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Joint estimation of supply and use tables*

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Abstract. We apply the RAS updating idea for the joint estimation of supply and use tables (SUTs). In contrast to standard input-output techniques, our approach does not require the availability of total outputs by product for the projection year(s). This condition is usually not met in practice. The algorithm, called the SUT-RAS method, jointly estimates SUTs that are immediately consistent. It is applicable to different settings of SUTs, such as the frameworks with basic prices and purchasers' prices, and a setting in which use tables are separated into domestic and imported uses. Our empirical evaluations show that the SUT-RAS method improves upon existing short-cut methods in projection of Spanish SUTs.

JEL classification: C61, C89, D57

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1 Introduction

Input-output (IO) tables provide a detailed picture of the interactions in an economy, summarizing the production and use of all goods and services, and of income generated in the production process. These tables are extensively used in a wide range of studies ranging from the analysis of international trade, productivity, efficiency, income inequality to ecological and environmental studies (see, e.g., ten Raa 2005; Miller and Blair 2009, and extensive references thereof). These analyses rely on the use of symmetric input-output tables (SIOTs) either from the product-by-product or industry-by-industry type. An IO table is often constructed from underlying supply and use tables (SUTs) by means of a particular technology assumption. In fact, SUTs provide more detailed and useful information, since they explicitly distinguish between commodities and industries that allows us to appropriately considering secondary

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products besides the main products of industries.[1] Also many IO analyses require the linking of an IO table to additional data sets such as international trade and employment statistics. While the first dataset is organized by product, the second is typically collected at an industry base. Linking these to a SIOT that is either from the product-by-product or industry-by-industry type is problematic. Instead, SUTs being industry-by-product provide a natural link to the additional data sources.

There exists, however, a problem of timeliness of SUTs (and SIOTs), which mainly has to do with the large financial costs and human efforts required to collect the appropriate data. Thus, the majority of countries provide benchmark tables based on the detailed surveys on mainly five years intervals. To fill the gap in-between the benchmark SUTs, it is necessary to use the so-called non-survey methods. Many different non-survey methods have been employed in updating the SIOTs. Jackson and Murray (2004) and Pavia et al. (2009) provide recent overviews and evaluations of the various methods.

The updating procedures for SIOTs could in principle also be used for projecting SUTs. In Temurshoev et al. (2010) eight different methods were studied and evaluated in the estimation of Dutch and Spanish SUTs and it was concluded that the well-known (Generalized) RAS algorithm (for a recent reference on (G)RAS see e.g., Lahr and de Mesnard 2004) was generally superior.[2] There is, however, one important drawback of using SIOTs updating methods in order to estimate SUTs: one has to have the row and column sums of both the use and supply tables for the projection year(s). This is largely impractical for SUTs estimation because, although outputs by industry are available from other sources (such as national accounts), outputs by product are typically not available for the projection year. One, of course, can project SUTs on the base of only column totals information using, for example, a one-sided RAS method that has to satisfy only one constraint, namely, the column sums condition, rather than both the column sums and row sums conditions. This is partially the basis of one of the few existing SUT-updating procedures which was used in the construction of intermediate inputs in the EUKLEMS database and known as the EUKLEMS method (Timmer et al. 2005).[3] It is obvious that such estimation of SUTs is inefficient as it does not use the full potential of the original RAS algorithm, and requires arbitrary adjustments to make SUTs consistent with respect to the derived outputs by products.

In this paper we consider simultaneous estimation of SUTs that does not require the availability of the use and supply totals by products, which are, instead, endogenously derived. We apply the traditional RAS procedure not separately to the use and the supply tables, but instead write the problem in terms of the joint estimation of SUTs such that the two requirements of the SUT framework are satisfied. These are the identities of total inputs and total outputs by industry, and total supply and total use by products. It is proved that, like the (G)RAS procedure, the estimates of SUTs are derived by biproportional adjustments of the original SUTs. However, unlike the standard (G)RAS algorithm, the process of updating SUTs are not independent: only three dependent (and not four pairwise independent) multipliers need to be computed to jointly estimate the supply and use tables. Because of its closeness to the original (G)RAS algorithm, we refer to this method as the SUT-RAS approach.

[1] The *System of National Accounts 2008*, for example, states: “Supply and use tables are a powerful tool with which to compare and contrast data from various sources and improve the coherence of the economic information system. They permit an analysis of markets and industries and allow productivity to be studied at this level of disaggregation” (URL: http://unstats.un.org/unsd/nationalaccount/sna2008.asp, p. 271).

[2] They also found that two other, less commonly used, methods proposed, respectively, by Harthoorn and van Dalen (1987) and Kuroda (1988) performed as well as (G)RAS.

[3] See O’Mahony and Timmer (2009) for a description of this database.

We show that the SUT-RAS method is rather flexible and can be used in a range of settings. In Section 2 we separately consider the situation in which SUTs are both given in basic prices, and when the supply table is in basic prices and includes the transformation into purchasers' prices, while the use table is given at purchasers' prices. The latter setting is most common in available datasets worldwide. Various interesting applications also require series in which the use table is separated into domestic and imported uses and this case is considered as well. Additionally, we also study the possibility of introducing extra projection data, besides the minimum information on the projection year(s) that are needed for the SUT-RAS implementation, including exogenous export and imports statistics, for example, from international trade sources.

Our approach is a particular case of a more general balancing problem described by Lenzen et al. (2006, 2009). They provide a general balancing problem of balancing a table under partial information of any kind (i.e., it may or may not include row and/or column totals and constraints can include any number of adjacent and/or non-adjacent elements).[4] The advantage of our approach is that we apply the problem explicitly to the SUTs framework. Thus, we: (*i*) provide analytical solutions in terms of supply and use matrices; (*ii*) derive an easy and simple algorithm for the multipliers computation; and (*iii*) consider four different settings of SUTs each of which can be separately useful for an analyst depending on what kind of dataset he/she has access to. This allows an analyst to avoid the first needed steps of data vectorization and constructing a (binary) constraint matrix in the case of the generalized matrix balancing approach, which in fact can be quite time consuming. Further, the derived multipliers from the SUT-RAS approach might be useful for certain analyses where the focus is on changes in total outputs by product and by industry in SUTs.

In Section 3 the performance of the SUT-RAS method is tested for a set of SUTs of Spain. Comparisons are made with the results obtained from the EUKLEMS and Euro methods. The last method is widely used in Europe and advocated in the Eurostat handbook for IO table compilation (see Eurostat 2008, Chapter 14).[5] This method relies on two assumptions: first, the shares of industries in the production of commodities remain constant, and second, the fixed input coefficients determine the relations of all product inputs to production of industries. In contrast, the SUT-RAS method is a theory-based approach, which minimizes the deviations of the projected SUTs structure from that of some benchmark year. And while joint estimation in SUT-RAS immediately guarantees the consistency of the SUTs, the Euro method needs an *ad hoc* assumption of the fixed product sales structure to make SUTs consistent. It is found that the SUT-RAS method greatly improves the EUKLEMS and Euro methods in estimating Spanish SUTs. Finally, Section 4 gives some concluding remarks.

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[4] Lenzen et al. (2006) call this extension of the standard RAS method as 'constrained RAS' (cRAS). In their later work they do not use this term, but instead consider it as "a completely general formulation of GRAS" (pers. comm. Manfred Lenzen). In fact, the word 'generalized' is confusing. For example, GRAS as used by Junius and Oosterhaven (2003) deals with both positive and negative elements, while the same term used by Thissen and Löfgren (1998) is meant to "endogenously determining the subtotals and the overall total of the [Social Accounting Matrix (SAM)] matrix" (p. 1992). But these two approaches differ significantly because the first is a biproportional approach, while the second is a nonlinear programming method. This difference is shown by Lemelin (2009) who uses exactly the same objective function (in terms of probabilities), as Thissen and Löfgren (1998). See also Robinson et al. (2001) who apply the cross-entropy methods to coefficients for estimating a SAM. Within the IO framework Gordon et al. (2009) use a RAS-type technique to account for short-term substitution effects of price changes. In fact, to be totally fair with respect to the early literature on matrix balancing, the generalized version of RAS as described by Lenzen et al. (2006, 2009) should be referred to as a "generalized Bregman's balancing method", since Lamond and Stewart (1981) had already showed that the traditional RAS method is, in fact, a special case of Bregman's (1967) balancing approach.

[5] The Euro method was originally devised for updating SIOTs, but is also used in a SUT-setting, see a report prepared by Joerg Beutel to the European Commission (e.g., contract number 1508302007 FISC-D, April 2008). Its detailed description is provided by Temurshoev et al. (2010).

2 The problems of joint estimation of supply and use tables

In this section we present the theoretical model of joint estimation of supply and use tables (SUTs) and provide its solution. Since use tables in base year can be available both at basic and purchasers' prices, we consider both cases. Thus, first we give the details of the updating method when the original benchmark SUTs are at basic prices, while the second case is analysed in one of the later sections.

2.1 Estimation of SUTs at basic prices

Let us first consider the case when the benchmark SUTs are both given at basic prices. For the projection year(s) the following data is available:[6] (*i*) $\mathbf{x}_b$ – output totals by industry; (*ii*) $\mathbf{v}_b$ – value-added totals by industry; (*iii*) $\mathbf{y}_b$ – totals of final demand categories; and (*iv*) M – overall sum of commodity imports. The subscript b indicates that the corresponding vector/matrix is expressed in basic prices. These data are typically available from the national accounts. The question is how the availability of this minimal information together with some benchmark SUTs of the earlier period can be used to estimate the consistent SUTs for the projection year(s). To do so, let us first present a framework of SUTs at basic prices. Define the vector of commodity outputs at basic prices by $\mathbf{q}_b$ and the vector of sectoral intermediate use totals by $\mathbf{u}_b$. The last vector can be easily derived, since by definition it equals the difference between the vectors $\mathbf{x}_b$ and $\mathbf{v}_b$. Imports vector $\mathbf{m}$ is given at CIF prices. Table 1 gives SUTs' framework, where p, s, f and m determine a member of, respectively, products, industries (or sectors), final demand categories and total imports sets. This notation will be shown to be useful below. Thus, Table 1 shows that $\mathbf{U}_b\boldsymbol{\imath} + \mathbf{Y}_b\boldsymbol{\imath} = \mathbf{q}_b = \mathbf{V}_b'\boldsymbol{\imath} + \mathbf{m}$, namely, total use by product is equal to total supply by product, and $\mathbf{U}_b'\boldsymbol{\imath} + \mathbf{v}_b = \mathbf{x}_b = \mathbf{V}_b\boldsymbol{\imath}$, that is, total intermediate input and value added by industry is equal to sectoral total output. Hence, the economic system given in Table 1 is balanced, namely, the economy-wide use of products is equal to the overall production.

Next we define the benchmark year (denoted by subscript 0) SUTs matrix as:

$$\mathbf{A} = \begin{pmatrix} \mathbf{O} & \bar{\mathbf{U}}_0 \\ \bar{\mathbf{V}}_0 & \mathbf{O} \end{pmatrix}, \tag{1}$$

where $\bar{\mathbf{U}}_0 = (\mathbf{U}_{b,0}, \mathbf{Y}_{b,0})$ and $\bar{\mathbf{V}}_0 = (\mathbf{V}_{b,0}', \mathbf{m}_0)'$ are, respectively, the use table and supply table at basic prices. The aim is to estimate the corresponding matrix for some projection year that is denoted by $\mathbf{X}$. Using the traditional RAS-type objective, we want $\mathbf{X}$ to be as close as possible

Table 1. A framework of SUTs at basic prices

	p	s	f	Σ
p	$\mathbf{O}$	$\mathbf{U}_b$ – intermediate use	$\mathbf{Y}_b$ – final demand	$\mathbf{q}_b$
s	$\mathbf{V}_b$ – make matrix	$\mathbf{O}$	$\mathbf{O}$	$\mathbf{x}_b$
m	$\mathbf{m}'$ – imports vector	$\mathbf{0}'$	$\mathbf{0}'$	M
Σ	$\mathbf{q}_b'$	$\mathbf{u}_b' = \mathbf{x}_b' - \mathbf{v}_b'$	$\mathbf{y}_b'$	

[6] Matrices are given in bold capital letters; vectors in bold lower case letters; and scalars in italicized capital case letters. Vectors are columns by definition, thus row vectors are obtained by transposition, indicated by a prime. $\hat{\mathbf{x}}$ denotes the $n \times n$ diagonal matrix with the elements of $\mathbf{x}$ on its main diagonal and zeros elsewhere. The null matrix and null vector of appropriate dimensions are denoted, respectively, by $\mathbf{O}$ and $\mathbf{0}$. Finally, the summation vector of ones is denoted by $\boldsymbol{\imath}$.

to **A**, but it should satisfy the two identities in the SUT framework. That is, in the estimated matrix total supply by product should be equal to total use by product, and total input by industry should be equal to total output by industry. Further, the overall sum of the estimated imports should match the given corresponding value $M > 0$.[7]

As in Junius and Oosterhaven (2003), we define $z_{ij} \equiv x_{ij} / a_{ij}$ whenever $a_{ij} \neq 0$, and set $z_{ij} = 1$ for $a_{ij} = 0$. This mathematical trick is useful in preserving the signs of the original elements in the estimated matrix. Next, define sets $I = \{p\}$, $II = \{\{s\}, \{f\}\}$, and $III = \{\{s\}, m\}$, and the expanded vectors of total outputs and total uses, respectively, as $\bar{\mathbf{x}} = (\mathbf{x}'_b, M)'$ and $\bar{\mathbf{u}} = (\mathbf{u}'_b, \mathbf{y}'_b)'$. We consider the following constrained optimization problem:

$$\min_{z_{ij}} \sum_i \sum_j |a_{ij}| \left(z_{ij} \ln\left(\frac{z_{ij}}{e}\right) + 1 \right) \tag{2}$$

such that

$$\sum_{j \in II} a_{pj} z_{pj} - \sum_{k \in III} a_{kp} z_{kp} = 0 \quad \text{for all } p \in I, \tag{3}$$

$$\sum_{k \in I} a_{kj} z_{kj} = \bar{u}_j \quad \text{for all } j \in II, \tag{4}$$

$$\sum_{p \in I} a_{ip} z_{ip} = \bar{x}_i \quad \text{for all } i \in III, \tag{5}$$

where $|a_{ij}|$ is the absolute value of a_{ij} and e is the base of the natural logarithm.

Function (2) is the objective used in the generalized RAS (GRAS) problem (see Lenzen et al. 2007; Huang et al. 2008). Minimizing this function implies that we want x_{ij} to be as close as possible to the original element a_{ij} for all i and all j. This is because for $z_{ij} = 1$ the value of (2) is zero, which is its minimum possible value. This objective function is somewhat similar to the well-known information based entropy measure. Employing Table 1, the three constraints of our problem can be easily interpreted. Condition (3) by using the definition of z_{ij} boils down to $\sum_s x_{ps} + \sum_f x_{pf} - \sum_s x_{sp} - x_{mp} = 0$ (note that $x_{mp} = m_p$), which in matrix form is $\mathbf{U}_b \boldsymbol{\imath} + \mathbf{Y}_b \boldsymbol{\imath} = \mathbf{V}'_b \boldsymbol{\imath} + \mathbf{m}$. Thus, this constraint ensures the identity of supply and use by products, and as a result the commodity output vector is endogenously determined. Constraint (4) guarantees that the column totals of the estimated intermediate use matrix and final demand matrix are equal to $\bar{\mathbf{u}}'$, while (5) requires that the row totals of the make matrix and commodity imports match their given totals, $\bar{\mathbf{x}}$. Thus, these two conditions will also guarantee that total input by industry is equal to total output by industry.[8]

From the SUTs construction we know that the intermediate use matrix, make matrix, and imports vector do not allow for negative elements, while the final demand matrix allows for negative entries as well. Thus, let us define $\mathbf{P}_0$ as a matrix with all non-negative entries of $\bar{\mathbf{U}}_0$, and $\mathbf{N}_0 \equiv \mathbf{P}_0 - \bar{\mathbf{U}}_0$ containing absolute values of the negative elements of $\bar{\mathbf{U}}_0$ (i.e., of $\mathbf{Y}_{b,0}$). Then, the associated Lagrangean of our problem is

[7] When $M = 0$, the entire import vector **m** is irrelevant for the SUTs estimation, and thus should be simply ignored (i.e., deleted from $\bar{\mathbf{V}}_0$).

[8] Note that we do not explicitly consider the zero matrices of **A** in the above problem, since implicitly they are already accounted for. This is because we set $z_{ij} = 1$ for all $a_{ij} = 0$, which implies that any cell that was zero in the original matrix **A** remains zero in the estimated matrix **X** as well.

$$\mathcal{L} = \sum_{(i,j)\notin \mathbf{N}_0} a_{ij}\left(z_{ij}\ln\left(\frac{z_{ij}}{e}\right)+1\right) - \sum_{(i,j)\in \mathbf{N}_0} a_{ij}\left(z_{ij}\ln\left(\frac{z_{ij}}{e}\right)+1\right) + \sum_{p\in I}\lambda_p\left(\sum_{k\in III} a_{kp}z_{kp} - \sum_{j\in II} a_{pj}z_{pj}\right)$$
$$+ \sum_{j\in II}\tau_j\left(\bar{u}_j - \sum_{k\in I} a_{kj}z_{kj}\right) + \sum_{i\in III}\mu_i\left(\bar{x}_i - \sum_{p\in I} a_{ip}z_{ip}\right),$$

where λ_p, τ_j and μ_i are the corresponding Lagrange multipliers of the constraints (3)–(5). The optimal solutions of this function can be easily derived as:

$$z_{pj} = \begin{cases} e^{\lambda_p}e^{\tau_j} & \text{if } a_{pj} \geq 0 \text{ for all } p \in I \text{ and all } j \in II, \\ e^{-\lambda_p}e^{-\tau_j} & \text{if } a_{pj} < 0 \text{ for all } p \in I \text{ and all } j \in II, \end{cases} \tag{6}$$

$$z_{ip} = e^{\mu_i}e^{-\lambda_p} \quad \text{for all } i \in III \text{ and all } p \in I. \tag{7}$$

Thus, expressions (6) and (7) give, respectively, the estimates of $\bar{\mathbf{U}}$ and $\bar{\mathbf{V}}$. Note that for the solution of the problem (2)–(5) it always holds that $z_{ij} > 0$, which means that the estimated matrices will preserve the signs of the original elements. For simplicity, denote $r_u(p) \equiv e^{\lambda_p}$, $s_u(j) \equiv e^{\tau_j}$ and $r_v(i) \equiv e^{\mu_i}$. Then, using the optimal solutions (6)–(7), we thus established the following result.

Theorem 1. *The solutions of the problem* (2)–(5) *of updating SUTs at basic prices are given by* $\bar{\mathbf{U}} = \hat{\mathbf{r}}_u\mathbf{P}_0\hat{\mathbf{s}}_u - \hat{\mathbf{r}}_u^{-1}\mathbf{N}_0\hat{\mathbf{s}}_u^{-1}$ *and* $\bar{\mathbf{V}} = \hat{\mathbf{r}}_v\bar{\mathbf{V}}_0\hat{\mathbf{r}}_u^{-1}$.

Theorem 1 clearly shows the similarity of the joint SUTs updating to the (G)RAS solutions: in this case in order to get the corresponding estimates, the semipositive supply table $\bar{\mathbf{V}}_0$ is scaled row- and column-wise similar to RAS procedure, while the use table $\bar{\mathbf{U}}_0$ is scaled also row- and column-wise, but the factors are different depending on whether one is updating its non-negative or strictly negative entries similar to the GRAS algorithm. Because of this closeness to the (G)RAS algorithm, we refer to our method as the SUT-RAS approach. However, the main difference now is that we join these two tables according to the SUTs framework from the outset, and do not consider 'RASing' each matrix separately. As a result, the estimates are dependent in the sense that we use only three dependent multipliers (i.e., $\boldsymbol{r}_u$, $\mathbf{s}_u$ and $\mathbf{r}_v$), for the joint estimation of the SUTs components, and not four multipliers, which would be pairwise independent, in the case of separate updating of $\bar{\mathbf{U}}_0$ and $\bar{\mathbf{V}}_0$. The last option, however, is largely unfeasible from the practical perspective, because the totals of outputs by products, $\mathbf{q}_b$, are not available for the majority of countries worldwide (except e.g., Japan that produces annual symmetric IO tables by products). But when the components of the SUTs are joined, we need not know this vector, which will be obtained endogenously, and furthermore, the consistency of SUTs is immediately guaranteed. Thus, in contrast, for example, to the Euro method (see Eurostat 2008, Chapter 14), no further steps are needed to equalize sectoral inputs and outputs.[9]

What is the intuition behind the updating procedure stated in Theorem 1? We answer this question assuming that the total use of product *i* becomes smaller than total supply of product *i* once the sectoral constraints (4) and (5) have been imposed. To make these SUTs consistent product-wise, all sectoral and final demand uses of product *i* need to be increased, while all possible sectoral supply of product *i* have to be decreased. This is exactly what the SUT-RAS

[9] The Euro method uses the so-called fixed product sales structure model to make SUTs consistent in the second step of each iteration of its algorithm. The assumption is that each product has its own specific sales structure, irrespective of the industry where it is produced.

algorithm does. In this case, Theorem 1 implies that the row multiplier of the use table corresponding to product i becomes larger than unity. That is, $r_u(i) > 1$ increases (resp. decreases) all the positive (resp. negative) uses of product i by a factor of $r_u(i)$ (resp. $1/r_u(i)$) resulting in a higher total use of i. At the same time Theorem 1 shows that the (sectoral) domestic and imported supply of product i will decrease by a factor of $1/r_u(i)$ giving lower total supply of product i. Consequently, the gap between total supply and total use of product i diminishes, and eventually, using the SUT-RAS algorithm, the identity of supply and use of product i will be reached.

Given Theorem 1, our task is now to find out how the row and column multipliers can be computed. By plugging the optimal solutions in the constraints (3)–(5), we are able to determine these vectors. First, constraint (3) implies that $\bar{\mathbf{U}}\boldsymbol{\imath} - \bar{\mathbf{V}}'\boldsymbol{\imath} = \mathbf{0}$. Using Theorem 1 we thus have $\hat{\mathbf{r}}_u\mathbf{P}_0\mathbf{s}_u - \hat{\mathbf{r}}_u^{-1}\mathbf{N}_0\hat{\mathbf{s}}_u^{-1}\boldsymbol{\imath} - \hat{\mathbf{r}}_u^{-1}\bar{\mathbf{V}}_0'\mathbf{r}_v = \mathbf{0}$. Premultiplying the last equation by the diagonal matrix $\hat{\mathbf{r}}_u$, yields $\hat{\mathbf{r}}_u^2\mathbf{P}_0\mathbf{s}_u - (\mathbf{N}_0\hat{\mathbf{s}}_u^{-1}\boldsymbol{\imath} + \bar{\mathbf{V}}_0'\mathbf{r}_v) = \mathbf{0}$. This is a quadratic equation in $\mathbf{r}_u$ without a linear term that admits two solutions, but for our purposes we only need its positive root. Thus,

$$\mathbf{r}_u = \sqrt{\widehat{\mathbf{P}_0\mathbf{s}_u}^{-1}\left(\mathbf{N}_0\hat{\mathbf{s}}_u^{-1}\boldsymbol{\imath} + \bar{\mathbf{V}}_0'\mathbf{r}_v\right)}. \tag{8}$$

Constraint (4) in matrix form is $\bar{\mathbf{U}}'\boldsymbol{\imath} = \bar{\mathbf{u}}$, hence $\hat{\mathbf{s}}_u\mathbf{P}_0'\mathbf{r}_u - \hat{\mathbf{s}}_u^{-1}\mathbf{N}_0'\hat{\mathbf{r}}_u^{-1}\boldsymbol{\imath} = \bar{\mathbf{u}}$. Premultiplying the last expression by $\hat{\mathbf{s}}_u$, we obtain $\hat{\mathbf{s}}_u^2\mathbf{P}_0'\mathbf{r}_u - \hat{\mathbf{s}}_u\bar{\mathbf{u}} - \mathbf{N}_0'\hat{\mathbf{r}}_u^{-1}\boldsymbol{\imath} = \mathbf{0}$. This is a quadratic equation in $\mathbf{s}_u$, and as in (8) we are interested only in its positive root. Denote $\circ$ and $\oslash$, respectively, the Hadamard element by element multiplication and division. Thus,

$$\mathbf{s}_u = 0.5 \times \widehat{\mathbf{P}_0'\mathbf{r}_u}^{-1}\left(\bar{\mathbf{u}} + \sqrt{\bar{\mathbf{u}} \circ \bar{\mathbf{u}} + 4 \times (\mathbf{P}_0'\mathbf{r}_u) \circ (\mathbf{N}_0'\hat{\mathbf{r}}_u^{-1}\boldsymbol{\imath})}\right). \tag{9}$$

And, finally, condition (5) states that $\bar{\mathbf{V}}\boldsymbol{\imath} = \bar{\mathbf{x}}$, thus using again Theorem 1 we obtain $\hat{\mathbf{r}}_v\bar{\mathbf{V}}_0\hat{\mathbf{r}}_u^{-1}\boldsymbol{\imath} = \hat{\bar{\mathbf{x}}}\boldsymbol{\imath}$. This can be rewritten as $\hat{\bar{\mathbf{x}}}^{-1}\bar{\mathbf{V}}_0\hat{\mathbf{r}}_u^{-1}\boldsymbol{\imath} = \hat{\mathbf{r}}_v^{-1}\boldsymbol{\imath}$, or equivalently,

$$\mathbf{r}_v = \boldsymbol{\imath} \oslash \left(\hat{\bar{\mathbf{X}}}^{-1}\bar{\mathbf{V}}_0\hat{\mathbf{r}}_u^{-1}\boldsymbol{\imath}\right). \tag{10}$$

Note from (10) that it must be always the case that $\mathbf{x}$ is a strictly positive vector. Also it follows from (9) that $\bar{\mathbf{u}}$ should be a strictly positive vector either, otherwise the diagonal matrix $\widehat{\mathbf{P}_0'\mathbf{r}_u}^{-1}$ is not defined.[10] We observe that Equations (9) and (10), besides the exogenously given data, depend directly only on $\mathbf{r}_u$. Thus, we propose the following algorithm for computing the required row and column multipliers.

- *Step* $t = 0$. Initialize $\mathbf{s}_u(0) = \boldsymbol{\imath}$ and $\mathbf{r}_v(0) = \boldsymbol{\imath}$.[11]
- *Step* $t = 1, \ldots, k$. Calculate $\mathbf{r}_u(t)$ on the base of $\mathbf{s}_u(t-1)$ and $\mathbf{r}_v(t-1)$, and then use $\mathbf{r}_u(t)$ to compute $\mathbf{s}_u(t)$ and $\mathbf{r}_v(t)$.
- *Step* $t = k$. Stop when $|\mathbf{r}_u(k) - \mathbf{r}_u(k-1)| < \varepsilon\boldsymbol{\imath}$ for sufficiently small $\varepsilon > 0$.
- *Step* $t = k + 1$. Derive the final estimates as $\bar{\mathbf{U}} = \hat{\mathbf{r}}_u(k)\mathbf{P}_0\hat{\mathbf{s}}_u(k) - \hat{\mathbf{r}}_u^{-1}(k)\mathbf{N}_0\hat{\mathbf{s}}_u^{-1}(k)$ and $\bar{\mathbf{V}} = \hat{\mathbf{r}}_v(k)\bar{\mathbf{V}}_0\hat{\mathbf{r}}_u^{-1}(k)$.

[10] A practical note on this issue, which is also true for other settings that will follow, is the following. In real computations it is not necessary to delete a zero row/column from the tables. In MATLAB, for example, one might write a very simple function that derives the inverse of positive elements while desregarding (say, equalizing to unity) the undefined ratios.

[11] Note that the two multipliers have different dimensions. In this case, $\mathbf{s}_u(0)$ has a row dimension equal to the number of industries and final demand categories, while that of $\mathbf{r}_v(0)$ is equal to the number of industries plus one (for total imports).

The above mentioned algorithm converges provided that the solution to the SUT-RAS problem exists.[12] The condition which must be checked before running the algorithm is the consistency of the external data for the projection year. From the SUTs framework in Table 1 it must be true that the economy-wide use is equal to the overall supply, that is, $\boldsymbol{\imath}'\bar{\mathbf{x}} = \boldsymbol{\imath}'\bar{\mathbf{u}}$.

2.2 SUTs estimation with additional information

Often it may happen that besides the column totals of intermediate and final uses, sectoral value-added and total outputs, and aggregate imports, there might be more data available for the projection year(s). Usually this extra information contains the vectors of exports and imports by product from international trade statistics, or, for example, households' consumption by product. It has been widely reported that, in general, introduction of accurate exogenous information into the RAS updating, besides the margins of a projection table, improves the resulting estimates (see e.g., de Mesnard and Miller 2006), although there are cases when this does not hold. In this section we consider the joint estimation of SUTs at basic prices with extra external data availability.

Table 2 presents SUTs framework for our estimation purposes when commodity imports, $\mathbf{m}$, and commodity exports vector, denoted by $\mathbf{e}$, are exogenously available.[13] Thus, the benchmark SUTs matrix (1) now has $\bar{\mathbf{V}}_0 = \mathbf{V}_0$ and $\bar{\mathbf{U}}_0 = (\mathbf{U}_{b,0}, \mathbf{Y}^r_{b,0})$. For simplicity's sake, we do not introduce additional notations, and assume that the reader notices that, for example, $\bar{\mathbf{u}} = (\mathbf{u}'_b, \mathbf{y}^{r\prime}_b)'$ here is different from $\bar{\mathbf{u}}$ in Section 2.1. Also the make matrix is now written without the subscript b; namely, instead of $\mathbf{V}_{b,0}$ in what follows we will simply write $\mathbf{V}_0$. The corresponding sets are $I = \{p\}$, $II = \{\{s\}, \{f^r\}\}$, and $III = \{s\}$. The minimization problem will be the same as in (2)–(5) with the following differences. The first constraint (3) becomes

$$\sum_{j \in II} a_{pj} z_{pj} - \sum_{k \in III} a_{kp} z_{kp} = m_p - e_p \quad \text{for all } p \in I, \tag{11}$$

the second constraint (4) is still valid, and instead of $\bar{x}_i$ we now have only the sectoral output x_i in constraint (5). It is easy to check that Theorem 1 still holds for this case, but the multipliers are now computed differently. So, the first constraint (11) implies that $\bar{\mathbf{U}}\boldsymbol{\imath} - \mathbf{V}'\boldsymbol{\imath} = \mathbf{f}$, where $\mathbf{f} \equiv \mathbf{m} - \mathbf{e}$. Using Theorem 1 we thus have $\hat{\mathbf{r}}_u \mathbf{P}_0 \mathbf{s}_u - \hat{\mathbf{r}}_u^{-1} \mathbf{N}_0 \hat{\mathbf{s}}_u^{-1} \boldsymbol{\imath} - \hat{\mathbf{r}}_u^{-1} \mathbf{V}'_0 \mathbf{r}_v = \mathbf{f}$. Premultiplying the last equation by the diagonal matrix $\hat{\mathbf{r}}_u$, yields $\hat{\mathbf{r}}_u^2 \mathbf{P}_0 \mathbf{s}_u - \hat{\mathbf{r}}_u \mathbf{f} - (\mathbf{N}_0 \hat{\mathbf{s}}_u^{-1} \boldsymbol{\imath} + \mathbf{V}'_0 \mathbf{r}_v) = \mathbf{0}$. This is a quadratic equation in $\mathbf{r}_u$ with linear term, hence its positive root is:

Table 2. A framework of SUTs at basic prices with more exogenous information

	p	s	f^r	Σ
p	$\mathbf{O}$	$\mathbf{U}_b$ – intermediate use	$\mathbf{Y}^r_b$ – reduced $\mathbf{Y}_b$	$\mathbf{q}_b - \mathbf{e}$
s	$\mathbf{V}_b$ – make matrix	$\mathbf{O}$	$\mathbf{O}$	$\mathbf{x}_b$
Σ	$\mathbf{q}'_b - \mathbf{m}'$	$\mathbf{u}'_b = \mathbf{x}'_b - \mathbf{v}'_b$	$\mathbf{y}^{r\prime}_b$	

[12] Function (2) is the sum of strictly convex functions, hence is itself strictly convex either. Constraints (3)–(5) are linear equality constraints, thus are convex as well as concave functions, but not strictly so. It is known that the sum of a strictly convex function and a convex function is a strictly convex function (see e.g., Chiang 1984, Theorems I-III, p. 342). Thus, our Lagrangian is a strictly convex function, which guarantees existence of a unique solution to the problem (2)–(5).

[13] The vector $\mathbf{e}$ can contain any other information from the final demand matrix. That is, it can also be consumption by households, investments, or the sum of any final demand categories by product.

$$\mathbf{r}_u = 0.5 \times \hat{\mathbf{p}}_u^{-1}\left(\mathbf{f} + \sqrt{\mathbf{f} \circ \mathbf{f} + 4 \times \mathbf{p}_u \circ \mathbf{n}_u}\right), \tag{12}$$

where $\mathbf{p}_u \equiv \mathbf{P}_0\mathbf{s}_u$ and $\mathbf{n}_u \equiv \mathbf{N}_0\hat{\mathbf{s}}_u^{-1}\boldsymbol{\imath} + \mathbf{V}_0'\mathbf{r}_v$. Note that (12) boils down to (8) whenever $\mathbf{f} = \mathbf{0}$.

Similarly, constraint (4) is $\bar{\mathbf{U}}'\boldsymbol{\imath} = \bar{\mathbf{u}}$, hence $\hat{\mathbf{s}}_u\mathbf{P}_0'\mathbf{r}_u - \hat{\mathbf{s}}_u^{-1}\mathbf{N}_0'\hat{\mathbf{r}}_u^{-1}\boldsymbol{\imath} = \bar{\mathbf{u}}$. This will yield exactly the same expression for the column multiplier $\mathbf{s}_u$ as in (9) (albeit its reduced row dimension due to accounting for exogenous data). In the same way, condition (5) now is $\mathbf{V}\boldsymbol{\imath} = \mathbf{x}$, thus using again Theorem 1 we obtain similar expression to (10) as:

$$\mathbf{r}_v = \boldsymbol{\imath} \oslash \left(\hat{\mathbf{x}}^{-1}\mathbf{V}_0\hat{\mathbf{r}}_u^{-1}\boldsymbol{\imath}\right). \tag{13}$$

Here again it is true that the vectors $\bar{\mathbf{u}}$ and $\mathbf{x}$ should be strictly positive, otherwise the multipliers $\mathbf{s}_u$ and $\mathbf{r}_v$ are not defined (which, however, can be easily dealt with in practical applications, see footnote 10). The algorithm of computing the three multipliers in the presence of extra exogenous data is the same as presented in Section 2.1.

2.3 *Updating SUTs with use tables in purchasers' prices*

Now we consider the case when the supply table has also the valuation adjustment matrix that translates outputs at basic prices into the total supply at purchasers' prices, and the use table is at purchasers' prices. This is currently the most common case for which SUTs are available.[14] This framework for our joint SUTs updating purposes is illustrated in Table 3.

This setting is different from Table 1 for SUTs at basic prices in only one respect (besides differences in prices), which is the inclusion of a valuation adjustment matrix. The last comprises the matrix of trade and transportation margins $\mathbf{T}$, and the vector of product taxes net of subsidies $\mathbf{n}$. Consistent with the official published SUTs, the totals of the trade and transportation margins sum to zero. To clarify, we consider the following example of a hypothetical four product economy in Table 4:[15]

For the above hypothetical valuation adjustment (Table 4) we thus have:

$$\mathbf{T} = \begin{pmatrix} 12 & 22 & -34 & 0 \\ 10 & 15 & 0 & -25 \end{pmatrix} \quad \text{and} \quad \mathbf{n}' = (-2 \quad 30 \quad 4 \quad 6).$$

Note that $\mathbf{U}$, $\mathbf{Y}$, $\mathbf{q}$, $\mathbf{u}$ and $\mathbf{y}$ are all expressed in purchasers' prices, thus do not have subscript b. To estimate SUTs we again define the original matrix $\mathbf{A}$ as in (1), but with

Table 3. Supply table at basic prices, including transformation into purchasers' prices, and use table at purchasers' prices

	p	*s*	*f*	Σ
p	$\mathbf{O}$	$\mathbf{U}_b$ – intermediate use	$\mathbf{Y}_b$ – final demand	$\mathbf{q}$
s	$\mathbf{V}_b$ – make matrix	$\mathbf{O}$	$\mathbf{O}$	$\mathbf{x}_b$
m	$\mathbf{m}'$ – imports vector	$\mathbf{0}'$	$\mathbf{0}'$	M
t	$\mathbf{T}$ – make matrix	$\mathbf{O}$	$\mathbf{O}$	$\mathbf{0}$
n	$\mathbf{n}'$ – net taxes vector	$\mathbf{0}'$	$\mathbf{0}'$	N
Σ	$\mathbf{q}'$	$\mathbf{u}' = \mathbf{x}_b' - \mathbf{v}_b'$	$\mathbf{y}'$	

[14] For example, the SUT database from Eurostat (URL: http://epp.ec.europa.eu), contains only use tables at purchasers' prices, not at basic prices.

[15] For simplicity we give only the total trade margins, but this column in reality is further divided into wholesale, retail, and motor trade margins.

Table 4. A hypothetical four product economy

Products	Trade margins	Transports margins	Net taxes
Agriculture	12	10	–2
Manufacturing	22	15	30
Trade	–34	0	4
Transportation	0	–25	6
Total	0	0	38

$\bar{\mathbf{U}}_0 = (\mathbf{U}_0, \mathbf{Y}_0)$ and $\bar{\mathbf{V}}_0 = (\mathbf{V}'_{b,0}, \mathbf{m}_0, \mathbf{T}', \mathbf{n}_0)'$. Similarly, we define $\bar{\mathbf{x}} = (\mathbf{x}'_b, M, \mathbf{0}', N)'$ and $\bar{\mathbf{u}} = (\mathbf{u}', \mathbf{y}')'$, and the corresponding sets are $I = \{p\}$, $II = \{\{s\}, \{f\}\}$, and $III = \{\{s\}, m, \{t\}, n\}$. Note that mathematically our problem is exactly the same as (2)–(5). The main distinction of this setting from the SUTs at basic prices is that the supply table $\bar{\mathbf{V}}_0$ allows for negative entries as well, e.g., net taxes can be negative when subsidies exceed taxes. Thus, besides (6), the second optimal condition instead of (7) is

$$z_{ip} = \begin{cases} e^{\mu_i} e^{-\lambda_p} & \text{if } a_{ip} \geq 0 \text{ for all } i \in III \text{ and all } p \in I, \\ e^{-\mu_i} e^{\lambda_p} & \text{if } a_{ip} < 0 \text{ for all } i \in III \text{ and all } p \in I. \end{cases} \tag{14}$$

As in Section 2.1, define $r_u(p) \equiv e^{\lambda_p}$, $s_u(j) \equiv e^{\tau_j}$, $r_v(i) \equiv e^{\mu_i}$, and $\bar{\mathbf{U}}_0 = \mathbf{P}_0 - \mathbf{N}_0$. Further, let $\mathbf{P}_0^v$ be a matrix with all non-negative elements of $\bar{\mathbf{V}}_0$ and $\mathbf{N}_0^v \equiv \mathbf{P}_0^v - \bar{\mathbf{V}}_0$. Thus, the optimal solutions (6) and (14) imply:

Theorem 2. *The solutions of the joint estimation of SUTs, where the supply table includes transformations into purchasers' prices, and use table is at purchasers' prices, are given by* $\bar{\mathbf{U}} = \hat{\mathbf{r}}_u \mathbf{P}_0 \hat{\mathbf{s}}_u - \hat{\mathbf{r}}_u^{-1} \mathbf{N}_0 \hat{\mathbf{s}}_u^{-1}$ *and* $\bar{\mathbf{V}} = \hat{\mathbf{r}}_v \mathbf{P}_0^v \hat{\mathbf{r}}_u^{-1} - \hat{\mathbf{r}}_v^{-1} \mathbf{N}_0^v \hat{\mathbf{r}}_u$.

Theorem 2 again confirms that in order to jointly estimate consistent SUTs one needs to compute only three dependent multipliers $\mathbf{r}_u$, $\mathbf{s}_u$ and $\mathbf{r}_v$. Their dependency reflects the fact that all the components of SUTs are estimated simultaneously. Thus, the commodity output at purchasers' prices, $\mathbf{q}$, is endogenously derived. Using the supply and use by products identity condition, $\bar{\mathbf{U}}\boldsymbol{\imath} - \bar{\mathbf{V}}'\boldsymbol{\imath} = \mathbf{0}$, together with Theorem 2, yields $\hat{\mathbf{r}}_u \mathbf{P}_0 \mathbf{s}_u - \hat{\mathbf{r}}_u^{-1} \mathbf{N}_0 \hat{\mathbf{s}}_u^{-1} \boldsymbol{\imath} - \hat{\mathbf{r}}_u^{-1} \mathbf{P}_0^{v\prime} \mathbf{r}_v + \hat{\mathbf{r}}_u \mathbf{N}_0^{v\prime} \hat{\mathbf{r}}_v^{-1} \boldsymbol{\imath} = \mathbf{0}$. Its premultiplication by $\hat{\mathbf{r}}_u$ gives a quadratic equation in $\mathbf{r}_u$, thus (compared to (8)):

$$\mathbf{r}_u = \sqrt{\hat{\mathbf{p}}_u^{-1} \mathbf{n}_u}, \tag{15}$$

where $\mathbf{p}_u \equiv \mathbf{P}_0 \mathbf{s}_u + \mathbf{N}_0^{v\prime} \hat{\mathbf{r}}_v^{-1} \boldsymbol{\imath}$ and $\mathbf{n}_u \equiv \mathbf{N}_0 \hat{\mathbf{s}}_u^{-1} \boldsymbol{\imath} + \mathbf{P}_0^{v\prime} \mathbf{r}_v$.

Condition (4) requires $\bar{\mathbf{U}}'\boldsymbol{\imath} = \bar{\mathbf{u}}$, hence $\hat{\mathbf{s}}_u \mathbf{P}'_0 \mathbf{r}_u - \hat{\mathbf{s}}_u^{-1} \mathbf{N}'_0 \hat{\mathbf{r}}_u^{-1} \boldsymbol{\imath} = \bar{\mathbf{u}}$. Its solution in terms of $\mathbf{s}_u$ is already given in (9) and is valid in the current setting as well. Finally, from (5) it follows that $\bar{\mathbf{V}}\boldsymbol{\imath} = \bar{\mathbf{x}}$. Thus, using Theorem 2, we obtain $\hat{\mathbf{r}}_v \mathbf{P}_0^v \hat{\mathbf{r}}_u^{-1} \boldsymbol{\imath} - \hat{\mathbf{r}}_v^{-1} \mathbf{N}_0^v \mathbf{r}_u = \bar{\mathbf{x}}$ or equivalently, $\hat{\mathbf{r}}_v^2 \mathbf{P}_0^v \hat{\mathbf{r}}_u^{-1} \boldsymbol{\imath} - \hat{\mathbf{r}}_v \bar{\mathbf{x}} - \mathbf{N}_0^v \mathbf{r}_u = \mathbf{0}$. Therefore,

$$\mathbf{r}_v = 0.5 \times \widehat{\mathbf{P}_0^v \hat{\mathbf{r}}_u^{-1} \boldsymbol{\imath}}^{-1} \left(\bar{\mathbf{x}} + \sqrt{\bar{\mathbf{x}} \circ \bar{\mathbf{x}} + 4 \times (\mathbf{P}_0^v \hat{\mathbf{r}}_u^{-1} \boldsymbol{\imath}) \circ (\mathbf{N}_0^v \mathbf{r}_u)} \right). \tag{16}$$

Note that when $\mathbf{N}_0^v$ is a zero matrix, (16) boils down to (10) as it should do.

Given our earlier discussions on the availability of extra exogenous information, the elaborations in Section 2.2 can be used in the current setting as well. First, one can prefer to have the totals of trade and transport margins to be exogenously given, and not estimated within the

SUT-RAS process. In that case negative figures in **T** are nullified, and instead of zero totals one will have positive total margins in Table 3. This case is explained in detail in Temurshoev and Timmer (2010) and will not be discussed here. Second, one may wish to use other sources, like trade statistics. Then the vectors of commodity imports **m** and exports **e** (see footnote 13) will not be estimated. In Table 3 these exogenously given vectors will be subtracted from the corresponding totals in the table margins similar to Table 2 and the reduced final demand matrix, $\mathbf{Y}^r$, will take the place of **Y**. The reader can easily confirm that in such a setting we will have $\bar{\mathbf{U}}_0 = (\mathbf{U}_0, \mathbf{Y}_0^r)$, $\bar{\mathbf{V}}_0 = (\mathbf{V}'_{b,0}, \mathbf{T}', \mathbf{n})'$, $\bar{\mathbf{x}} = (\mathbf{x}'_b, \mathbf{0}', N)'$ and $\bar{\mathbf{u}} = (\mathbf{u}', \mathbf{y}^{r\prime})'$. Again the problem is similar to (2)–(5), with only exception that instead of (3) the right constraint (11) is imposed that guarantees the supply and use by product identity, namely, $\bar{\mathbf{U}}\boldsymbol{\imath} + \mathbf{e} = \bar{\mathbf{V}}'\boldsymbol{\imath} + \mathbf{m}$. Hence, Theorem 2 holds for the setting with extra information. The multiplier $\mathbf{r}_u$ is computed using (12) with $\mathbf{p}_u \equiv \mathbf{P}_0\mathbf{s}_u + \mathbf{N}_0^{v\prime}\hat{\mathbf{r}}_v^{-1}\boldsymbol{\imath}$ and $\mathbf{n}_u \equiv \mathbf{N}_0\hat{\mathbf{s}}_u^{-1}\boldsymbol{\imath} + \mathbf{P}_0^{v\prime}\mathbf{r}_v$ (different $\mathbf{p}_u$ and $\mathbf{n}_u$ in (12) and here simply reflect the difference between basic and purchasers' price settings). As before, the other two multipliers $\mathbf{s}_u$ and $\mathbf{r}_v$ are defined, respectively, by (9) and (16).

The algorithm of computing the three multipliers is similar to that presented in Section 2.1, with the only difference that in step $t = k + 1$ the final estimate of $\bar{\mathbf{V}}$ is derived using the corresponding expression from Theorem 2. An important remark again is that it must be true that the exogenous given data are consistent. That is, the economy-wide use should be equal to the overall production (i.e., with exogenous data, $\boldsymbol{\imath}'\mathbf{u} + \boldsymbol{\imath}'\mathbf{y} + \boldsymbol{\imath}'\mathbf{e} = \boldsymbol{\imath}'\mathbf{x}_b + N + M$). Otherwise, the SUT-RASing might still produce the required estimates, but the error, representing the gap in this identity, will appear as the overall difference of the endogenized vectors of total use and supply by products.[16]

2.4 SUT-RASing with domestic and imported use tables

In this section we provide a somewhat different SUTs estimation method, which distinguishes the intermediate and final uses into domestic and imported use tables. Having use tables separately for domestic and imported uses is important, since many economic analyses are based only on the domestic input structure of the economy, rather than on its entire technology that includes also imported inputs.

If we want to use the setting analysed in Sections 2.1–2.3, then the most obvious way of producing domestic and imported use tables would be as follows. The SUT-RAS endogenously

[16] The final note is that if we were to multiply the first constraint of our optimization problem, which links supply and use tables and guarantees their consistency product-wise, by minus one (–1), then we would obtain alternative expressions of SUTs estimates. It can be shown that then the estimated SUTs instead of those given in Theorem 2 would be given by

$$\bar{\mathbf{U}} = \hat{\mathbf{r}}_u^{-1}\mathbf{P}_0\hat{\mathbf{s}}_u - \hat{\mathbf{r}}_u\mathbf{N}_0\hat{\mathbf{s}}_u^{-1} \quad \text{and} \quad \bar{\mathbf{V}} = \hat{\mathbf{r}}_v\mathbf{P}_0^v\hat{\mathbf{r}}_u - \hat{\mathbf{r}}_v^{-1}\mathbf{N}_0^v\hat{\mathbf{r}}_u^{-1}, \tag{17}$$

where the corresponding multipliers (in a general case with extra exogenous information) are

$$\mathbf{r}_u = 0.5 \times \hat{\mathbf{n}}_u^{-1}\left(\mathbf{f} + \sqrt{\mathbf{f} \circ \mathbf{f} + 4 \times \mathbf{p}_u \circ \mathbf{n}_u}\right),$$

$$\mathbf{r}_v = 0.5 \times \widehat{\mathbf{P}_0^v\mathbf{r}_u}^{-1}\left(\bar{\mathbf{x}} + \sqrt{\bar{\mathbf{x}} \circ \bar{\mathbf{x}} + 4 \times (\mathbf{P}_0^v\mathbf{r}_u) \circ (\mathbf{N}_0^v\hat{\mathbf{r}}_u^{-1}\boldsymbol{\imath})}\right),$$

$$\mathbf{s}_u = 0.5 \times \widehat{\mathbf{P}_0'\hat{\mathbf{r}}_u^{-1}\boldsymbol{\imath}}^{-1}\left(\bar{\mathbf{u}} + \sqrt{\bar{\mathbf{u}} \circ \bar{\mathbf{u}} + 4 \times (\mathbf{P}_0'\hat{\mathbf{r}}_u^{-1}\boldsymbol{\imath}) \circ (\mathbf{N}_0'\mathbf{r}_u)}\right),$$

with $\mathbf{n}_u$ and $\mathbf{p}_u$ defined in (15). Certainly, the two sets of expressions are equivalent in their outputs, thus it is only a matter of taste which one of them to use.

Table 5. SUTs with domestic and imported use tables

	p^d	p^m	s	f	Σ
p^d	$\mathbf{O}$	$\mathbf{O}$	$\mathbf{U}^d$	$\mathbf{Y}^d$	$\mathbf{q}-\mathbf{m}$
p^m	$\mathbf{O}$	$\mathbf{O}$	$\mathbf{U}^m$	$\mathbf{Y}^m$	$\mathbf{m}$
s	$\mathbf{V}_b$	$\mathbf{O}$	$\mathbf{O}$	$\mathbf{O}$	$\mathbf{x}_b$
t	$\mathbf{T}$	$\mathbf{O}$	$\mathbf{O}$	$\mathbf{O}$	$\mathbf{0}$
n	$\mathbf{n}'$	$\mathbf{0}'$	$\mathbf{0}'$	$\mathbf{0}'$	N
m	$\mathbf{0}'$	$\mathbf{m}'$	$\mathbf{0}'$	$\mathbf{0}'$	M
Σ	$\mathbf{q}-\mathbf{m}'$	$\mathbf{m}'$	$\mathbf{u}'=\mathbf{x}_b'-\mathbf{v}_b'$	$\mathbf{y}'$	

computes the vector of total outputs by product, $\mathbf{q}$, be it at basic or purchasers' prices. We denote the entire domestic use table (i.e., intermediate and final uses) by $\bar{\mathbf{U}}^d$, and the corresponding imported table by $\bar{\mathbf{U}}^m$, which are going to be estimated on the base of the available corresponding matrices of some benchmark year. Then from our first SUT-RAS step we already know their column and row sums. Thus, we estimate the matrix $(\bar{\mathbf{U}}^{d\prime}, \bar{\mathbf{U}}^{m\prime})'$ such that it has the row totals of $(\mathbf{q}'-\mathbf{m}', \mathbf{m}')'$ and the column totals of $\bar{\mathbf{u}}$, which was used in the SUT-RAS step as well. Since we know these row and column totals for the projection year, one can in the second step use the GRAS algorithm to jointly estimate $\bar{\mathbf{U}}^d$ and $\bar{\mathbf{U}}^m$. Of course, there is no guarantee that the matrix $\bar{\mathbf{U}}^d+\bar{\mathbf{U}}^m$ is equal to the total use table $\bar{\mathbf{U}}$ from the first step SUT-RAS result.[17] Hence, in such cases the total use table should be taken from the second step GRAS procedure, namely, $\bar{\mathbf{U}}^d+\bar{\mathbf{U}}^m$, implying that the first step SUT-RAS outcome for the use table was used only to compute the vectors of outputs and imports by product (if the last was not available). The projected supply table is taken from the first step SUT-RAS approach.

Now we pose a question whether instead of the two-step procedure discussed above, can we provide a SUT-RAS setting, where the supply table and the domestic and imported use tables are estimated within one framework. In what follows, we provide such framework of SUTs estimation that is valid for both basic and purchasers' prices settings. Let us start with the case when use tables are expressed in purchasers' prices. This 'more' general SUTs setting is illustrated in Table 5, which is an adjusted version of Table 3 that makes distinction between domestic products, p^d, imported products, p^m, and also separates the intermediate and final use tables into the domestic and imported parts. This framework divides the supply and use identity into two equalities: one identity for domestic products only, and another supply and use identity for imported products only. Thus now we place the vector of imports separately from the supply table of domestic products.

We define the expanded vectors of total outputs and total uses as $\bar{\mathbf{x}}=(\mathbf{x}_b', \mathbf{0}', N)'$ and $\bar{\mathbf{u}}=(\mathbf{u}', \mathbf{y}')'$. Further, we denote the non-negative and absolute values of negative entries of the benchmark domestic use table, $\bar{\mathbf{U}}_0^d$, by matrices $\mathbf{P}_0^d$ and $\mathbf{N}_0^d$, respectively, that is, $\bar{\mathbf{U}}_0^d=\mathbf{P}_0^d-\mathbf{N}_0^d$. The corresponding matrices for the imported use table, $\bar{\mathbf{U}}_0^m$, are $\mathbf{P}_0^m$ and $\mathbf{N}_0^m$. The benchmark supply matrix $\bar{\mathbf{V}}_0=(\mathbf{V}_{b,0}', \mathbf{T}_0', \mathbf{n}_0)'$ (in comparison to Section 2.3 it excludes $\mathbf{m}_0$), is also separated into $\mathbf{P}_0^v$ and $\mathbf{N}_0^v$ similarly, namely, $\bar{\mathbf{V}}_0=\mathbf{P}_0^v-\mathbf{N}_0^v$. These distinctions are made because positive elements have a different role in the estimation procedure of SUTs than strictly negative entries. The use tables can have negative entries because of changes in inventories, while the existence of net taxes can result in negative entries in the supply table. Imports vector is by definition non-negative. We skip here the mathematical details of the corresponding optimization problem, and refer the interested reader to Temurshoev and Timmer (2010). The following result can be proved.

[17] In fact, this can be taken as an additional constraint with the GRAS algorithm, but we will not explore this issue here.

Theorem 3. *The solutions of the joint estimation of SUTs that distinguishes between domestic and imported use of products are given by* $\bar{\mathbf{U}}^d = \hat{\mathbf{r}}_d \mathbf{P}_0^d \hat{\mathbf{s}}_u - \hat{\mathbf{r}}_d^{-1} \mathbf{N}_0^d \hat{\mathbf{s}}_u^{-1}$, $\bar{\mathbf{U}}^m = \hat{\mathbf{r}}_m \mathbf{P}_0^m \hat{\mathbf{s}}_u - \hat{\mathbf{r}}_m^{-1} \mathbf{N}_0^m \hat{\mathbf{s}}_u^{-1}$, $\bar{\mathbf{V}} = \hat{\mathbf{r}}_v \mathbf{P}_0^v \hat{\mathbf{r}}_d^{-1} - \hat{\mathbf{r}}_v^{-1} \mathbf{N}_0^v \hat{\mathbf{r}}_d$, *and* $\mathbf{m}' = r\mathbf{m}_0' \hat{\mathbf{r}}_m^{-1}$.

Theorem 3 shows that in order to estimate three matrices and one vector, we need to compute only five dependent multipliers, which are computed as follows:

$$\mathbf{r}_d = 0.5 \times \mathbf{p}_d^{-1}\left(-\mathbf{c} + \sqrt{\mathbf{c} \circ \mathbf{c} + 4 \times \mathbf{p}_d \circ \mathbf{n}_d}\right), \tag{18}$$

$$\mathbf{r}_m = \sqrt{\widehat{\mathbf{P}_0^m \mathbf{s}_u}^{-1} \left(\mathbf{N}_0^m \hat{\mathbf{s}}_u^{-1} \boldsymbol{\imath} + r\mathbf{m}_0\right)}, \tag{19}$$

$$\mathbf{r}_v = 0.5 \times \widehat{\mathbf{P}_0^v \mathbf{r}_d^{-1} \boldsymbol{\imath}}^{-1} \left(\bar{\mathbf{x}} + \sqrt{\bar{\mathbf{x}} \circ \bar{\mathbf{x}} + 4 \times \left(\mathbf{P}_0^v \mathbf{r}_d^{-1} \boldsymbol{\imath}\right) \circ \left(\mathbf{N}_0^v \mathbf{r}_d\right)}\right), \tag{20}$$

$$\mathbf{s}_u = 0.5 \times \hat{\mathbf{p}}_s^{-1}\left(\bar{\mathbf{u}} + \sqrt{\bar{\mathbf{u}} \circ \bar{\mathbf{u}} + 4 \times \mathbf{p}_s \circ \mathbf{n}_s}\right), \tag{21}$$

$$r = M / \left(\mathbf{m}_0' \hat{\mathbf{r}}_m^{-1} \boldsymbol{\imath}\right), \tag{22}$$

where $\mathbf{p}_d \equiv \mathbf{P}_0^d \mathbf{s}_u + \mathbf{N}_0^{v\prime} \hat{\mathbf{r}}_v^{-1} \boldsymbol{\imath}$, $\mathbf{n}_d \equiv \mathbf{N}_0^d \hat{\mathbf{s}}_u^{-1} \boldsymbol{\imath} + \mathbf{P}_0^{v\prime} \mathbf{r}_v$, $\mathbf{p}_s = \mathbf{P}_0^{d\prime} \mathbf{r}_d + \mathbf{P}_0^{m\prime} \mathbf{r}_m$, and $\mathbf{n}_s \equiv \mathbf{N}_0^{d\prime} \hat{\mathbf{r}}_d^{-1} \boldsymbol{\imath} + \mathbf{N}_0^{m\prime} \hat{\mathbf{r}}_m^{-1} \boldsymbol{\imath}$.

From equations (18)–(22) follows that if $\mathbf{r}_d$ and $\mathbf{r}_m$ converge, so do the other three multipliers.[18] Thus we propose the following algorithm similar to the one presented in Section 2.1:

- *Step* $t = 0$. Initialize $\mathbf{s}_u(0) = \boldsymbol{\imath}$, $\mathbf{r}_v(0) = \boldsymbol{\imath}$ and $r = 1$.
- *Step* $t = 1, \ldots, k$. Calculate $\mathbf{r}_d(t)$ and $\mathbf{r}_m(t)$ on the base of $\mathbf{s}_u(t-1)$, $\mathbf{r}_v(t-1)$ and $r(t-1)$, and then use $\mathbf{r}_d(t)$ and $\mathbf{r}_m(t)$ to compute $\mathbf{s}_u(t)$, $\mathbf{r}_v(t)$, and $r(t)$.
- *Step* $t = k$. Stop when $|\mathbf{r}_d(k) - \mathbf{r}_d(k-1)| < \varepsilon\boldsymbol{\imath}$ and $|\mathbf{r}_m(k) - \mathbf{r}_m(k-1)| < \varepsilon\boldsymbol{\imath}$ for sufficiently small $\varepsilon > 0$.
- *Step* $t = k + 1$. Derive the final estimates using Theorem 3 and multipliers from step k.

3 Empirical assessment

We apply the SUT-RAS method to the Spanish benchmark SUTs for 2000 and 2005, which are available from the National Statistics Institute of Spain both at basic and purchasers' prices. The data were disaggregated into 72 products and 72 industries. We symmetrized the tables because we want to compare the results of the SUT-RAS algorithm with those of two other methods of updating SUTs. These are the Euro method (Beutel 2002; Eurostat 2008), which works for symmetric SUTs only, and EUKLEMS method (Timmer et al. 2005). The Euro method requires that the use table at basic prices is distinguished between domestic and imported intermediate and final uses. Otherwise all the approaches require almost the same data availability for the projection year tables except for the Euro method that does not use outputs by industry and

[18] Theorem 3 and the multipliers in (18)–(22) also hold for the case when use tables are given in basic prices. Then one must not consider the valuation adjustments given by **T** and **n**. Finally, in case of availability of additional information, such as imports and exports vectors, one can easily adapt the SUT-RAS problem of this section, similar to that described in the last part of Section 2.3.

computes them endogenously.[19] For detailed description of the Euro and EUKLEMS methods the reader is referred to Temurshoev et al. (2010).

To assess the relative performance of the methods, we use the following criteria:

1. Mean absolute percentage error (Butterfield and Mules 1980):

$$MAPE = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{|x_{ij}-x_{ij}^{true}|}{|x_{ij}^{true}|}\times 100,$$

where x_{ij}^{true} is the true element, while x_{ij} is its estimate. Thus, *MAPE* shows the average percentage by which each estimated element is larger or smaller than its true value. Note that we take the denominator in absolute value as well so that it does not allow to reduce the actual error when $x_{ij}^{true} < 0$.

2. Weighted absolute percentage error (Mínguez et al. 2009):

$$WAPE = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{|x_{ij}^{true}|}{\sum_k \sum_l x_{kl}^{true}}\right)\frac{|x_{ij}-x_{ij}^{true}|}{|x_{ij}^{true}|}\times 100,$$

which weights each percentage deviation of x_{ij} from x_{ij}^{true} by the relative size of the corresponding true element in the overall sum of the actual elements.

3. Standardized weighted absolute difference (Lahr 2001):

$$SWAD = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}|x_{ij}^{true}|\times|x_{ij}-x_{ij}^{true}|}{\sum_k \sum_l (x_{kl}^{true})^2},$$

which is somewhat similar to *WAPE* with the difference that the absolute deviations are weighted by the size of the true transactions.

4. The *psi statistic* (Kullback 1959; Knudsen and Fotheringham 1986):

$$\hat{\psi} = \frac{1}{\sum_k \sum_l x_{kl}^{true}}\sum_i \sum_j \left[|x_{ij}^{true}|\times\left|\ln\left(\frac{x_{ij}^{true}}{s_{ij}}\right)\right| + |x_{ij}|\times\left|\ln\left(\frac{x_{ij}}{s_{ij}}\right)\right|\right],$$

where $s_{ij} = \left(|x_{ij}^{true}| + |x_{ij}|\right)/2$.[20] Knudsen and Fotheringham (1986) conclude that the psi statistic is one of the most useful goodness-of-fit measures for matrix comparative purposes.

5. *RSQ* (or coefficient of determination) – the square of the correlation coefficient between the elements of the actual matrix, $\mathbf{X}^{true}$, and the predicted matrix, $\mathbf{X}$, when at least one of the elements is different from zero.

The results of the estimation of Spanish SUTs at basic prices are given in Table 6. We use the 2000 SUTs as benchmarks and the required totals vectors from 2005 SUTs in order to estimate the 2005 tables, and then compare the estimates with the true 2005 SUTs. The final demand matrix for this exercise consists of total consumption, gross capital formation, and total exports.

[19] This is, in fact, one of the drawbacks of the Euro method, since it does not use total outputs by industry that are also available from the national accounts. We believe it is important to use this extra information rather than estimating it.

[20] This information-based statistic has a lower limit of zero when $\mathbf{X}^{true} = \mathbf{X}$, and upper bound of $mn \ln 2$ when the non-zero elements of $\mathbf{X}^{true}$ correspond to the zero elements of $\mathbf{X}$, and vice versa. Unlike *MAPE*, *WAPE* and *SWAD*, the psi statistic is insensitive to the change in the positions of x_{ij}^{true} and x_{ij}, and it offers the advantage of considering the case when $x_{ij}^{true} = 0$ and $x_{ij} \neq 0$ (next to the reverse situation). When $x_{ij}^{true} = 0$, we set the corresponding element of *MAPE*, *WAPE* and *SWAD* to zero, and when $x_{ij}^{true} = x_{ij} = 0$, the corresponding entry of $\hat{\psi}$, along other goodness-of-fit measures, is nullified as well.

Table 6. Results of updating Spanish SUTs at basic prices

	MAPE	R.	WAPE	R.	SWAD	R.	$\hat{\psi}$	R.	RSQ	R.	CmR.
					Make matrix (72 × 72)						
EURO	21.62	3	8.45	3	0.109	3	0.083	3	0.9922	3	3
EUKLEMS	19.44	1	2.52	1	0.004	1	0.024	1	0.9998	1	1
SUT-RAS	20.78	2	3.03	2	0.010	2	0.029	2	0.9998	1	2
					Supply table (make matrix + imports, 72 × 73)						
EURO	21.65	3	8.77	3	0.108	3	0.086	3	0.9916	3	3
EUKLEMS	19.47	1	3.69	1	0.007	1	0.035	1	0.9995	1	1
SUT-RAS	20.78	2	4.21	2	0.013	2	0.041	2	0.9994	2	2
					Total intermediate use (72 × 72)						
EURO	38.99	3	20.90	3	0.349	3	0.206	3	0.9205	3	3
EUKLEMS	35.78	1	16.35	2	0.150	2	0.161	2	0.9829	2	2
SUT-RAS	36.27	2	16.07	1	0.131	1	0.158	1	0.9880	1	1
					Total final demand (72 × 3)						
EURO	254.02	2	8.40	2	0.050	3	0.085	2	0.9963	2	2
EUKLEMS	753.38	3	9.78	3	0.042	2	0.125	3	0.9955	3	3
SUT-RAS	111.10	1	7.22	1	0.028	1	0.073	1	0.9973	1	1

Note: The rank of each indicator is given in column R., while CmR. is the combined rank of the averages of all the five rankings. The indicators provide the comparison of the true 2005 SUTs with the 2005 estimates benchmarked on the 2000 SUTs.

The second column of Table 6, for example, shows that the Euro method produces the estimate of the make matrix, whose elements are, on average, 21.6 percent larger or smaller than the true make matrix entries according to *MAPE*. Similarly, the EUKLEMS and SUT-RAS methods are on average, respectively, 19.4 percent and 20.8 percent ‘in error’ according to *MAPE*. However, we should note that *MAPE* is not a good measure of goodness-of-fit, since it gives identical weights to all elements of a matrix of deviations of the estimated and true matrices.[21] In this respect, the *WAPE* indicator is much more relevant as it takes into account the relative size of each element of the true matrix, which, for example, strongly suggests that the Euro method is producing much worse predictions of the make matrix and supply table than those estimated by the EUKLEMS and SUT-RAS methods. In general, Table 6 shows that the EUKLEMS method outperforms the other two methods in estimating the supply table in basic prices, although note that its average errors are quite close to those of the SUT-RAS algorithm. However, when we compare the estimates of the use tables, we find that the SUT-RAS algorithm outperforms both the Euro and EUKLEMS methods. In particular, one can easily observe that the EUKLEMS is performing worst in the final demand matrix estimation. The reason for this large deviation is that the EUKLEMS method considers the column of changes in inventories as a residual between the commodity output vectors obtained from the estimated SUTs in order to make them consistent. This procedure turns out to have a rather large negative impact on the overall quality of estimation of the final demand matrix. This is confirmed in Figure 1, which illustrates *WAPE*s by final demand categories, whose sum for each method equals the overall *WAPE* given in Table 6. Figure 1 clearly illustrates another important issue: compared to the SUT-RAS approach, the Euro and EUKLEMS methods produce extra errors of 0.87 percent and 0.46 percent, respectively, in the consumption vector estimation according to *WAPE*. These percentages, in fact, do not indicate small errors, because total consumption accounted for 32.2

[21] This can be seen in the strange figures of *MAPE* for the final demand matrix estimation results in Tables 6 and 7. Further, we have to note that the coefficient of determination, *RSQ*, is a weak statistic for matrix comparison purposes either (see also Knudsen and Fotheringham 1986). We, however, provide the values of the *MAPE* and *RSQ* measures here only for the sake of completeness, as these indicators are often reported in the studies on updating IO matrices.

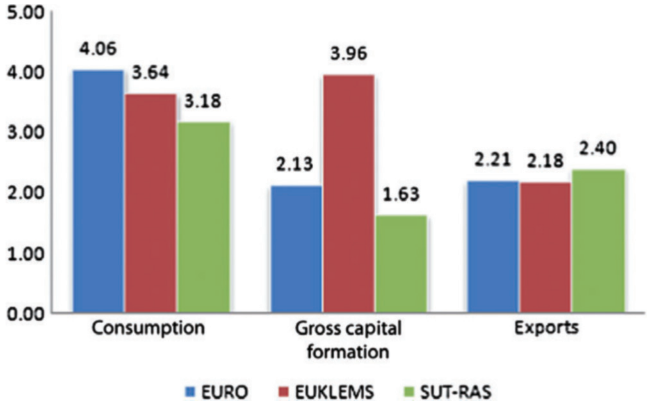

Fig. 1. *WAPE*s by final demand categories

Table 7. Results of updating Spanish SUTs: Purchasers' prices setting

	MAPE	R.	WAPE	R.	SWAD	R.	$\hat{\psi}$	R.	RSQ	R.	CmR.
					Make matrix (72 × 72)						
EUKLEMS	19.44	1	2.52	1	0.004	1	0.024	1	0.9998	1	1
SUT-RAS	20.27	2	2.89	2	0.009	2	0.027	2	0.9998	1	2
					Supply table (make matrix + imports + 3 valuation items, 72 × 76)						
EUKLEMS	19.85	1	6.44	2	0.009	1	0.495	2	0.9985	2	2
SUT-RAS	20.30	2	5.79	1	0.014	2	0.064	1	0.9986	1	1
					Total intermediate use (72 × 72)						
EUKLEMS	41.23	2	19.99	2	0.071	1	0.194	2	0.9878	1	2
SUT-RAS	35.18	1	15.91	1	0.148	2	0.157	1	0.9842	2	1
					Total final demand (72 × 6)						
EUKLEMS	5,989.00	2	33.75	2	0.196	2	0.361	2	0.8853	2	2
SUT-RAS	125.92	1	6.60	1	0.026	1	0.068	1	0.9977	1	1

percent of total product use in Spain for 2005. Since the corresponding total consumption was 655,496 million euros, these additional errors roughly mean that, compared to the SUT-RAS method, the Euro and EUKLEMS wrongly estimate consumption components at the amount of, respectively, 5,714 and 3,011 million euros. For the same year, total intermediate use, gross capital formation and total exports comprised, respectively, 46.0 percent, 12.1 percent, and 9.7 percent of the economy-wide product use. Thus, the same reasoning also holds for gross capital formation estimation as it is the third largest consumer of product uses.

As we have mentioned earlier, the majority of countries do not have the entire SUTs available at basic prices. Thus, in our second examination, we consider the case when the use tables are at purchasers' prices, and supply tables include also the valuation adjustment matrix that transforms total supply at basic prices into supply at purchasers' prices. The last in our Spanish case includes trade margins, transportation margins, and taxes less subsidies on products. The final demand matrix now consists of final consumption expenditure by households, and by nonprofit institutions serving households (NPISH), government expenditure, gross fixed capital formation, change in inventories and total exports.

The results of the estimation are presented in Table 7 (based on the analysis in Section 2.3). We cannot evaluate the Euro method, since it requires the necessary data to be in basic prices and estimates SUTs at basic prices only. Table 7 shows that in the estimation of only the 72 × 72

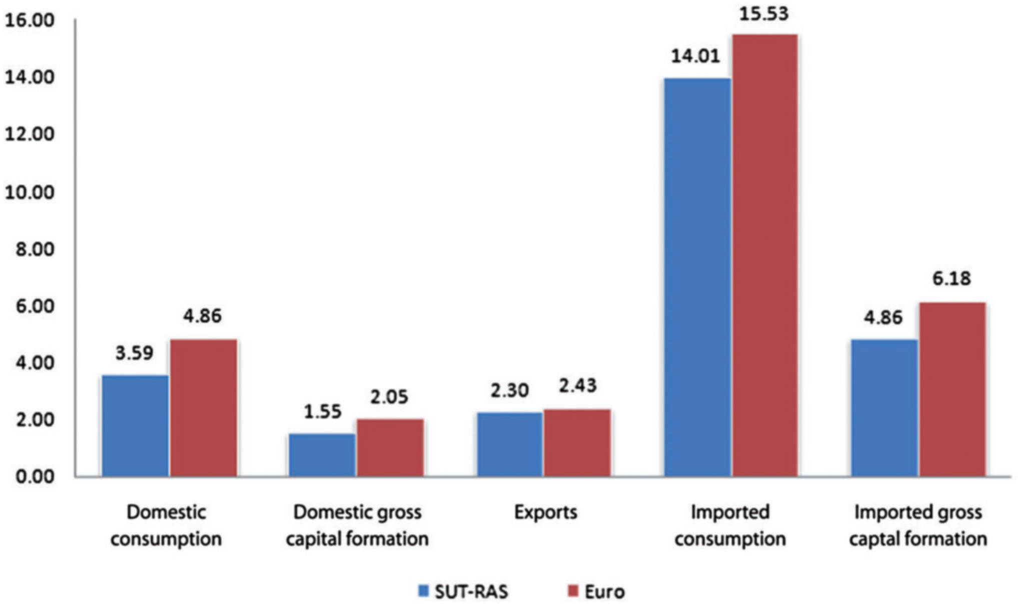

Fig. 2. *WAPE*s by final demand categories for domestic and imported uses

make matrix, EUKLEMS outperforms the SUT-RAS method. Notice that the corresponding numbers for EUKLEMS are exactly those from Table 6, since the make matrix in both considered SUTs settings is expressed in basic prices. But now if we also consider imports, trade margins, transport margins, and net taxes on products (hence the supply table has 72×76 dimension), overall the SUT-RAS is performing better in predicting the 2005 supply table. The difference in estimation between the two approaches becomes more apparent in updating the use tables. So, according to *WAPE*, the SUT-RAS estimate of the intermediate use table is, on average, 15.9 percent 'in error', while that for the EUKLEMS estimate is 20.0 percent. In particular, again we can confirm that the EUKLEMS prediction of the final demand matrix is far worse than that of the SUT-RAS method (i.e., the corresponding *WAPE*s are 33.8% and 6.6%).

Next, we consider the SUT-RAS approach when use tables are separated into domestic and imported uses (see Section 2.4), and make use of the same dataset. This distinction is made for SUTs at basic prices, thus we are also able to compare the SUT-RAS estimation with the Euro method separately for imported and domestic use tables. The results of SUT-RAS vs. Euro estimation of the make matrix and imports vector are, respectively, 3.07 percent vs. 8.45 percent, and 9.18 percent vs. 10.77 percent according to *WAPE*. The corresponding figures for the domestic and imported intermediate use tables are, respectively, 21.39 percent vs. 26.86 percent, and 41.43 percent and 42.58 percent.[22] For the estimated domestic and imported final demand matrices the corresponding *WAPE*s of SUT-RAS vs. Euro estimation are, respectively, 7.45 percent vs. 9.34 percent, and 18.87 percent vs. 21.71 percent. Hence, in all cases the SUT-RAS method produces better estimates than those of the Euro method, and apparently there is much scope for improvement with the SUT-RAS estimation. The composition of *WAPE*s for the domestic and imported final demand categories is shown in Figure 2. It clearly demonstrates the

[22] If we sum the derived domestic and imported intermediate use tables and compare it to the true total intermediate use table, we get the overall *WAPE* of 16.15 percent, which is slightly higher than that presented in Table 6. The last number of 16.07 percent was the result of SUT-RASing when the use table was not distinguished between domestic and imported uses.

Table 8. Added information in the SUT-RAS procedure

	MAPE	WAPE	SWAD	$\hat{\psi}$
	Make matrix			
SUT-RAS	20.777	3.025	0.0102	0.0287
SUT-RAS + imports	20.932	3.024	0.0099	0.0287
SUT-RAS + imports + exports	21.307	3.070	0.0101	0.0292
	Total intermediate use			
SUT-RAS	36.271	16.074	0.1312	0.1582
SUT-RAS + imports	35.919	15.940	0.1320	0.1569
SUT-RAS + imports + exports	36.050	15.593	0.1276	0.1534
	Total reduced final demand			
SUT-RAS	111.096	7.221	0.0283	0.0732
SUT-RAS + imports	109.190	6.754	0.0283	0.0685
SUT-RAS + imports + exports	157.315	5.778	0.0224	0.0592
	Make + intermediate and final use			
SUT-RAS	30.209	7.460	0.0230	0.0735
SUT-RAS + imports	30.074	7.292	0.0229	0.0719
SUT-RAS + imports + exports	30.441	7.009	0.0212	0.0690

point made earlier on the severity of errors for final demand categories as they comprise a large portion of the total product use, in particular, consumption of domestic and imported products.[23]

The final exercise we want to consider is how introducing extra accurate information into the SUT-RAS procedure affects its final estimates.[24] One can find examples in which using extra correct information leads the traditional RAS algorithm to produce poorer estimates. However, de Mesnard and Miller (2006) state that "[a]s a general rule, introduction of accurate exogenous information into RAS improves the resulting estimates, and counterexamples should probably not be taken too seriously" (p. 517). Using the Spanish SUTs at basic prices, we consider two cases where information for commodity imports and exports were used exogenously for the projection of 2005 SUTs. We consider these cases because in reality international trade statistics provide an alternative source for time series of exports and imports. The results, given in Table 8, show that, indeed, there are a few cases when adding true additional information produces poorer estimates. For example, having both exports and imports exogenous results in slightly poorer estimate of the make matrix according to the *MAPE* and *WAPE* indicators. That is, they increase from 20.78 percent to 21.31 percent and from 3.03 percent to 3.07 percent,

[23] That is, for example, 1.28 percent extra error of the Euro method in comparison to the SUT-RAS approach is, in fact, significant because consumption of domestic products in Spain for 2005 accounted for 33.7 percent of all domestic uses. The same is true for the consumption of imported products, which overall comprised 22.6 percent of total imported uses, and the Euro method produces extra 1.52 percent error according to *WAPE*.

[24] Often it is required to estimate taxes net of subsidies on products in the framework of SUTs at basic prices. This can be easily incorporated in the analysis of Section 2.1 by defining the expanded SUTs as (ignoring the subscripts 0s):

$$\bar{\mathbf{V}} = \begin{pmatrix} \mathbf{V}_b & \mathbf{0} \\ \mathbf{m}' & 0 \\ \mathbf{0}' & N \end{pmatrix} \quad \text{and} \quad \bar{\mathbf{U}} = \begin{pmatrix} \mathbf{U}_b & \mathbf{Y}_b \\ \mathbf{n}_1' & \mathbf{n}_2' \end{pmatrix},$$

where $\mathbf{n}_1$ and $\mathbf{n}_2$ are, respectively, the vectors of net taxes of industries and final demand categories, and N is the overall sum of net taxes in the economy, i.e., $(\mathbf{n}_1'\mathbf{n}_2')\boldsymbol{\imath} = N$. Having access to the official dataset of Belgian SUTs at basic prices for the years 1995–2004, we have also examined a series of backward extrapolations using both the Euro and SUT-RAS methods. That is, using the 2004 SUTs as a benchmark, we projected SUTs for nine years of 1995–2003, and the obtained estimates were compared with the corresponding official SUTs using *WAPE* measure. However, because of the space constraints we do not provide the results here, and refer the interested reader to Temurshoev and Timmer (2010). In short, the results suggested that the SUT-RAS approach greatly improves upon the Euro method in projection of Belgium SUTs.

respectively. The same also holds for the estimated use tables according to *MAPE* indicator, which as we know is not a good measure anyway. However, in general, we find that adding extra true exogenous data produces better projections. So, in the overall evaluations of the make matrix together with total intermediate and final use matrices, we observe that all goodness-of-fit measures, except *MAPE*, constantly decrease with added true exogenous information. This also confirms the viewpoint of de Mesnard and Miller (2006) stated above for the case of SUT-RASing in the example of Spain. Therefore, it is recommended to make use of the extra correct exogenous data for SUT-RASing, if they are available.

4 Conclusion

In this paper we applied the standard RAS updating idea specifically for the estimation of supply and use tables (SUTs). The characterizing features of the derived SUT-RAS method are as follows: (*i*) it does not require the availability of total outputs by product for the projection year(s), which are instead endogenously derived; (*ii*) it jointly estimates the SUTs; (*iii*) the estimated SUTs are immediately consistent, and thus, unlike the Euro method (Eurostat 2008) and EUKLEMS approach (Timmer et al. 2005), no additional assumptions are needed to make them consistent; (*iv*) it is biproportional and theory-based method; (*v*) it is general enough to handle both basic and purchasers' price settings of SUTs; (*vi*) it is also appropriate for cases when intermediate and final use tables are distinguished between domestic and imported uses; (*vii*) one can easily consider introduction of an extra accurate information into the SUT-RAS algorithm; and (*viii*) unlike the Euro method, the SUTs do not have to be square.

Our empirical assessment of the method for the Spanish SUTs data showed that the SUT-RAS method is performing quite well, where we made a detailed comparison with the outcomes of the Euro and EUKLEMS methods. Thus, we conclude that the SUT-RAS method may be used for SUTs estimation, especially because it is theory-based approach. The economic theory behind this approach is similar to the well-known standard (G)RAS method. That is, one estimates the new SUTs that are as close as possible to the benchmark SUTs, but they have to satisfy certain restrictions on the SUTs structure and on the available information of the projection year. For interpolation when two benchmarks for the beginning and ending period are available, we suggest to use a varying benchmarks scheme that gives higher (resp. lower) weight to closer (resp. further) benchmark SUTs depending on a given projection year. This procedure ensures that structural change, if indeed happened during the interpolation period, will be more or less taken into account. We should also mention that in the SUT-RAS estimation procedure, large elements are given a higher weight than small transactions. From a practical point of view this feature is desirable, because statisticians always try to estimate large entries of SUTs as accurate as possible, while they might give less attention to the small transactions accuracy. Finally, we found that using extra available exogenous information generally improves the quality of the SUT-RAS estimates, confirming results for traditional RAS.

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Resumen. Aplicamos la idea de actualización con RAS para la estimación conjunta de cuadros de oferta y utilización (COU). En contraste con las técnicas estándar de input-output, nuestro enfoque no requiere la disponibilidad del total de outputs por producto para el año(s) proyectado(s). En la práctica, esta condición normalmente no se satisface. El algoritmo, denominado método COU-RAS, estima conjuntamente aquellos COU que son inmediatamente uniformes. Es aplicable a diferentes contextos de COU, tales como marcos con precios básicos y precios de comprador, y el caso en el que los cuadros de uso están separados entre usos interiores e importados. Nuestras evaluaciones empíricas muestran que el método SUT-RAS representa una mejora respecto a métodos abreviados en la proyección de COU españoles.

要約　我々は供給使用表（SUT: supply and use tables）の同時推計に対し RAS 法のアップデートというアイデアを適用する。標準的な産業関連分析のテクニックとは対照的に、我々のアプローチでは予測年の財別総生産を必要としない。通常、この条件は実際には満たされない。SUT-RAS 法と呼ばれるアルゴリズムで同時一致 SUT 推計を行う。基準価格と購入者価格を用いたフレームワークや国内用と輸入用に分割された表を使う設定など、これは様々な設定の SUT に応用できる。我々の実証検証によるとSUT-RAS 法はスペインの SUT の推計における既存の簡便法よりも優れている。