ŘETĚZOVÉ ZLOMKY

$$T(x) = \frac{1}{x - \lfloor x \rfloor} \qquad \boxed{x = \frac{p + \sqrt{d}}{q}}$$

QUADRATIC IRRATIONAL

$$\boxed{a = \lfloor x \rfloor}$$

FLOOR

$p \in \mathbb{Z}$

$q \in \mathbb{Z} \setminus \{0\}$

~~$d \in \mathbb{N} \setminus \{k;$~~

$d \in \mathbb{N} \setminus \{k;\ \exists m \in \mathbb{N}: k = m^2\}$

$\Leftrightarrow$ d is not perfect square

1) ODVOZENÍ VZORCŮ PRO p_{k+1} a q_{k+1}:

$$x_k = \frac{p_k + \sqrt{d}}{q_k}$$

$$x_k - \lfloor x_k \rfloor = x_k - a_k = \frac{p_k + \sqrt{d}}{q_k} - a_k = \frac{\sqrt{d} - \overbrace{(a_k q_k - p_k)}^{\text{call it } p'}}{q_k}$$

$$x_{k+1} = \frac{1}{x_k - \lfloor x_k \rfloor} = \frac{q_k}{\sqrt{d} - p'} = \frac{q_k(\sqrt{d} + p')}{d - p'^2} = \frac{p' + \sqrt{d}}{\frac{d - p'^2}{q_k}}$$

$$\Rightarrow p' = \boxed{p_{k+1} = a_k q_k - p_k} \in \mathbb{Z} \quad (\mathbb{Z} \cdot \mathbb{Z} - \mathbb{Z} \text{ produces } \mathbb{Z})$$

$$\boxed{q_{k+1} = \frac{d - p_{k+1}^2}{q_k}} \in \mathbb{Z} \quad \left(\text{NEEDS FORMAL PROOF}\right)$$

2) PROOF $q_{k+1} \in \mathbb{Z}$ BY INDUCTION:

$\underline{k = 0}$: $p_0 = 0 \quad q_0 = 1 \qquad q_1 = d - a_0 \in \mathbb{Z}$ ✓

INDUCTION (PROVE THIS IMPLICATION): $q_{k+1} = \frac{d - p_{k+1}^2}{q_k} \in \mathbb{Z} \Rightarrow q_{k+2} = \frac{d - p_{k+2}^2}{q_{k+1}} \in \mathbb{Z}$

$$p_{k+2} = a_{k+1} q_{k+1} - p_{k+1}$$

$$d - p_{k+2}^2 = d - a_{k+1}^2 q_{k+1}^2 + 2 a_{k+1} q_{k+1} p_{k+1} - p_{k+1}^2 = \underbrace{d - p_{k+1}^2}_{\text{DIVISIBLE BY } q_{k+1}} + q_{k+1}(2 a_{k+1} p_{k+1} - \ldots)$$

$\square$