

Next-to-Leading Order Debye Mass for the Quark-Gluon Plasma

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The Debye screening mass for a quark-gluon plasma at high temperature is calculated to next-to-leading order in the QCD coupling constant from the correlator of two Polyakov loops. The result agrees with the screening mass defined by the location of the pole in the gluon propagator as calculated by Rebhan. It is logarithmically sensitive to nonperturbative effects associated with the screening of static chromomagnetic fields.

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One of the fundamental properties of a plasma is the Debye screening mass m_D , whose inverse is the screening length for electric fields in the plasma. The definition of the Debye mass is conventionally given in terms of the small-momentum ($k \rightarrow 0$) limit of the static ($\omega = 0$) Coulomb propagator $[k^2 + \Pi_{00}(0, k)]^{-1} = [k^2 \epsilon(0, k)]^{-1}$, where $\Pi_{00}(\omega, k)$ is the longitudinal photon self-energy function and $\epsilon(\omega, k)$ is the corresponding dielectric function,

$$m_D^2 = \lim_{k \rightarrow 0} \Pi_{00}(0, k). \quad (1)$$

the Debye mass can alternatively be defined in terms of the location of the pole in the static propagator for complex k ,

$$k^2 + \Pi_{00}(0, k) = 0 \quad \text{at } k^2 = -m_D^2. \quad (2)$$

At leading order in the coupling constant, the static self-energy has the simple form $\Pi_{00}(0, k) = m_D^2$ independent of k , and the definitions (1) and (2) are equivalent. Beyond leading order in the coupling constant, they no longer need be equivalent. A fundamental question of plasma physics is then this: What is the correct general definition of the Debye mass?

This question applies equally well to a quark-gluon plasma, where the Debye mass describes the screening of chromoelectric fields. Because the coupling constant of quantum chromodynamics (QCD) is relatively large, higher order corrections to the Debye mass are probably not negligible. The question of the correct definition therefore becomes one of practical importance. In the case of QCD, the longitudinal gluon self-energy function $\Pi_{00}(\omega, k)$ is gauge dependent. If the Debye mass is relevant to the screening of chromoelectric fields, then it must be gauge invariant. Thus, gauge invariance can be used as a guide to the correct definition of the Debye mass. Formal arguments due to Kobes, Kunstatter, and Rebhan indicate that, in spite of the gauge dependence of the gluon propagator, the locations of its poles are gauge invariant [1]. This suggests that (2) is the correct general definition. However, it is important to back up these formal arguments with explicit calculations.

There have been a number of calculations of the next-to-leading order correction to the Debye mass using the

conventional definition (1) [2,3]. The results are infrared finite but gauge dependent. The naive application of the pole definition (2) gives a gauge-dependent result, which is also infrared-divergent. However, it was recently shown by Rebhan [4] that, with a careful treatment of infrared effects, the pole definition does in fact give a gauge-invariant result.

Since the calculation of Rebhan involves subtle interchanges of limits, it is desirable to have an independent verification of this result. It is also desirable to calculate the next-to-leading order Debye mass directly from a gauge-invariant quantity, to provide assurance that it is relevant to physical quantities. The simplest such quantity is the correlation function of two Polyakov loops. A previous calculation of this correlator to next-to-leading nontrivial order by Nadkarni [3] gave results that seemed to be incompatible with simple Debye screening. In this Letter, we reexamine this calculation and show that, by careful treatment of infrared effects, it can be used to extract the Debye mass to next-to-leading order. The result agrees with that obtained by Rebhan from the pole in the gluon propagator. These results, together with the general arguments of Ref. [1], provide compelling evidence that the pole definition (2) is the correct definition of the Debye mass beyond leading order in the coupling constant.

We begin by reviewing the calculation by Rebhan. At leading order in the QCD coupling constant g , the Debye mass is given by a straightforward perturbative calculation, $m_D = \sqrt{(2N_c + N_f)/6} gT$, where $N_c = 3$, N_f is the number of flavors of light quarks, and T is the temperature. At next-to-leading order, it is necessary to resum perturbation theory by including the Debye mass in the Coulomb propagator. The contribution of order g^3 to the static longitudinal gluon self-energy is

$$\begin{aligned} \delta \Pi_{00}(0, k) = N_c g^2 T \int \frac{d^3 p}{(2\pi)^3} \\ \times \left[\frac{1}{\mathbf{p}^2 + m_D^2} + \frac{2(m_D^2 - k^2)}{\mathbf{p}^2(\mathbf{q}^2 + m_D^2)} \right. \\ \left. - \xi \frac{(k^2 + m_D^2)(\mathbf{p}^2 + 2\mathbf{p} \cdot \mathbf{k})}{(\mathbf{p}^2)^2(\mathbf{q}^2 + m_D^2)} \right], \quad (3) \end{aligned}$$

where $\mathbf{q} = \mathbf{p} + \mathbf{k}$. In (3), ultraviolet divergences are understood to be removed using dimensional regularization. Integrating over \mathbf{p} , we obtain the result

$$\delta\Pi_{00}(0, k) = \frac{N_c g^2 T}{2\pi} \times m_D \left[\frac{m_D^2 - k^2}{2im_D k} \ln \frac{m_D + ik}{m_D - ik} - \frac{\xi + 1}{2} \right]. \quad (4)$$

Upon taking the limit $k \rightarrow im_D$, we find a correction that is logarithmically divergent and gauge dependent. However, as pointed out by Rebhan, for k near im_D , the integral in (3) is extremely sensitive to the infrared region $\mathbf{p} \rightarrow 0$. He regularized the infrared region by replacing $1/\mathbf{p}^2$ by $1/(\mathbf{p}^2 + m_{\text{mag}}^2)$. Taking the limit $k \rightarrow im_D$ in the presence of the regulator and then taking the limit $m_{\text{mag}} \rightarrow 0$, he obtained the gauge-invariant result

$$\delta m_D = \frac{N_c g^2 T}{4\pi} \left[\ln \frac{2m_D}{m_{\text{mag}}} - \frac{1}{2} \right]. \quad (5)$$

The logarithm indicates that the Debye mass at next-to-leading order is weakly sensitive to the screening of the static chromomagnetic interaction, which is dominated by nonperturbative effects. The particular regulator used by Rebhan corresponds to a model for these nonperturba-

tive effects in which they simply shift the pole in the transverse gluon propagator from $p = 0$ to the imaginary values $p = \pm im_{\text{mag}}$. One could equally well use dimensional regularization as the infrared regulator. Again the pole definition (2) gives a gauge-invariant result which diverges as the regulator is removed.

We next review Nadkarni's calculation of the next-to-leading order correction to the correlator of Polyakov loops. The calculation was carried out in static gauge, where the Polyakov loop reduces to a simple exponential, $\Omega(\mathbf{x}) = \exp[-igA_4(\mathbf{x})/T]$. The correlator of two such operators separated by a distance R is $C_{PL}(R) = \langle \text{Tr}\Omega(0)\text{Tr}\Omega(\mathbf{R}) \rangle / N_c^2$. Nadkarni calculated this correlator to order g^5 and found

$$C_{PL}(R) = 1 + \frac{(N_c^2 - 1)g^4}{8N_c^2 T^2} \times V(R)^2 \left[1 + \frac{N_c g^2 T}{m_D} [f_1(R) + f_2(R)] \right], \quad (6)$$

where $V(R) = e^{-m_D R}/4\pi R$ is the Debye-screened Coulomb potential. The complete expressions for the functions $f_1(R)$ and $f_2(R)$ are given in Ref. [3]. As in the Appendix of Ref. [3], the integrals can be simplified by expanding out the numerators and canceling propagator factors wherever possible. Keeping only those terms that grow at least linearly with R , the functions reduce to

$$f_1(R) = -2m_D V(R)^{-1} \int \frac{d^3 k}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{R}} \frac{1}{(k^2 + m_D^2)^2} \int \frac{d^3 p}{(2\pi)^3} \left(\frac{1}{\mathbf{p}^2 + m_D^2} + \frac{4m_D^2}{\mathbf{p}^2(\mathbf{q}^2 + m_D^2)} \right), \quad (7)$$

$$f_2(R) = -2m_D V(R)^{-2} \int \frac{d^3 k}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{R}} (k^2 + 2m_D^2) \int \frac{d^3 p_1}{(2\pi)^3} \times \int \frac{d^3 p_2}{(2\pi)^3} \frac{1}{(\mathbf{p}_1^2 + m_D^2)(\mathbf{q}_1^2 + m_D^2)(\mathbf{p}_2^2 + m_D^2)(\mathbf{q}_2^2 + m_D^2)(\mathbf{p}_1 - \mathbf{p}_2)^2}, \quad (8)$$

where $\mathbf{q}_i = \mathbf{p}_i + \mathbf{k}$. Ultraviolet divergences are understood to be regularized using dimensional regularization. Evaluating the integrals in the limit $R \rightarrow \infty$, Nadkarni obtained

$$f_1(R) \rightarrow \frac{m_D R}{4\pi} [-2 \ln(2m_D R) + 3 - 2\gamma], \quad (9)$$

$$f_2(R) \rightarrow \frac{m_D R}{4\pi} [\ln(m_D R) + \gamma], \quad (10)$$

where γ is Euler's constant. The terms proportional to R can be absorbed into an order- g^2 correction to the Debye mass in the g^4 term in (6), which is proportional to

$\exp(-2m_D R)$. The terms proportional to $R \ln R$, however, cannot be absorbed into the Debye mass. Nadkarni concluded that the next-to-leading order correction to the Debye mass is undefined.

We now show how the next-to-leading order Debye mass can be extracted from the calculation of the correlator of Polyakov loops. The $R \log R$ terms in (9) and (10) indicate that the integrals in (7) and (8) are sensitive to the infrared region. As a regularization of the infrared region, we replace the massless propagator $1/\mathbf{p}^2$ in (7) by $1/(\mathbf{p}^2 + m_{\text{mag}}^2)$ and similarly for $1/(\mathbf{p}_1 - \mathbf{p}_2)^2$ in (8). If we first set $m_{\text{mag}} = 0$ and then evaluate the integrals in the limit $R \rightarrow \infty$, we recover Nadkarni's results (9) and (10). However, if we take the limit $R \rightarrow \infty$ in the presence of the regulator, we obtain a different result.

We first consider $f_1(R)$ in (7). Evaluating the first integral over \mathbf{p} with dimensional regularization, we obtain the constant value $-m_D/4\pi$. The second integral over \mathbf{p} gives a function of \mathbf{k} that has branch points at $k = \pm i(m_D + m_{\text{mag}})$. The integral over \mathbf{k} can be evaluated by expressing it as an integral over k along the entire real axis and then deforming the contour into the upper-half complex plane. It receives contributions from the double pole at $k = im_D$ and from the branch cut beginning at $k = i(m_D + m_{\text{mag}})$. In the limit $R \rightarrow \infty$, the contribution from the branch cut is exponentially suppressed by a factor of $e^{-m_{\text{mag}}R}$ and can therefore be neglected. The second integral over \mathbf{p} in (7) can therefore be replaced by its value at $k = im_D$, and it reduces to $\ln(2m_D/m_{\text{mag}})/(8\pi m_D)$ in the limit $m_{\text{mag}} \ll m_D$. Evaluating the remaining contour integral over k , we find that the asymptotic behavior of $f_1(R)$ as $R \rightarrow \infty$ is

$$f_1(R) \rightarrow \frac{m_DR}{4\pi} \left[-2 \ln \frac{2m_D}{m_{\text{mag}}} + 1 \right]. \quad (11)$$

We next consider $f_2(R)$ in (8). In the limit $m_{\text{mag}} \rightarrow 0$, the integral over \mathbf{p}_1 and \mathbf{p}_2 gives $\ln[(k^2 + 4m_D^2)/4m_D^2]/[16\pi^2 k^2(k^2 + 4m_D^2)]$, which has coincident poles and logarithmic branch points at $k = \pm 2im_D$. In the presence of the regulator, the integral has branch points at $k = \pm 2im_D$ and $k = \pm i(2m_D + m_{\text{mag}})$. The integral over \mathbf{k} can again be evaluated by expressing it as an integral over k along a contour that is deformed into the upper-half complex plane. The contribution from the branch cut beginning at $k = i(2m_D + m_{\text{mag}})$ is suppressed by a factor of $e^{-m_{\text{mag}}R}$ and can be neglected in the limit $R \rightarrow \infty$. The contribution from the branch point beginning at $k = 2im_D$ has the asymptotic behavior

$$f_2(R) \rightarrow \frac{m_D^2}{2\pi m_{\text{mag}}^2} \ln R. \quad (12)$$

There is a quadratic infrared divergence that is cut off by the magnetic mass, but there are no terms linear in R .

Inserting (11) into (6), we find that the correction proportional to R has precisely the form $-2\delta m_D R$, where δm_D is given by Rebhan's expression (5). This correction can be absorbed into the order- g^4 term in (6) by making the substitution $m_D \rightarrow m_D + \delta m_D$ in the potential $V(R)$. It therefore represents a correction to the Debye mass. This result was obtained by using a magnetic mass as an infrared regulator, which separated the poles associated with the Debye mass from the branch cuts associated with the emission of transverse gluons. We could equally well have used a different infrared regulator, such as dimensional regularization. As long as the same regulator is used in the calculation of the correlator of Polyakov loops and in the calculation of the pole in the gluon propagator, the next-to-leading order correction to the Debye mass will be the same. This is evident from

comparing the relevant parts of the integrals over \mathbf{p} in (3) and (7).

The fact that the Debye mass depends on an infrared cutoff at next-to-leading order should not be too disturbing. At large R , the correlator of Polyakov loops is dominated by a sum of terms that fall exponentially like $\exp(-MR)$, with M being the mass of a color-singlet bound state in $(2+1)$ -dimensional extended QCD [5]. This field theory is an $SU(3)$ gauge theory describing magnetic gluons, coupled to an adjoint scalar field describing electric gluons. One of the bound states consists predominantly of a pair of electric gluons, and its mass is $2m_D$ plus a small binding energy. It is the contribution of this bound state that corresponds to the perturbative contribution to the Polyakov loop correlator calculated above. The infrared divergence in the next-to-leading order correction to $2m_D$ is a signal that the binding energy of this state cannot be calculated in perturbation theory.

As an aside, it should be pointed out that the truly asymptotic behavior of the Polyakov loop correlator as $R \rightarrow \infty$ is dominated not by the lowest bound state of electric gluons but by a magnetic glueball [3]. Its coupling to the Polyakov loop is suppressed by extra powers of the coupling constant, but its mass is much less than $2m_D$ at weak coupling, so it dominates at sufficiently large R .

We have calculated the Debye mass for a quark-gluon plasma to next-to-leading order in the QCD coupling constant from the gauge-invariant correlation function of Polyakov loops. The result agrees with that obtained by Rebhan from the pole in the gluon propagator. It is gauge invariant, but depends logarithmically on nonperturbative effects associated with the screening of static chromomagnetic fields. This result provides support for the general definition of the Debye mass in terms of the location of the pole in the propagator of the gauge field.

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