SEQUENCES

- Ranked Sequences
- Positions
- Positional Sequences
- General Sequences
- Bubble Sort Algorithm

The Ranked Sequence ADT

- A ranked sequence S (with n elements) supports the following methods:

  - elemAtRank(r):
    Return the element of S with rank r; an error occurs if $r < 0$ or $r > n - 1$
    Input: Integer; *Output*: Object

  - replaceElemAtRank(r,e):
    Replace the element at rank r with e and return the old element; an error condition occurs if $r < 0$ or $r > n - 1$
    Input: Integer r, Object e; *Output*: Object

  - insertElemAtRank(r,e):
    Insert a new element into S which will have rank r; an error occurs if $r < 0$ or $r > n - 1$
    Input: Integer r, Object e; *Output*: Object

  - removeElemAtRank(r):
    Remove from S the element at rank r; an error occurs if $r < 0$ or $r > n - 1$
    Input: Integer; *Output*: Object

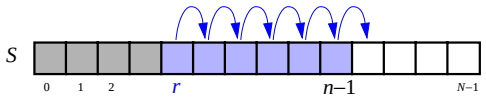
Array-Based Implementation

- Some Pseudo-Code:

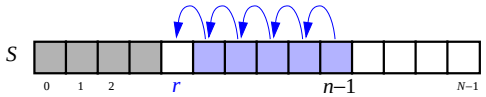
```
Algorithm insertElemAtRank(r,e):
  for i = n - 1, n - 2, ... , r do
    S[i+1] ← s[i]
  S[r] ← e
  n ← n + 1

Algorithm removeElemAtRank(r):
  e ← S[r]
  for i = r, r + 1, ... , n - 2 do
    S[i] ← S[i + 1]
  n ← n - 1
  return e
```

- A Graphical Representation

insertElemAtRank(r,e):



removeElemAtRank(r):



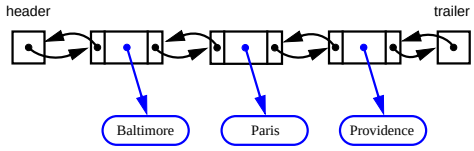
Array-Based Implementation (contd.)

- Time complexity of the various methods:

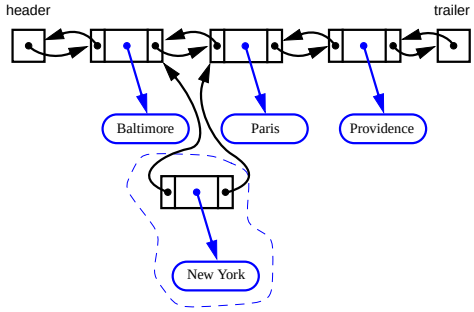
Method	Time
size	$O(1)$
isEmpty	$O(1)$
elemAtRank	$O(1)$
replaceElemAtRank	$O(1)$
insertElemAtRank	$O(n)$
removeElemAtRank	$O(n)$

Implementation with a Doubly Linked List

- the list before insertion:

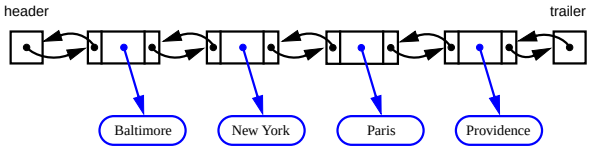


- creating a new node for insertion:

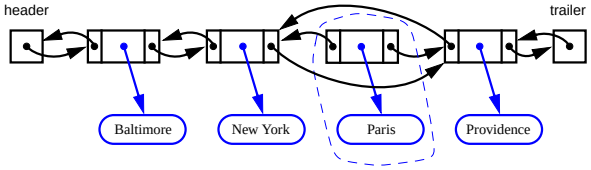


Implementation with a Doubly Linked List

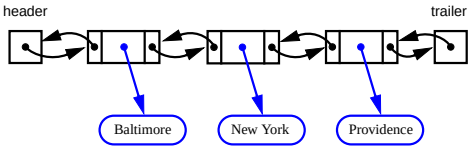
- the list after insertion and before deletion:



- deleting a node:



- after deletion:



Java Implementation

```
public class NodeRankedSequence
  extends MyDeque implements Deque,
  RankedSequence{
public void insertElemAtRank (int rank, Object element)
   throws BoundaryViolationException {
    if (rank != size()) //rank size() is OK for
                        //insertion
     checkRank(rank);
     DLNode next = nodeAtRank(rank); // the new node
                        //will be right before this
     DLNode prev = next.getPrev(); // the new node
                        //will be right after this
     DLNode node = new DLNode(element, prev, next);
     next.setPrev(node);
     prev.setNext(node);
     size++;
  }
  public Object removeElemAtRank (int rank)
   throws BoundaryViolationException {
    checkRank(rank);
    DLNode node = nodeAtRank(rank);  // node to
                        //be removed
     DLNode next = node.getNext(); //node before it
```


Java Implementation (cont.)

```
     DLNode prev = node.getPrev(); // node after it
    prev.setNext(next);
    next.setPrev(prev);
    size--;
    return node.getElement();
  }
   private DLNode nodeAtRank (int rank) {
    // auxiliary method to find the node of the
    //element with the given rank
    DLNode node;
    if (rank <= size()/2) { //scan forward from head
       node = header.getNext();
       for (int i=0; i < rank; i++)
       node = node.getNext();
    }
    else { // scan backward from the tail
       node = trailer.getPrev();
       for (int i=0; i < size()-rank-1 ; i++)
       node = node.getPrev();
    }
    return node;
   }
```

Nodes and Positions

- Node-Based Operations:
  - Node specific methods, e.g. removeAtNode(Node v) and insertAfterNode(Node v, Object e), would be O(1).
  - However, node-based operations are not meaningful in an array-based implementation because there are no nodes in an array.
  - **Dilemma**:
    - If we do not include the-node based operations int the generic sequence ADT, we are not taking full advantage of doubly-linked lists.
    - If we do include them, we violate the generality of object oriented design.

Nodes and Positions (cont.)

- Positions:
  - Inituitve notion of “place” of an element.
  - This concept allows us to enjoy doubly-linked list without violating object-oriented design.
  - Positions have 2 methods:
    element(): Return the element at the Position
    Input: none; Output: Object
    container():Return the sequence that contains this position.
    Input: none; Output: sequence
  - Positions are defined relatively.
  - Positions are not tied to an element or rank
  - A Sequence is a container of elements that are each stored in a position

The Positional Sequence ADT

- The methods are:
  - first()
  - last()
  - before()
  - after()
  - size()
  - isEmpty()
  - replace(p,e)
  - swap(p, q)
  - insertFirst(e)
  - insertLast(e)
  - insertBefore(p,e)
  - insertAfter(p,e)
  - remove(p)
  - isFirst(p)
  - isLast(p)

Doubly Linked List Implementation

- Implementation of a node using Positions

```
class NSNode implements Position {
  private NSNode prev, next; // References to the
                             //nodes before and after
  private Object element; //Element stored in this
                          //position
  private Container cont; //Container of this
                          //position
  NSNode(NSNode newPrev, NSNode newNext,
      Container container, Object elem) { //Initialize
                                          //the node
    prev = newPrev;
    next = newNext;
    cont = container;
    element = elem;
  }
  public Container container()
    throws InvalidPositionException {
    if (cont == null)
      throw new InvalidPositionException
        (“Position has no container!”);
```

Doubly Linked List Implementation(cont.)

```
        return cont;
    }
    public Object element()
      throws InvalidPositionException {
      if (cont == null)
        throw new InvalidPositionException
          ("Position has no container!");
      return element;
    }
    // Accesor methods
    NSNode getNext() { return next; }
    NSNode getPrev() { return prev; }
    void setNext(NSNode newNext) { next = newNext; }
    // Update methods
    void setPrev(NSNode newPrev) { prev = newPrev; }
    void setElement(Object newElement) { element =
                                          newElement; }
    void setContainer(Container newCont) { cont =
                                          newCont; }
}
```


Doubly Linked List Implementation

- Code for other methods of a Doubly Linked List:
  - checkPosition

```
protected NSNode checkPosition(Position p) throws
  InvalidPositionException{
    if (p==head)
      throw new InvalidPositionException("Head of
        the sequence is not a valid position");
    if (p==tail)
      throw new InvalidPositionException ("Tail of the
        sequence is not a valid position");
}
```

  - first

```
public Position first() throws
  EmptyContainerException {
    if(isEmpty())
      throw new EmptyContainerException
        ("Sequence is empty");
    return head.getNext();
  }
```


Doubly Linked List Implementation (cont.)

- before

```
public Position before(Position p) throws
  InvalidPositionException, BoundaryViolationException{
    NSNode n = checkPosition(p);
    NSNode prev = n.getPrev();
    if(prev==head)
      throw new BoundaryViolationException ("Cannot
        go past the beginning of the sequence");
    return prev;
  }
```

- insertAfter

```
  public Position insertAfter (Position p, Object element)
    throws InvalidPositionException{
      NSNode n = checkPosition(p);
      numElts++;
      NSNode newNode = new NSNode(n, n.getNext(),
        this, element);
      n.getNext().setPrev(newNode);
      n.setNext(newNode);
      return newNode;
    }
```


Doubly Linked List Implementation (cont.)

- remove

```
public Object remove(Position p) throws
  InvalidPositionException {
    NSNode n = checkPosition(p);
    numElts--;
    NSNode nPrev = n.getPrev();
    NSNode nNext = n.getNext();
    nPrev.setNext(nNext);
    nNext.setPrev(nPrev);
    Object nElem = n.element();
    // unlink the position from the list
    //and make it invalid
    n.setNext(null);
    n.setPrev(null);
    n.setContainer(null);
    return nElem;
  }
```

The Sequence ADT in Java

```
public interface PositionalSequence extends
PositionalContainer {

/* ***********  Accessor Methods
*********** */

  public Position first()
   throws EmptyContainerException;

  public Position last()
   throws EmptyContainerException;

  public Position before (Position p)
   throws InvalidPositionException,
      BoundaryViolationException;

  public Position after (Position p)
   throws InvalidPositionException,
      BoundaryViolationException;
```


The Sequence ADT in Java (contd.)

```
/* ***********  Information Methods
*********** */

  public boolean isEmpty();

  public boolean size();

  public boolean isFirst (Position p)
   throws InvalidPositionException;

  public boolean isLast (Position p)
   throws InvalidPositionException;

/* ***********  Update Methods
*********** */

  public Position insertFirst (Object element);

  public Position insertLast (Object element);
```


The Sequence ADT in Java (contd.)

```
/* ***********  More Update Methods
*********** */

  public Position insertBefore (Position p, Object
                                element)
   throws InvalidPositionException;

  public Position insertAfter (Position p, Object element)
   throws InvalidPositionException;

  public Object remove (Position p)
   throws InvalidPositionException;

  public Object replace (Position p, Object element)
    throws InvalidPositionException;

  public void swap (Position p, Position q)
    throws InvalidPositionException;
}
```


Comparison of Sequence Implementations

- **Is replaceElemAtRank O(1) in a list?????**

Operations	Array	List
size, isEmpty	$O(1)$	$O(1)$
atRank, rankOf, elemAtRank	$O(1)$	$O(n)$
first, last	$O(1)$	$O(1)$
before, after	$O(1)$	$O(1)$
replace, replaceElemAtRank, swap	$O(1)$	$O(1)$
insertElemAtRank, removeElemAtRank	$O(n)$	$O(n)$
insertFirst, insertLast	$O(1)$	$O(1)$
insertAfter, insertBefore	$O(n)$	$O(1)$
remove	$O(n)$	$O(1)$

Bubble Sort

- A Bubble Sort works by scanning through a sequence and swapping a given element with the next one if the former is smaller than the latter:

Pass	Swaps	Sequence
		(5, 7, 2, 6, 9, 3)
1st	$7 \leftrightarrow 2$, $7 \leftrightarrow 6$, $9 \leftrightarrow 3$	(5, 2, 6, 7, 3, 9)
2nd	$5 \leftrightarrow 2$, $7 \leftrightarrow 3$	(2, 5, 6, 3, 7, 9)
3rd	$6 \leftrightarrow 3$	(2, 5, 3, 6, 7, 9)
4th	$5 \leftrightarrow 3$	(2, 3, 5, 6, 7, 9)

- Here is implementation of a Bubble Sort for an Array-Based Sequence:

```
public void static bubbleSort1(IntegerSequence s) {
  int n = s.size();
  for (int i=0; i<n; i++)      // i-th pass
    for (int j=0; j<n-i; j++)
      if (s.atRank(j).element().intValue() >
s.atRank(j+1).element().intValue())
        swap(s.positionAtRank(j), s.positionAtRank(j+1));
}
```


Bubble Sort (contd.)

- This Implementation is designed for a sequence based on a doubly linked list.

```
public void static bubbleSort2(IntegerSequence s) {
  int n = s.size();
  IntegerSequencePosition prec, succ;

  for (int i=0; i<n; i++) {    // i-th pass
    prec = s.firstPosition();
    for (int j=0; j<n-i; j++) {
      succ = s.after(prec);
      if (prec.element().intValue() >
succ.element().intValue())
        swap(prec,succ);
      else
        prec = succ;
    }
  }
}
```