

Advanced Statistic Models A 5786 – List of Topics

1. Review of eigenvalues, eigenvectors, and the spectral decomposition of a symmetric square matrix.
2. Cholesky decomposition of a positive definite matrix.
3. Review of operations with random vectors: expectation and covariance matrix.
4. Review of multivariate normal (MVN) distribution and its properties.
5. Definition of chi-square, t, and F distributions.
6. $W \sim N_d(0, V) \Rightarrow W^T V^{-1} W \sim \chi_d^2$.
7. If $W \sim N_d(0, I)$ and P is a projection matrix of rank k, $\| PW \|_F^2 \sim \chi_k^2$.
8. Maximum likelihood estimate (MLE) of μ for a single iid sample of $N_d(\mu, V)$ data, V known.
(includes derivatives of inner products and quadratic forms)
9. Likelihood ratio test (LRT) for $H_0: \mu = \mu^0$.
10. Confidence region for μ .
11. Noncentral chi-square distribution and its properties.
12. Power of the LRT.
13. Derivatives of expressions involving matrices and determinants.
14. MLE of μ and V for a single iid sample of $N_d(\mu, V)$ data, μ and V both unknown.
- 14a. MLE of μ and V for a single iid sample of $N_d(\mu, V)$ data for parametric submodels.
15. Likelihood ratio statistic for testing $H_0: \mu = \mu^0$ when μ and V are both unknown.
16. Wishart distribution and its properties.
17. Joint distribution of sample mean vector and sample covariance matrix for iid $N_d(\mu, V)$ data.
18. Hotelling's T^2 statistic and its distribution (including noncentral F distribution).
19. Inference for two independent samples of MVN data with same covariance matrix, test of $H_0: \mu_1 = \mu_2$, confidence region for $\Delta = \mu_1 - \mu_2$.
- 19a. MLE's for two independent samples of MVN data for parametric submodels.
20. Inference on means of two independent samples with different covariance matrices.
21. Test of equality of 2 (or more) covariance matrices using Wilks's theorem (including refinements of Bartlett and Box).
22. Generalized least squares (GLS): Maximum likelihood estimation, Newton-Raphson and Fisher scoring algorithms for computing estimates.
23. Balanced random line model: Maximum likelihood estimation and inference for fixed effect parameters.
24. Linear mixed models (LMM): model formulation.
25. LMM's: maximum likelihood estimation.
26. LMM's: inference for fixed effect and random effect parameters based on asymptotic normal theory (including the Satterthwaite method of computing degrees of freedom).
27. Running LMM's in R.
28. Estimation of random effects in LMM's and related confidence intervals.
29. Logistic regression with random intercept.
30. Logistic regression with multiple random effects.
31. Principal components analysis (PCA): derivation using minimum MSE approach, percent variance explained
32. Multiple comparisons – introduction
33. Multiple comparisons – familywise error rate (FWER), weak and strong control.
34. Bonferroni method, Fisher LSD method, Holm method, adjusted p-values.
35. Multiple comparisons procedure based on the multivariate normal distribution.
36. False Discovery Rate (FDR) – definition and elementary properties.
37. Benjamini-Hochberg (BH) procedure – FDR control with independent hypotheses (including proof).
38. FDR bound for BH procedure under general dependence (including proof).
39. FDR control of BH procedure under Positive Regression Dependence on a Subset (PRDS) (not including proof).
40. Post-selection inference – introduction, conditional error rates.
41. Simultaneous confidence intervals with FWER control: Bonferroni and multivariate normal methods
42. False Coverage Rate (FCR) – definition and elementary properties.
43. Benjamini-Yekutieli (BY) procedure for FCR control (with proof).
44. Empirical process view of BH procedure, Storey's procedure.