Assignment ∅

① Vectors + Matrices

1) $y^T z = (1 \cdot 2) + (3 \cdot 3) = 2 + 9 = 11$

2) $Xy = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} (2 \cdot 1) + (4 \cdot 3) \\ (1 \cdot 1) + (3 \cdot 3) \end{bmatrix} = \begin{bmatrix} 14 \\ 10 \end{bmatrix}$

3) $\det(X) = (2 \cdot 3) - (4 \cdot 1) = 6 - 4 = 2$

$\det(X) \neq 0$, so X is invertible

$$X^{-1} = \frac{1}{\det(X)} \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1.5 & -2 \\ -0.5 & 1 \end{bmatrix}$$

4) $\det(X) \neq 0$ means that rows + columns are linearly independent.

2 independent rows $\rightarrow$ rank = 2

② Calculus

1) $y = x^3 + x - 5 \qquad \frac{dy}{dx} = 3x^2 + 1$

2) $f(x_1, x_2) = x_1 \sin(x_2) e^{-x_1} \qquad \nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}$

$\partial x_1 = \sin(x_2) \frac{\partial}{\partial x_1} (x_1 e^{-x_1})$

$= \sin(x_2)(e^{-x_1} - x_1 e^{-x_1})$

$\partial x_2 = (x_1 e^{-x_1}) \frac{\partial}{\partial x_2} (\sin(x_2))$

$= (x_1 e^{-x_1}) \cos(x_2)$

$$\nabla f(x) = \frac{\sin(x_2)(e^{-x_1} - x_1 e^{-x_1})}{x_1 e^{-x_1} \cos(x_2)}$$

③ PROBABILITY + STATISTICS

$S = \{1, 1, 0, 1, 0\}$

1) $\bar{x} = \frac{3}{5} = 0.6$

2) $s^2 = \frac{(1-0.6)^2 + (1-0.6)^2 + (0-0.6)^2 + (1-0.6)^2 + (0-0.6)^2}{4}$

$= \frac{0.16 + 0.16 + 0.36 + 0.16 + 0.36}{4}$

$= \frac{1.2}{4} = 0.3$

3) $P(S) = (0.5)^5 = 0.03125$

4) Since S has a sample size of 5, with three occurrences of $x=1$, the probability $P(x=1)$ that maximizes the probability of the observed outcome is

$P(x=1) = \frac{3}{5} = 0.6$

5) $P(z=T \text{ AND } y=b) = 0.1$ (from table)

$P(z=T \mid y=b) = \frac{P(z=T \cap y=b)}{P(y=b)}$

$= \frac{0.1}{0.25} = 0.4$

6)
- False
- True
- True
- False
- True

7) Multivariate Gaussian:

$$\frac{1}{\sqrt{(2\pi)^d |\Sigma|}} \exp\left\{\left(-\frac{1}{2} \cdot (x-\mu)^T \Sigma^{-1} (x-\mu)\right)\right\}$$

Bernoulli:

$$p^x (1-p)^{1-x}$$

Uniform: $\frac{1}{b-a}$, when $a \le x \le b$; 0 otherwise

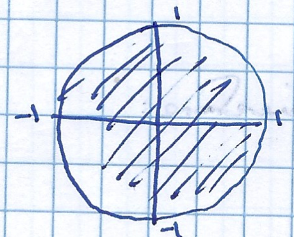
Binomial:

$$\binom{n}{x} p^x (1-p)^{n-x}$$

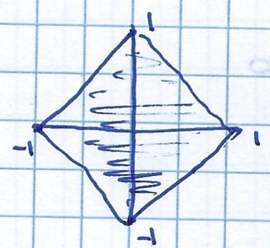
④ LINEAR ALGEBRA

VECTOR NORMS:

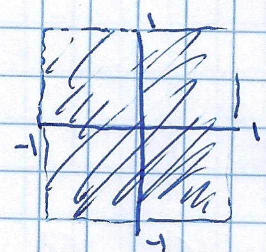
1)



2)



3)



GEOMETRY:

1) w is orthogonal to the line $w^T x + b = 0$ if its dot product with any vector on that line is $\varnothing$

Two points x_1 and x_2 that lie on that line will satisfy

① $w^T x_1 + b = 0$ and $w^T x_2 + b = 0$

and form a vector that lies on the line: $x_1 - x_2$

From ①, we determine $w^T x_1 = -b$ and $w^T x_2 = -b$

We compute the dot product: $w^T(x_1 - x_2) = w^T x_1 - w^T x_2$

Substituting, we get $w^T(x_1 - x_2) = -b - (-b) = -b + b = 0$

Since $w^T(x_1 - x_2) = 0$, w is orthogonal to the vector, and is therefore orthogonal to the line.

2) The distance from a point p to the line $w^T x + b = 0$ is given by

$$d = \frac{|w^T p + b|}{\|w\|}$$

The point p is the origin, $(\emptyset, \emptyset)$. Therefore:

$$d = \frac{|w^T(\emptyset) + b|}{\|w\|}$$

Since $w^T(\emptyset) = \emptyset$, this simplifies to

$$d = \frac{|b|}{\|w\|}$$