Now to find defining differential equation for V_s, we need

$$\int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\vec{r}}\, \frac{p^2+m_D^2}{p^2+Re(\Pi_R)}$$

First write

$$\frac{p^2+m_D^2}{p^2+Re(\Pi_R)} = 1 + \frac{m_D^2-Re(\Pi_R)}{p^2+Re(\Pi_R)}$$

then

$$\int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\vec{r}}\left(1+\frac{m_D^2-Re(\Pi_R)}{p^2+Re(\Pi_R)}\right) = \delta(\vec{r}) + \int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\vec{r}}\, \frac{m_D^2-Re(\Pi_R)}{p^2+Re(\Pi_R)}$$

Consider second term:

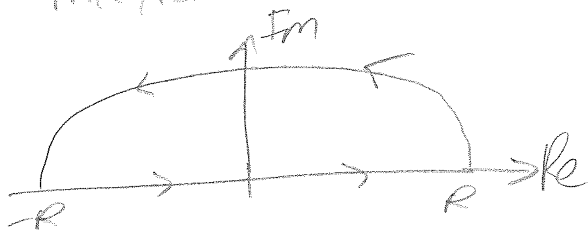
$$\frac{2\pi}{(2\pi)^3}\int_0^{\pi} d\theta \sin\theta \int_0^{\infty} dp\, p^2\, \frac{m_D^2-Re(\Pi_R)}{p^2+Re(\Pi_R)}\, e^{ipr\cos\theta}$$

same trick as before

$$= \frac{1}{(2\pi)^2}\int_0^{\pi/2} d\theta \sin\theta \int_{-\infty}^{\infty} dp\, p^2\, \frac{m_D^2-Re(\Pi_R)}{p^2+Re(\Pi_R)}\, e^{ipr\cos\theta}$$

Perform momentum integration via contour integration:

$$\oint_{C_R} dz\, z^2\, \frac{m_D^2-Re(\Pi_R)}{z^2+Re(\Pi_R)}\, e^{izr\cos\theta}, \quad C_R:$$



$$\oint_{C_R} dz(\dots) = \lim_{R\to\infty}\Big[\int_{-R}^{R} dz(\dots) + \underbrace{\int dz\,(z=Re^{i\varphi})}_{A}\Big]$$

$$= 2\pi i \sum Res$$

For (A) let $z=Re^{i\varphi} \Rightarrow dz = iRe^{i\varphi}d\varphi$ and we have

$$\lim_{R\to\infty}\int_0^{\pi} d\varphi\, iRe^{i\varphi}\left(R^2e^{2i\varphi}\right)\frac{m_D^2-Re(\Pi_R)}{R^2e^{2i\varphi}+Re(\Pi_R)}\, e^{iRe^{i\varphi}r\cos\theta}$$

$$\approx \lim_{R\to\infty} i\int_0^{\pi} d\varphi\, Re^{i\varphi}\left(M_D^2 - Re(\Pi_R)\right) e^{iRe^{i\varphi} r\cos\theta}$$

according to Mathematica

$$= \lim_{R\to\infty} \frac{2(Re(\Pi_R) - M_D^2)\,\sin(R r\cos\theta)}{r\cos\theta}$$

Now, recall that $Re(\Pi_R(\theta))$ is symmetric around $\frac{\pi}{2}$, and $\sin(\cos\theta))$ is anti-symmetric, and $\cos(\theta)$ is symmetric.

→ Not important.

↳ From looks of above expression evaluated numerically (also with $d\theta$ integral) in Mathematica, does not go to zero or some easily perceptable combination of r, T, M_D.

Return to

$$\frac{1}{(2\pi)^2}\int_0^{\pi/2} d\theta \sin\theta \underbrace{\int_{-\infty}^{\infty} dp\, p^2 \frac{M_D^2 - Re(\Pi_R)}{p^2 + Re(\Pi_R)} e^{ipr\cos\theta}}_{\sqrt{Re(\Pi_R)}\,(Re(\Pi_R) - M_D^2)\left[\pi\cosh\left(\sqrt{Re(\Pi_R)}\, r\cos\theta\right) - i\,Ci\left(-i\sqrt{Re(\Pi_R)}\, r\cos\theta\right)\cdot \sinh\left(\sqrt{Re(\Pi_R)}\, r\cos\theta\right) + i\,Ci\left(i\sqrt{Re(\Pi_R)}\, r\cos\theta\right)\sinh\left(\sqrt{Re(\Pi_R)}\, r\cos\theta\right)\right]}$$

→ according to Mathematica when damped with e^{-ap}.

$$= g_{Re}(r, M_D, T)\cdot \frac{1}{(2\pi)^2}$$