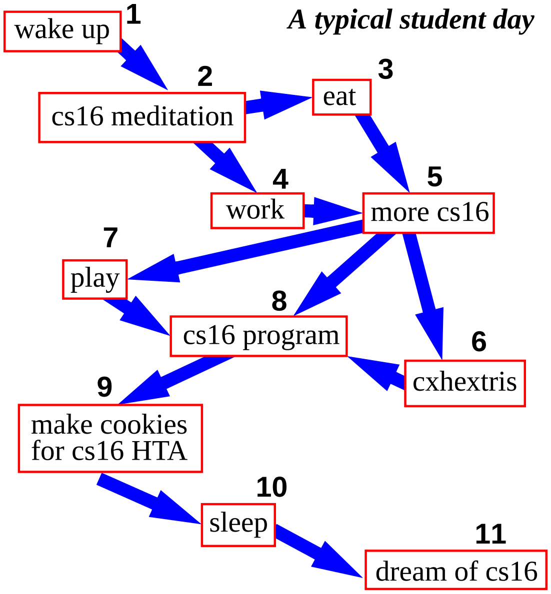
DIGRAPHS

A typical student day



What's a Digraph?

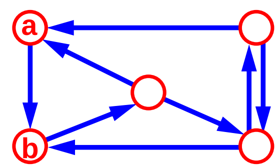
a) A small burrowing animal with long sharp teeth and a unquenchable lust for the blood of computer science majors

b) A distressed graph

c) A directed graph

Each edge goes in one direction

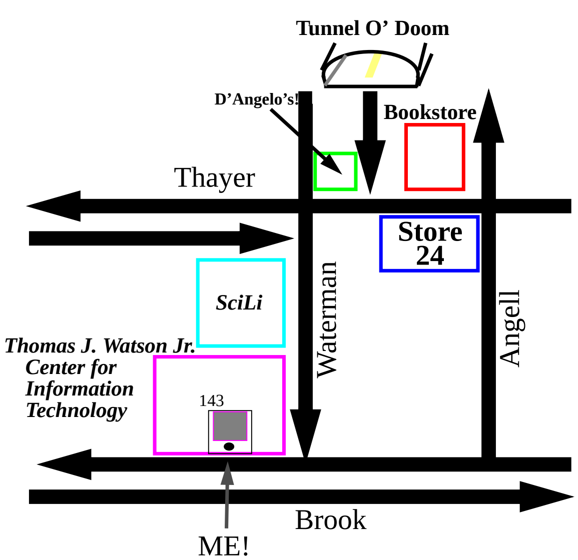
Edge (a,b) goes from a to b, but not b to a



You're saying, "Yo, how about an example of how we might be enlightened by the use of digraphs!!" – Well, if you insist. . .

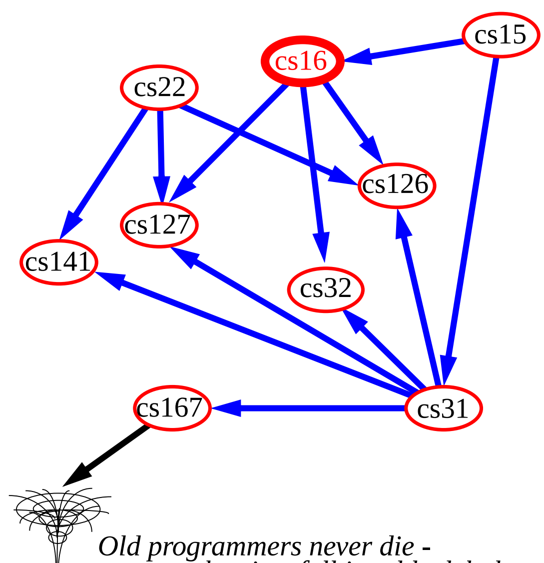
Applications

Maps: digraphs handle one-way streets
(especially helpful in Providence)



Another Application

Scheduling: edge (a,b) means task a must be completed before b can be started



Old programmers never die -
they just fall into black holes

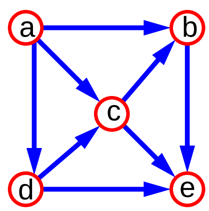
DAG’s

dag: (noun) dÂ-g

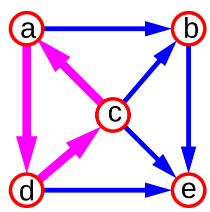
1. **D**i-**A**cyl-**G**lycerol – My favorite snack!
2. “~~man’s~~ best friend”
   person’s
3. **directed acyclic graph**

Say What?!

directed graph with no directed cycles



DAG

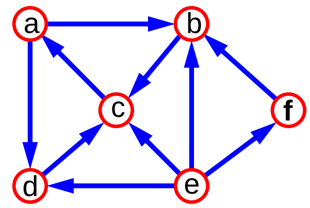


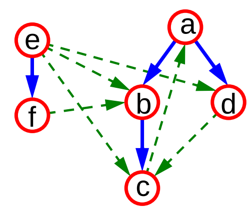
not a DAG

Depth-First Search

Same algorithm as for undirected graphs

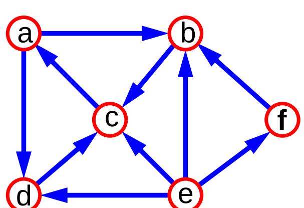
On a connected digraph, may yield unconnected DFS trees (i.e., a DFS forest)

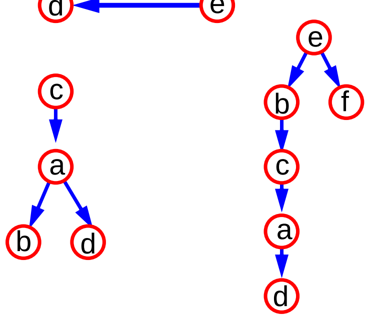




Reachability

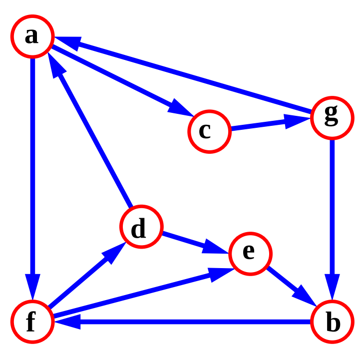
DFS tree rooted at v: vertices reachable from v via directed paths



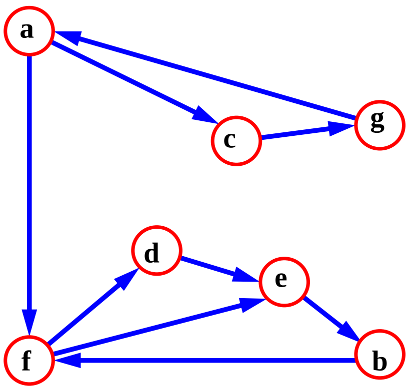


Strongly Connected Digraphs

Each vertex can reach all other vertices



Strongly Connected Components



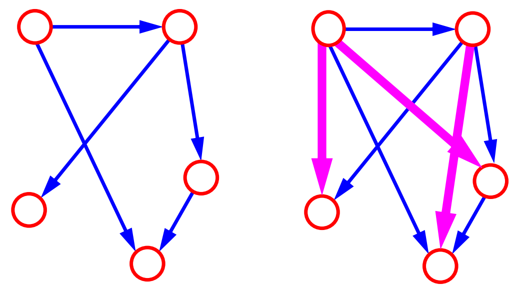
{ a , c , g }

{ f , d , e , b }

Transitive Closure

Digraph G^* is obtained from G using the rule:

If there is a directed path in **G** from a to b, then add the edge (a,b) to **G^***

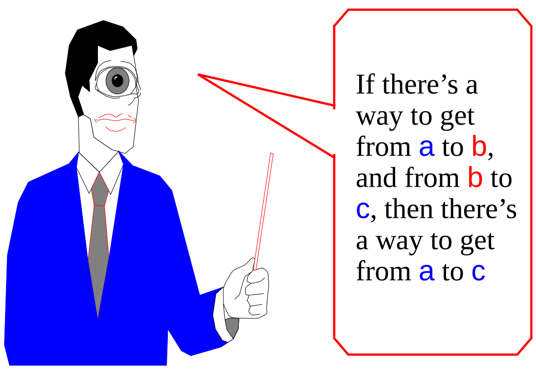


G **G^***

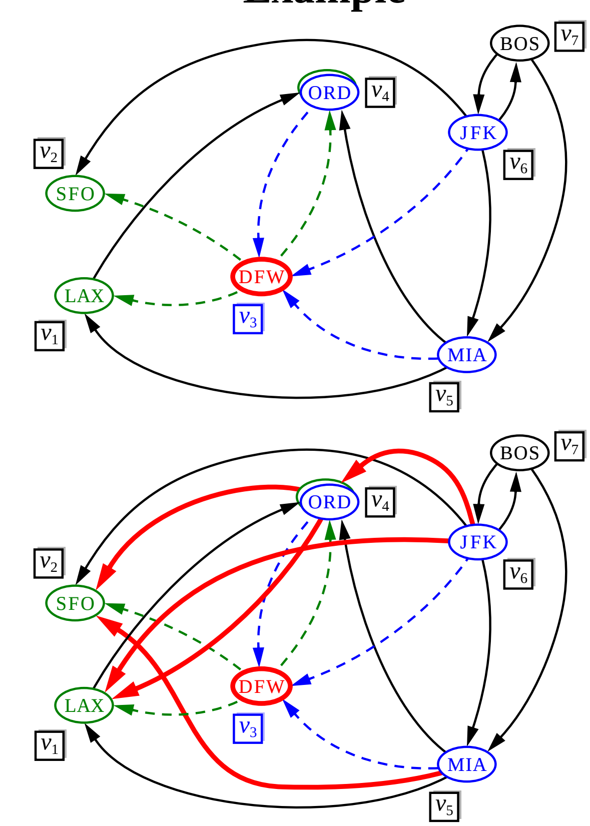
Computing the Transitive Closure

We can perform DFS starting at each vertex
Time: **O**(**n**(**n**+**m**))

Alternatively ... Floyd-Warshall Algorithm:



Example



Floyd-Warshall Algorithm

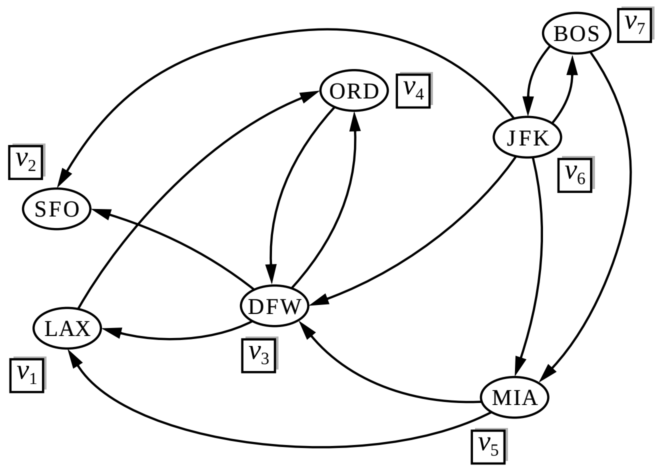
- this algorithms assumes that methods areAdjacent and insertDirectedEdge take O(1) time (e.g., adjacency matrix structure)

Algorithm FloydWarshall(G)
  let v_1 ... v_n be an arbitrary ordering of the vertices
  $G_0 = G$
  for $k = 1$ **to** n **do**
    // consider all possible routing vertices v_k
    $G_k = G_{k-1}$
    for each $(i, j = 1, ..., n)$ $(i != j)$ $(i, j != k)$ **do**
      // for each pair of vertices v_i and v_j
      if G_{k-1}.areAdjacent(v_i,v_k) **and**
        G_{k-1}.areAdjacent(v_k,v_j) **then**
          G_k.insertDirectedEdge(v_i,v_j,null)
    return G_0

- digraph G_k is the subdigraph of the transitive closure of G induced by paths with intermediate vertices in the set $\{ v_1, ..., v_k \}$
- running time: $O(n^3)$

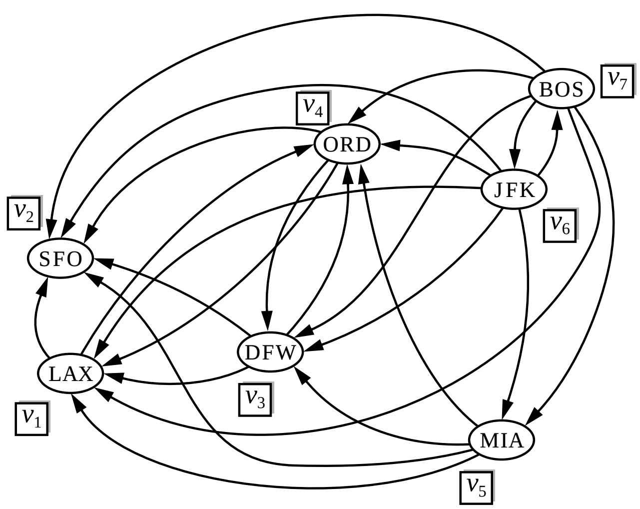
Example

- digraph G



Example

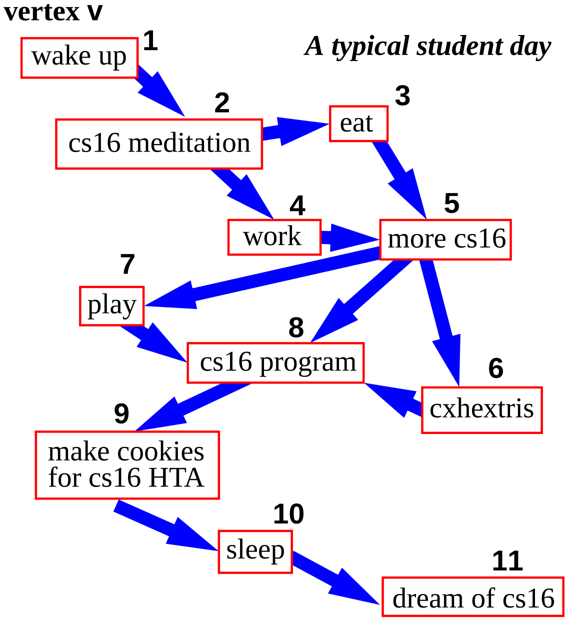
- digraph G*



Topological Sorting

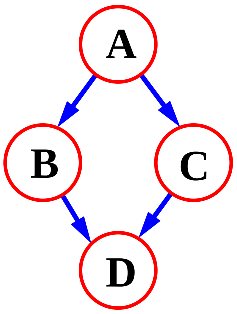
For each edge (u,v), vertex u is visited before vertex v

A typical student day



Topological Sorting

Topological sorting may not be unique



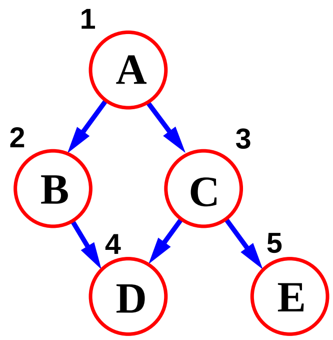
A B C D
or
A C B D

– ***You*** *make the call!*

Topological Sorting

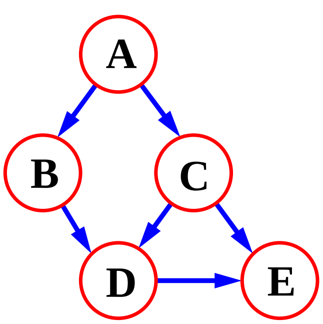
Labels are increasing along a directed path

A digraph has a topological sorting *if and only if* it is acyclic (i.e., a dag)



Algorithm for Topological Sorting

method TopologicalSort
  if there are more vertices
    let *v* be a source;
      // a vertex w/o incoming edges
    label and remove *v*;
    TopologicalSort;



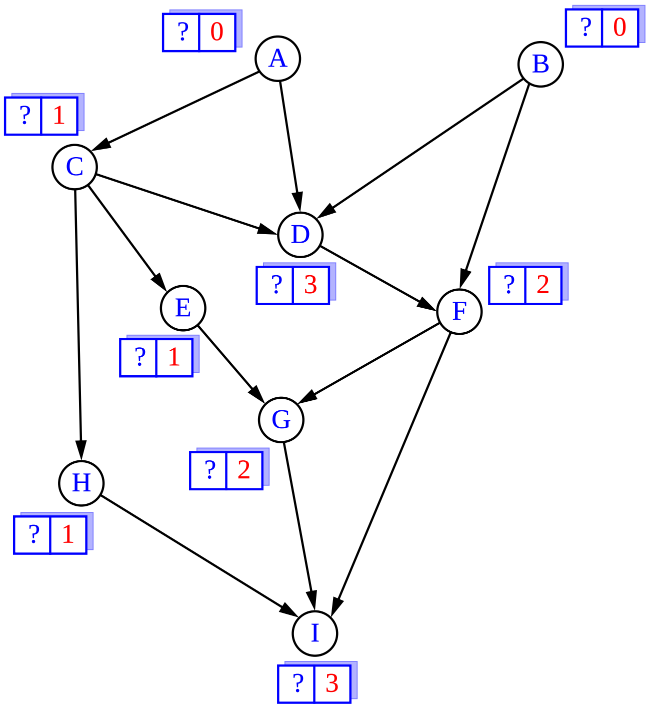
Algorithm (continued)

Simulate deletion of sources using indegree counters

TopSort(Vertex v);
  label v;
  foreach edge(v,w)
    indeg(w) = indeg(w) – 1;
    if indeg(w) = 0
      TopSort(w);

1. Compute indeg(**v**) for all vertices
2. Foreach vertex **v** do
   if **v** not labeled and indeg(**v**) = 0
     then TopSort(**v**)

Example



Reverse Topological Sorting

```
RevTopSort(Vertex v)
  mark v;
  foreach edge(v,w)
      if v not marked
            RevTopSort(w);
  label v;
```