In classical mechanics, things were either particles or waves, and could never be both.

A particle's motion may be described with

- $m$, its mass,
- $\vec{r}$, its displacement
- $\vec{v}$, its velocity
- $\vec{a}$, its acceleration

$\rightarrow$ Momentum, $\vec{p} = m\vec{v}$

In quantum mechanics, the momentum, $\vec{p}$, is of the greatest interest.

its energy is described with

$$E = \tfrac{1}{2}mv^2 + U_{PE}, \quad \text{where } U_{PE} = \frac{kx^2}{2} \text{ for a spring, } -\frac{q^2}{r}\left(\frac{1}{4\pi\epsilon_0}\right) \text{ for charges, ...}$$

Relating energy and momentum, we find that

$$\vec{p} = m\vec{v}, \quad \therefore \vec{p}^2 = m^2\vec{v}^2$$

and

$$\tfrac{1}{2}m\vec{v}^2 = \frac{\vec{p}^2}{2m}.$$

All of these physical properties are deterministic — that is, if we know the mass, the force, then we must also know the position, velocity, and speed, through

$$\vec{F} = m\vec{a} \quad \text{or} \quad \vec{F} = -k\vec{x}.$$

A wave's motion is described as periodic perturbations which propagate through time and space. Typically, described using a harmonic function, like $\sin(x)$ or $\cos(x)$.

The velocity of a wave is expressed as $\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

And the wave vector is defined as:

$$A(x) = \underset{\text{Amplitude}}{A_0} \sin\Big(\underbrace{\frac{2\pi x}{\lambda} + \phi_0}_{\text{Phase}}\Big) \quad \text{where } \frac{2\pi}{\lambda} = k$$

Related to position.

For a wave vector in time, the definition is

$$A(t) = A_0 \sin\left(2\pi\frac{t}{T} + \phi_0\right) = A_0 \sin(\omega t + \phi)$$

where the angular velocity, $\omega$, is represented as $\omega = 2\pi\nu = \frac{2\pi}{T}$

$$\nu = \frac{1}{T}$$

$T$: Period

The two forms of a wave vector may be combined to form the following:

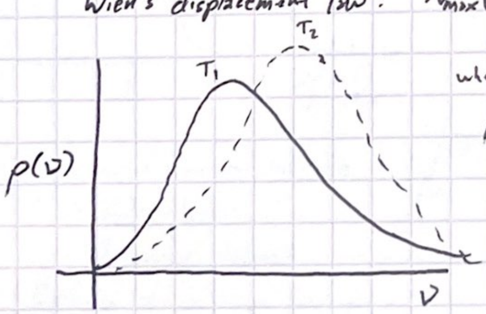
$$A(t,x) = A_0(\sin(kx - \omega t + \phi_0))$$

The distinctive difference between waves and particles are that

1) Waves are not localized, unlike particles
2) Waves colliding results in either constructive or destructive interference
3) When particles collide, the momentum of each particle is affected.

<u>Blackbody Radiation</u>: When an object absorbs light, but does not reflect it.

Wien's displacement law: $\lambda_{max}(T) = \frac{1.44}{5}\ K\cdot cm^{-1}$




where $T_2 > T_1$

$\rho(\nu)$ is the density of the electromagnetic wave.

$$\rho(\nu)_{d\nu} = \bar{E}\nu^2 d\nu$$

$\bar{E}$: Average energy $= k_B T$

"Ultraviolet catastrophe"

- Classical mechanics was unable to explain why $\lambda_{max}$ would not keep increasing and increasing, as would be expected of experiments due to the relationship of $\rho(\nu)_{d\nu}$ as shown to the left.

Max Planck's solution:

- Proposed that the energy of harmonic oscillations is quantized in proportion to $\nu$, discretely.

$$E = \underbrace{h\nu}_{\text{"One quanta"}} \cdot n$$

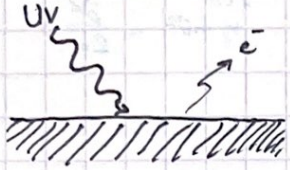
Where $h = 6.626\times10^{-34}\ J/s$, planck's constant

$n$ = an integer, 0, 1, 2, 3...

- Broke classical physics: $n \neq 1.5$.

Albert Einstein: Photoelectric effect

- Described how UV light ejects electrons




To describe this process, physicists used the relation:

$$E_{light} = \underset{\text{(work)}}{\phi_{e^-}} + \underset{\text{(kinetic energy of electron)}}{KE_{e^-}}$$

In order for an electron to be ejected, $E_{light} > \phi_{e^-}$:

$$KE_{e^-} = E_{light} - \phi_{e^-}$$

Einstein also used Planck's ideas to describe light as traveling in quanta as well:

$$KE_{e^-} = \rho V - \phi$$ , and found that his $\rho = h$, planck's constant

**Bohr** assumed angular momentum, $l$, was also quantized for his model of the atom.

$$l = m \cdot v \cdot r = \alpha n.$$ Bohr found that $\alpha = \hbar$, or $\frac{h}{2\pi}$.

Louis de Broglie then proposed wave particle duality — if waves can act as particles, then the opposite, that particles act like waves, must also be true:

$$\lambda = \frac{h}{p}$$ , where $\lambda$ = wavelength
$p$ = particle momentum
$h$ = planck's constant

In this course, how particles behave as waves will be the focus.

$\boxed{\text{There are 5 essential postulates for quantum mechanics}}$

1) The wave function
2) Q.M. operators and measurements
3) Outcome of a single measurement
4) Outcome of many measurements
5) The Schrödinger Equation