Discussion on Thermal Equilibrium Models

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Abstract

Thermal equilibrium of the atmosphere is very complicated in reality. The Earth absorbs the radiation from the sun and emits long-wave radiation back. The uneven distribution of the temperature (energy) caused convection. We simulate the physical process of this actual atmosphere by constructing simple numerical models focusing on pure radiative equilibrium. We get the temperature of each layer by adjusting emissivity in each layer, albedo and solar constant, finally adding influence of convection to make the model closer to the reality.

Introduction

- **The concept of thermal equilibrium**

Considering earth as a system, earth's energy budget demonstrates the balance between the energy receives from the sun and the outgoing long-wave radiation from the earth, which is thermal equilibrium (**Figure 1**).

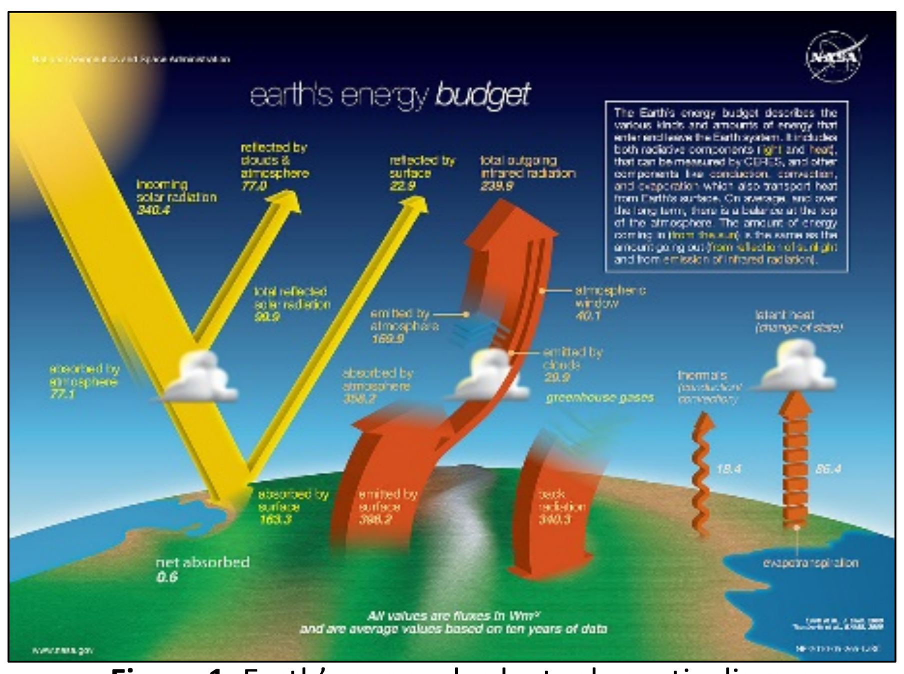


Figure 1. Earth's energy budget schematic diagram

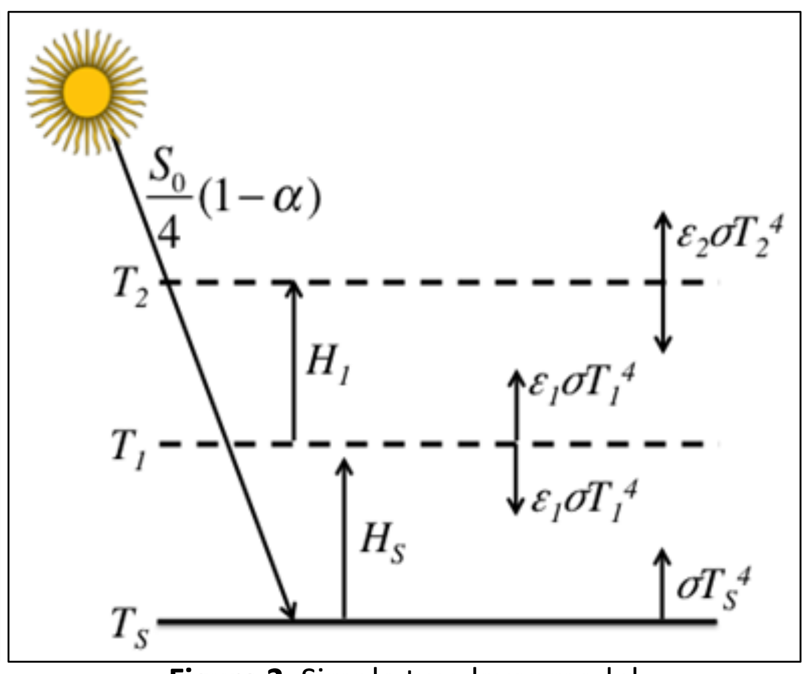


Figure 2. Simple two-layer model

Processes and Results

1. Constructing thermal equilibrium model

We construct a simple thermal equilibrium model, supposing that surface and atmosphere can only gain energy by absorbing radiation flux from the sun and release energy by emitting radiation.

$$Q_{absorb} + Q_{reflect} + Q_{transmit} = Q_{total}$$

In the model, we only assume that there will only be absorption and transmission happening when energy was project to each layer in atmosphere. Effect of the reflection will only be considered when the surface is not a blackbody.

When it comes to heat transfer, Kirchhoff's law of thermal radiation is applied to describe :

$$\varepsilon = \alpha = \frac{E}{E_0} \qquad \alpha : \text{albedo} ; \varepsilon : \text{emissivity}$$

Albedo is equivalent to emissivity corresponding to the temperature of the layer. Assuming that surface is a blackbody, which its $\varepsilon_0 = 1$, two-layer atmosphere model (**Figure 2**) can be shown as ternary simultaneous equations below:

$$\frac{S_0(1-a)}{4} = \varepsilon_2\sigma T_2^{\,4} + (1-\varepsilon_2)\varepsilon_1\sigma T_1^{\,4} + (1-\varepsilon_2)(1-\varepsilon_1)\sigma T_0^{\,4}$$
$$2\varepsilon_2\sigma T_2^{\,4} = (1-\varepsilon_1)\varepsilon_2\sigma T_0^{\,4} + \varepsilon_1\varepsilon_2\sigma T_1^{\,4}$$
$$2\varepsilon_1\sigma T_1^{\,4} = \varepsilon_1\sigma T_0^{\,4} + \varepsilon_1\varepsilon_2\sigma T_2^{\,4}$$

Simplified by constructing a matrix corresponding to ternary simultaneous equations above:

$$\begin{bmatrix} (1-\varepsilon_1)(1-\varepsilon_2) & \varepsilon_1(1-\varepsilon_2) & \varepsilon_2 \\ \varepsilon_2(1-\varepsilon_1) & \varepsilon_1\varepsilon_2 & -2\varepsilon_2 \\ \varepsilon_1 & -2\varepsilon_1 & \varepsilon_1\varepsilon_2 \end{bmatrix} \begin{bmatrix} T_0^{\,4} \\ T_1^{\,4} \\ T_2^{\,4} \end{bmatrix} = \begin{bmatrix} \frac{S_0(1-a)}{4\sigma} \\ 0 \\ 0 \end{bmatrix}$$

And so on, the n-layer model can be constructed as matrix, the main part was shown:

$$\begin{bmatrix} (1-\varepsilon_1)(1-\varepsilon_2)\dots(1-\varepsilon_n) & \varepsilon_1(1-\varepsilon_2)\dots(1-\varepsilon_n) & \cdots & \varepsilon_n \\ \varepsilon_n(1-\varepsilon_1)(1-\varepsilon_2)\dots(1-\varepsilon_{n-1}) & \varepsilon_n\varepsilon_1(1-\varepsilon_2)\dots(1-\varepsilon_{n-1}) & \cdots & -2\varepsilon_n \\ \vdots & & & \vdots \\ \varepsilon_1 & -2\varepsilon_1 & \cdots & \varepsilon_n\varepsilon_1(1-\varepsilon_2)\dots(1-\varepsilon_{n-1}) \end{bmatrix}$$

In the calculation, we can adjust the albedo of the surface corresponding to different type of landform and emissivity of layers corresponding to different composition of air in order to make the results closer to real atmosphere temperature.

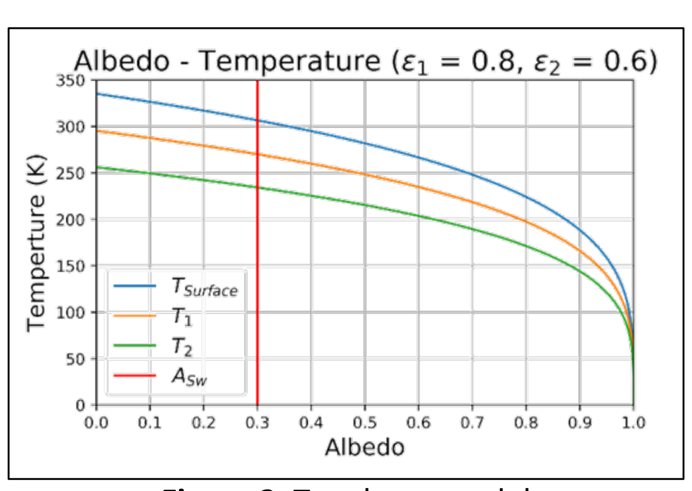


Figure 3. Two-layer model

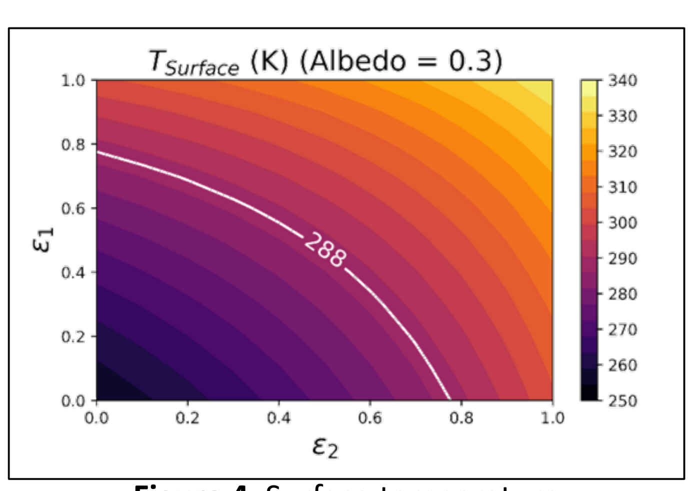


Figure 4. Surface temperature

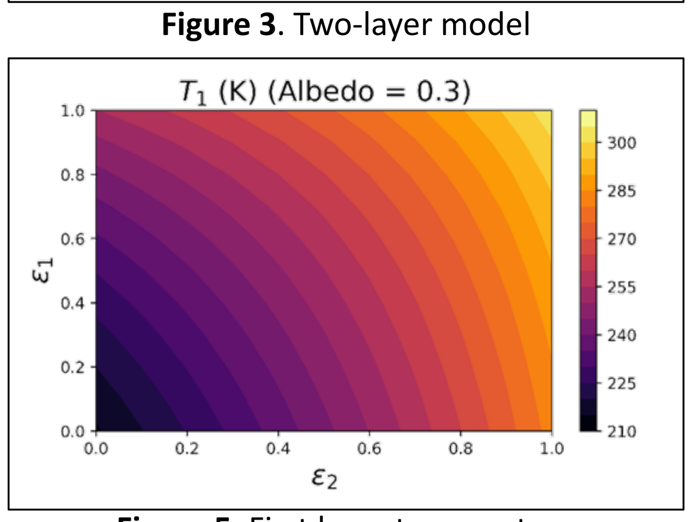


Figure 5. First layer temperature

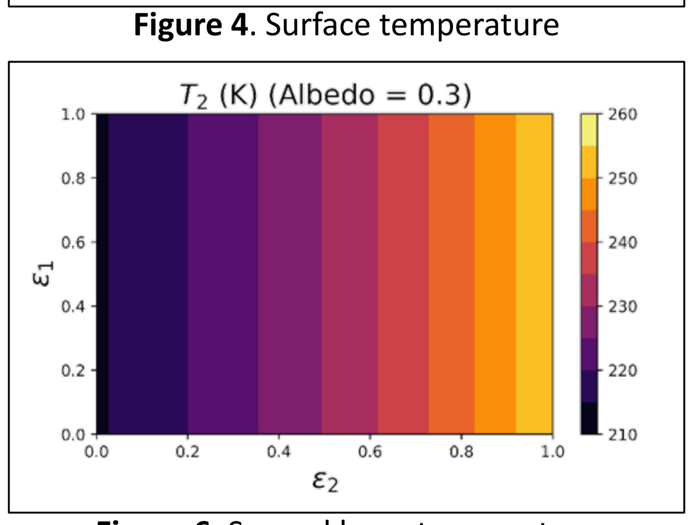


Figure 6. Second layer temperature

2. Investigating the intensity of the convection

Due to the uneven distribution of the energy, temperature declines from surface to higher layer, and this situation causes the whole atmosphere be in an unstable status.

Convection, is one way to balance out the uneven distribution by transferring extra energy to higher layer. In the model, only convection is considered since the differences of temperature in each layer are significant.

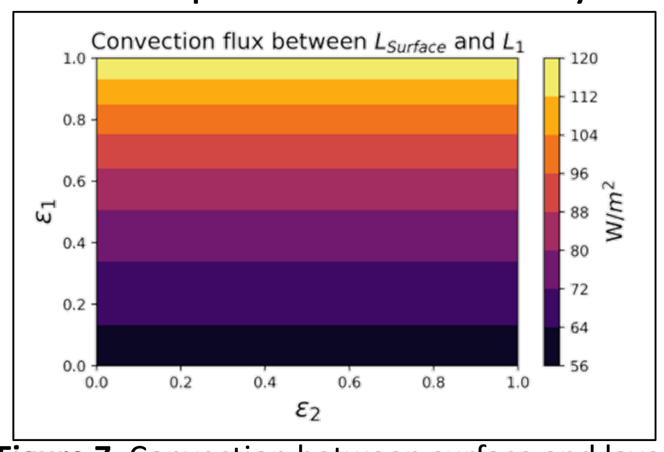


Figure 7. Convection between surface and layer1

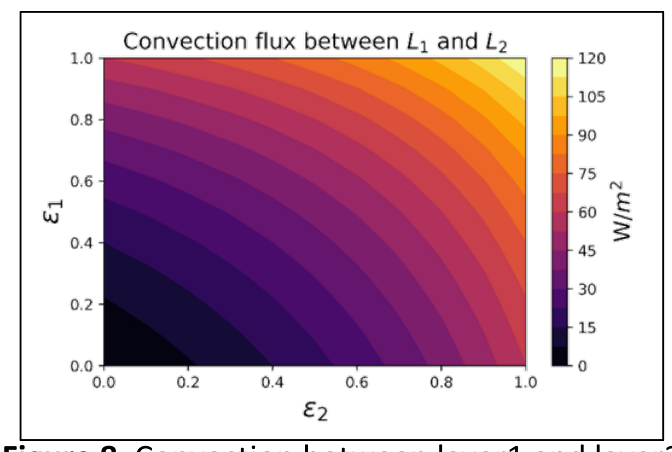


Figure 8. Convection between layer1 and layer2

3. Situation in graybody

The results above have considered the surface a blackbody. However, there are many different landforms result in different ε_0. Thus, we have to consider the effect of reflection ($Q_{reflect}$) since the radiation won't transmit through the ground.

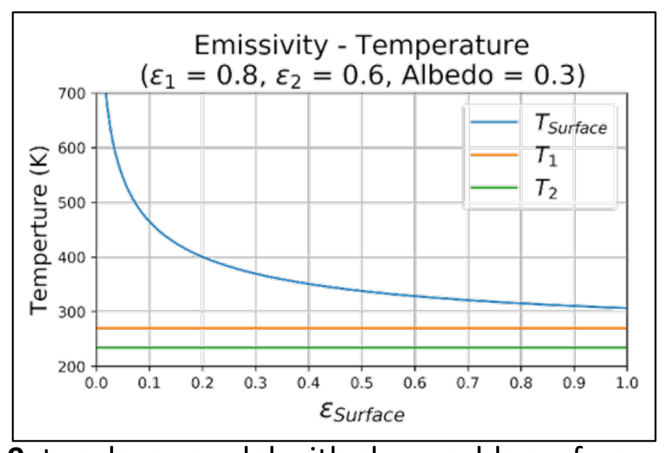


Figure 9. two-layer model with changeable surface emissivity

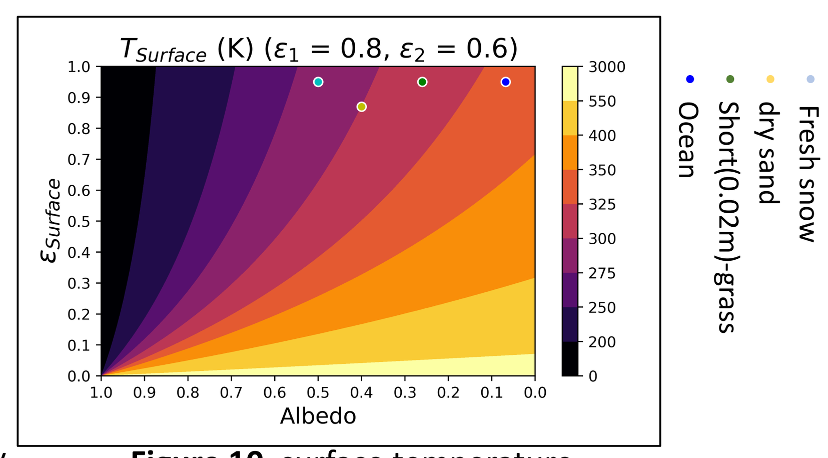


Figure 10. surface temperature

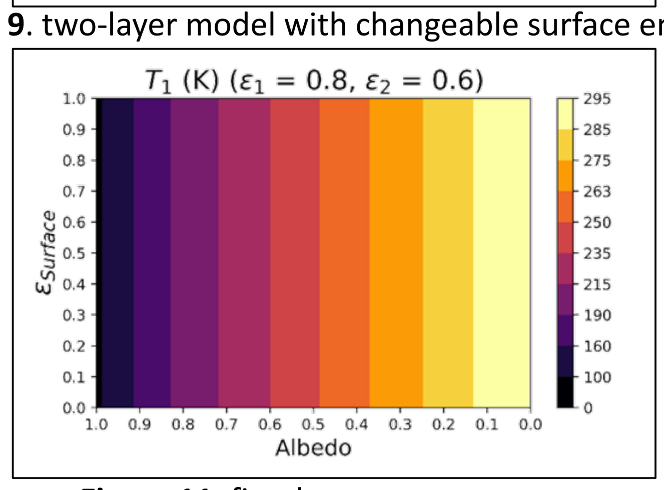


Figure 11. first-layer temperature

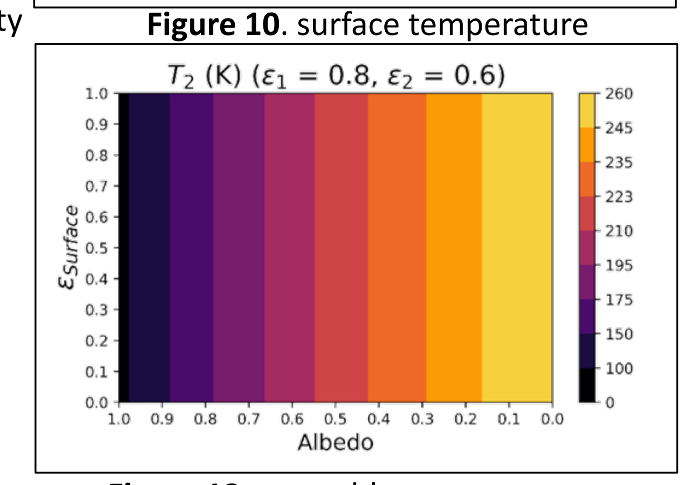


Figure 12. second-layer temperature

While plotting the convection flux between different layers, we assume that ε_1 and ε_2 are constants, 0.8 and 0.6, respectively. We can get the result of figure 13 by taking figure 9 as a contrast.

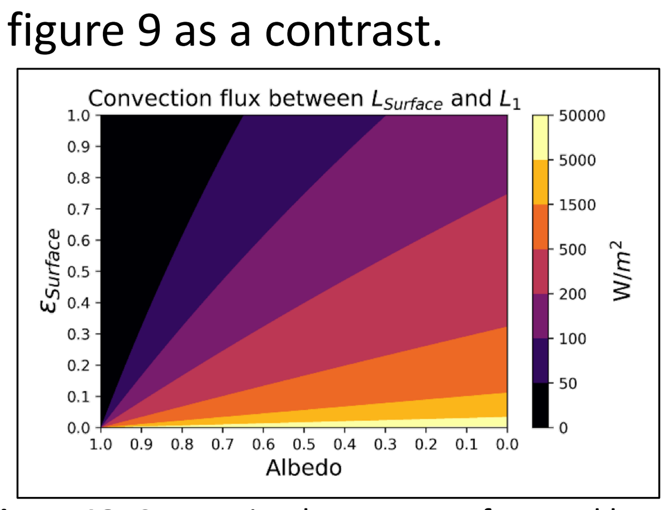


Figure 13. Convection between surface and layer1

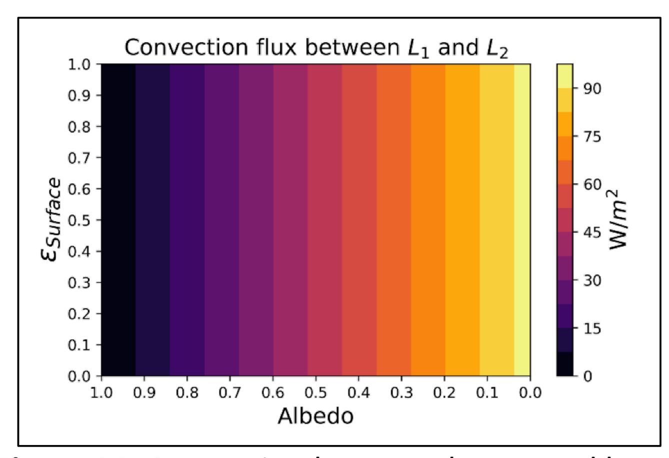
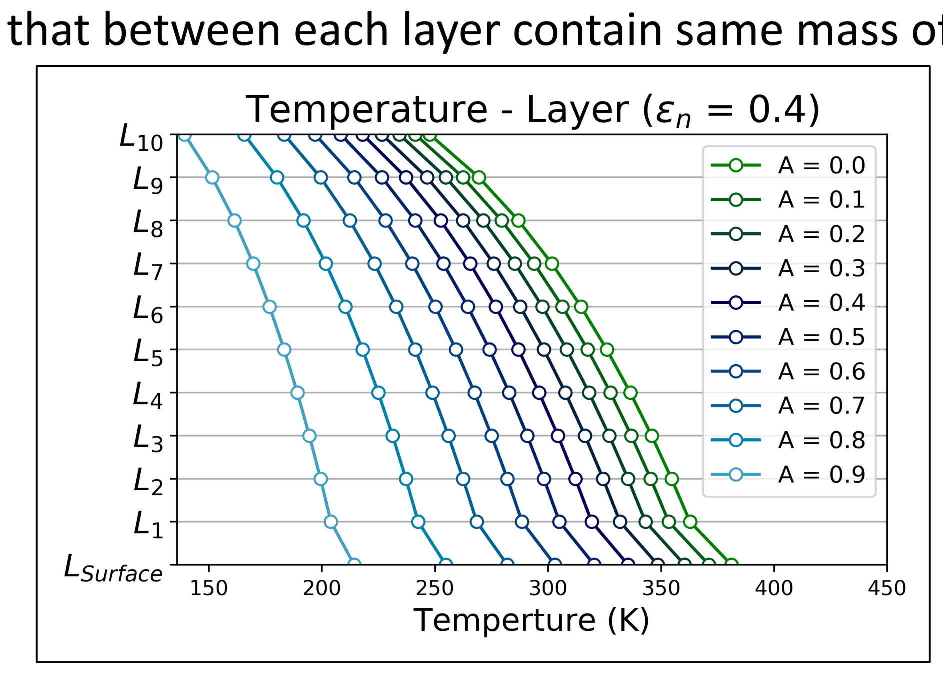


Figure 14. Convection between layer1 and layer2

Conclusion

- **n-layer model**

As we know, the whole atmosphere is a continuous fluid and most of the air parcels are accumulated near the surface. Thus, we construct a multiple layer model to see that if it will become closer to the real atmosphere. By using the matrix for n-layer model and assuming that the surface is a blackbody and assuming that between each layer contain same mass of air, now let n = 10, then:



Future Works

1. Increasing the value of n in n-layer model
2. Convert isobaric system into contour interval
3. Consider the existence and coverage ratio of clouds (other object which able to absorb and radiate energy)

Reference

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