987 | 0.99691 |
| **12** | 0.99997 | 0.99949 | 0.99999 | 0.99967 | 0.99999 | 0.99974 | 0.99996 | 0.99961 |
| **13** | 0.99957 | 0.98861 | 0.99922 | 0.98867 | 0.99966 | 0.99266 | 0.99973 | 0.99201 |
| **14** | 0.93461 | 0.99608 | 0.94572 | 0.99588 | 0.96810 | 0.99750 | 0.96865 | 0.99590 |
| **15** | 0.99599 | 0.99851 | 0.99470 | 0.99823 | 0.99855 | 0.99866 | 0.99953 | 0.99925 |
| **16** | 0.99936 | 0.98159 | 0.99966 | 0.97201 | 0.99986 | 0.99011 | 0.99992 | 0.99785 |
| **17** | 0.99979 | 0.98179 | 0.99980 | 0.98988 | 0.99974 | 0.97250 | 0.99996 | 0.98976 |
| **18** | 0.99999 | 0.99494 | 0.99999 | 0.99668 | 0.99985 | 0.99562 | 0.99991 | 0.99431 |
| **19** | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| **20** | 0.99990 | 0.98865 | 0.99983 | 0.99804 | 0.99986 | 0.99358 | 0.99996 | 0.98772 |
| **21** | 1.00000 | 0.99289 | 1.00000 | 0.99778 | 0.99998 | 0.99806 | 0.99999 | 0.99615 |
| **22** | 1.00000 | 0.99512 | 1.00000 | 0.99385 | 1.00000 | 0.98760 | 0.99999 | 0.98334 |
| **23** | 1.00000 | 0.99641 | 1.00000 | 0.99701 | 1.00000 | 0.99564 | 1.00000 | 0.99419 |
| **24** | 1.00000 | 0.99988 | 1.00000 | 0.99991 | 1.00000 | 0.99964 | 1.00000 | 0.99971 |
| **25** | 0.99982 | 0.99674 | 0.99998 | 0.99681 | 0.99945 | 0.98995 | 0.99988 | 0.99020 |
| **26** | 1.00000 | 0.98722 | 0.99999 | 0.98161 | 0.99999 | 0.98983 | 0.99999 | 0.99467 |
| **27** | 1.00000 | - | 1.00000 | - | 0.99999 | - | 0.99999 | - |
| **28** | 0.99993 | 0.99705 | 0.99987 | 0.99766 | 0.99976 | 0.98882 | 0.99979 | 0.99463 |
| **29** | 1.00000 | 0.99943 | 0.99943 | 0.99868 | 0.99899 | 0.99939 | 0.99958 | 0.99905 |
| **30** | 1.00000 | 0.99660 | 1.00000 | 0.99528 | 1.00000 | 0.98831 | 1.00000 | 0.99306 |
| **31** | 0.99999 | 0.99962 | 0.99996 | 0.99966 | 0.99997 | 0.99848 | 0.99993 | 0.99796 |
| **32** | 0.99995 | 0.97585 | 0.99982 | 0.97080 | 0.99973 | 0.95773 | 0.99972 | 0.94793 |
| **33** | 0.99934 | 0.99745 | 0.99890 | 0.99527 | 0.99869 | 0.98478 | 0.99832 | 0.99008 |
| **34** | 0.99985 | 0.99214 | 0.99992 | 0.98942 | 0.99988 | 0.98858 | 0.99979 | 0.98991 |
| **35** | 0.99924 | 0.99718 | 0.99754 | 0.99920 | 0.99614 | 0.99959 | 0.99808 | 0.99954 |
| **36** | 0.99989 | 0.96772 | 0.99966 | 0.95684 | 0.99951 | 0.94556 | 0.99899 | 0.94179 |
| **37** | 0.99996 | 0.99874 | 0.99983 | 0.99593 | 0.99986 | 0.99463 | 0.99992 | 0.99667 |
| **38** | 1.00000 | 0.99737 | 0.99999 | 0.99583 | 0.99999 | 0.99559 | 0.99999 | 0.99363 |
| **39** | 0.99857 | 0.96972 | 0.99867 | 0.95708 | 0.99933 | 0.98517 | 0.99800 | 0.97334 |
| **40** | 1.00000 | 0.99911 | 1.00000 | 0.99383 | 1.00000 | 0.99582 | 1.00000 | 0.98540 |
| **41** | 1.00000 | 0.99649 | 1.00000 | 0.99688 | 1.00000 | 0.99090 | 1.00000 | 0.99347 |
| **42** | 1.00000 | 0.99986 | 1.00000 | 0.99962 | 1.00000 | 0.99963 | 1.00000 | 0.99933 |
| **43** | 1.00000 | 0.99883 | 1.00000 | 0.99804 | 1.00000 | 0.99690 | 1.00000 | 0.99683 |
| **44** | 0.99999 | 0.99677 | 0.99991 | 0.99614 | 0.99997 | 0.99196 | 0.99998 | 0.99158 |
| **45** | 0.99994 | 0.98997 | 0.99997 | 0.98909 | 0.99998 | 0.98660 | 0.99991 | 0.98442 |
| **46** | 0.92996 | 1.00000 | 0.87006 | 1.00000 | 0.95830 | 1.00000 | 0.99112 | 1.00000 |
| **47** | 0.99746 | 0.99719 | 0.99022 | 0.99331 | 0.98930 | 0.98974 | 0.99007 | 0.98893 |
| **48** | 0.99945 | 0.96401 | 0.99934 | 0.96512 | 0.99934 | 0.96475 | 0.99955 | 0.96006 |
| **49** | 0.99998 | 0.69598 | 0.99992 | 0.74451 | 0.99991 | 0.72003 | 0.99987 | 0.69246 |
| **50** | 0.99999 | 0.99893 | 0.99999 | 0.99914 | 0.99998 | 0.99793 | 0.99999 | 0.99770 |
| **51** | 1.00000 | 0.99883 | 1.00000 | 0.99811 | 1.00000 | 0.99700 | 1.00000 | 0.99340 |
| **52** | 0.99996 | 0.98351 | 0.99940 | 0.98239 | 0.99858 | 0.97562 | 0.99967 | 0.97629 |
| **53** | 0.99979 | 0.99902 | 0.99908 | 0.99673 | 0.99155 | 0.98719 | 0.99685 | 0.98673 |
| **54** | 0.99997 | 0.99708 | 0.99992 | 0.99221 | 0.99993 | 0.99019 | 0.99989 | 0.98833 |
| **55** | 0.99980 | 0.99458 | 0.99991 | 0.99177 | 0.99991 | 0.99102 | 0.99979 | 0.98833 |
| **56** | 0.99994 | 0.98713 | 0.99976 | 0.97996 | 0.99989 | 0.98269 | 0.99989 | 0.98100 |
| **57** | 1.00000 | 0.99738 | 1.00000 | 0.99572 | 1.00000 | 0.99384 | 1.00000 | 0.98050 |
| **58** | 1.00000 | 0.71353 | 1.00000 | 0.71733 | 1.00000 | 0.70347 | 1.00000 | 0.69509 |
| **59** | 0.99975 | - | 0.99975 | - | 0.99975 | - | 0.99975 | - |
| **60** | 1.00000 | 0.99891 | 1.00000 | 0.99976 | 1.00000 | 0.99970 | 1.00000 | 0.99814 |
| **61** | 0.99995 | - | 0.99995 | - | 0.99995 | - | 0.99995 | - |
| **62** | 0.99997 | 0.99602 | 0.99998 | 0.97067 | 0.99997 | 0.96734 | 0.99990 | 0.92913 |
| **63** | 0.92920 | - | 0.92914 | - | 0.92814 | - | 0.92693 | - |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **64** | 0.99934 | 0.99928 | 0.99939 | 0.99415 | 0.99917 | 0.97698 | 0.99930 | 0.98287 |
| **65** | 0.99954 | 0.99913 | 0.99865 | 0.98974 | 0.99466 | 0.97055 | 0.99821 | 0.97644 |
| **66** | 0.99995 | 0.99971 | 0.99963 | 0.99150 | 0.99781 | 0.96647 | 0.99949 | 0.97368 |
| **67** | 0.99654 | 0.97478 | 0.99898 | 0.97192 | 0.99899 | 0.96798 | 0.99958 | 0.96332 |
| **68** | 0.93928 | 0.97964 | 0.96385 | 0.97683 | 0.95215 | 0.96579 | 0.94955 | 0.96014 |
| **69** | 0.99999 | 0.99820 | 0.99996 | 0.99700 | 0.99995 | 0.99932 | 0.99994 | 0.99919 |
| **70** | 1.00000 | - | 1.00000 | - | 1.00000 | - | 1.00000 | - |

# 5 IOT TRANSFORMATION IN IBESTAT (2015-2018)

As mentioned in various manuals (United Nations and Eurostat), the starting point for transforming SUTs to IOTs is to have both destination and origin tables at basic prices. However, with the SUT-RAS model with additional information, after the estimates, the origin tables are generally obtained at basic prices with transformation to acquisition prices and the destination table at acquisition prices. Therefore, the destination table must be transformed beforehand.

## 5.1 USE TABLE AT BASIC PRICES

To obtain the target tables at basic prices from 2015 to 2018, it is necessary to subtract the corresponding portion of commercial and transport margins and taxes from each value in use and add subsidies. In this case, as only estimated data are available for the target years for total margins and TLS by product, these two categories are used.

However, the problem arises that the specific values for each combination of product and industry are not available for the margin use table or TLS for the projected years. Therefore, to obtain these values, we refer to the base year 2014, where these values are available and from which distribution coefficients can be extracted.

Going into more detail, for 2014, the difference between the value of use at acquisition prices and the value at basic prices is obtained for each combination of industry and product. This produces a matrix representing the distribution of total margins and TLS for each combination of product use by industry. Thus, by calculating the proportion of each value in relation to its total per row, the coefficients used for the rest of the projected years are obtained.

In this way, the basic price destination table would be obtained for each year as the difference between the value for each product and industry of use at acquisition prices minus the corresponding value of imports per product for that same year distributed on the basis of the coefficients for the base year.

## 5.2 IOT TRANSFORMATION (2015-2018)

In the construction of symmetric input-output tables (IOTs) from supply and use tables (SUTs), Models B (product-by-product) and D (industry-by-industry) were applied, following the methodological framework outlined by Eurostat and the United Nations. Given the absence of a decomposition of the use table into domestic and imported components, both IOTs were derived as total symmetric tables, encompassing the entire supply regardless of origin.

Model D was applied to transform the use table into an industry-by-industry IOT. Due to the lack of disaggregation by origin, and in order to ensure consistency with the total supply, the output vector $\boldsymbol{x}$ was defined as the sum of domestic production and imports per product (i.e., row-wise). This adjustment was necessary to align the technical coefficient matrix and final demand with the observed totals, since using only domestic production would have led to imbalances under the total use framework.

Model B was employed to derive the product-by-product IOT, under the assumption of fixed product technology. In this case, the transformation matrix was computed using the total supply by product (again, including imports) to reflect the complete input structure. As with Model D, the absence of origin-based decomposition necessitated the use of total flows, thus producing a symmetric IOT that reflects both domestic and imported inputs.

Prior to the application of the transformation models, the use table—originally expressed at purchasers' prices—was converted to basic prices. This was accomplished by redistributing trade and transport margins (MARG) and net taxes on products (IMPU) across rows (products), using the structural distribution observed in the base year. The weights derived from this base-year structure were held constant and applied to the projected years.

Despite the theoretical property of Models B and D to avoid generating negative values when starting from strictly non-negative input data, the use table at basic prices displayed isolated negative entries. These originated from the fixed proportional allocation of MARG and IMPU across projected years. Given the complex and evolving distribution patterns produced by the SUT-RAS method, applying a static set of weights has, in some instances, resulted in implausible corrections—particularly where the structure of demand and margins shifted significantly over time. These anomalies were subsequently propagated into the IOTs, despite the models' design to maintain non-negativity.

# 6 Conclusion

Despite the methodological rigor applied through SUT-RAS projections and the formal application of Models B and D for symmetric IOT derivation, the lack of import-use decomposition and the static redistribution of margins and taxes based on the base-year structure imposed inherent limitations. The resulting IOTs reflect total flows, integrating domestic and imported components, and rely on a non-standard output vector to ensure matrix consistency. Consequently, while structural integrity has been largely preserved across years, the fixed allocation of price adjustment components has occasionally introduced negative entries, propagated through the IOTs despite the intrinsic non-negativity constraints of the transformation models. These outcomes underline the trade-off between model formalism and data availability in regional IOT construction.

# 7 References

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