A06458

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Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

THE UNIVERSITY OF BIRMINGHAM

Degree of B.Sc. with Honours
Artificial Intelligence and Computer Science. Second Examination
Computer Science. Second Examination
Computer Science/Software Engineering. Second Examination
Computer Science/Software Engineering with Business Studies. Second Examination

Degree of BEng/MEng with Honours
Computer Science/Software Engineering. Second Examination

Joint Degree of B.Sc. with Honours
Mathematics and Computer Science. Second Examination

Joint Degree of B.A. with Honours
Computer Studies and English. Second Examination

Occasional Computer Science/Software Engineering

Occasional American Studies

06 02504

Graphics 1

Thursday 16th May 2002 0930 - 1130

[Answer Question 1 and TWO other Questions]

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[Answer Question 1 and TWO other Questions]

1 (a) Let

$$A = \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 \\ 7 \end{bmatrix} \qquad D = [\ 3 \ \ -3\ ]$$

Provide the answers, together with justifications, to the following questions:

(i) Is the matrix X below the result of multiplication of A*B or B*A?

$$X = \begin{bmatrix} 4 & 5 \\ 0 & -11 \end{bmatrix}$$

(ii) Which of the operations is undefined: A*C or C*A?

(iii) What is the result of multiplying D*C?

[4%]

(b) A viewing coordinate system is specified by the View Reference Point (VRP) and three vectors:

- $\bar{\mathbf{N}}$, from VRP to a target point, P_T, on the object
- $\bar{\mathbf{U}}$, the “up” vector
- $\bar{\mathbf{V}}$, the “handedness” vector

Given VRP = (30, 50, -100) and P_T = (10, 10, 10), calculate the $\bar{\mathbf{U}}$ vector.

[8%]

Question 1 continues over the page

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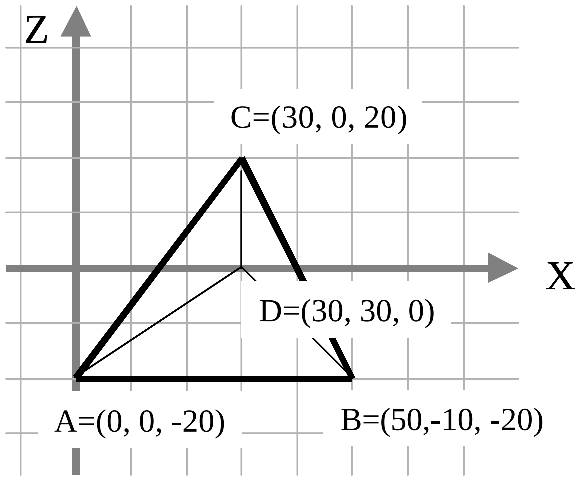
Question 1 continued

(c) Define a complete vertex table for the pyramid whose projection on to the ZX plane is shown in the figure below.

[6%]



(d) Given the following viewing parameters:

VRP = (30, 10, -100)

$$\bar{\mathbf{N}} = \begin{bmatrix} 0 \\ 4 \\ 10 \\ 1 \end{bmatrix} \qquad \bar{\mathbf{U}} = \begin{bmatrix} 0 \\ 10 \\ -4 \\ 1 \end{bmatrix} \qquad \bar{\mathbf{V}} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

viewing distance D = 20.

Construct all the transformation matrices necessary for computing the perspective projection of the pyramid defined in (c). DO NOT carry out any calculations. The generic transformation matrices are given in the Appendix.

[10%]

Question 1 continues over the page

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Question 1 continued

(e) How would you compute a single transformation matrix which combines all the transformations which need to be carried out in (d)? Name the matrix operation and specify the ORDER in which the elementary transformation matrices have to be combined using this operation. DO NOT carry out any calculations.

[4%]

(f) The combined transformation matrix for computing the perspective projection specified in (d) is defined as follows.

$$M = \begin{bmatrix} 1 & 0 & 0 & -30 \\ 0 & 0.93 & -0.37 & -46.42 \\ 0 & 0.37 & 0.93 & 89.13 \\ 0 & 0.02 & 0.05 & 4.46 \end{bmatrix}$$

Using this matrix compute the projected coordinates of the vertex B of the pyramid defined in (c).

[8%]

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2. (a) Briefly explain the following terms:
   (i) DDA
   (ii) a cosine function
   (iii) a Bézier curve
   (iv) a convex hull

[6%]

(b) The following fragment of pseudo-code specifies a part of Bresenham's midpoint circle algorithm for generating a circle centred on the origin. Modify this code so that it correctly generates a circle centred on the point (x_c, y_c)

[8%]

1. Input radius r
2. Plot a point at (0, r)
3. Calculate the initial value of the decision parameter as $p_0 = \frac{5}{4} - r$
4. At each position x_k, starting at k = 0, perform the following test:

   if $p_k < 0$
      plot point at (x_{k+1}, y_k)
      compute new $p_{k+1} = p_k + 2x_{k+1} + 1$
   else
      plot point at ($x_k + 1$, $y_k - 1$)
      compute new $p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$
   where $x_{k+1} = x_k + 1$ and $y_{k+1} = y_k - 1$

5. Determine symmetry points in the other seven octants and plot points
6. Repeat steps 4 and 5 until $x \geq y$

Question 2 continues over the page.

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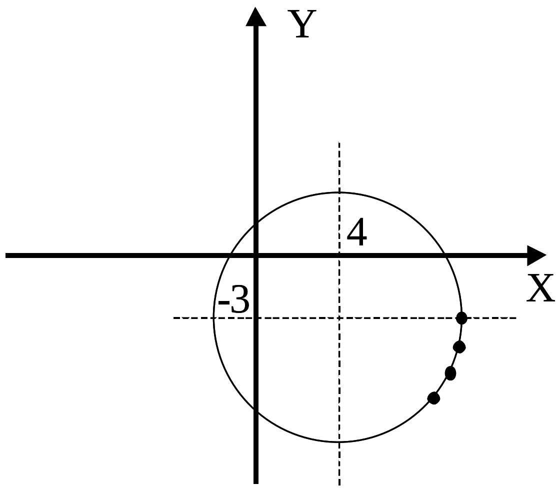
Question 2 continued

(c) Use the modified algorithm to compute the positions of the first 4 pixels shown schematically in the diagram below. The circle is centred on $(x_c, y_c) = (4,-3)$ and has radius r = 6.

[10%]



(d) Explain why the condition

$x \geq y$

is used as the termination criterion for Bresenham's midpoint circle algorithm.

[6%]

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3 (a) Briefly explain the following terms:
- (i) a local coordinate system
- (ii) constructive solid geometry
- (iii) sweep representations
- (iv) 3D viewport

[6%]

(b) Sketch the following:
- (i) Left-handed coordinate system seen from the negative end of the X axis
- (ii) Left-handed coordinate system seen from the positive end of the Y axis
- (iii) Left-handed coordinate system seen from the negative end of the Z axis
- (iv) Right-handed coordinate system seen from the negative end of the Y axis

[8%]

(c) On each of the sketches above, indicate the direction of the positive angle of rotation about the respective axis (e.g. axis X in (i)).

[4%]

(d) A 2-dimensional triangle is defined in a 2D coordinate system W1 by the following (homogeneous) vertices:

$$\bar{\mathbf{V}}\mathbf{1} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \qquad \bar{\mathbf{V}}\mathbf{2} = \begin{bmatrix} 9 \\ 2 \\ 1 \end{bmatrix} \qquad \bar{\mathbf{V}}\mathbf{3} = \begin{bmatrix} 7 \\ 10 \\ 1 \end{bmatrix}$$

Compute the coordinates of the triangle in another 2D coordinate system, W2, whose axes are rotated by $+90^{o}$ about the centre of W1.

You MUST make use of transformation matrices. Specify the numerical values in each matrix that you use. The results of calculations must be numerical. Show every step of your calculations.

[12%]

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4. (a) Provide a brief description of the following terms:
   (i) Event driven interaction
   (ii) Vector display
   (iii) Primaries
   (iv) In-betweening

[6%]

(b) Define the following colours using the CMY model:
   (i) fully saturated red
   (ii) 50% saturated yellow

[6%]

(c) Outline the key features of the Indexed Colour Model.

[4%]

(d) Explain how to achieve animation effects using a Colour Look-up Table and without making any changes to a frame buffer.

[5%]

(e) During an image acquisition the red camera sensor was faulty and the resulting colour image contains only half of the red component. Sketch approximately the shape of the mapping functions for RGB which would compensate for this fault.

[4%]

(f) Convolution is a generic image processing operation which changes an image in different ways, depending on the values of a convolution kernel.

   (i) Give an example of a sharpening kernel.
   (ii) In what way are pixel values changed by this kernel to give an impression of increased image sharpness?

[5%]

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THE APPENDIX
3-dimensional transformations

Translation $$T = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling $$S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation

about Z axis $$R_z = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 & 0 \\ \sin\varphi & \cos\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

about X axis $$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi & 0 \\ 0 & \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

about Y axis $$R_y = \begin{bmatrix} \cos\varphi & 0 & \sin\varphi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\varphi & 0 & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Combined rotation matrix

$$R_{xyz} = \begin{bmatrix} A_x & A_y & A_z & 0 \\ B_x & B_y & B_z & 0 \\ C_x & C_y & C_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $C = \dfrac{\overline{N}}{|\overline{N}|}$ $A = \dfrac{\overline{U} \times \overline{N}}{|\overline{U} \times \overline{N}|}$ $B = \dfrac{\overline{C} \times \overline{A}}{|\overline{C} \times \overline{A}|}$

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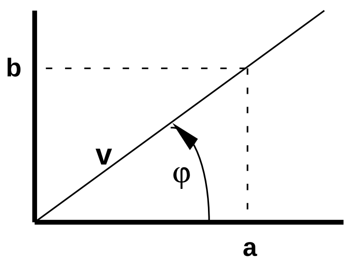
THE APPENDIX (cont)

Perspective projections

$$P_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & \mathbf{0} \end{bmatrix} \qquad P_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/D & \mathbf{1} \end{bmatrix}$$

Basic trigonometric definitions



$\cos \varphi = \dfrac{a}{v}$

$\sin \varphi = \dfrac{b}{v}$

$v = \sqrt{a^2 + b^2}$

$\sin 0° = 0$ $\quad \sin 90° = 1$
$\cos 0° = 1$ $\quad \cos 90° = 0$

$\sin 45° = \cos 45° = \dfrac{1}{v2}$

$\sin(-\varphi) = -\sin(\varphi)$
$\cos(-\varphi) = \cos(\varphi)$

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THE APPENDIX (cont)

Bézier curves

Bézier function $P(u)$ in parametric form:

$$P(u) = \sum_{k=0}^{n} p_k B_{k\,n}(u)$$

Blending functions:

$$B_{k\,n}(u) = C(n, k)\, u^k (1 - u)^{n-k}$$

where

$$C(n, k) = \frac{n!}{k!\,(n-k)!}$$

$n! = n \times (n-1) \times \ldots \times 1, \quad 0! = 1$

END OF APPENDIX