

## Non-Abelian Debye mass at next-to-leading order

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It is shown that, after a resummation of leading high-temperature contributions, a complete and gauge-independent result for the non-Abelian Debye screening mass at next-to-leading order can be extracted from the static gluon propagator. In contrast with previous, incomplete results, the correction to the Debye mass is found to be logarithmically sensitive to the nonperturbative magnetic mass and positive, in accordance with recent high-statistics results from lattice calculations.

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Over the last few years it has become clear that a complete calculation of perturbative corrections to the dispersion laws of quasiparticles in high-temperature QCD requires a resummation of the leading-order terms with characteristic scale  $gT$ , where  $g$  is the coupling constant and  $T$  the temperature. An improved perturbation theory has been developed in particular by Braaten and Pisarski [1] and has been applied in the first place to derive damping effects of collective excitations [2], where it turned out to be indispensable in order to obtain accurate and gauge-independent results for the leading-order term  $\sim g^2T$ . Most recently it has also been employed successfully in a calculation of the next-to-leading-order term of the QCD plasma frequency  $\delta\omega_{pl}^2 \sim g\omega_{pl}^2$  [3].

All these applications are still within the limits of a perturbative barrier set by the presumably incalculable screening of static magnetic fields [4], which causes a breakdown of perturbation theory at a certain order. In the case of the gluon self-energy from which corrections to dispersion laws are extracted this incalculability is expected at the order  $g^4T^2$ , whereas the above-mentioned results are derived from this quantity evaluated up to order  $g^3T^2$ . However, even at this order occasionally a sensitivity to the behavior of static transverse gluons at momentum scale  $\ll gT$  has been found, giving rise to a logarithmic enhancement  $\sim \ln(1/g)$ .

In addition to the dispersion law of propagating collective modes, a quantity of singular importance which can be derived from the gauge boson self-energy is the static (chromo)electric screening (Debye) mass [5]. At leading order  $\Pi_{00}(k_0 = 0, \mathbf{k}) = m_0^2 \sim g^2T^2$ , which gives rise to a pole in the static gluon propagator at  $\mathbf{k}^2 = -m_0^2$ , and, consequently, to exponential screening in the potential

$$\begin{aligned}\Phi(r) &= Q \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\mathbf{k}\mathbf{r}}}{k^2 + \Pi_{00}(k_0 = 0, k)} \\ &= \frac{Q}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{\sin(kr)}{r} \frac{k dk}{k^2 + \Pi_{00}(0, k)} \\ &= \frac{Q}{4\pi r} e^{-m_0 r}\end{aligned}\quad (1)$$

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with

$$m_0^2 = \frac{g^2(N + N_f/2)T^2}{3}, \quad (2)$$

for gauge group  $SU(N)$  and  $N_f$  fermions. In linear response theory, the gradient of  $\Phi$  gives the longitudinal electric field generated by a static charge  $Q$ .

Again, one would expect perturbative calculability of the screening mass at the next-to-leading order  $\delta \equiv \delta m^2/m_0^2 \sim g$ . Early attempts (using temporal gauge) [6,7] gave  $\delta = -cg$ , with  $c$  a positive constant depending on the particular partial resummation employed, whereas in general axial gauge even a different sign was obtained [8]. In covariant gauges, Toimela [9] found that the time-time component of the gluon self-energy  $\Pi_{00}(k_0 = 0, \mathbf{k} \rightarrow 0)$  is gauge dependent at the order  $g^3T^2$ , which was confirmed by Nadkarni [10] on the basis of a dimensionally reduced effective theory. This was taken to mean [10] that the Debye mass could not be extracted from the electrostatic propagator.

However, all these investigations concentrated on the limit  $\mathbf{k} \rightarrow 0$  of the static time-time component  $\Pi_{00}$ , guided by the fact that this limit indeed yields the leading-order contribution to the Debye mass. But at leading order  $g^2T^2$ ,  $\Pi_{00}(k_0 = 0, \mathbf{k})$  happens to be independent of  $\mathbf{k}$ , whereas the pole of the corrected electrostatic propagator is located at finite  $|\mathbf{k}| \sim gT$ . Gauge dependence at  $\mathbf{k} \rightarrow 0$ , away from the physical pole of the propagator, is thus only to be expected in a non-Abelian gauge theory. Conversely, formal arguments exist that the relevant poles of a self-consistently corrected propagator should be gauge independent also in the non-Abelian theory [11]. And it is just the pole in Eq. (1) that determines the exponential decay.

Incidentally, in QED it is equally important to define the electric screening mass in a self-consistent manner by the location of the pole rather than the  $\mathbf{k} \rightarrow 0$  limit of  $\Pi_{00}$  as almost invariably done in the literature. This in fact modifies the QED Debye mass squared at and above the order  $e^4T^2$  [12].

According to the resummation program of Ref. [1], a complete calculation of the next-to-leading-order term in the non-Abelian Debye mass should be possible and requires the resummation of all hard-thermal-loop contributions. In fact, because only a static quantity is to

be calculated, this task can be greatly simplified. In the problem of determining next-to-leading-order corrections to the effective potential in thermal field theories, such a simplified scheme has been employed recently by Arnold and Espinosa [13]. They have found that in gauge theories it is algebraically much simpler to resum only static modes. This does not touch the completeness of the resummation because, in the imaginary-time formalism, nonstatic modes always imply  $K^2 \equiv -[(2\pi nT)^2 + k^2] \gtrsim T^2 \gg (gT)^2$ , so that hard-thermal-loop corrections are truly perturbative. Separating the static modes does, however, give away the possibility of a straightforward

analytic continuation of any external frequencies. The full resummation scheme of Braaten and Pisarski thus is mandatory when external frequencies  $\sim gT$  are to be considered as in dynamical properties of quasiparticles.

The next-to-leading-order contributions to static quantities of relative order  $O(g)$  are determined by resummation of one-loop diagrams, and for them it suffices to take into account only the static modes (which also precludes fermionic contributions). A further major simplification arises in that with all gluon lines being static the hard-thermal-loop corrections to the vertices vanish. Only the static gluon propagator is thus needed, which reads

$$\Delta_{\mu\nu} \Big|_{p_0=0} = \left[ \frac{1}{\mathbf{p}^2 + m_0^2} \delta_\mu^0 \delta_\nu^0 + \frac{1}{\mathbf{p}^2} \left( \eta_{\mu\nu} - \delta_\mu^0 \delta_\nu^0 + \frac{P_\mu P_\nu}{\mathbf{p}^2} \right) + \alpha \frac{P_\mu P_\nu}{(\mathbf{p}^2)^2} \right]_{p_0=0}, \quad (3)$$

where  $P = (p_0, \mathbf{p})$  and  $\alpha$  is the gauge parameter of covariant gauges.

Evaluating  $\Pi_{00}$  at relative order  $O(g)$  then yields

$$\begin{aligned} \delta\Pi_{00}(k_0 = 0, \mathbf{k}) = & gmN \sqrt{\frac{6}{2N + N_f}} \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{1}{\mathbf{p}^2 + m^2} + \frac{1}{\mathbf{p}^2} \right. \\ & \left. + \frac{4m^2 - (\mathbf{k}^2 + m^2)[3 + 2\mathbf{p} \cdot \mathbf{k}/\mathbf{p}^2]}{\mathbf{p}^2(\mathbf{q}^2 + m^2)} + \alpha(\mathbf{k}^2 + m^2) \frac{\mathbf{p}^2 + 2\mathbf{p} \cdot \mathbf{k}}{\mathbf{p}^4(\mathbf{q}^2 + m^2)} \right\}, \end{aligned} \quad (4)$$

where  $\mathbf{q} = \mathbf{p} + \mathbf{k}$  and dimensional regularization is understood [13] (yielding no pole terms because of the odd integration dimension). Here and in the following the index 0 on  $m$  has been dropped, the difference in  $\delta\Pi$  being formally of higher order.

This result shows that  $\delta\Pi_{00}(k_0, \mathbf{k} \rightarrow 0)$  is indeed gauge dependent, as found in Ref. [10] in a one-loop calculation in the dimensionally reduced effective theory, to which the above reasoning has in fact boiled down as far as the  $O(g)$  correction is concerned. It also shows that this gauge dependence is to disappear on an algebraic level when going to the imaginary pole  $\mathbf{k}^2 = -m^2$ .

Closer inspection reveals, however, that with  $\mathbf{k}^2 \rightarrow -m^2$  there appear “mass-shell” singularities caused by the massless denominators in Eq. (4). A simple introduction of a magnetic mass in the denominator in front of the second term of the gluon propagator, Eq. (3), provides a physical cutoff, and because the singularities at hand are only logarithmic, the coefficient of the ensuing logarithm should be insensitive to the detailed structure of the infrared limit of the transverse propagators [14,15]. All the massless denominators figuring in the  $\alpha$ -independent part of Eq. (4) are indeed associated with the magnetic sector (as opposed to pure gauge modes). Assuming  $m_{\text{magn}} \sim g^2 T$ , the leading contribution to  $\delta\Pi_{00}$  is determined by the logarithmically divergent pieces and reads

$$\begin{aligned} \delta\Pi_{00}(k_0, \mathbf{k}) \Big|_{\mathbf{k}^2 = -m^2} \\ \equiv \delta m^2 = gm^2 \frac{N}{2\pi} \sqrt{\frac{6}{2N + N_f}} \ln \frac{1}{g} + O(g), \end{aligned} \quad (5a)$$

giving an unexpectedly large and positive correction to

the non-Abelian Debye mass. The positive sign seems to be a genuine non-Abelian effect, for the next-to-leading-order correction of the Debye mass, in, e.g., scalar QED is negative [16]. Moreover, no logarithmic enhancement occurs in the latter. [In spinor QED there are no  $O(g)$  corrections at all due to Pauli suppression.]

The sublogarithmic terms of course do depend on the detailed structure of the infrared limit of the transverse propagators, and so cannot be determined completely in the present resummed one-loop calculation. However, adopting the hypothesis that this infrared limit just amounts to a finite contribution  $-\frac{1}{2}\Pi_{ii}(k=0, \mathbf{k} \rightarrow 0) = m_{\text{magn}}^2 \sim g^4 T^2$  one may go on to estimate these sublogarithmic terms from Eq. (4).

Here one encounters a subtle difficulty with the  $\alpha$ -dependent term in Eq. (4), because by approaching the imaginary pole  $\mathbf{k}^2 \rightarrow -m^2$ , the explicit factor that apparently ensures gauge independence gets canceled by a linear singularity in the momentum integral. Exactly the same phenomenon was encountered in the recalculation of plasmon damping rates in general covariant gauges in Refs. [17], while being absent in homogeneous (“strict”) gauges. In Ref. [18] I have argued that this behavior just reflects a singular, gauge-dependent behavior of the *residue* of the propagator rather than an actual gauge dependence of the pole determining the dispersion laws. Indeed, introducing an (unphysical) cutoff again moves the gauge dependence seemingly afflicting the pole position into the residue, while the correction to the pole position becomes independent of this infrared regularization. In this way, the sublogarithmic terms are determined [19] and give rise to

$$\delta m^2 = gm^2 N \sqrt{\frac{6}{2N + N_f}} \frac{1}{2\pi} \left( \ln \frac{2m}{m_{\text{magn}}} - \frac{1}{2} \right) + O(g^2). \quad (5b)$$

Quite recently, lattice simulations have been performed which permit the extraction of corrections to the classical Debye screening length. A rather high precision has been reached in simulations of pure SU(2) gauge theory at temperatures up to nearly 8 times the critical temperature [20], with the finding of a *positive* excess in Debye mass squared of  $\delta = +0.30(9)$  at  $\beta = 3$ , corresponding to  $T \approx 7.8T_c$  and  $g_R^2 \approx 1.19$ . Unfortunately, the coupling is rather large so that a quantitative comparison with just the logarithmically enhanced result of Eq. (5a) is out of question. However, taking the result of Eq. (5b) seriously and inserting a value for the magnetic mass as suggested by lattice simulations [22],  $m_{\text{magn}} \approx 0.273g^2T$  for SU(2), Eq. (5b) yields  $\delta = +0.51$ , which comes remarkably close considering the largeness of  $g$ .

A significantly increased Debye mass has been found previously also in lattice simulations of pure SU(3) [21], however with larger statistical errors.

It should be noted that the previous, incomplete results for corrections to the non-Abelian Debye mass [6–9] have mostly yielded a negative value for  $\delta$ , albeit mutually disagreeing in numerical magnitude. The complete gauge-independent result presented here differs from the former in that all relevant hard-thermal-loop contributions are identified and resummed, and also in that the Debye mass is defined through the pole of the electrostatic propagator rather than the (gauge-dependent) zero-momentum limit of the gluon self-energy.

It is the pole that determines the exponential screening in Eq. (1), whereas the preexponential factor therein will generally be gauge dependent. At the next-to-leading order considered here, gauge-dependent contributions to the latter indeed arise from the residue of the pole of the

propagator as well as from a logarithmic branch cut contribution, which even gives rise to a (gauge-dependent) modification of the preexponential  $1/r$  behavior.

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## APPENDIX

In this appendix I give the complete contributions to  $\Pi_{\mu\nu}(k_0 = 0, \mathbf{k})$  at order  $g^3T^2$  for  $k \gtrsim m \sim gT$ , in general covariant gauge. The static limit of  $\Pi_{\mu\nu}$  is transverse, so there are exactly two independent structure functions:  $\Pi_{00}$  and  $\Pi_{ii}$ .

The correction to  $\Pi_{00}$  as given by Eq. (4) is found to be

$$\delta\Pi_{00}(0, k) = \frac{g^2NmT}{2\pi} \left[ \frac{m^2 - k^2}{mk} \arctan \frac{k}{m} + \frac{\alpha - 2}{2} \right] \quad (\text{A1})$$

for  $k^2 > 0$ . This can be immediately continued analytically to  $k^2 < 0$ , but for  $-k^2 \equiv \kappa^2 \rightarrow m^2$ , it has a logarithmic singularity. As argued in the text, the massless denominators in Eq. (4) require an infrared cutoff. Let this for simplicity be a mass term  $\lambda \ll m$  uniformly for transverse and gauge modes. In the case of the transverse gluons this can eventually be identified with the magnetic screening mass, whereas in the case of the massless gauge modes it is just a regulator which drops out from the end results, Eqs. (5). Then for  $\kappa \approx m$  Eq. (A1) gets replaced by

$$\delta\Pi_{00}(0, k) = \frac{g^2NmT}{4\pi} \left\{ \frac{m^2 + \kappa^2}{m\kappa} \ln \frac{(m + \kappa)^2 - \lambda^2}{m^2 - (\kappa - \lambda)^2} - 1 - (1 - \alpha)(m^2 - \kappa^2) \frac{1}{(m - \lambda)^2 - \kappa^2} \right\}. \quad (\text{A2})$$

In the limit  $\kappa \rightarrow m$  this gives the gauge independent result for the Debye mass discussed in the text. For general  $k$  it remains gauge-dependent, however, so that beyond leading order there is no gauge invariant meaning to be attributed to the dielectricity  $\epsilon = 1 - \Pi_{00}(0, k)/k^2$  as defined from the gluon propagator.

For  $k \gg m$ ,

$$\delta\Pi_{00}(0, k) \rightarrow -\frac{1}{4}g^2NkT \quad (\text{A3})$$

becomes independent of the gauge parameter, but in fact would be different in a background covariant gauge [23]. This term would give rise to a modification of the Debye screening in a gluon plasma [24], if such a behavior persisted for soft momentum  $k \lesssim gT$ . However, the complete result of Eq. (A1) reveals that it does not.

The next-to-leading-order correction to the other structure function,  $\Pi_{ii}(0, k)$ , can be derived in a similar manner, and is found to be

$$\delta\Pi_{ii}(0, k) = g^2NmT \left\{ \frac{(\alpha + 1)^2 + 10}{16} \frac{k}{m} + \frac{1}{4\pi} \left[ 2 - \frac{k^2 + 4m^2}{mk} \arctan \frac{k}{2m} \right] \right\}. \quad (\text{A4})$$

Again, the magnetic permeability defined by  $1/\mu = 1 - \frac{1}{2}\Pi_{ii}/k^2$  is a gauge-dependent quantity beyond leading order. The gauge-dependent terms vanish only at the location of the pole of the transverse gluon propagator, which is at  $\mathbf{k} = 0$ . There the correction term vanishes completely, which means that there is no magnetic mass squared of the order  $g^3T^2$ . The magnetic mass must therefore be  $\ll g^{3/2}T$ .

For small  $k \ll m$ , Eq. (A4) has a linear behavior with gauge-dependent but positive definite coefficient. The transverse propagator therefore has the form  $1/(k^2 - ck)$  with  $c \sim g^2T$ . This corresponds to a pole at space-like momentum with  $k \sim g^2T$  [25]. However, in this regime the perturbatively incalculable contributions  $\sim g^4T^2$  to  $\Pi_{\mu\nu}$  become relevant and are expected to remove this pathology by the generation of a magnetic mass term.

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