

Qustion on proof of de Morgans laws

David

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1 Inclusion both ways

Prove:

$$(A \cup B)^c = A^c \cap B^c \quad (1)$$

Inclusion going from $(A \cup B)^c$ to $A^c \cap B^c$:

$$(A \cup B)^c = \{x \in \mathbb{R} : x \notin A \cup B\} \therefore x \notin A \text{ and } x \notin B \quad (2)$$

$$\implies x \in A^c \text{ and } x \in B^c \therefore x \in A^c \cap B^c \quad (3)$$

Including going from $A^c \cap B^c$ to $(A \cup B)^c$:

$$A^c \cap B^c = \{x \in \mathbb{R} : x \in A^c \text{ and } x \in B^c\} \therefore x \notin A \text{ and } x \notin B \quad (4)$$

$$\implies x \notin A \cup B \therefore x \in (A \cup B)^c \quad (5)$$