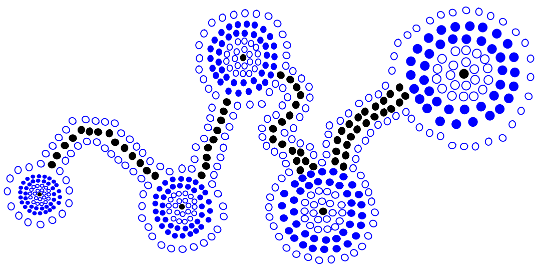
SORTING

- Review of Sorting
- Merge Sort
- Sets



Sorting Algorithms

- Selection Sort uses a priority queue P implemented with an unsorted sequence:
  - **Phase 1**: the insertion of an item into P takes $O(1)$ time; overall $O(n)$
  - **Phase 2**: removing an item takes time proportional to the number of elements in P $O(n)$: overall $O(n^2)$
  - Time Complexity: $O(n^2)$
- Insertion Sort is performed on a priority queue P which is a sorted sequence:
  - **Phase 1**: the first insertItem takes $O(1)$, the second $O(2)$, until the last insertItem takes $O(n)$: overall $O(n^2)$
  - **Phase 2**: removing an item takes $O(1)$ time; overall $O(n)$.
  - Time Complexity: $O(n^2)$
- Heap Sort uses a priority queue K which is a heap.
  - insertItem and removeMinElement each take $O(\log k)$, k being the number of elements in the heap at a given time.
  - **Phase 1**: n elements inserted: $O(n\log n)$ time
  - **Phase 2**: n elements removed: $O(n \log n)$ time.
  - Time Complexity: $O(n\log n)$

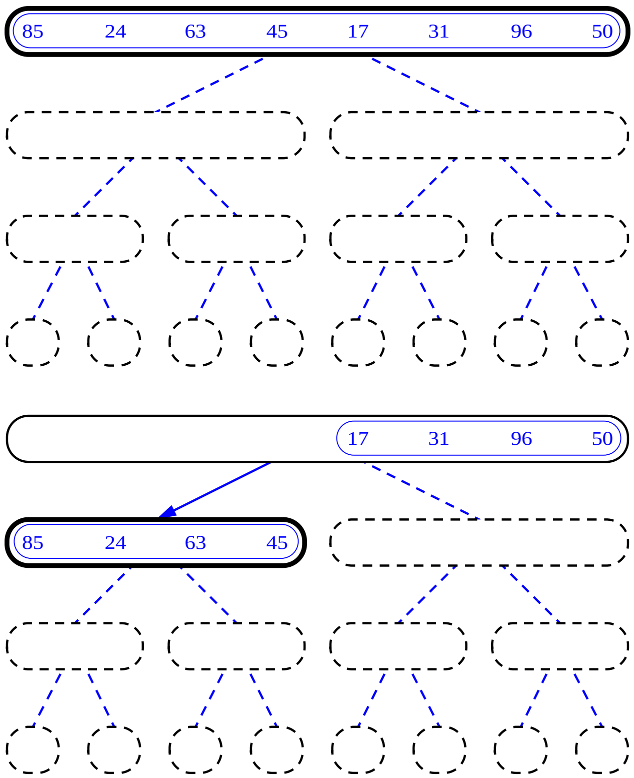
Divide-and-Conquer

- *Divide and Conquer* is more than just a military strategy, it is also a method of algorithm design that has created such efficient algorithms as Merge Sort.
- In terms or algorithms, this method has three distinct steps:
  - **Divide**: If the input size is too large to deal with in a straightforward manner, divide the data into two or more disjoint subsets.
  - **Recurse**: Use divide and conquer to solve the subproblems associated with the data subsets.
  - **Conquer**: Take the solutions to the subproblems and “merge” these solutions into a solution for the original problem.

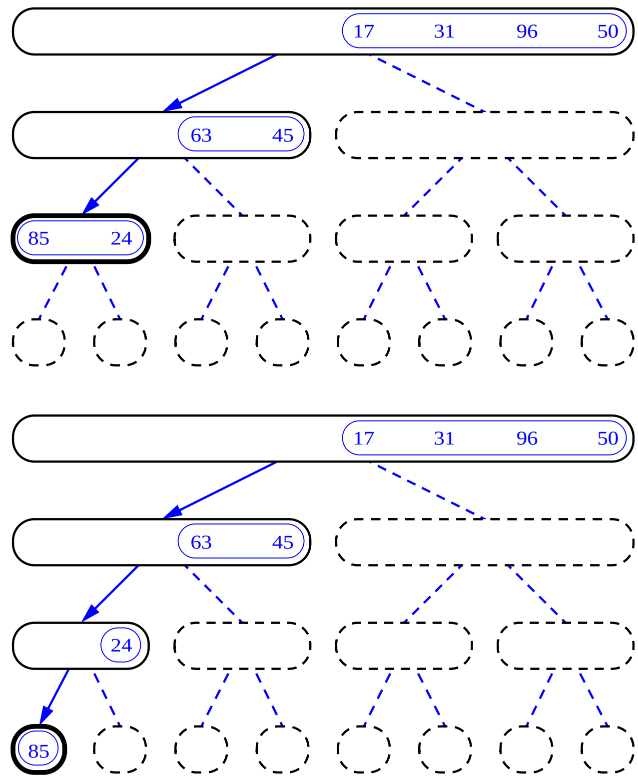
Merge-Sort

- Algorithm:
  - **Divide**: If S has at leas two elements (nothing needs to be done if S has zero or one elements), remove all the elements from S and put them into two sequences, S_1 and S_2, each containing about half of the elements of S. (i.e. S_1 contains the first $\lceil n/2 \rceil$ elements and S_2 contains the remaining $\lfloor n/2 \rfloor$ elements.
  - **Recurse**: Recursive sort sequences S_1 and S_2.
  - **Conquer**: Put back the elements into S by merging the sorted sequences S_1 and S_2 into a unique sorted sequence.
- Merge Sort Tree:
  - Take a binary tree T
  - Each node of T represents a recursive call of the merge sort algorithm.
  - We assocoate with each node v of T a the set of input passed to the invocation v represents.
  - The external nodes are associated with individual elements of S, upon which no recursion is called.

Merge-Sort



Merge-Sort(cont.)



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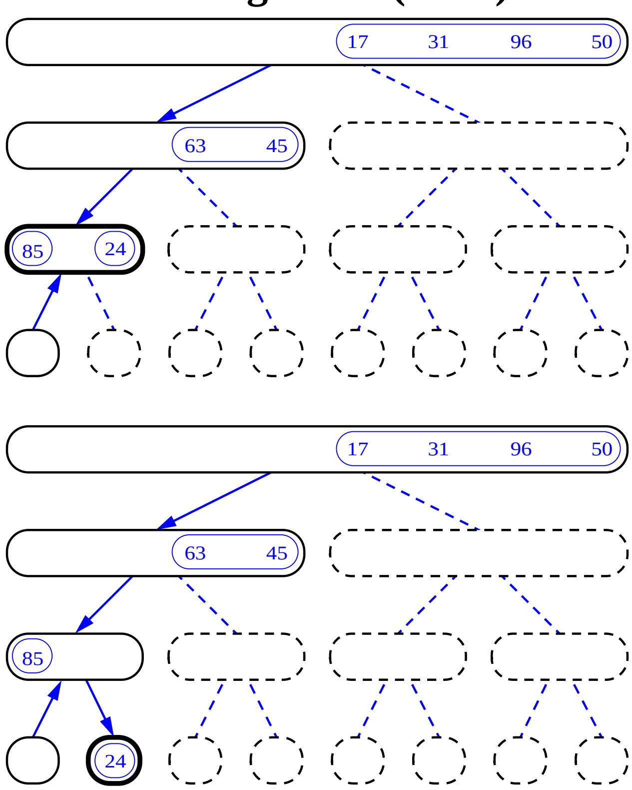

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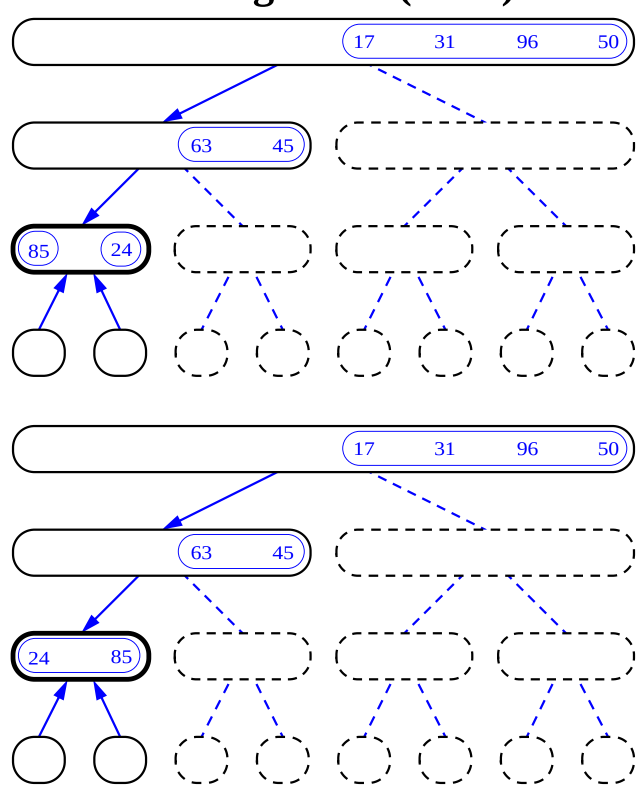

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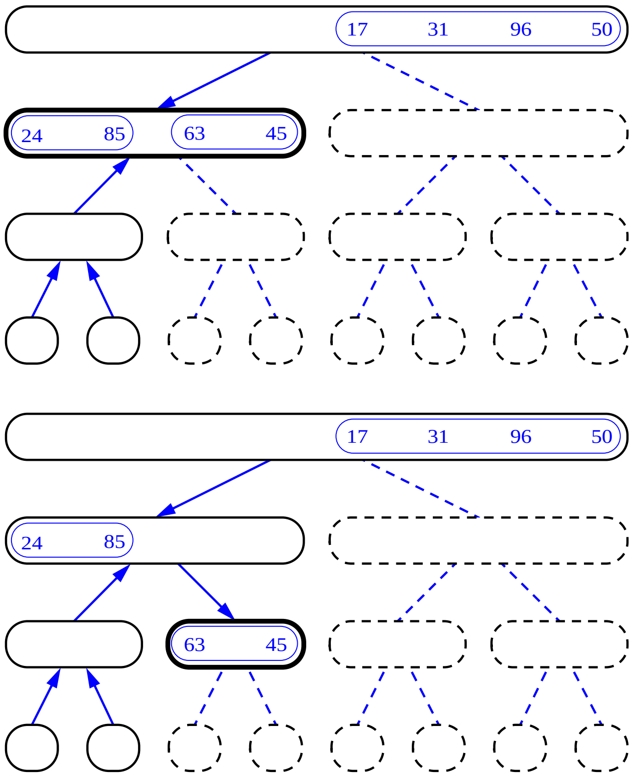

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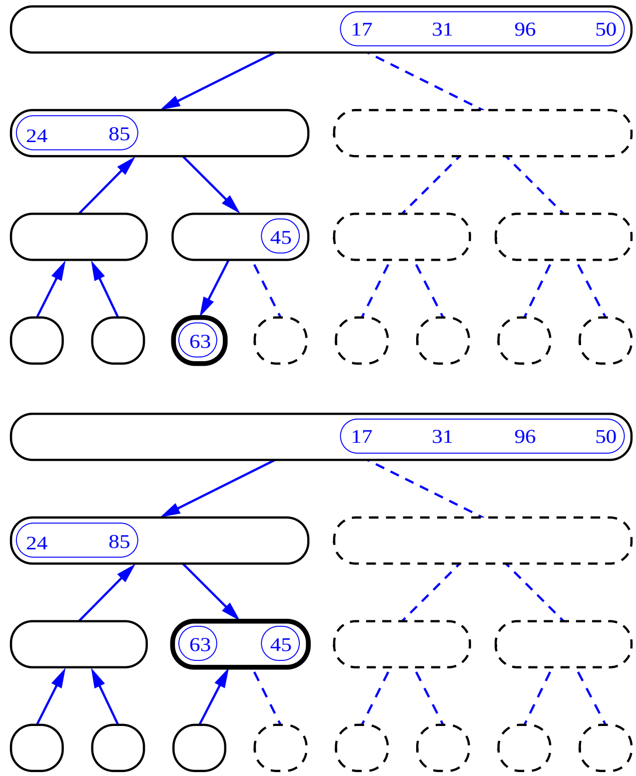

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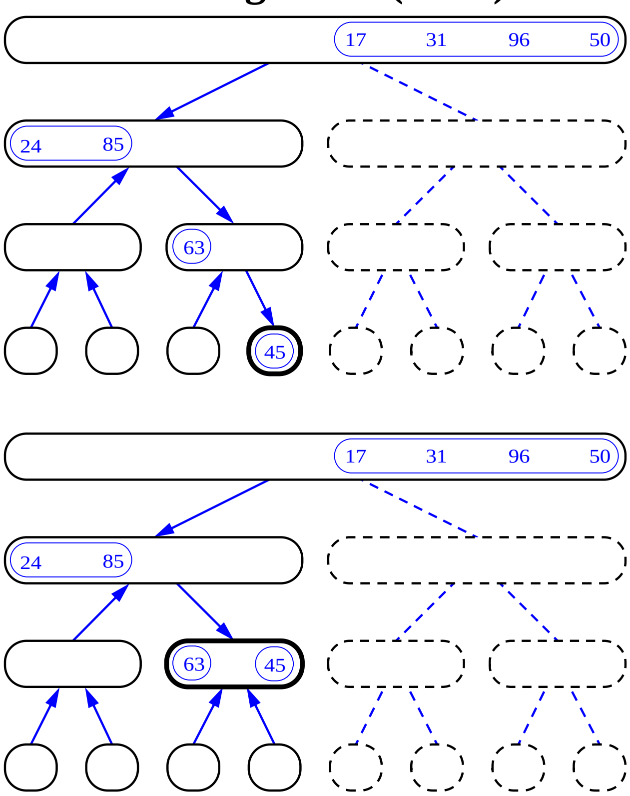
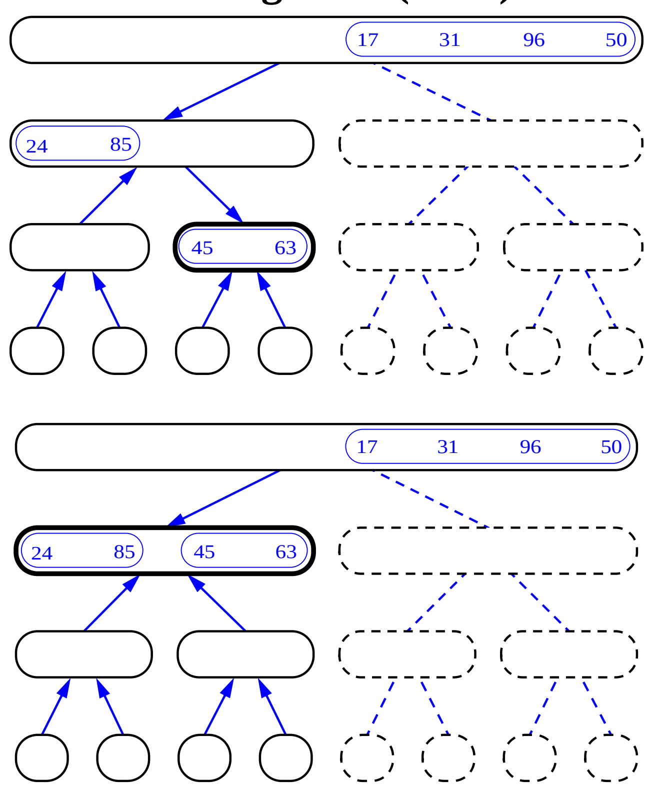
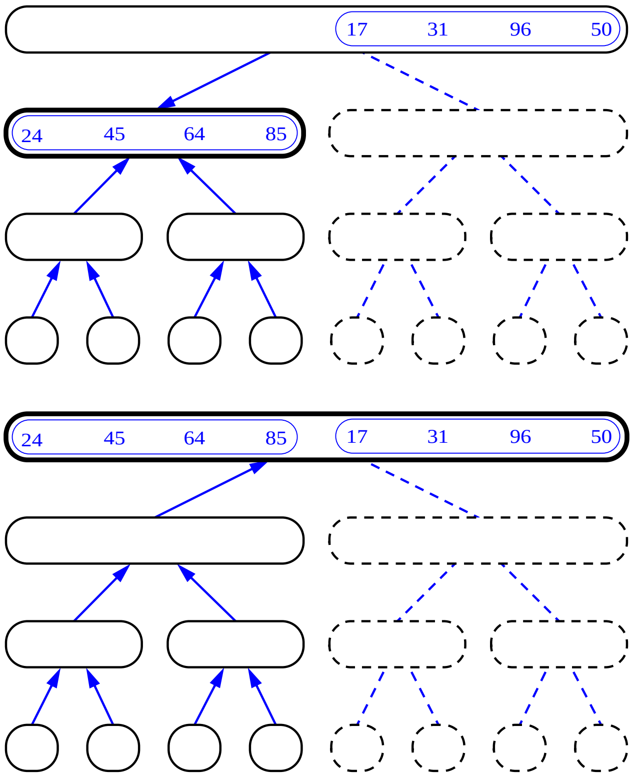


Merge-Sort(cont.)



Merge-Sort (cont.)



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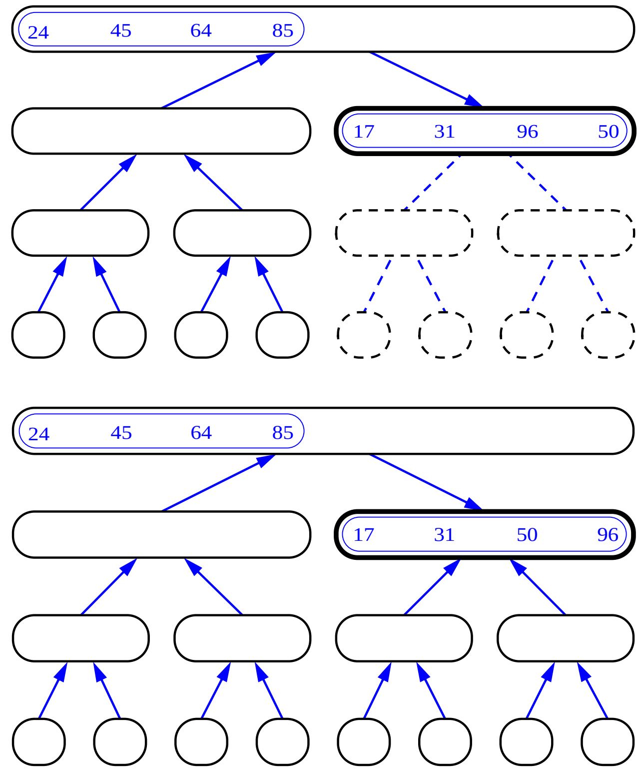

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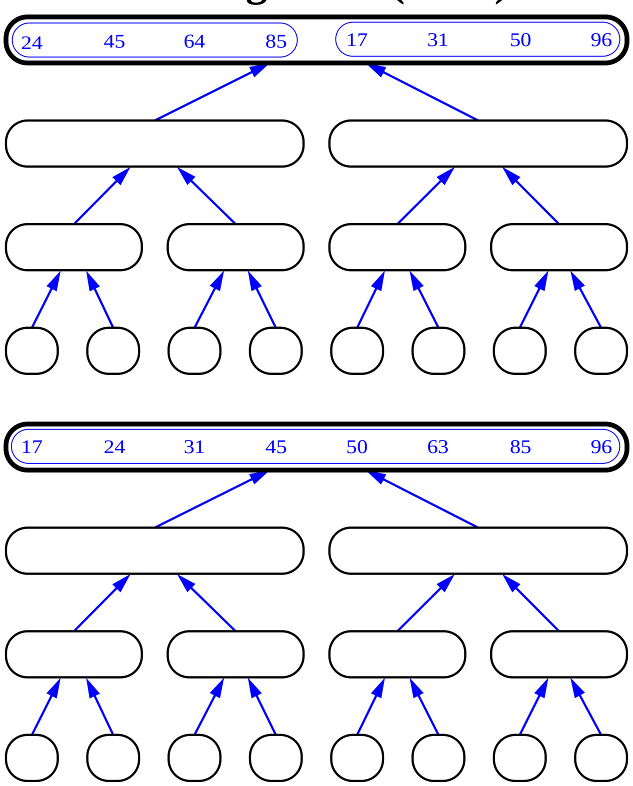


Merging Two Sequences

- Pseudo-code for merging two sorted sequences into a unique sorted sequence

Algorithm merge (S1, S2, S):

Input: Sequence *S1* and *S2* (on whose elements a total order relation is defined) sorted in nondecreasing order, and an empty sequence *S*.

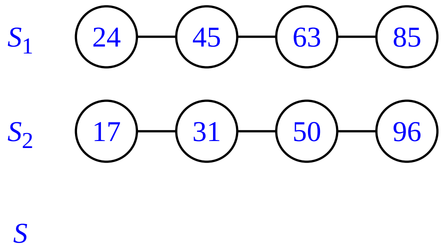
Ouput: Sequence *S* containing the union of the elements from *S1* and *S2* sorted in nondecreasing order; sequence *S1* and *S2* become empty at the end of the execution

```
while S1 is not empty and S2 is not empty do
  if S1.first().element() ≤ S2.first().element() then
    {move the first element of S1 at the end of S}
    S.insertLast(S1.remove(S1.first()))
  else
    { move the first element of S2 at the end of S}
    S.insertLast(S2.remove(S2.first()))
while S1 is not empty do
  S.insertLast(S1.remove(S1.first()))
  {move the remaining elements of S2 to S}
while S2 is not empty do
  S.insertLast(S2.remove(S2.first()))
```

Merging Two Sequences (cont.)

- Some pictures:

a)



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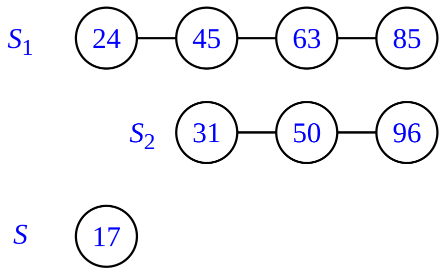
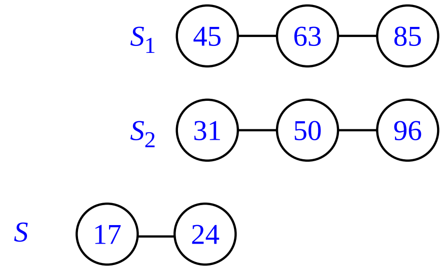
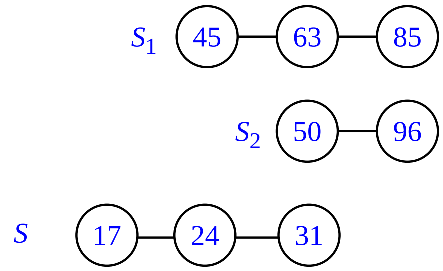


Merging Two Sequences (cont.)

c)

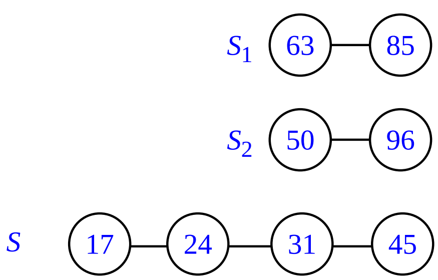


d)



Merging Two Sequences (cont.)

e)



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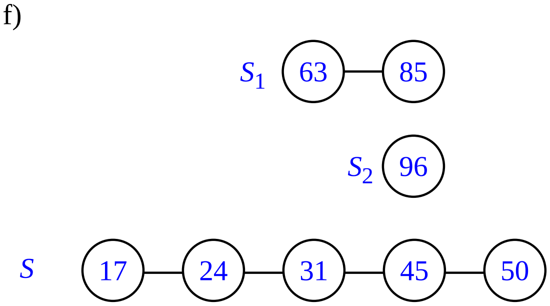
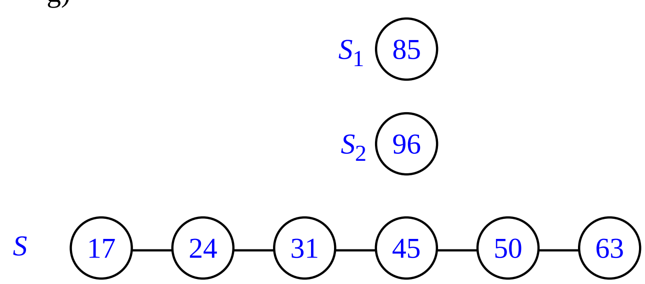


Merging Two Sequences (cont.)

g)



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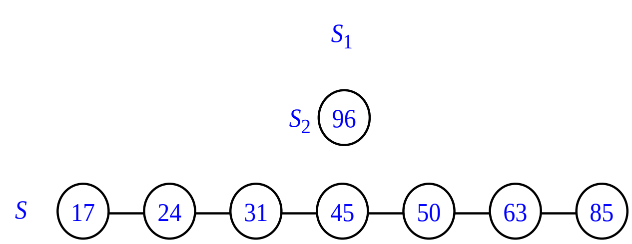
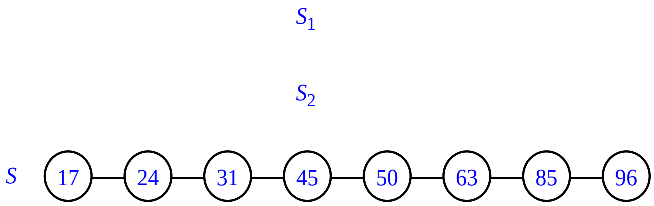


Merging Two Sequences (cont.)

i)



Java Implementation

- Interface SortObject

```
public interface SortObject {
  //sort sequence S in nondecreasing order
   using compartor c
  public void sort (Sequence S, Comparator c);
}
```


Java Implementation (cont.)

```
public class ListMergeSort implements SortObject {
  public void sort(Sequence S, Comparator c) {
    int n = S.size();
    // a sequence with 0 or 1 element is
      already sorted
    if (n < 2) return;
    // divide
    Sequence S1 = (Sequence)S.newContainer();
    for (int i=1; i <= (n+1)/2; i++) {
     S1.insertLast(S.remove(S.first()));
  }
  Sequence S2 = (Sequence)S.newContainer();
  for (int i=1; i <= n/2; i++) {
    S2.insertLast(S.remove(S.first()));
}
   // recur
  sort(S1,c);
  sort(S2,c);
  //conquer
  merge(S1,S2,c,S);
 }
```


Java Implementation (cont.)

```
public void merge(Sequence S1, Sequence S2,
    Comparator c, Sequence S) {
while(!S1.isEmpty() && !S2.isEmpty()) {
  if(c.isLessThanOrEqualTo(S1.first().element(),
    S2.first().element())) {
    S.insertLast(S1.remove(S1.first()));
  }
  else
    S.insertLast(S2.remove(S2.first()));
}
   if(S1.isEmpty()) {
     while(!S2.isEmpty()) {
  S.insertLast(S2.remove(S2.first()));
     }
}
   if(S2.isEmpty()) {
     while(!S1.isEmpty()) {
  S.insertLast(S1.remove(S1.first()));
     }
   }
 }
```

Running Time of Merge-Sort

- **Proposition 1**: The merge-sort tree associated with the execution of a merge-sort on a sequence of n elements has a height of $\lceil \log n \rceil$
- **Proposition 2**: A merge sort algorithm sorts a sequence of size n in $O(n\log n)$ time
- We assume only that the input sequence S and each of the sub-sequences created by each recursive call of the algorithm can access, insert to, and delete from the first and last nodes in $O(1)$ time.
- We call the time spent at node v of merge-sort tree T the running time of the recusive call associated with v, excluding the recursive calls sent to v's children.
- If we let i represent the depth of node v in the merge-sort tree, the time spent at node v is $O(n/2^i)$ since the size of the sequence associated with v is $n/2^i$.
- Observe that T has exactly 2^i nodes at depth i. The total time spent at depth i in the tree is then $O(2^i n/2^i)$, which is $O(n)$. We know the tree has height $\lceil \log n \rceil$
- Therefore, the time complexity is $O(n\log n)$

Set ADT

- A Set is a data structure modeled after the mathematical notation of a set. The fundamaental set operations are *union*, *intersection*, and *subtraction*.
- A brief aside on mathemeatical set notation:
  - $A \cup B = \{ x: x \in A \text{ or } x \in B \}$
  - $A \cap B = \{ x: x \in A \text{ and } x \in B \}$
  - $A - B = \{ x: x \in A \text{ and } x \notin B \}$
- The specific methods for a Set A include the following:
  - size():
    Return the number of elements in set A
    Input: None; **Output**: integer.
  - isEmpty():
    Return if the set A is empty or not.
    Input: None; **Output**: boolean.
  - insertElement(*e*):
    Insert the element *e* into the set A, unless *e* is already in A.
    Input: Object; **Output**: None.

Set ADT (contd.)

- elements():
  Return an enumeration of the elements in set A.
  Input: None; **Output**: Enumeration.
- isMember(*e*):
  Determine if *e* is in A.
  Input: Object; **Output**: Boolean.
- union(B):
  Return $A \cup B$.
  Input: Set; **Output**: Set.
- intersect(B):
  Return $A \cap B$.
  Input: Set; **Output**: Set.
- subtract(B):
  Return $A - B$.
  Input: Set; **Output**: Set.
- isEqual(B):
  Return true if and only if A = B.
  Input: Set; **Output**: boolean.

Generic Merging

```
Algorithm genericMerge(A, B):
  Input: Sorted sequences A and B
  Output: Sorted sequence C
  let A' be a copy of A { We won't destroy A and B}
  let B' be a copy of B
  while A' and B' are not empty do
    a←A'.first()
    b←B'.first()
    if a<b then
      firstIsLess(a, C)
      A'.removeFirst()
    else if a=b then
      bothAreEqual(a, b, C)
      A'.removeFirst()
      B'.removeFirst()
      else
        firstIsGreater(b, C)
        B'.removeFirst()
  while A' is not empty do
      a←A'.first()
        firstIsLess(a, C)
      A'.removeFirst()
  while B' is not empty do
      b←B'.first()
      firstIsGreater(b, C)
      B'.removeFirst()
```

Set Operations

- We can specialize the generic merge algorithm to perform set operations like union, intersection, and subtraction.
- The generic merge algorithm examines and compare the current elements of *A* and *B*.
- Based upon the outcome of the comparision, it determines if it should copy one or none of the elements *a* and *b* into *C*.
- This decision is based upon the particular operation we are performing, i.e. union, intersection or subtraction.
- For example, if our operation is union, we copy the smaller of *a* and *b* to *C* and if *a*=*b* then it copies either one (say *a*).
- We define our copy actions in firstIsLess, bothAreEqual, and firstIsGreater.
- Let's see how this is done ...

Set Operations (cont.)

- For union

```
public class UnionMerger extends Merger {
  protected void firstIsLess(Object a, Object b,
Sequence
       C) {
    C.insertLast(a);
  }
  protected void bothAreEqual(Object a, Object b,
       Sequence C) {
    C.insertLast(a);
  }
  protected void firstIsGreater(Object b, Sequence C) {
    C.insertLast(b);
  }
```

- For intersect

```
public class IntersectMerger extends Merger {
  protected void firstIsLess(Object a, Object b, Sequence
C) { } // null method
  protected void bothAreEqual(Object a, Object b,
Sequence C) {
    C.insertLast(a);
}
```


Set Operations (cont.)

```
  protected void firstIsGreater(Object b, Sequence C) { }
       // null method
```

- For subtraction

```
public class SubtractMerger extends Merger {
  protected void firstIsLess(Object a, Object b,
Sequence
       C) {
    C.insertLast(a);
  }
  protected void bothAreEqual(Object a, Object b,
       Sequence C) { } // null method
  protected void firstIsGreater(Object b, Sequence C) {
}
    // null method
```