DFG Deutsche Forschungsgemeinschaft

ISOQUANT SFB1225

The in-medium heavy-quark potential and its phenomenology

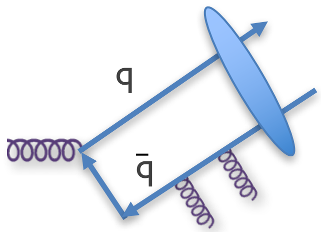
Alexander Rothkopf
Institute for Theoretical Physics
Collaborative Research Center SFB1225 ISOQUANT
Heidelberg University

References:

with Y. Burnier and O.Kaczmarek	JHEP 1512 (2015) 101, JHEP 1610 (2016) 032
with Y. Burnier	PRD95 (2017) 054511
with S. Kajimoto, Y. Akamatsu, M. Asakawa	arXiv:1705.03365
with P. Petreczky and J. Weber	in preparation
with B. Krouppa and M. Strickland	in preparation

Motivation: Heavy-ion collisions

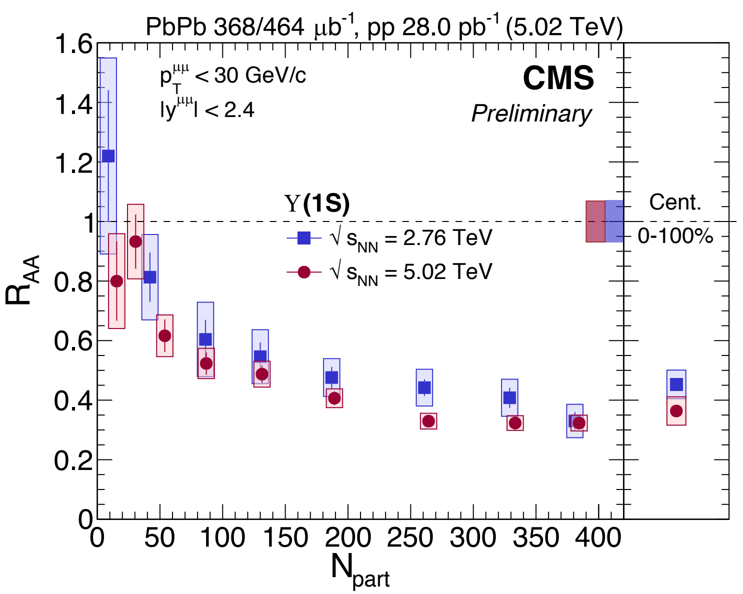
- Hard probes: susceptible to medium but distinguishable from it $Q_{probe} > T_{med}$



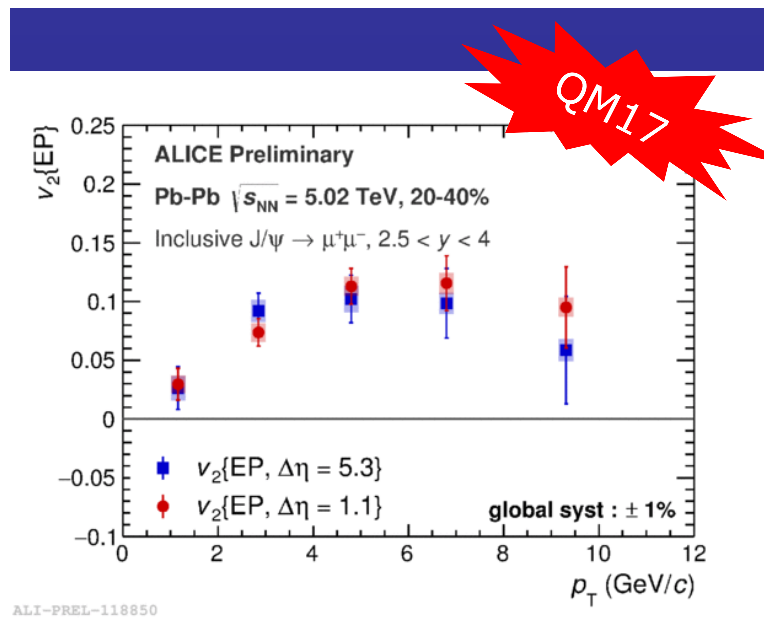
Bound states of $c\bar{c}$ or $b\bar{b}$: **Heavy quarkonium** $M_Q > T_{med}$

In vacuum: $m^{\Upsilon}=9.460\text{GeV}$, $\Gamma^{\Upsilon} = 54(1)\text{keV}$; $m^{J/\psi}=3.096\text{ GeV}$, $\Gamma^{J/\psi} = 93(3)\text{keV}$

see QM2017 summary
by E. Scomparin arXiv:1705.05810



bb: sampling the full QGP evolution



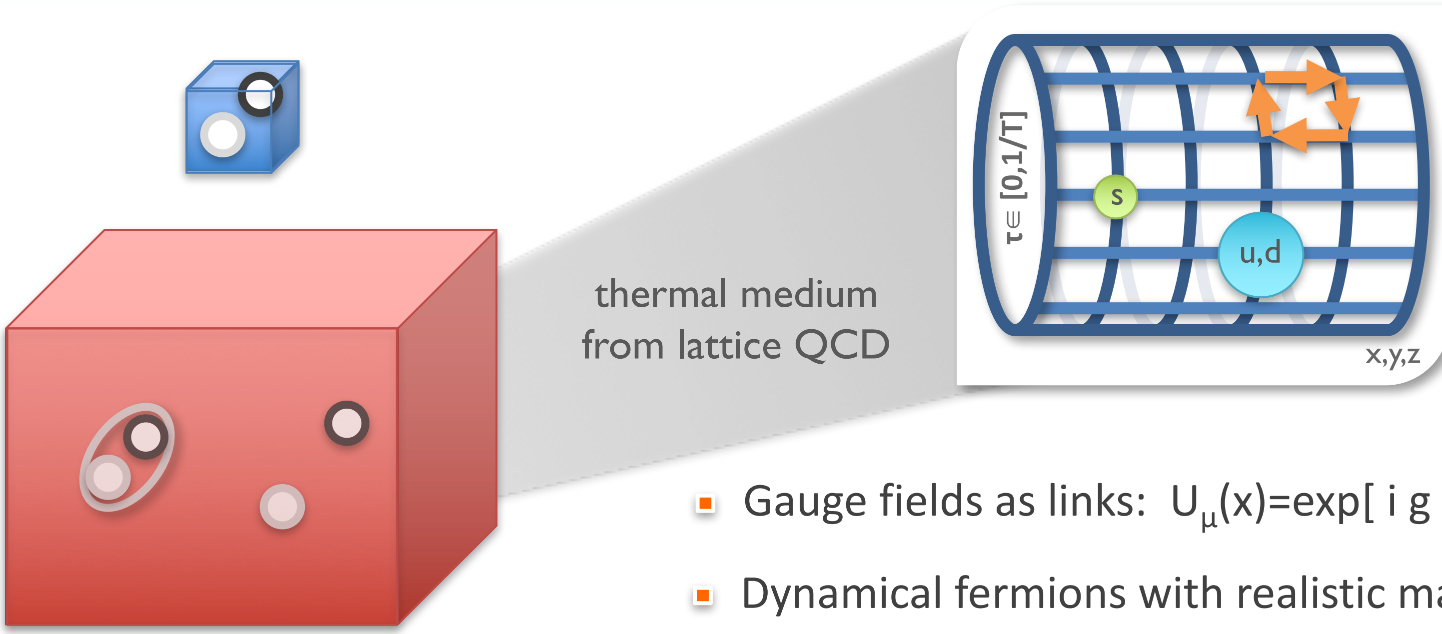
cc: thermal probe of the late stages

- Goal: Non-relativistic potential for in-medium QQbar from first principles

Outline

- Motivation: Relativistic heavy-ion collisions
- The in-medium heavy-quark potential from lattice QCD
  - Current status of the potential extraction from the lattice
  - Gauss-Law parametrization and Debye mass parameter
- Quarkonium phenomenology from the in-medium potential
  - Charmonium production at the phase boundary at LHC
  - Bottomonium suppression at RHIC and LHC
- Summary

A first-principles scenario



T. Matsui and H. Satz: Phys.Lett. B178 (1986) 416

Kinetically equilibrated heavy quarks

Static medium at finite temperature

- Gauge fields as links: $U_\mu(x)=\exp[\, i\, g\, \Delta x_\mu\, A_\mu(x)\,]$
- Dynamical fermions with realistic masses
- Finite extend in imaginary time: $1/T = \beta = N_\tau\, a_\tau$

$$\langle O(U)\rangle = \int \mathcal{D}U\, O(U)\, e^{-S_E^{QCD}[U]}$$

$$P(U_i) \sim e^{-S[U_i]} \Rightarrow \langle O(U)\rangle \approx \frac{1}{N}\sum_{i=1}^{N} O(U_i)$$

How to establish a non-perturbative potential picture for $Q\bar{Q}$ using lattice QCD?

Towards a potential for $Q\bar{Q}$

- **Intuition**: Interactions with medium via a non-relativistic potential description

$$\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{\mathbf{p}}{m_Q} \ll 1$$

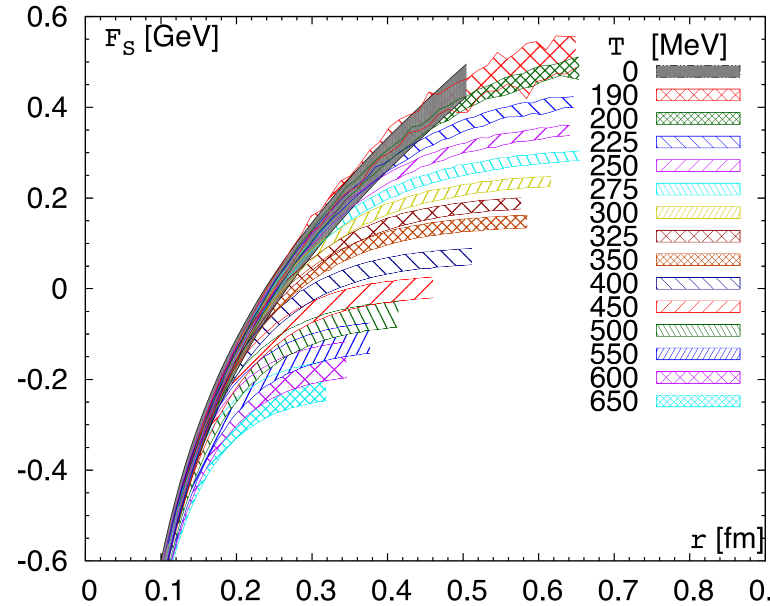
- Potential at T=0 well understood from lattice QCD: Cornell $V(R) = -\alpha_s/R + \sigma R$

- Potential models at T>0:
  - Ad hoc identification of V(r) with color singlet free energies in Coulomb Gauge

$$F^{(1)}(R) = -\frac{1}{\beta}\log\left[\langle P(R)P^{\dagger}(0)\rangle\right]$$

Nadkarni, PRD 34,(1986)

  - Many other proposals: internal energy U^{I}, linear combinations
C.Y.Wong, PRC 72, (2005),H. Satz JPG 36,(2009), …



J.H.Weber, P. Petreczky, A. Bazavov POSLAT2015 177

- A QCD derived Schrödinger equation required to define first principles potential

Defining the T>0 $Q\bar{Q}$ potential

- Effective field theory $\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{p}{m_Q} \ll 1$

Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

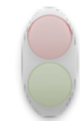
Relativistic thermal field theory

QCD	NRQCD	pNRQCD
Dirac fields	Pauli fields	Singlet/Octet
$\bar{Q}(x), Q(x)$	$\chi^\dagger(x), \chi(x)$	$\psi_S(R,t), \psi_O(R,t)$

$\bar{q}(x), q(x), A^\mu(x)$

$$i\partial_t \psi_S = \Big(V^{QCD}(R) + \mathcal{O}(m_Q^{-1})\Big)\psi_S$$

Quantum mechanics

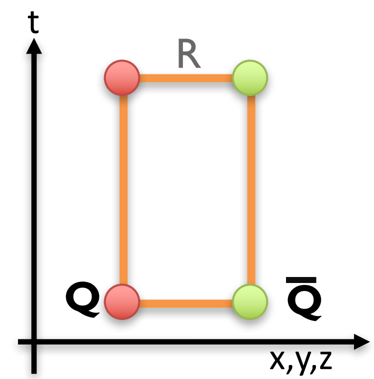


- Matching between QCD and pNRQCD in the static limit

$$\langle \psi_S(R,t)\psi_S^*(R,0)\rangle_{pNRQCD} \equiv W_\square(R,t) = \mathrm{Tr}\Big(\exp\Big[-i\int_\square dx^\mu A_\mu(x)\Big]\Big)$$

$$i\partial_t W_\square(R,t) \overset{t>>t_{med}}{=} V^{QCD}(R) W_\square(R,t)$$

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{i\partial_t W_\square(R,t)}{W_\square(R,t)} \in \mathbb{C}$$

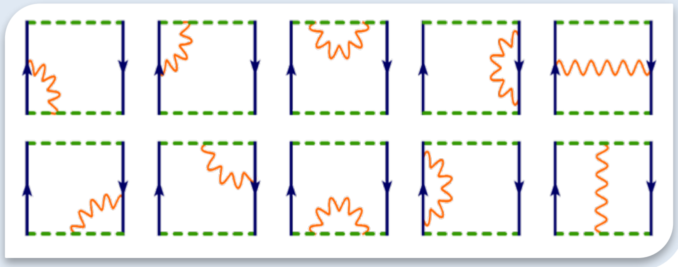


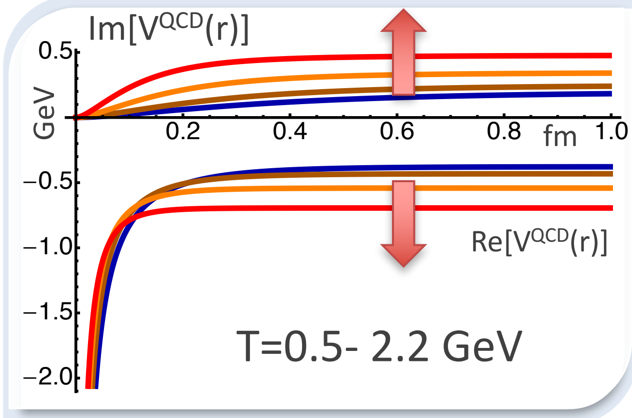
The potential at high temperature

- $T \gg T_C$: Asymptotic freedom of QCD allows weak coupling evaluation (HTL)

Laine et al. JHEP03 (2007) 054; Beraudo et. al. NPA 806:312,2008

$$W_\square(R,t) =$$





$$V^{QCD}_{HTL}(R) = -\frac{gC_F}{4\pi}\left[m_D + \frac{e^{-m_D R}}{R}\right] - i\frac{g^2 T C_F}{4\pi}\phi(m_D R)$$

$$\phi(x) = \int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin[zx]}{zx}\right]$$

Debye screening

Landau damping
Singlet-Octet transitions

  - Re[V] from Debye screening: presence of deconfined color charges
  - Im[V] from Landau damping: scattering with medium partons (decoherence)

- In the phenomenologically relevant regime $T \sim T_C$ non-perturbative contributions

Extracting V^{QCD} from lattice QCD

ISOQUANT SFB1225

- On the lattice real-time observables not directly accessible!
- How to connect to the Euclidean domain: **spectral functions**

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\, \rho_\square(R,\omega) \quad \Longleftrightarrow \quad W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau}\, \boxed{\rho_\square(R,\omega)}$$

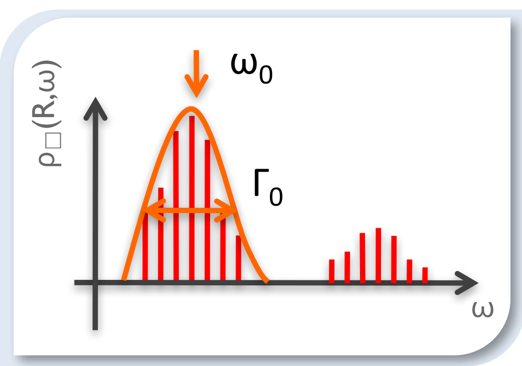
A.R., T.Hatsuda & S.Sasaki
PRL 108 (2012) 162001

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{-i\omega t}\, \rho_\square(R,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\, \rho_\square(R,\omega)}$$

Improved Bayesian spectral reconstruction

Y.Burnier, A.R. PRL 111 (2013) 182003

- Relation between spectrum and potential from the symetries of $W_\square$(R,t)



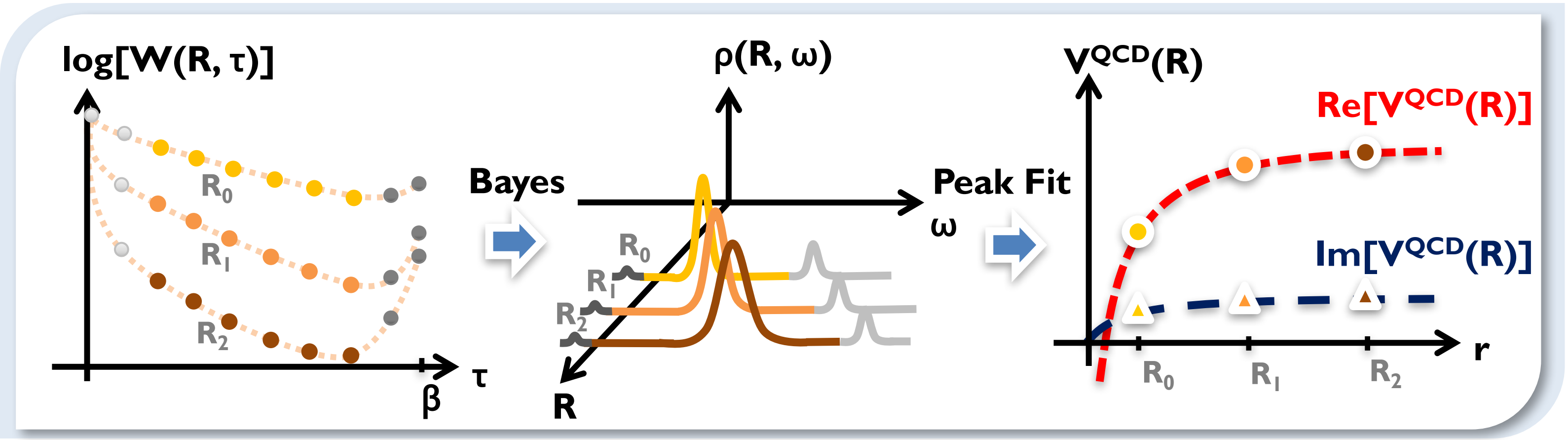
$$\rho_\square(R,\omega) = \frac{1}{\pi} e^{\gamma_1(R)} \frac{\Gamma_0(R)\cos[\gamma_2(R)] - (\omega_0(R)-\omega)\sin[\gamma_2(R)]}{\Gamma_0^2(R) + (\omega_0(R)-\omega)^2} + \kappa_0(R) + \kappa_1(R)(\omega_0(R)-\omega) + \dots$$

$$V^{QCD}(R) = \omega_0(R) + i\Gamma_0(R)$$

technical details: Y.Burnier, A.R. Phys.Rev. D86 (2012) 051503

Summary: V^{QCD} from the lattice

- From lattice QCD correlators to the complex heavy quark potential

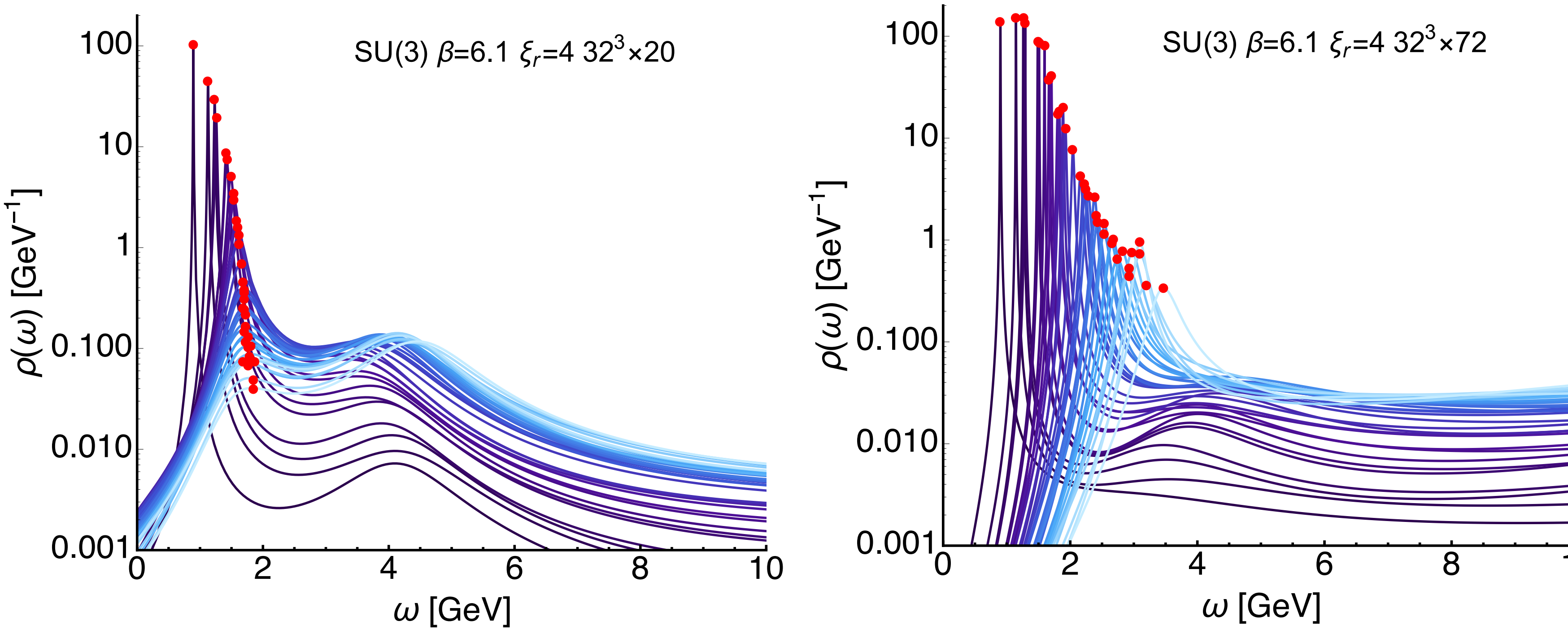


A.R. Mod. Phys. Lett. A, 28, 1330005 (2013)

- Technical detail: Wilson Line correlators in Coulomb gauge instead of Wilson loops
  Practical reason: Absence of cusp divergences, hence less suppression along τ

T>0 potential in quenched QCD (I)

- Naïve Wilson action SU(3) on 32^3xN_T β=6.1 ξ=4 N_T=[72..20]   T=[113-406]MeV

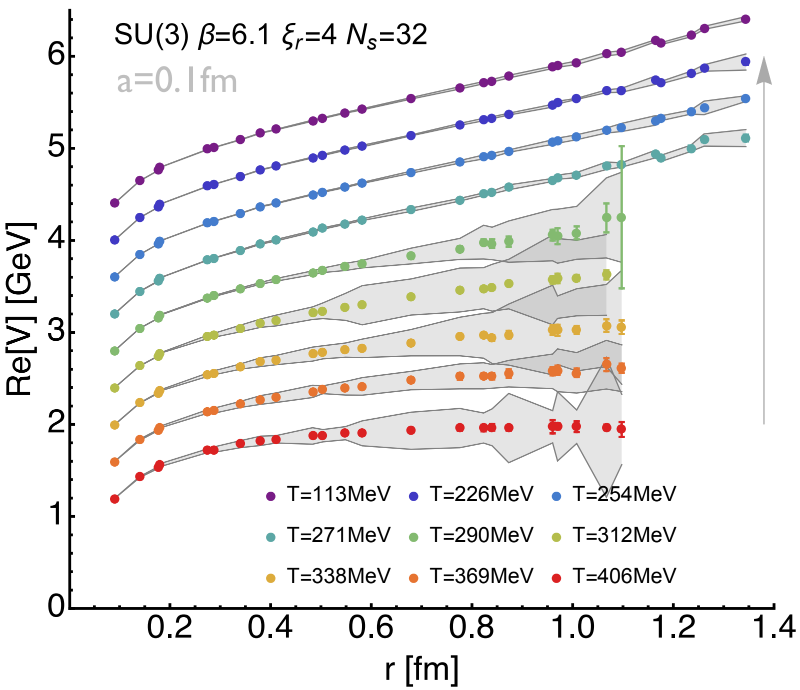


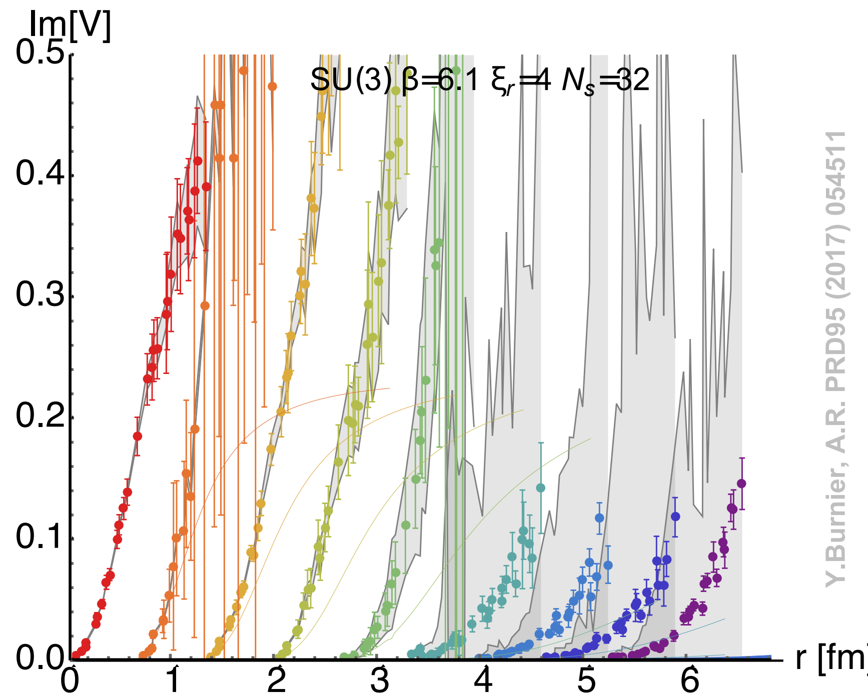
Y.Burnier, A.R. PRD95 (2017) 054511

- Spectral reconstruction successful due to simple structure: peak and shoulder
- Clear signal for well defined lowest lying peak at all T: potential picture valid

T>0 potential in quenched QCD (II)

- Naïve Wilson action SU(3) on $32^3 \mathrm{x} N_\tau$ β=6.1 ξ=4 N_τ=[72..20]   T=[113-406]MeV

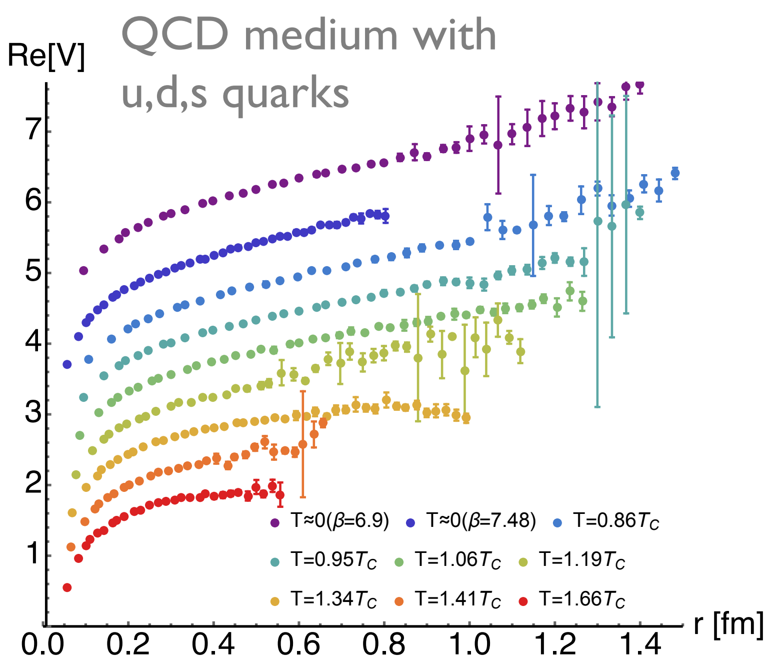


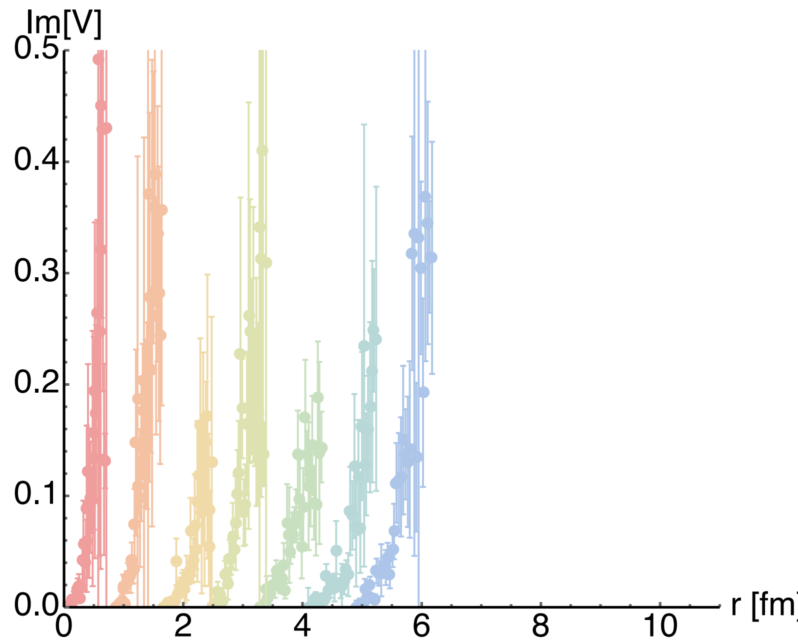


- New results closer to thermodynamic limit: clear signs of the phase transition
- Re[V] shows no T-dep. in confined phase, changes significantly in deconfined
- Im[V] below T_C compatible with zero, finite in the deconfined phase

T>0 static potential in full LQCD (I)

- N_f=2+1 AsqTad 48^3x12 m_π~300MeV T_C=173MeV β=[6.8..7.48] T=[148-286]MeV





Y.Burnier, O.Kaczmarek, A.R.
PRL 114 (2015) 082001

- Re[V] shows smooth transition from confining to Debye screened behavior
- Re[V] compatible with color singlet free energies in Coulomb gauge F^1
- Im[V] shows finite values above T_{PC} but errors are still large

The Pade approximation

- For Wilson line correlators: exploit analyticity in case of high statistics
- Find the "best" rational approximation R^m_n to Matsubara correlator data $D(\omega_n)$

$$R^m_n(\omega_n) = \frac{\sum_{j=0}^{m} a_j \omega_n^j}{1 + \sum_{k=0}^{n} b_k \omega_n^k} \qquad C(\omega) = \cfrac{z_0}{1 + \cfrac{z_1(\omega - \omega_1)}{1 + \cfrac{z_2(\omega - \omega_2)}{\vdots}}}$$

  - z_i chosen such that $C(\omega_n)=D(\omega_n)$ - approximation to R^m_n

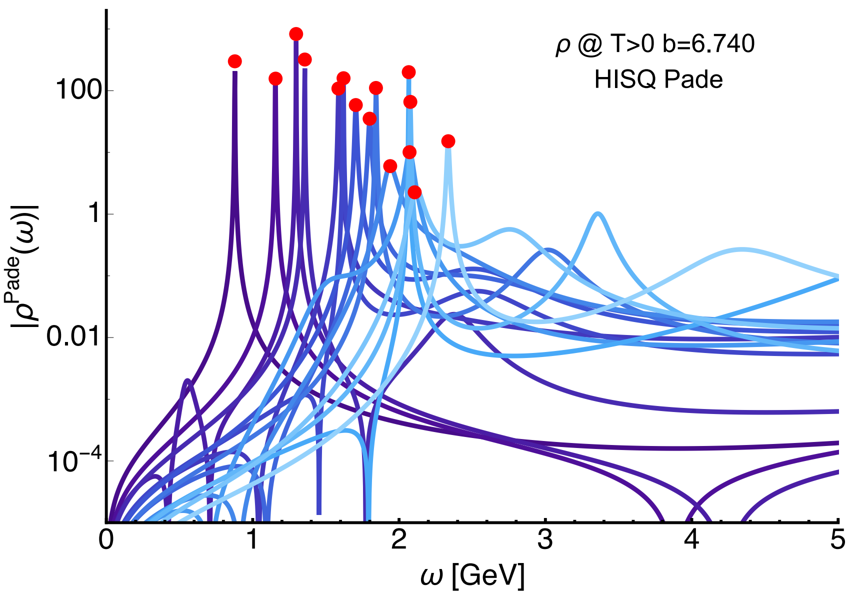
- Corresponds to a projection method onto rational basis functions
- Spectral functions from analytic continuation -Im[C(iω)]

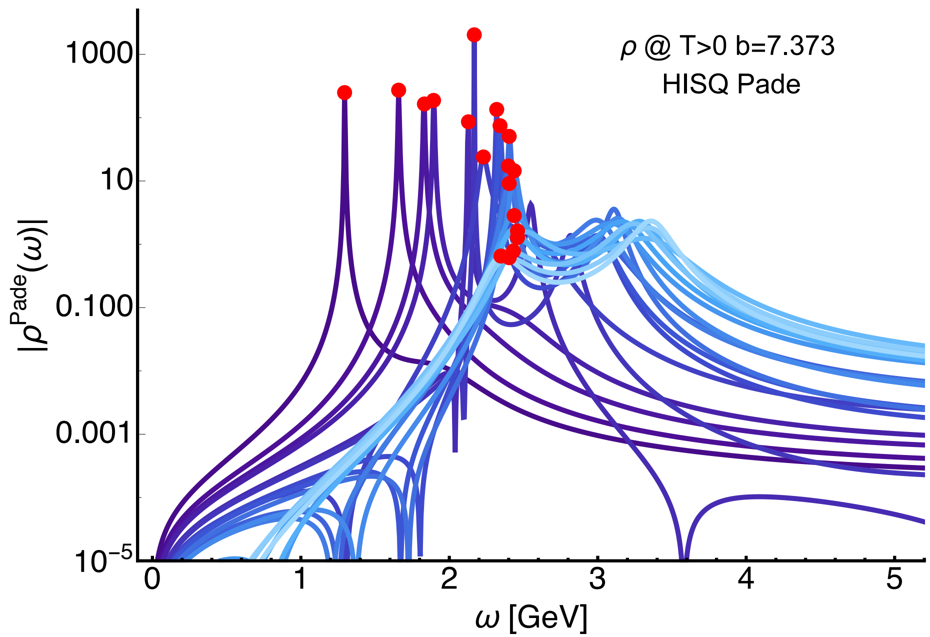
Not a Bayesian method: no prior information needed

In practice for high precision data robust results

T>0 static potential in full LQCD (II)

- N_f=2+1 HISQ 48^3x12 m_π~161MeV T_C=159MeV β=[6.74..7.825] T=[150-408]MeV

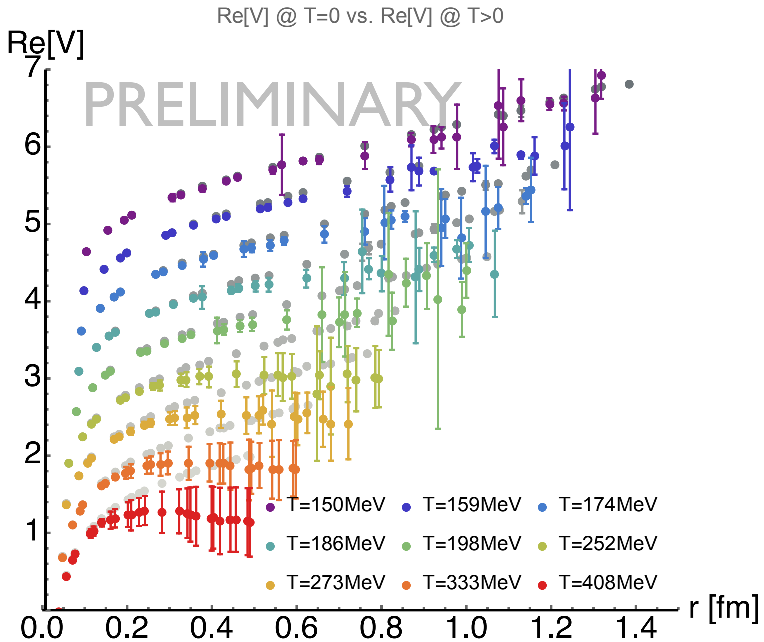


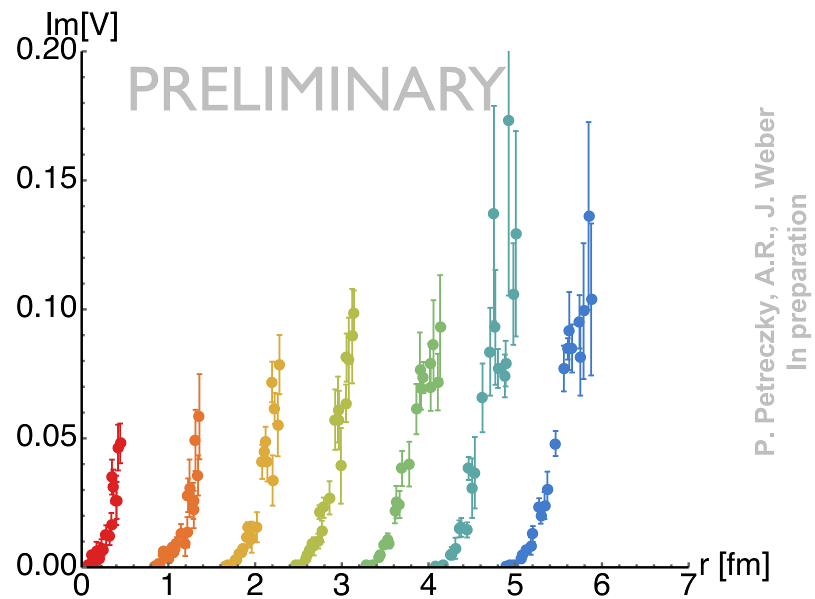


- High statistics on HotQCD ensembles (>5x10^3): also Pade for spectral reconstruction

T>0 static potential in full LQCD (II)

- N_f=2+1 HISQ 48^3x12 m_π~161MeV T_C=159MeV β=[6.74..7.825] T=[150-408]MeV

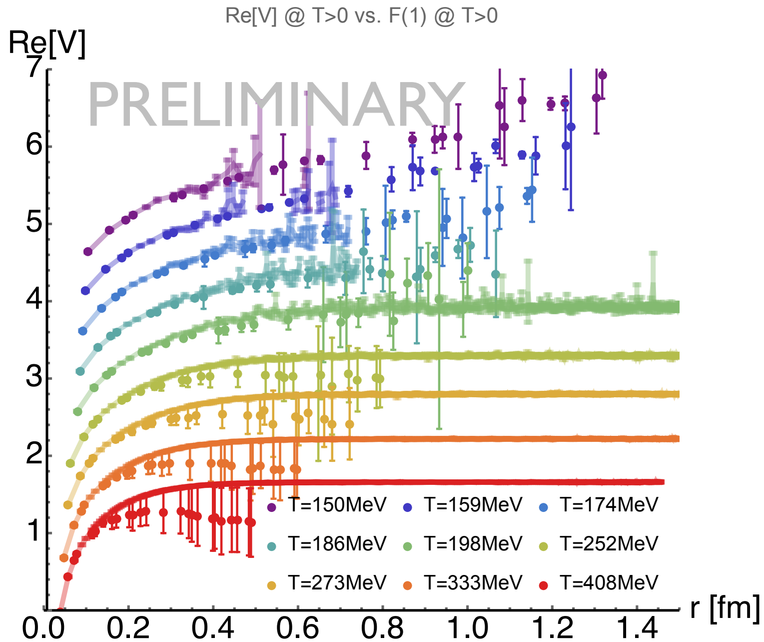


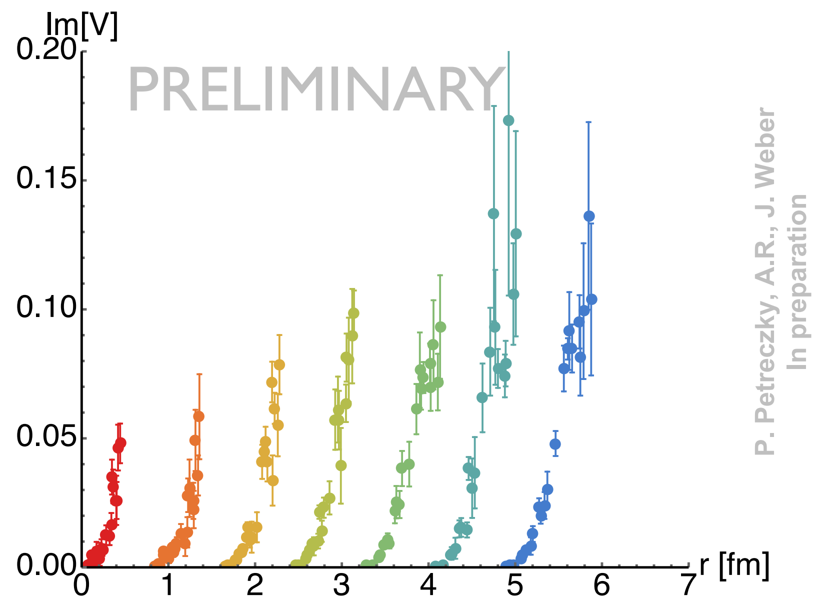


- High statistics on HotQCD ensembles (>5x10³): also Pade for spectral reconstruction
- Smooth changes in Re[V] but large uncertainty at high T, compatible with F[1]
- Investigation of systematic errors still ongoing

T>0 static potential in full LQCD (II)

- N_f=2+1 HISQ 48^3x12 m_π~161MeV T_C=159MeV β=[6.74..7.825] T=[150-408]MeV





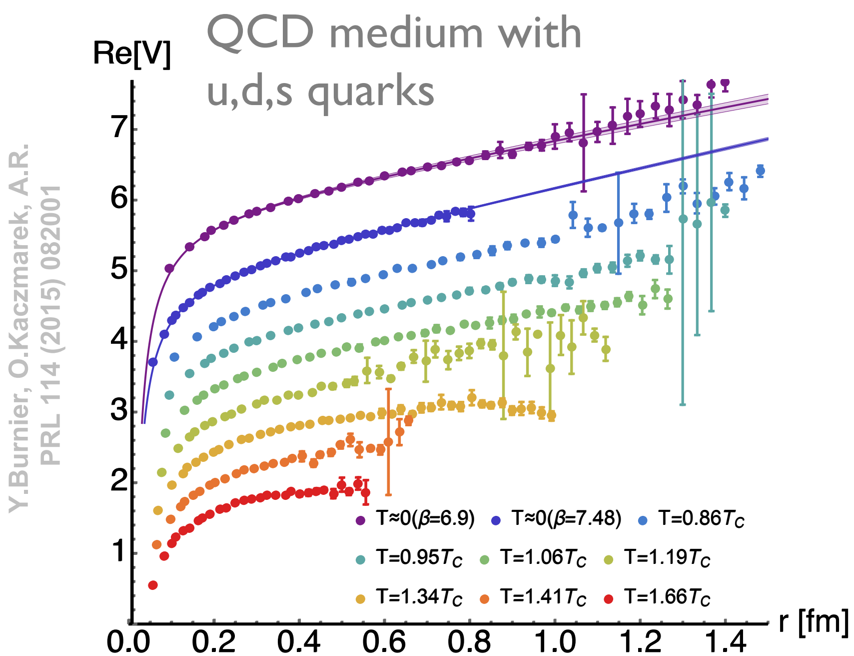
- High statistics on HotQCD ensembles (>5x10³): also Pade for spectral reconstruction
- Smooth changes in Re[V] but large uncertainty at high T, compatible with F[1]
- Investigation of systematic errors still ongoing

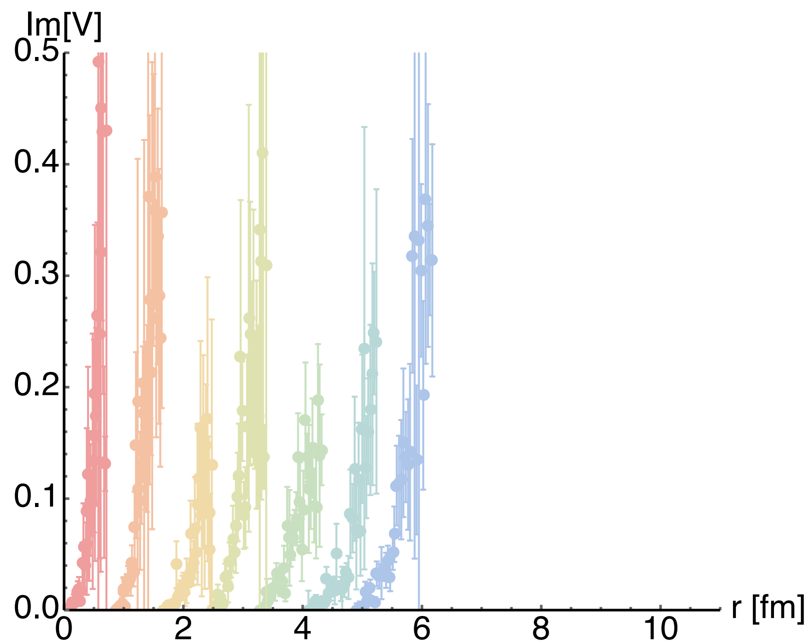
Outline

- Motivation: Relativistic heavy-ion collisions
- The in-medium heavy-quark potential from lattice QCD
  - Current status of potential extraction from the lattice
  - Gauss-Law parametrization and Debye mass parameter
- Quarkonium phenomenology from the in-medium potential
  - Charmonium production at the phase boundary at LHC
  - Bottomonium suppression at RHIC and LHC
- Summary

Towards QQbar phenomenology

- N_f=2+1 AsqTad 48^3x12 m_π~300MeV T_C=173MeV β=[6.8..7.48] T=[148-286]MeV

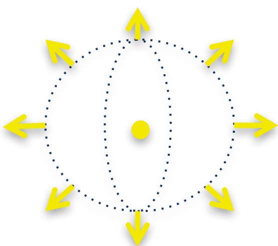




- At T~0 Re[V] well described by naïve Cornell ansatz: V= -α/r + σr + c

Generalized Gauss law and V^{QCD}

- Towards phenomenology: Analytic expression for Re[V^{QCD}] and Im[V^{QCD}] needed



$$V^{T=0}_{Q\bar{Q}} = V_C(r) + V_S(r) = -\frac{\alpha_S}{r} + \sigma r + c$$

$$\boxed{\vec{\nabla}\left(\frac{\vec{\nabla}V(r)}{r^{a+1}}\right) = -4\pi\, q\, \delta(\vec{r})}$$

$$V(r) = a\, q\, r^a$$

$$\vec{E} = -\vec{\nabla}V(r)$$

Strategy:

α_s,σ and c are vacuum prop. and do not change with T

At r‘s relevant for bb and cc running of α_S is not essential

Coulombic: a=-1 q=α_s

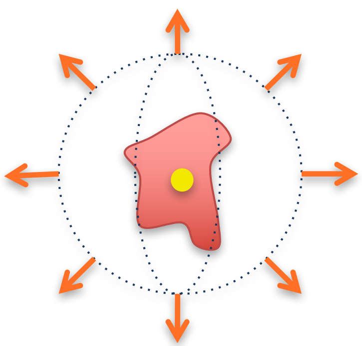
$$\vec{\nabla}\left(\vec{\nabla}V_C(r)\right) = -4\pi\, \alpha_S\, \delta(\vec{r})$$

String-like: a=+1 q=σ

$$\vec{\nabla}\left(\frac{\vec{\nabla}V_S(r)}{r^2}\right) = -4\pi\, \sigma\, \delta(\vec{r})$$

V. V. Dixit,
Mod. Phys. Lett. A 5, 227 (1990)

Introducing medium effects



In the classical theory of Debye: Boltzmann distr. backgr. charges <ρ>

$$\vec{\nabla}\left(\vec{\nabla}V_C(r)\right) = -4\pi\,\alpha\left(\delta(\vec{r}) + \langle\rho(\vec{r})\rangle\right)$$

P. Debye, H. Hückel, Phys.Z. 24, 185-206 (1923)

Here instead: Introduce medium via weak coupling HTL permittivity ε

$$p^2 V_C(\vec{p}) = 4\pi\frac{\alpha_s}{\varepsilon(\vec{p}, m_D)} \qquad \varepsilon^{-1}(\vec{p}, m_D) = \frac{p^2}{p^2+m_D^2} - i\pi T\frac{p\,m_D^2}{(p^2+m_D^2)^2}$$

linear response form where m_D->0 is possible

$$g(x) = 2\int_0^\infty dp\frac{\sin(px)}{px}\frac{p}{p^2+1}$$

Y.Burnier, A.R.: arXiv:1506.08684

$$-\nabla^2 V_C(r) + m_D^2 V_C(r) = \alpha_s\left(4\pi\delta(\vec{r}) - iTm_D^2\, g(m_D r)\right)$$

solving for Re[V_C] and Im[V_C]: reproduces Laine's HTL potential

V_S(r): Gauss Law operator not diagonal in Fourier space: assume validity of linear response

$$-\frac{1}{r^2}\frac{d^2V_S(r)}{dr^2} + \mu^4 V_S(r) = \sigma\left(4\pi\delta(\vec{r}) - iTm_D^2\, g(m_D r)\right) \qquad \mu^4 = m_D^2\frac{\sigma}{\alpha_S}$$

$$\mathrm{Re}V_S(r) = \frac{\Gamma[\frac{1}{4}]}{2^{\frac{3}{4}}\sqrt{\pi}}\frac{\sigma}{\mu}D_{-\frac{1}{2}}(\sqrt{2}\mu r) + \frac{\Gamma[\frac{1}{4}]}{2\Gamma[\frac{3}{4}]}\frac{\sigma}{\mu}$$

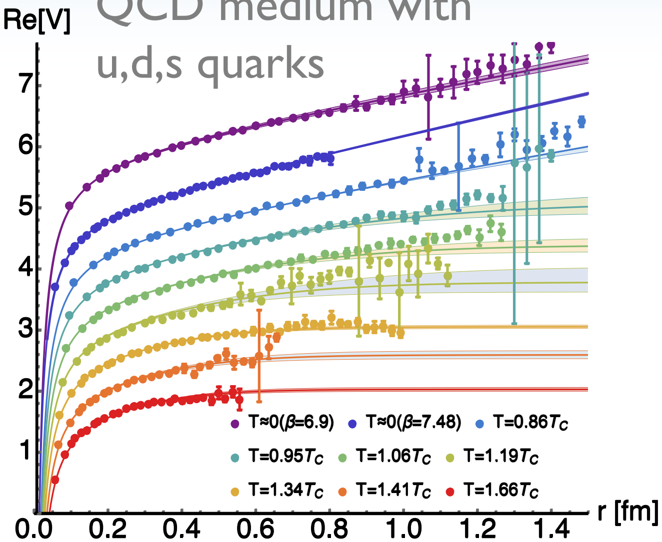
Im[V_S] as integral expression using Wronskian

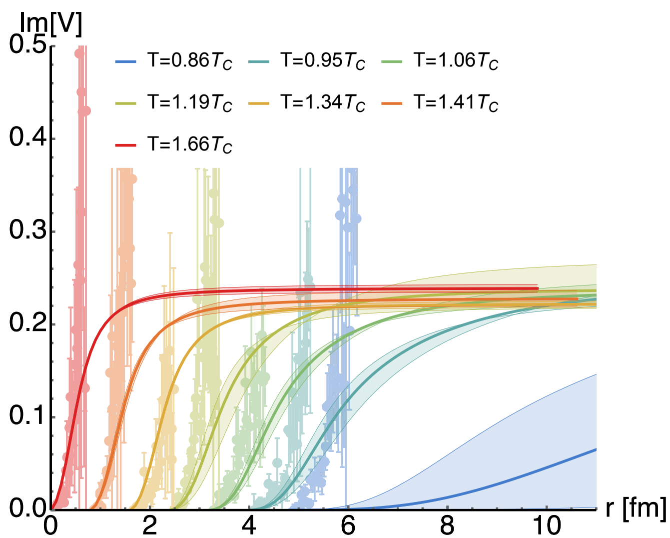
D_ν parabolic cylinder function

T>0 potential for phenomenology

QCD medium with u,d,s quarks

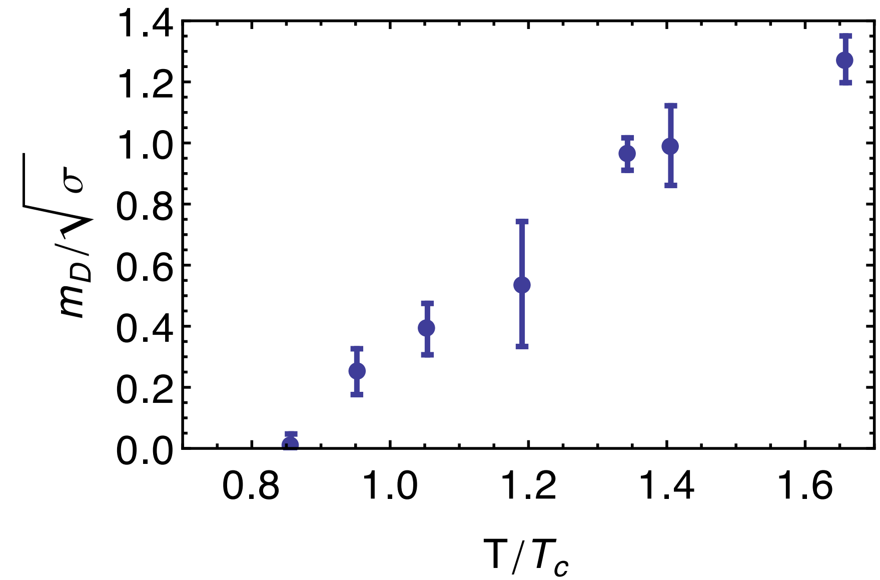
Y.Burnier, O.Kaczmarek, A.R. PRL 114 (2015) 082001





Nf=2+1, 48^3x12, asqtad action, m_π~300MeV

Y.Burnier, O.Kaczmarek, A.R. JHEP 1512 (2015) 101



- For realistic phenomenology a continuum extrapolated potential is needed but not yet available

Outline

- Motivation: Relativistic heavy-ion collisions
- The in-medium heavy-quark potential from lattice QCD
  - Current status of potential extraction from the lattice
  - Gauss-Law parametrization and Debye mass parameter
- Quarkonium phenomenology from the in-medium potential
  - Charmonium production at the phase boundary at LHC
  - Bottomonium suppression at RHIC and LHC
- Summary

Towards phenomenology

- Start with Bottomonium, since velocity- and radiative corrections small
- Bottom quark mass from renormalon subtracted scheme (RG flow to lower scale)

$$m_b^{\bar{MS}}(m_b^{\bar{MS}}) = 4.21(31)\mathrm{GeV} \Rightarrow m_b^{RS'}(E_{bind}) = 4.882(41)\mathrm{GeV}$$

A. Pineda, JHEP 0106 (2001) 022

- Continuum T=0 values for α_s, σ and c : Cornell fit to PDG Bottomonium masses

$$\alpha_S = 0.50 \pm 0.03, \ \sqrt{\sigma} = 0.415 \pm 0.015\mathrm{GeV}, \ c = -0.177 \pm 0.021$$

- Solve a Schrödinger equation for the spectral function in Fourier space:

Y. Burnier, M. Laine and M. Vepsalainen, JHEP 0801, 043 (2008)

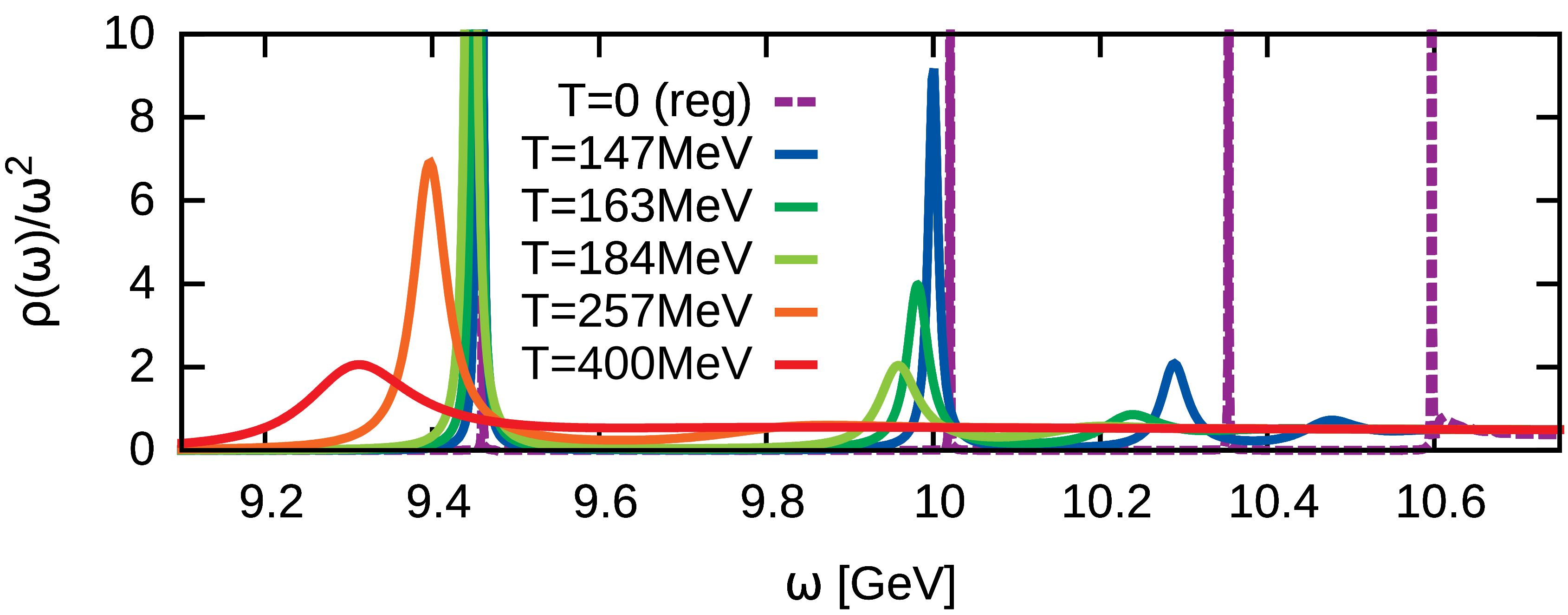
$$\Big[\tilde{H} \mp i|\mathrm{Im}V(r)|\Big] D^>(t,\mathbf{r},\mathbf{r}') = i\partial_t D^>(t,\mathbf{r},\mathbf{r}'), \quad t \gtrless 0$$

$$\tilde{H} = 2m_Q - \frac{\nabla^2}{2m_q} + \mathrm{Re}[V](r) + \frac{l(l+1)}{m_b r^2}$$

$$\tilde{D}(\omega,\mathbf{r},\mathbf{r}') \equiv \int_{-\infty}^{\infty} dt e^{i\omega t} D^>(t,\mathbf{r},\mathbf{r}') \Rightarrow \rho^V(\omega) = \lim_{\mathbf{r},\mathbf{r}'\to 0} \frac{1}{2}\tilde{D}(\omega,\mathbf{r},\mathbf{r}')$$

Reminder:
This complex potential is NOT the potential to evolve the wavefct. ψ(t,r)

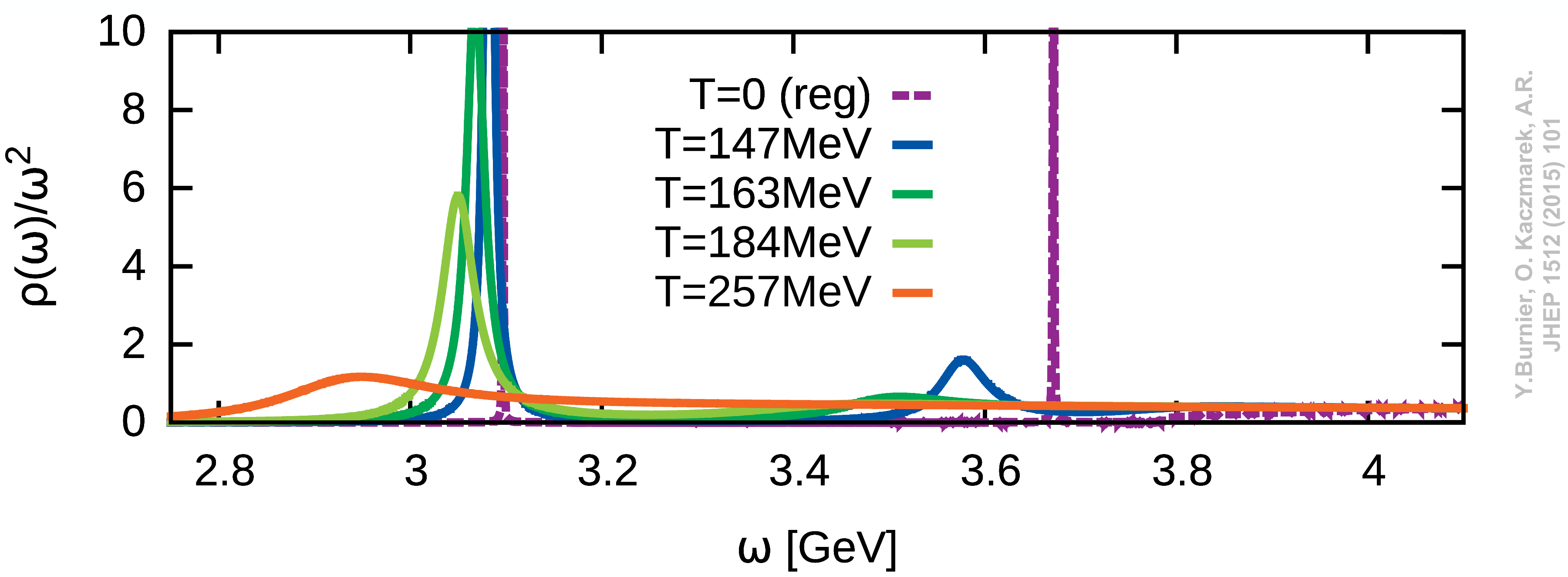
In-medium Bottomonium (S-wave)



Y.Burnier, O. Kaczmarek, A.R.
JHEP 1512 (2015) 101

- Only at T=0: small artificial Im[V^{QCD}] added, to give peaks a finite width
- Clear sequential melting pattern visible
- Systematic shift to lower mass: weakened bound state has less binding energy
  (opposite to fundamental particle which gains a thermal mass)

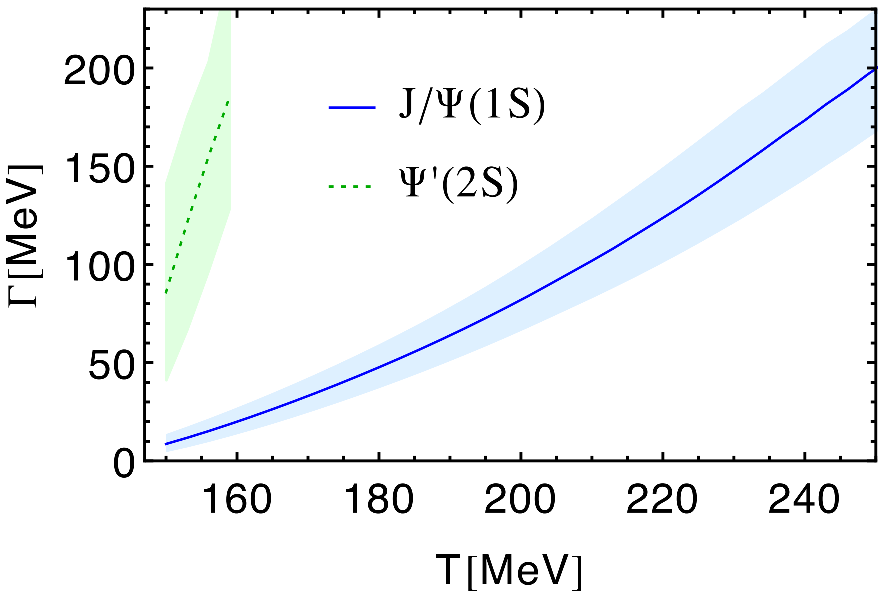
In-medium Charmonium (S-wave)



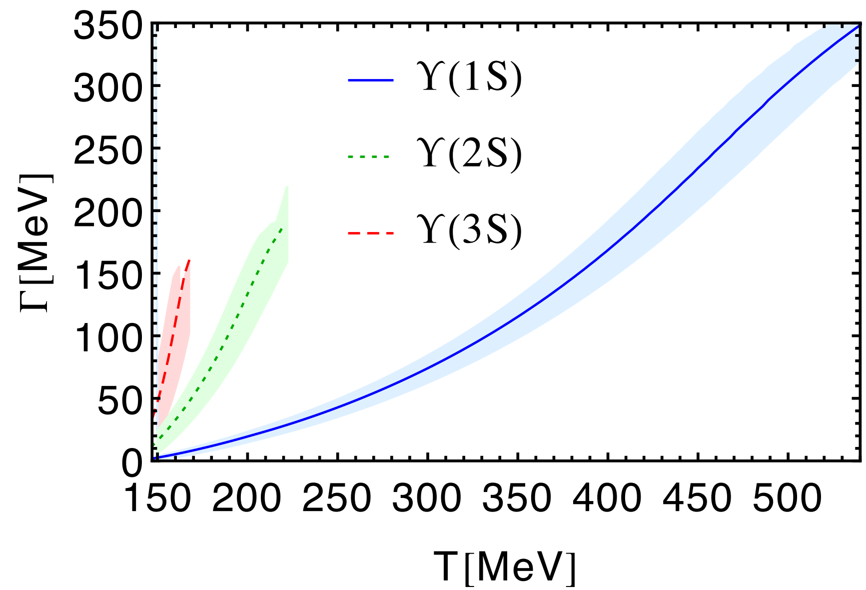
- Charmonium uses the same $V^{QCD}(r)$ as bb but m_c=1.47GeV from fit to PDG states
- Characteristic shift to lower masses, excited state quickly dissolves

Bound state spectral widths

Charmonium



Bottomonium



Y.Burnier, O. Kaczmarek, A.R. JHEP 1512 (2015) 101

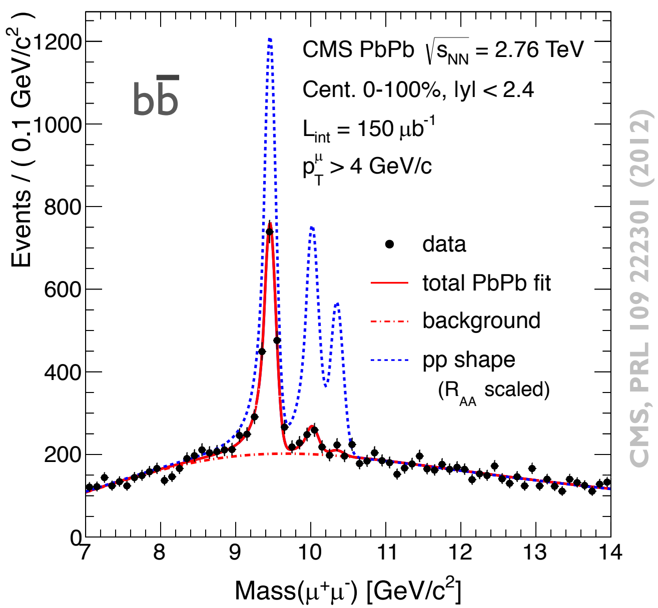
- Size of width significantly influenced by the presence of an imaginary part in V^{QCD}
- Scanning the T range to observe where Γ = E_{bind} estimates melting temperature

state	$J/\Psi(1S)$	$\Psi'(2S)$	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\Upsilon(4S)$
$T_{\rm melt}$	213^{+13}_{-11}	< 147	412^{+76}_{-22}	193^{+26}_{-8}	157^{+5}_{-4}	< 147

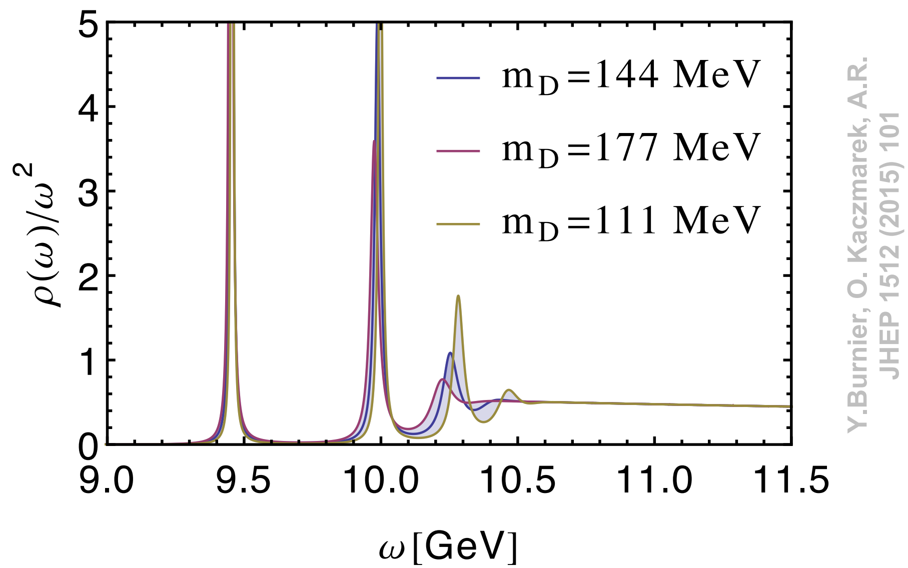
[MeV]

Towards a comparison with data

- Measured Di-leptons are not thermal but originate in the decay of vacuum states



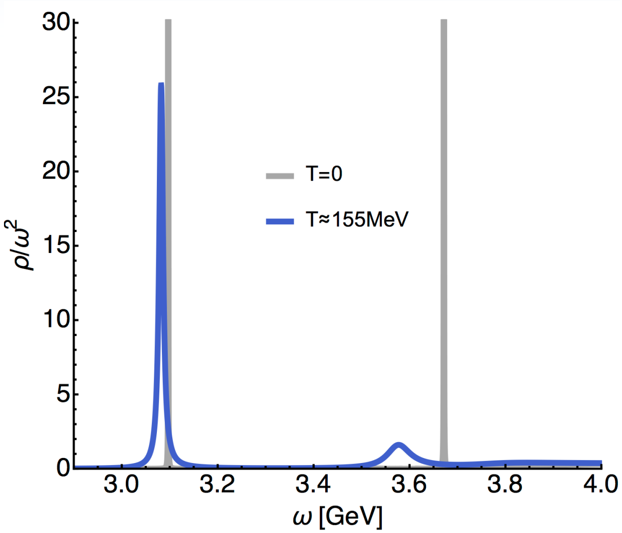
$\neq$



- How to take into account hadronization: possible for instantaneous freezeout at T_C
- Assume that states described by a spectral peak do form real quarkonium at freezeout

ψ‘ to J/ψ ratio from T>0 spectra

Y.Burnier, O.Kaczmarek, A.R.
JHEP 1512 (2015) 101



- Assume instantaneous freezeout: T>0 states convert to real vacuum particles at around T_C
- In-medium dilepton emission from area under spectral resonance peaks

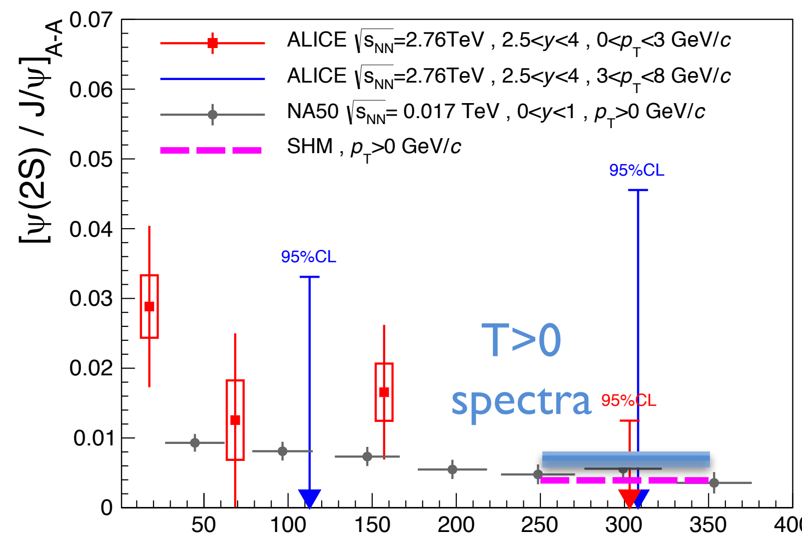
$$R_{\ell\bar{\ell}} \propto \int dp_0 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\rho(P)}{P^2} n_B(p_0)$$

(to leading order $\rho(P) = \rho(p_0^2 - \mathbf{p}^2)$)

- ”How many vacuum states do the in-medium peaks correspond to?”
- Number density: divide in-medium by T=0 dimuon emission rate:

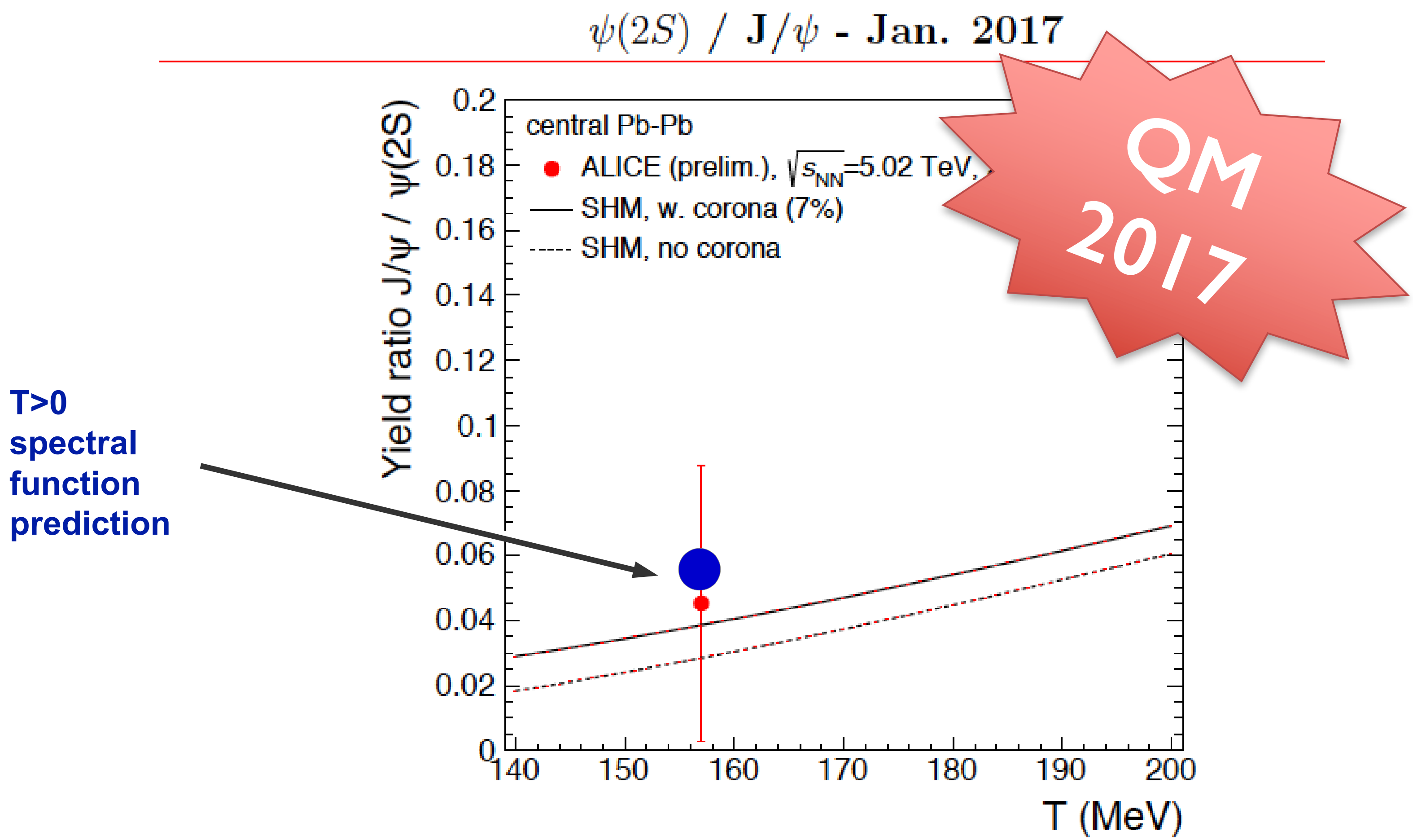
$$\frac{N_{\Psi'}}{N_{J/\Psi}} = \frac{R^{\Psi'}_{\ell\bar{\ell}}}{R^{J/\Psi}_{\ell\bar{\ell}}} \frac{M^2_{\Psi'}|\Phi_{J/\Psi}(0)|^2}{M^2_{J/\Psi}|\Phi_{\Psi'}(0)|^2}$$

Y.Burnier, O. Kaczmarek, A.R. JHEP 1512 (2015) 101



ALICE Collaboration JHEP05(2016)179

First preliminary ALICE data



ALICE data, Pb–Pb: Quark Matter 2017; pp: arXiv:1702.00557

slide adapted from
Peter Braun-Munzinger

Outline

- Motivation: Relativistic heavy-ion collisions
- The in-medium heavy-quark potential from lattice QCD
  - Current status of potential extraction from the lattice
  - Gauss-Law parametrization and Debye mass parameter
- Quarkonium phenomenology from the in-medium potential
  - Charmonium production at the phase boundary at LHC
  - Bottomonium suppression at RHIC and LHC
- Summary

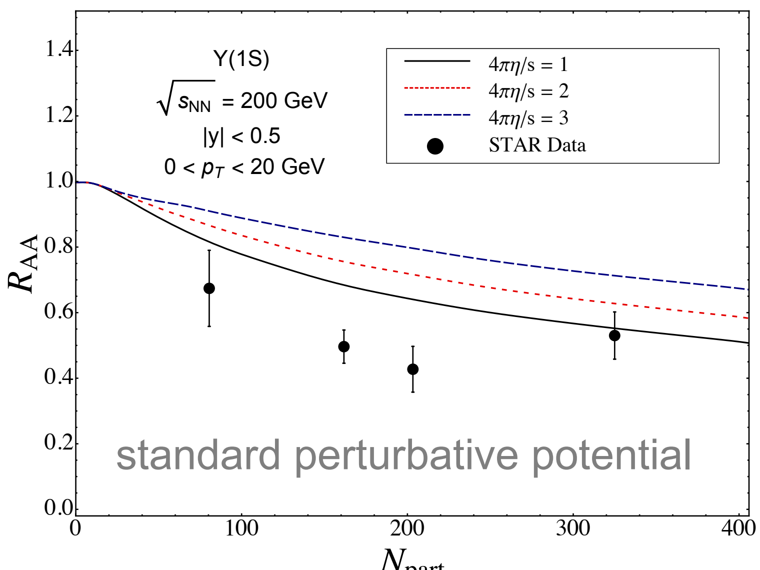
Bottomonium suppression

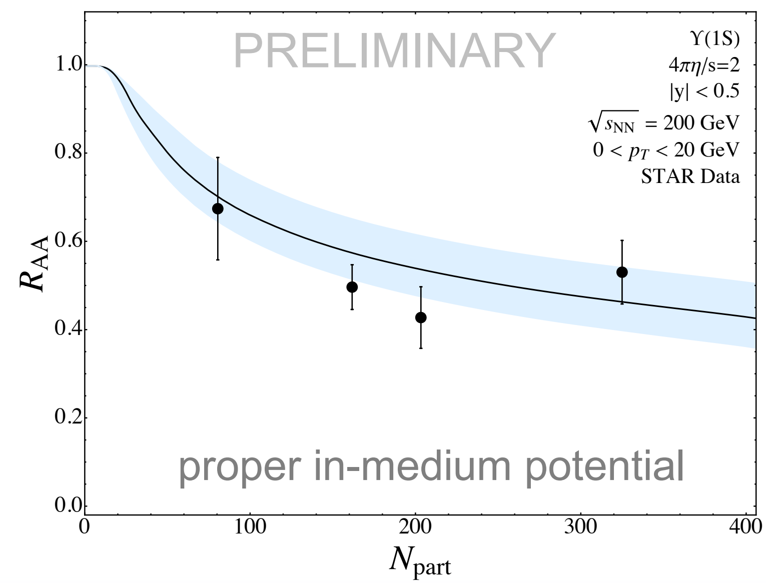
- With M. Strickland: aim to reduce modeling input and give uncertainties for V_{QQ}

Anisotropic Hydro ($m_D(\tau,x)$) & Glauber initial pure state QQ prod. & V_{QQ} in Schrödinger Eq. E_{bind} and $\Gamma(\tau,x)$

Folding together over real-time evolution gives R_{AA}

- No regeneration contributions included: our first try at RHIC data

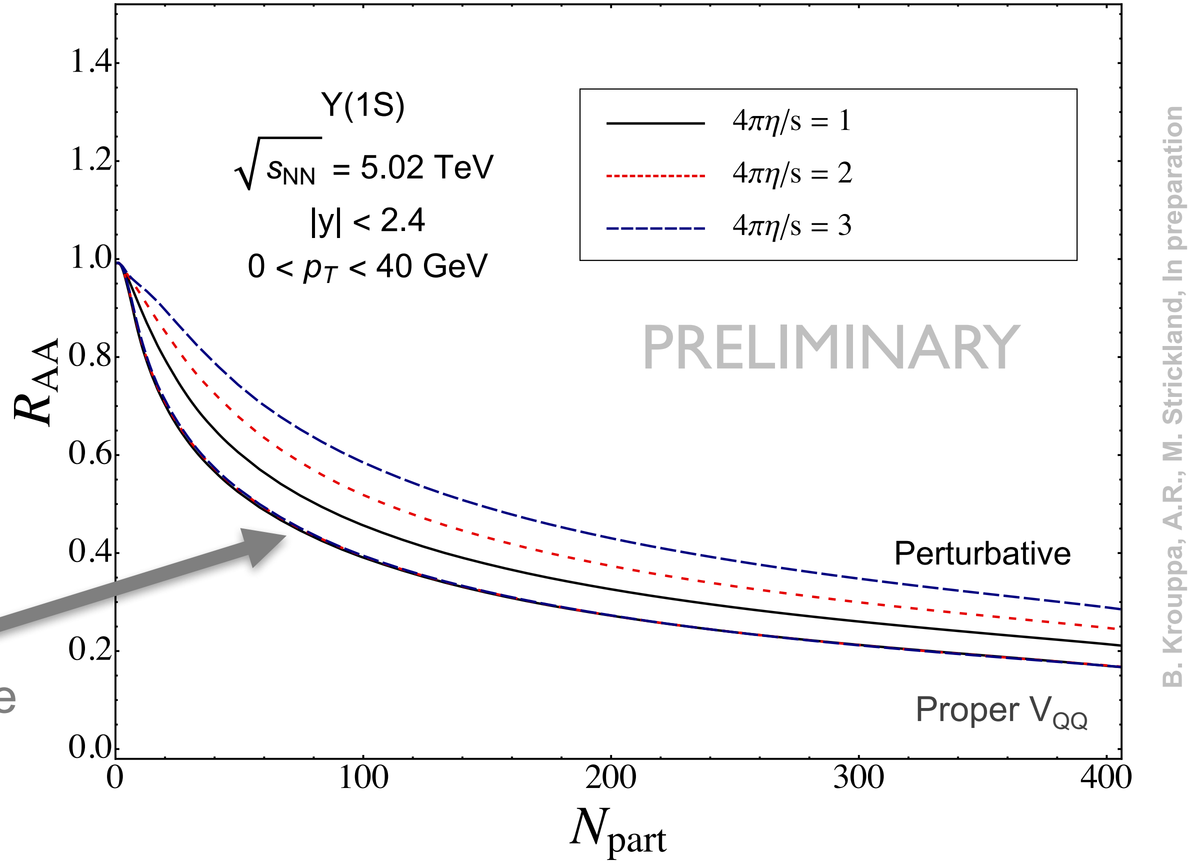




B. Krouppa, A.R., M. Strickland
In preparation

Viscosity depencence of R_{AA}

- With the proper potential dependence on η/S significantly reduced

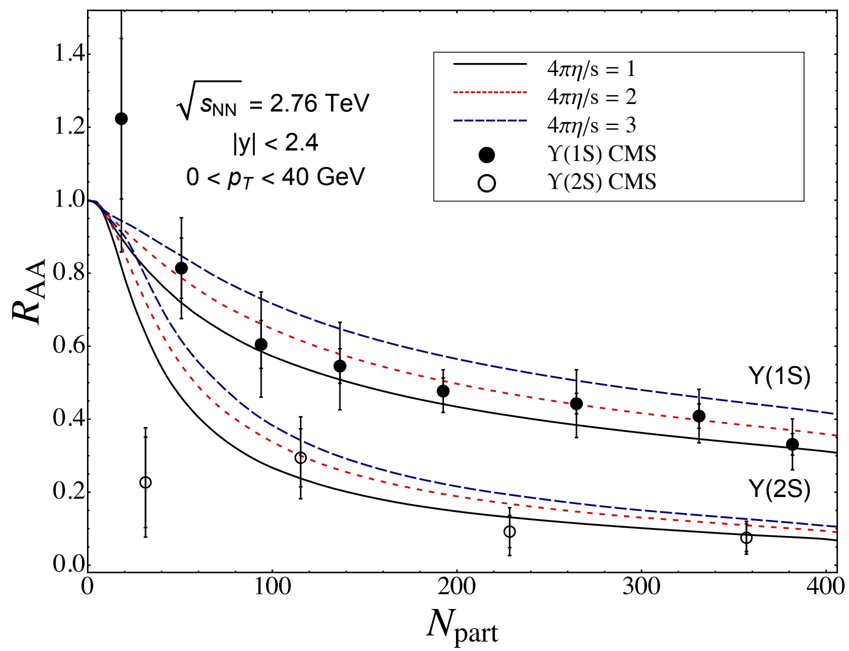


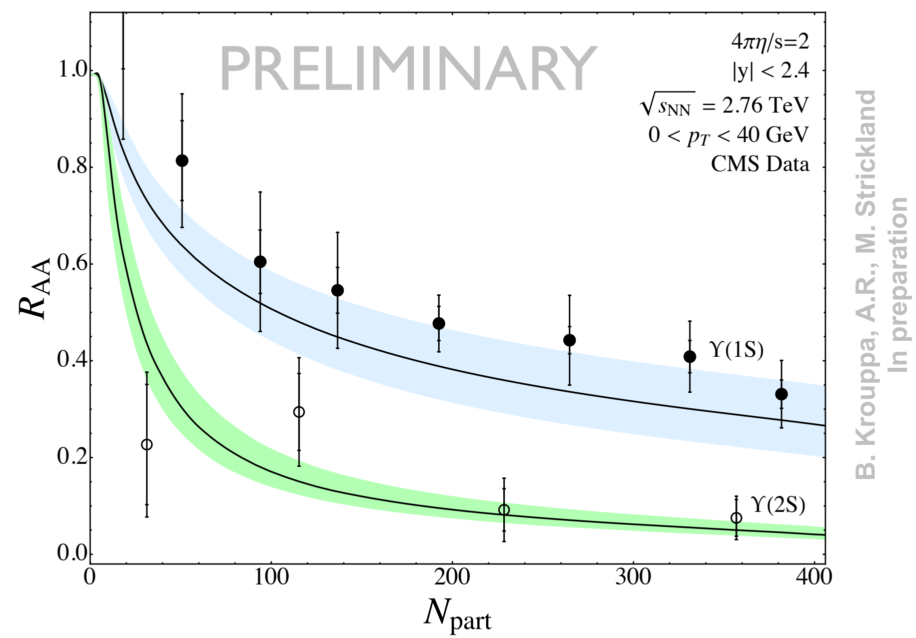
Three curves for different η/s virtually indistinguishable

- Reinforces the picture of Bottomonium as a dynamical thermometer

Bottomonium at LHC energies

- Perturbative potential appears to work better at LHC than at RHIC
- Proper in-medium potential slightly low on Υ(1S), excellent agreement with Υ(2S)





- Recent finding: Naïve treatment of V_{QQ} in Schrödinger eq. underestimates GS R_{AA}

S.Kajimoto, Y.Akamatsu, M. Asakawa, A.R., arXiv 1705.03365

- Outstanding question: What is the role of regeneration (work in progress)

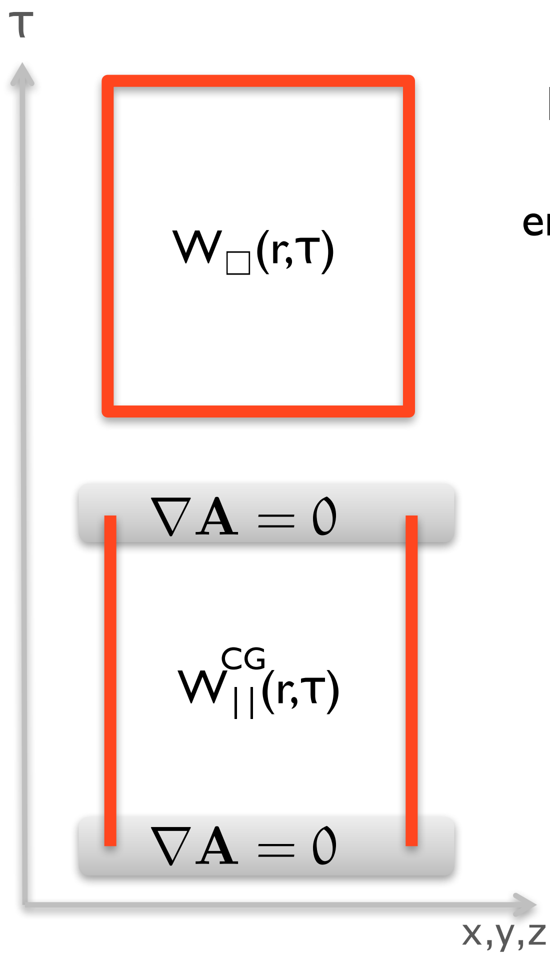
Summary

- ***QCD derived static potential V_{QQ}*** for in-medium heavy-QQ available
- ***Lattice QCD progress*** in determining Re & Im of V_{QQ}
  - Clear signs for phase transition in quenched QCD on coarse lattices
    Y.Burnier, A.R. PRD95 (2017) 054511
  - Ongoing work to extract the potential on current generation HISQ lattices
    P. Petreczky, A.R., J. Weber in progress
- ***Lattice QCD vetted*** Gauss-Law ***parametrization*** of V_{QQ}
- Promising results on ***quarkonium phenomenology*** from the potential
  - Predictions for ***ψ' / J/ ψ ratio*** agree very well with preliminary ALICE data
    Y.Burnier, O.Kaczmarek, A.R. JHEP 1512 (2015) 101, JHEP 1610 (2016) 032
  - ***R_{AA} of Y(1S, 2S)*** in HIC promising for STAR, at LHC question of regeneration?
    B.Krouppa, M. Strickland, A.R. In progress

Thank you for your attention

Wilson Loop vs. Wilson Lines

- Wilson loop contains cusp divergencies: supresses the signal exponentially

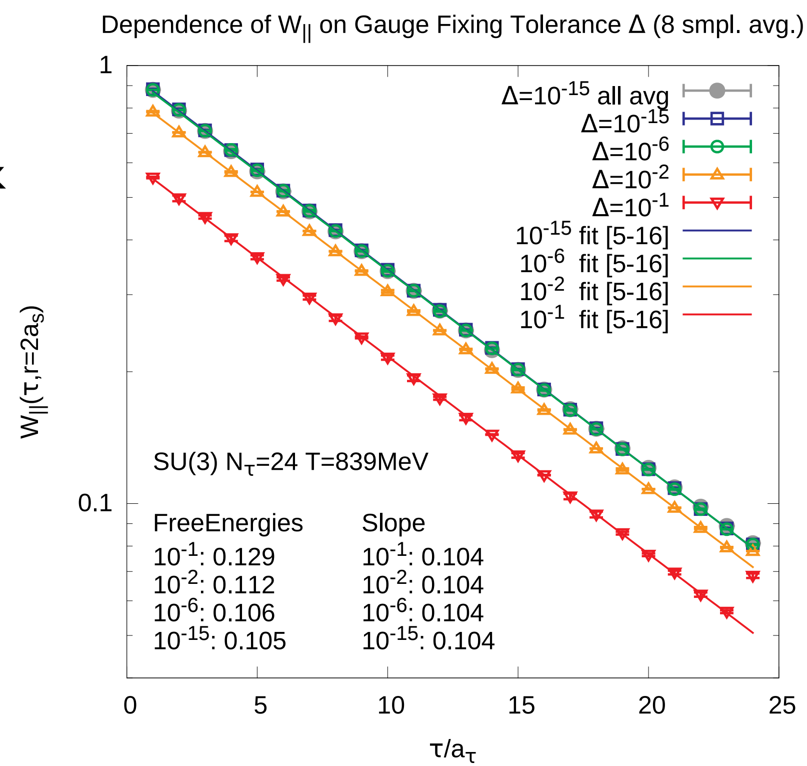


In LO HTL perturbation theory: $W_\square$ and $W_{||}^{CG}$ encode same spectral peak

Is there gauge invariant information in $W_{||}$?

Relax the gauge fixing

$$\nabla \mathbf{A} < \Delta$$



- $W_{||}$: potential information remains gauge independent