We start with the *Cornell + String Breaking* potential that is often employed in phenomenological studies,

$$V^{vac}(r) = \begin{cases} \sigma r - \frac{\tilde{\alpha}_s}{r}; & r < r_{sb}, \\ \sigma r_{sb} - \frac{\tilde{\alpha}_s}{r}; & r \geq r_{sb}, \end{cases} \tag{1}$$

where $r_{sb} \simeq 1.1$GeV is the string-breaking radius. Following Thakur, Kakade, and Patra [?], we want to transform this into Fourier space, multiply by the in-medium complex permittivity and then transform back to real space. In Fourier transforming Eq. (1), the angular parts can be carried out as usual and the radial integration must be split into two regions $[0, r_{sb}]$ and $[r_{sb}, \infty]$. The second of these must be regulated in a suitable manner to ensure convergence; as in [?] we damp the integrand with an exponential factor e^{-ar} before taking the limit $a \to 0$. The result for the string part of the potential is

$$V_S^{vac}(p) = 4\pi\sigma r_{sb}\left(\frac{-2+(2-p^2r_{sb}^2)\cos(pr_{sb})+2pr_{sb}\sin(pr_{sb})}{r_{sb}\,p^4} + \frac{pr_{sb}\cos(pr_{sb})-\sin(pr_{sb})}{p^3}\right). \tag{2}$$

The medium modifications are accounted for by multiplying by the complex permittivity as follows:

$$V_S(r) = \int \frac{\mathrm{d}^3p}{(2\pi)^3} V_S^{vac}(p)\,\varepsilon^{-1}(\mathbf{p}, m_D)\,e^{i\mathbf{pr}}, \tag{3}$$

where

$$\varepsilon^{-1}(\mathbf{p}, m_D) = \frac{p^2}{p^2+m_D^2} - i\pi T\frac{pm_D^2}{(p^2+m_D^2)^2}. \tag{4}$$

For the real part, this integration can be performed analytically, giving:

$$\mathrm{Re}V_S(r) = \begin{cases} \dfrac{\sigma e^{-m_D r}}{m_D^2 r}[2 + e^{m_D r}(-2 + e^{-m_D r_{sb}}(2+m_D r_{sb})\sinh(m_D r))] + \mathrm{Re}\,c_S; & r < r_{sb}, \\ \dfrac{\sigma e^{-m_D r}}{m_D^2 r}\,[2 - 2\cosh(m_D r_{sb}) + m_D r_{sb}\sinh(m_D r)] + \mathrm{Re}\,c_S; & r \geq r_{sb}, \end{cases} \tag{5}$$

where $\mathrm{Re}\,c_S$ is chosen to ensure $\mathrm{Re}V_S(r \to 0) = 0$ and $\lim_{m_D\to 0}\mathrm{Re}V_S(r) = \sigma r$, namely

$$\mathrm{Re}\,c_S = \sigma r_{sb} - \frac{e^{-m_D r_{sb}}}{mD}\left[2 + m_D r_{sb} + \sigma e^{m_D r_{sb}}(m_D r_{sb} - 2)\right]. \tag{6}$$

For the imaginary part, Eq. (3) can not be calculated analytically. However the angular integrations are performed to give the 'most-analytical' form:

$$\mathrm{Im}V_S(r) = -\frac{2\sigma}{\pi}\int_0^\infty \mathrm{d}p\;p^2\frac{\sin(pr)}{pr}V_S^{vac}(p)\,\mathrm{Im}\,\varepsilon^{-1}(p, m_D) + \mathrm{Im}\,c_S, \tag{7}$$

where again the constant term is chosen to ensure $\mathrm{Im}V_S(r \to 0) = 0$:

$$\mathrm{Im}\,c_S = \frac{2\sigma}{\pi}\int_0^\infty \mathrm{d}p\;p^2 V_S^{vac}(p)\,\mathrm{Im}\,\varepsilon^{-1}(p, m_D). \tag{8}$$

Eq. (7) is straightforward to evaluate numerically.

After performing the Debye-mass fitting, the real part is able to reproduce the lattice data and the imaginary part flattens to a constant as required. See Figures.

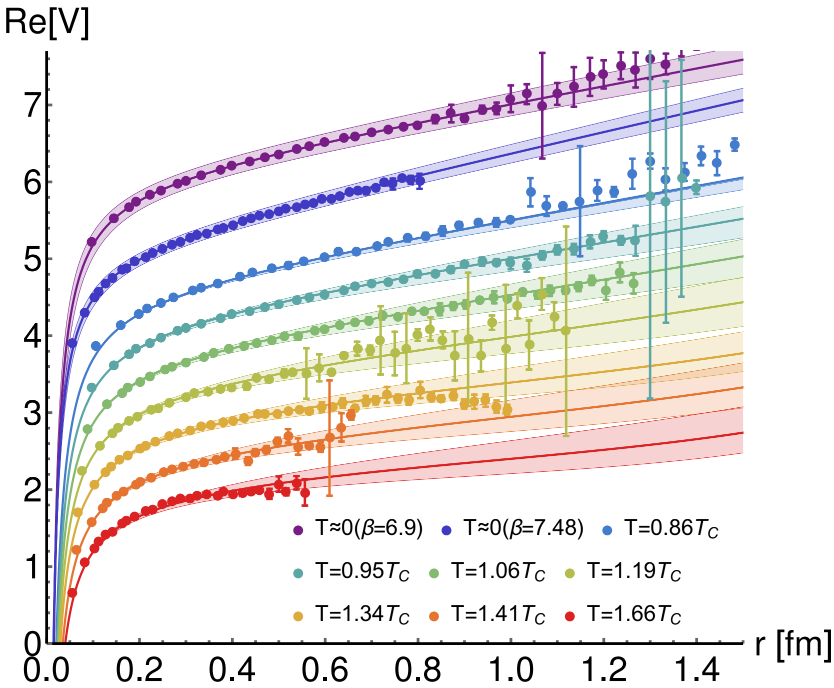

Figure 1: Fitting of the real part.

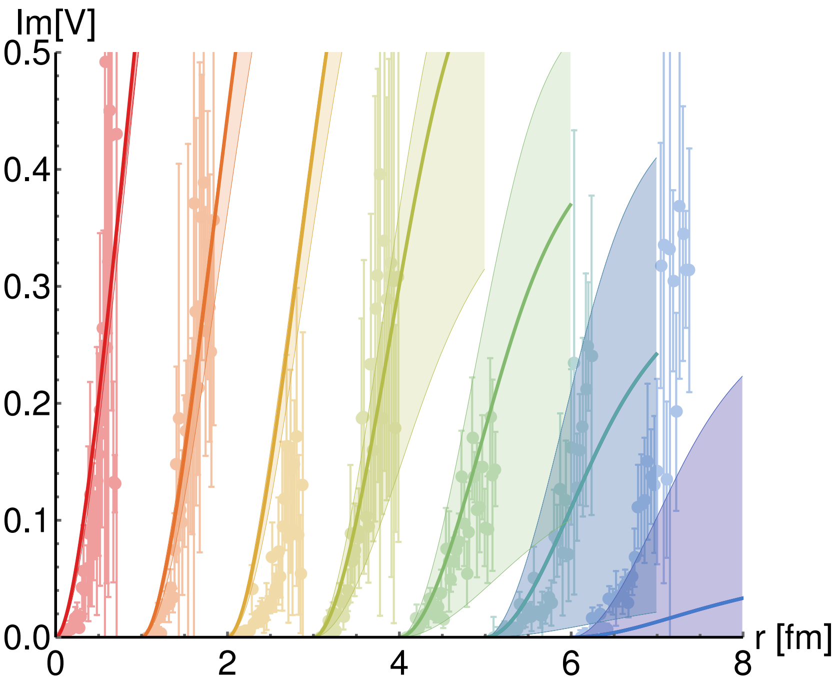

Figure 2: Resulting imaginary part.

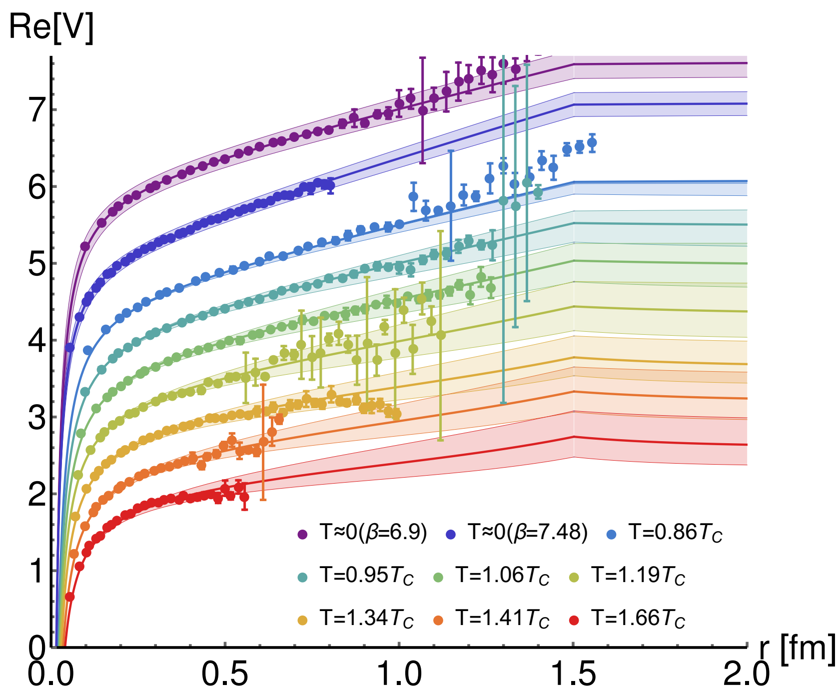

Figure 3: The real part contains a jump in the first derivative when $r = r_{sb}$. For $r > r_{sb}$ it quickly asymptotes to a constant.

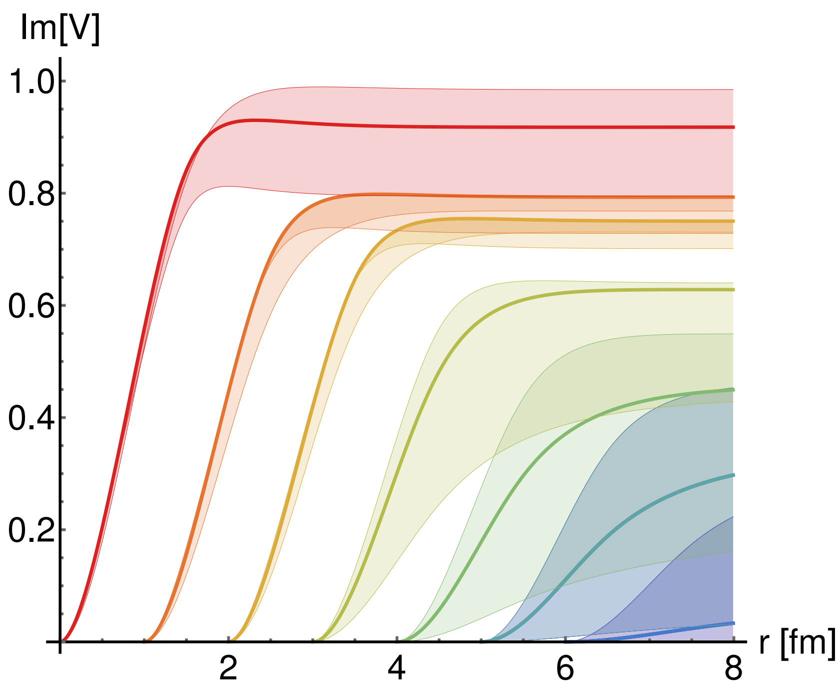

Figure 4: The imaginary part flattens off as required.