Asymptotic Behavior of the Correlator for Polyakov Loops

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(Received 5 October 1994)

The correlator for Polyakov loop operators in high temperature QCD is studied by using dimensionally reduced effective field theories. The Polyakov loop operator is expanded in terms of local gauge-invariant operators constructed out of the magnetostatic gauge field, with coefficients that can be calculated using resummed perturbation theory. The asymptotic behavior of the correlator is $\exp(-MR)/R$, where M is the mass of the lowest-lying glueball in $(2+1)$-dimensional QCD. This result implies that existing lattice calculations of the Polyakov loop correlator at the highest temperatures available do not probe the true asymptotic region.

PACS numbers: 12.38.Mh, 11.10.Wx, 12.38.Gc

One of the basic characteristics of a plasma is the screening of electric fields. The field created by a static charge falls off exponentially beyond the screening radius, whose inverse is called the Debye mass m_D. In a quark-gluon plasma at high temperature T, chromoelectric fields are believed to be screened in a similar way. However, it has proven to be difficult to give a precise definition to the Debye mass in perturbation theory. At leading order in the coupling constant g, m_D is proportional to gT. The next-to-leading order correction to m_D has been calculated using a resummed perturbation theory in which gluon propagator corrections of order g^2T^2 are summed up to all orders [1,2]. The correction is gauge invariant but infrared divergent, indicating a sensitivity to nonperturbative effects involving the scale g^2T.

Debye screening can also be studied nonperturbatively using lattice simulations [3–5]. One of the simplest probes of Debye screening is the correlator of two Polyakov loop operators as a function of their separation R. At lowest order in resummed perturbation theory, the behavior of the correlator is predicted to be $\exp(-2m_DR)/R^2$. By fitting the measured correlator to this form, one can extract a value for m_D. However, the resummed perturbation expansion is known to break down at higher orders, due to contributions that involve the exchange of magnetostatic gluons [6,7]. These contributions, which are inherently nonperturbative, can be expected to dominate at distances R much greater than $1/m_D$. This raises questions about the utility of determining m_D by fitting the correlator to a leading order expression.

In this paper, we determine the true asymptotic behavior of the Polyakov loop correlator at large R. We use effective field theory methods to express the Polyakov loop operator in terms of operators in 3-dimensional QCD. The asymptotic behavior of the Polyakov loop correlator is then given by a simple correlator in this effective theory. It has the form $\exp(-m_HR)/R$, where m_H is the mass of the lowest glueball state in $(2+1)$-dimensional QCD. The coefficient of the exponential is of order g^{12}. Our result implies that the asymptotic behavior of the Polyakov loop correlator is dominated by magnetostatic effects which have little to do with Debye screening.

We wish to study the correlator of Polyakov loop operators in thermal QCD in 4 space-time dimensions with temperature T. The fundamental fields are the gauge field $A_\mu(\mathbf{x},\tau)$, which takes values in the $SU(N_c)$ algebra, and the quark fields $\psi^i(\mathbf{x},\tau)$, whose indices range over N_c colors and N_f flavors. The gluon fields satisfy periodic boundary conditions in the Euclidean time τ with period $\beta = 1/T$, while the quark fields obey antiperiodic boundary conditions. The Polyakov loop operator is given by the trace of a path-ordered exponential:

$$L(\mathbf{x}) = \frac{1}{N_c}\,\mathrm{tr}\,\mathcal{P}\exp\left(-ig\int_0^\beta d\tau\, A_0(\mathbf{x},\tau)\right). \qquad (1)$$

The connected part of the correlator of two Polyakov loop operators separated by a distance R is

$$C(R) = \langle L^\dagger(\mathbf{0})\,L(\mathbf{R})\rangle - \langle L(\mathbf{0})\rangle^2. \qquad (2)$$

The Polyakov loop operator (1) creates two or more electric gluons (A_0 fields). The diagram for the Polyakov loop correlator (2) which is leading order in a naive expansion in powers of g is the one-loop diagram in which two electrostatic gluons are exchanged. It gives the asymptotic behavior $C(R) \sim 1/R^2$. This is not the correct asymptotic behavior for two reasons. First, thermal loop corrections generate a Debye screening mass for electrostatic gluons, so that the potential due to the exchange of an electrostatic gluon actually falls off exponentially, rather than like $1/R$ as in naive perturbation theory. Secondly, the true asymptotic behavior at sufficiently large temperature actually comes from higher order diagrams that involve the exchange of magnetostatic gluons. Our problem is to determine this asymptotic behavior.

An elegant way to solve this problem is to construct a sequence of two effective field theories which reproduce static correlation functions at successively longer distances. Thermal QCD can be used to calculate the static correlator (2) for any separation R. Ordinary perturbation theory in g^2 is accurate only for R of order $1/T$ or smaller, but the correlator can be calculated for larger

 0031-9007/95/74(18)/3530(4)$06.00

R by using nonperturbative methods such as lattice gauge theory simulations. The first of the two effective field theories is constructed so that it reproduces static correlators at distances R of order $1/(gT)$ or larger. In this effective theory, perturbation theory in g can be used to calculate correlators at distances of order $1/(gT)$, but lattice simulations are required at larger R. The second effective field theory is constructed so that it reproduces correlators at distances of order $1/(g^2T)$ or larger. This field theory is inherently nonperturbative. It is a confining theory whose correlators can be calculated by lattice simulations [8]. Nevertheless, we can use this field theory to determine unambiguously the asymptotic behavior of the Polyakov loop correlator.

The first effective field theory, which we call electrostatic QCD (EQCD), is the 3-dimensional Euclidean field theory obtained by dimensional reduction [9]. The fields in this theory are the electrostatic gluon field $A_0(\mathbf{x})$ and the magnetostatic gluon field $A_i(\mathbf{x})$. Up to a normalization, they can be identified with the zero-frequency modes of the gluon field $A_\mu(\mathbf{x}, \tau)$ for thermal QCD in a static gauge [10]. The action for the effective field theory is

$$S_{\mathrm{EQCD}} = \int d^3x \left\{ \frac{1}{2}\,\mathrm{tr}(G_{ij}G_{ij}) + \mathrm{tr}(D_iA_0D_iA_0) + m_{\mathrm{el}}^2 \mathrm{tr}(A_0A_0) + \delta\mathcal{L}_{\mathrm{EQCD}} \right\}, \tag{3}$$

where $G_{ij} = \partial_i A_j - \partial_j A_i + i g_E[A_i, A_j]$ is the magnetostatic field strength, $D_iA_0 = \partial_i A_0 + i g_E[A_i, A_0]$, and g_E is the coupling constant of the 3-dimensional gauge theory. The action for this effective field theory is invariant under static $\mathrm{SU}(N_c)$ gauge transformations. If the fields A_0 and A_i are assigned dimension 1/2, then the operators shown explicitly in (3) have dimensions 3, 3, and 1. The term $\delta\mathcal{L}_{\mathrm{EQCD}}$ in (3) includes all other local gauge-invariant operators of dimension 2 and higher that can be constructed out of A_0 and A_i. The effective theory EQCD is completely equivalent to thermal QCD at distance scales of order $1/(gT)$ or larger. The gauge coupling constant g_E, the mass parameter m_{el}^2, and the parameters in $\delta\mathcal{L}_{\mathrm{EQCD}}$ can be tuned as functions of T so that correlators of gauge-invariant operators in EQCD agree with the corresponding static correlators in thermal QCD to any desired accuracy for $R \gg 1/T$ [11]. By matching the two theories at tree level, we find that the gauge coupling constant is $g_E = g(T)\sqrt{T}$, where $g(T)$ is the running coupling constant at the momentum scale T. The mass parameter m_{el}^2 in (3) is the contribution to the square of the Debye screening mass from short distances of order $1/T$. At leading order in g, it is

$$m_{\mathrm{el}}^2 = \frac{2N_c + N_f}{6}\, g^2(T)\, T^2. \tag{4}$$

The coefficients of some of the higher-dimension operators in $\delta\mathcal{L}_{\mathrm{EQCD}}$ were recently calculated to leading order by Chapman [12]. Since the parameters in EQCD only take into account the effects of the momentum scale T, they can be calculated as perturbation series in $g^2(T)$. For example, the next-to-leading order correction to m_{el}^2 is of order $g^4(T)$. The Debye screening mass m_D defined by the location of the pole in the gluon propagator is also given at leading order by (4), but m_D^2 has corrections of order g^3 that arise from the momentum scale gT [1,2].

The effective field theory EQCD was used by Nadkarni to study the Polyakov loop correlator beyond leading order in the coupling constant [6]. In EQCD, the Polyakov loop operator is given by a simple exponential:

$$L(\mathbf{x}) = \frac{1}{N_c}\,\mathrm{tr}\,\exp\!\left[-igA_0(\mathbf{x})/\sqrt{T}\right]. \tag{5}$$

The one-loop diagram involving the exchange of two electrostatic gluons gives a correlator that falls exponentially due to electric screening:

$$C(R) \longrightarrow \frac{(N_c^2 - 1)\, g^4}{8N_c^2T^2}\left(\frac{e^{-m_{\mathrm{el}}R}}{4\pi R}\right)^2. \tag{6}$$

The corrections to this correlator of order g^6 were calculated by Nadkarni [6]. They have the asymptotic behavior $e^{-2m_{\mathrm{el}}R}\ln(R)/R$. In a recent reexamination of this calculation [2], it has been shown that one can extract from it the correction of order g^3 to the square of the Debye mass that was first obtained by Rebhan from the pole in the gluon propagator [1].

As pointed out by Nadkarni [6], the asymptotic behavior of the Polyakov loop correlator comes not from the one-loop diagram in which two electrostatic gluons are exchanged, but instead from higher-loop diagrams that involve the exchange of magnetostatic gluons. The simplest such diagrams are the 3-loop diagrams in Fig. 1. In perturbation theory, magnetostatic gluons remain massless in EQCD, and the diagrams in Fig. 1 give a contribution to the correlator that falls like $1/R^6$. However, this is not the correct asymptotic behavior, since nonperturbative effects become important at a distance R of order $1/(g^2T)$.

In order to determine the true asymptotic behavior of the Polyakov loop correlator, it is useful to construct a second effective field that reproduces static correlators at distances R of order $1/(g^2T)$ or larger. Implicit in this construction is the assumption that the gauge symmetry

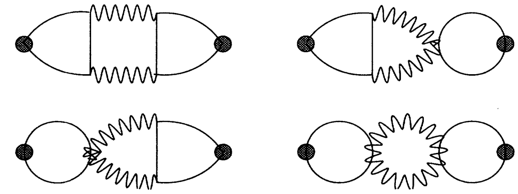

FIG. 1. Three-loop diagrams in EQCD that contribute to the correlator of two Polyakov loop operators. The solid lines are electrostatic gluons and the wavy lines are magnetostatic gluons.

of EQCD is realized in a confining phase and not in a Higgs phase. The second effective theory, which we call magnetostatic QCD (MQCD), is a pure SU(3) gauge theory in 3 space dimensions. The only fields are the magnetostatic gluon fields $A_i(\mathbf{x})$. The action is

$$S_{\mathrm{MQCD}} = \int d^3x \{\tfrac{1}{2}\,\mathrm{tr}(G_{ij}G_{ij}) + \delta\mathcal{L}_{\mathrm{MQCD}}\}, \tag{7}$$

where $\delta\mathcal{L}_{\mathrm{MQCD}}$ includes all local gauge-invariant operators that can be constructed out of A_i. The gauge coupling constant g_M of MQCD and the parameters in $\delta\mathcal{L}_{\mathrm{MQCD}}$ can be tuned so that MQCD is completely equivalent to EQCD, and therefore to thermal QCD, at distances of order $1/(g^2T)$ or larger. If $g(T)$ is sufficiently small, the parameters of MQCD can be obtained by perturbative calculations in EQCD. The expansion parameter is g_E^2/m_{el}, which is of order $g(T)$. At leading order, we have $g_M = g_E$. The gauge theory MQCD itself is inherently nonperturbative. Any perturbative expansion in powers of g_M is hopelessly plagued with infrared divergences. Thus the correlation functions in this theory must be calculated nonperturbatively using lattice simulations.

The static correlators of gauge-invariant operators in EQCD are reproduced at distances $R \gg 1/m_{\mathrm{el}}$ by the corresponding operators in MQCD. In EQCD, the Polyakov loop operator (5) creates electrostatic gluons only. It couples to magnetostatic gluons through loop diagrams involving electrostatic gluons, such as the one-loop diagrams in Fig. 2. Because of screening, electrostatic gluons can only propagate over distances of order $1/m_{\mathrm{el}}$. Thus, for magnetostatic gluons with wavelengths much greater than $1/m_{\mathrm{el}}$, the Polyakov loop operator behaves like a pointlike operator that creates magnetostatic gluons. It can therefore be expanded out in terms of local gauge-invariant operators constructed out of the field A_i:

$$L(\mathbf{x}) = \lambda_1(g)\,1 + \frac{\lambda_{G^2}(g)}{m_{\mathrm{el}}^3}\,G^2(\mathbf{x}) + \dots, \tag{8}$$

where $G^2 \equiv \mathrm{tr}(G_{ij}G_{ij})$ and ... represents operators of dimension 5 or larger, such as $G^3 \equiv g_M \mathrm{tr}(G_{ij}G_{jk}G_{ki})$.

Like the parameters in the effective action (7), the coefficients in the operator expansion (8) are computable in terms of the parameters of the EQCD action using perturbation theory in g. Both EQCD and MQCD reproduce the nonperturbative dynamics of thermal QCD at distances much greater than $1/m_{\mathrm{el}}$. Their perturbation expansions also give equivalent (although incorrect) descriptions of the long-distance dynamics. Since the coefficients in the operator expansion (8) are insensitive to the long-distance dynamics, the equivalence between perturbative EQCD and perturbative MQCD can be exploited as a device to compute the coefficients.

We proceed to calculate the coefficient λ_{G^2} in (8) to leading order in g. The simplest quantity that can be used to calculate λ_{G^2} is the coupling of the Polyakov loop operator to two long-wavelength magnetostatic gluons, which we denote by $\langle 0|L(\mathbf{0})|gg\rangle$. We take the gluons to have momenta $\mathbf{k}_1$ and $\mathbf{k}_2$, vector indices i and j, and color indices a and b. In perturbative MQCD, we can read off the coupling of the operator $L(\mathbf{0})$ to the two gluons directly from the expression (8) for the Polyakov loop operator:

$$\langle 0|L(\mathbf{0})|gg\rangle = \frac{2\lambda_{G^2}}{m_{\mathrm{el}}^3}\,\delta^{ab}(-\mathbf{k}_1\cdot\mathbf{k}_2\delta^{ij} + k_2^ik_1^j). \tag{9}$$

In perturbative EQCD, the coupling is given by the sum of the one-loop diagrams in Fig. 2:

$$\langle 0|L(\mathbf{0})|gg\rangle = \frac{g^4}{2}\,\delta^{ab}\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{\mathbf{p}^2 + m_{\mathrm{el}}^2}\,\frac{1}{(\mathbf{p}+\mathbf{k}_1+\mathbf{k}_2)^2 + m_{\mathrm{el}}^2}\left(\delta^{ij} + \frac{(2\mathbf{p}+\mathbf{k}_1)^i\,(2\mathbf{p}+2\mathbf{k}_1+\mathbf{k}_2)^j}{(\mathbf{p}+\mathbf{k}_1)^2 + m_{\mathrm{el}}^2}\right). \tag{10}$$

Expanding the integrand out to second order in $\mathbf{k}_1$ and $\mathbf{k}_2$ and evaluating the loop integrals, we find that (10) reduces to

$$\langle 0|L(\mathbf{0})|gg\rangle = \frac{g^4}{192\pi m_{\mathrm{el}}^3}\,\delta^{ab}(-\mathbf{k}_1\cdot\mathbf{k}_2\delta^{ij} + k_2^ik_1^j). \tag{11}$$

Comparing (9) and (11), we can read off the coefficient λ_{G^2}:

$$\lambda_{G^2} = \frac{g^4}{384\pi}. \tag{12}$$

Having determined the coefficient λ_{G^2} in the operator expansion (8) to leading order in g, we can now express the Polyakov loop correlator (2) in terms of correlators in MQCD:

$$C(R) = \left(\frac{\lambda_{G^2}}{m_{\mathrm{el}}^3}\right)^2\langle G^2(\mathbf{0})\,G^2(\mathbf{R})\rangle + \dots. \tag{13}$$

The ... in (13) represents the contributions of higher-dimension operators in the operator expansion (8), such as $G^3(\mathbf{x})$.

The asymptotic behavior of a correlator in MQCD, such as the one in (13), is related to the spectrum of QCD in (2 + 1) space-time dimensions. This is a confining gauge theory with a dynamically generated mass gap M_H between the vacuum and the state of next lowest energy. The single particle states in the spectrum are bound states of gluons (glueballs), and M_H is the mass of the lightest glueball. Assuming that the lowest glueball H is a scalar particle, the Fourier transform of the correlator $\langle G^2(\mathbf{0})\,G^2(\mathbf{R})\rangle$ has a pole at $k^2 = -M_H^2$, as well as poles

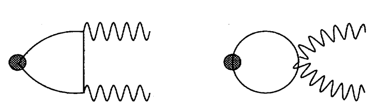

FIG. 2. One-loop diagrams in EQCD that couple a Polyakov loop operator to two magnetostatic gluons.

and branch cuts that are farther from the real k axis. The asymptotic behavior in R is dominated by the pole at $k^2 = -M_H^2$. Denoting the residue of the pole by $|\langle 0|G^2(\mathbf{0})|H\rangle|^2$, the asymptotic behavior is

$$C(R) \to \left(\frac{\lambda_{G^2}}{m_{\rm el}^3}\right)^2 |\langle 0|G^2(\mathbf{0})|H\rangle|^2 \frac{e^{-M_H R}}{4\pi R}. \qquad (14)$$

Both the mass M_H and the coupling strength $\langle 0|G^2(\mathbf{0})|H\rangle$ can be calculated using lattice simulations of MQCD. By dimensional analysis, M_H is proportional to g_M^2, which is of order g^2T. The matrix element $\langle 0|G^2(\mathbf{0})|H\rangle$ is proportional to g_M^5, and therefore the overall coefficient of the exponential in (14) is of order g^{12}. The result (14) is in accord with a conjecture by DeTar that the asymptotic behavior of correlators in high temperature QCD is dominated by color-singlet excitations [13].

It is interesting to compare the asymptotic behavior (14) with the result one would obtain at leading order in perturbation theory in MQCD. The leading order diagram is the one-loop diagram in which two magnetostatic gluons are exchanged. The possibility of a dynamically generated magnetic screening mass $m_{\rm mag}$ can be taken into account by replacing the propagators $1/\mathbf{p}^2$ of the gluons by $1/(\mathbf{p}^2 + m_{\rm mag}^2)$. The resulting expression for the correlator is

$$C(R) \approx 2(N_c^2 - 1)\left(\frac{\lambda_{G^2}}{m_{\rm el}^3}\right)^2 \left(\frac{e^{-m_{\rm mag}R}}{4\pi R^3}\right)^2 \times [6 + 12m_{\rm mag}R + 10(m_{\rm mag}R)^2 + 4(m_{\rm mag}R)^3]. \qquad (15)$$

The perturbative result, which is obtained by setting $m_{\rm mag} = 0$, falls like $1/R^6$. In the presence of magnetic screening, the asymptotic behavior is $e^{-2m_{\rm mag}R}/R^3$. Thus this model does not give the same asymptotic behavior as (14), even if we make the identification $m_H = 2m_{\rm mag}$.

The correlator of Polyakov loops has been calculated nonperturbatively using lattice simulations [3–5]. Recent studies have found that, at temperatures well above the quark-gluon plasma phase transition, the behavior of the correlator at large R is consistent with the form $e^{-\mu R}/R^n$ with n in the range $1 < n < 2$. At the highest temperatures available, the preferred value of n is close to 2, as predicted by the leading electrostatic contribution (6). At lower temperatures, the preferred value of n is closer to 1, consistent with the true asymptotic result (14). These numerical results have a simple interpretation. The electrostatic contribution (6) falls off like $e^{-2m_{\rm el}R}/(TR)^2$, with a coefficient proportional to g^4. The magnetostatic contribution (14) falls off more slowly like $e^{-M_H R}/(TR)$, but its coefficient is proportional to g^{12}. At the highest temperatures that have been probed by lattice simulations, the running coupling constant $g(T)$ is quite small. Because it has a very small coefficient, the magnetostatic contribution will probably not dominate over the electrostatic contribution until R is much larger than the size of present lattices. Thus the measured correlator behaves like $e^{-\mu R}/R^2$, and the mass μ extracted by fitting the correlator to this form can be interpreted as twice the Debye screening mass. At lower temperatures, the magnetostatic contribution is not so strongly suppressed and the true asymptotic behavior $e^{-\mu R}/R$ is probably observed on the lattice. If this interpretation of the numerical simulations is correct, then the mass μ extracted from fitting the correlator to the form $e^{-\mu R}/R$ has nothing to do with Debye screening, but instead is related to magnetostatic effects. This interpretation could be verified by calculating m_H and $\langle 0|G^2(\mathbf{0})|H\rangle$ using a lattice simulation of MQCD. One could then use (14) to predict quantitatively how large R must be in order to reach the truly asymptotic region of the Polyakov loop correlator.

This work was supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Grant No. DE-FG02-91-ER40684, and by the Ministerio de Educación y Ciencia of Spain. We are grateful to A. Rebhan for pointing out an error in a draft of this paper which significantly altered the conclusions.

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