

Heavy Quark Interactions in Finite Temperature QCD

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Abstract. We study the free energy of a heavy quark-antiquark pair in a thermal medium. We construct a simple ansatz for the free energy for two quark flavors motivated by the Debye-Hückel theory of screening.

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1 Introduction

The theory of quantum chromodynamics (QCD) predicts that at high temperature and density there is phase transition from hadron gas to quark gluon plasma (QGP). This QGP phase is believed to be formed in the collision of very high energy heavy-ions. Various probes have been proposed to detect the formation of QGP in high energy heavy-ion collision experiments. One of the clean signatures of QGP formation is the suppression of the J/Ψ [1]. At high temperatures and density, medium effects screen the interaction between the heavy quark and antiquark pairs ($\bar{Q}Q$). The Debye screening radius characterizes the distance beyond which significant thermal modification arises in the heavy quark potential. Qualitatively, one may expect that the $\bar{Q}Q$ dissociates in the medium when the Debye radius becomes smaller than the radius of $\bar{Q}Q$ bound state. Quantitative predictions, such as dissociation temperatures or energy densities, however, require the study of quarkonia in a medium. Recently there have been some finite temperature studies of $\bar{Q}Q$ spectra in quenched lattice QCD [2]. Previously, the medium dependence of quarkonia had been studied in the Schrödinger equation formalism. For these calculations, one needs the potential between the heavy quark and antiquark in the medium. However, this potential is not yet well understood. Only for very small distances or very high temperatures one can calculate it perturbatively. On the other hand, the free energy $F(r, T)$ of a static quark-antiquark pair can be calculated in lattice QCD, and using thermodynamic relations, one can then derive the change in energy of a thermal medium due to the presence of a $\bar{Q}Q$ pair from this free energy. This may be more closely related to the desired potential. In any case, it is desirable to understand how the free energy $F(r, T)$ depends on r and T . So far, a functional form of $F(r, T)$ in accord with the lattice data over wide range of temperatures is still missing. In this work we construct such a form of $F(r, T)$, based

on studies of screening in the Debye-Hückel theory[5]. Although there are only two parameters in our analytical form of $F(r, T)$, we can fit the lattice data quite well for all r and for a wide range of temperatures, $0 \leq T \leq 2 T_c$.

This paper is organized as follows. In section 2 we discuss the screening of the free energies of static quark anti-quark pair in the presence of medium and derive our ansatz for $F(r, T)$. In section 3 we fit our form of free energy $F(r, T)$ to the lattice results, thus obtaining the temperature dependence of the free parameters in $F(r, T)$. Section 4 contains discussions and conclusions.

2 Medium effects on the interaction of $\bar{Q}Q$

The free energy of a static quark-antiquark at zero temperature is given by [3]

$$F(r, T = 0) = \sigma r - \frac{\alpha}{r}, \quad (1)$$

where σ is the string tension and α is the gauge coupling constant. The first term is due to formation of flux tube or string between the Q and the \bar{Q} when they are pulled apart. This string free energy increases linearly with the separation of the $\bar{Q}Q$. The second term in the above equation corresponds to the Coulomb interaction between the static charges. This free energy $F(r, T = 0)$ describes the charmonium and bottomonium spectrum quite well in terms of α and σ .

The free energy $F(r, T)$ increases linearly with r until at some r_0 , it becomes energetically favorable for the string to break into two heavy-light mesons (D, B) rather than stretching further. After the string breaks at r_0 , the free energy remains constant for all separations r larger than r_0 . The value the free energy $F(r_0, T = 0)$ at the string breaking point r_0 can be calculated. Thus, the masses

of the J/Ψ , the D and $F(r_0, T=0)$ are related by

$$\begin{aligned} 2M_D &= M_{J/\Psi} + EB \\ &= 2m_c + F(r_0, T=0). \end{aligned} \quad (2)$$

Here EB denotes the binding energy and M_D , $M_{J/\Psi}$ and m_c the masses of the D , the J/Ψ and the charm quark, respectively. One can write a similar equation for the bottomonium system. Rearranging the terms, one arrives at the following relation

$$F(r_0, T=0) \simeq 2(M_D - m_c) \simeq 2(M_B - m_b). \quad (3)$$

Assuming universal behavior of the string one will get a universal value of r_0 , as both $2(M_D - m_c)$ and $2(M_B - m_b)$ are roughly equal to 1.2 GeV. Using the conventional value of $\sigma \simeq (0.4 \text{ GeV})^2$ for the string tension, we get

$$r_0 \simeq \frac{2(M_{D,B} - m_{c,b})}{\sigma} \simeq \frac{1.2 \text{ GeV}}{\sigma} \simeq 1.5 \text{ fm}.$$

At finite temperature, medium effects screen the heavy quark-antiquark interactions. The mechanism is quite well understood for Coulombic systems and leads to the well-known Debye-screened Coulomb potential and the corresponding Coulombic free energies [4]. In general we may separate the free energy $F(r, T)$ of a pair of static charges into contributions arising from its potential energy at zero temperature $F(r, T=0)$, and a screening function $f(r, T)$,

$$F(r, T) = F(r, 0)f(r, T). \quad (4)$$

This can be formulated in the framework of Debye-Hückel theory [4] and can easily be solved for Coulombic systems. The Debye-Hückel ansatz has been generalized to a large class of potentials in arbitrary dimensions [5]. In the following we briefly discuss the formalism of the Debye-Hückel theory and will then come back to the problem of finding an appropriate screening ansatz for the heavy quark potential.

2.1 Debye-Hückel Theory

In the framework of Debye-Hückel theory, the free energy $F(r, T)$ can be derived from a corresponding Poisson equation [5]. For example, a free energy F at $T=0$ with the form,

$$F = \beta r^\eta \quad (5)$$

in three space dimensions can be obtained by solving the Poisson equation

$$-\frac{\nabla^2 F}{r^{\eta+1}} + \frac{(\eta+1)}{r^{\eta+2}} \nabla F \cdot \hat{r} = 4\pi\beta\delta(r). \quad (6)$$

In the medium, the interaction between the heavy quark-antiquark changes because the heavy quarks polarize the

medium. In linear response theory this leads to the following substitution for the source/charge term in the Poisson equation

$$\delta(r) \longrightarrow \delta(r) + AF. \quad (7)$$

All effects of the medium are contained in the factor A . In the case of abelian charges, A is function of the density of charges in the medium. In a non-abelian medium, in which we are interested, making an estimate of the density of color charges is very difficult. However, if the free energy is known, one can give an estimate of A . This is what we basically do in this work. A has the mass dimension of 2 in 3-dimensional physical space. Defining the screening mass $\mu = (4\pi\beta A)^{1/(\eta+3)}$, one arrives at the following modified Poisson equation,

$$\frac{1}{r^{\eta+1}} \frac{d^2 F}{dr^2} + \frac{1-\eta}{r^{\eta+2}} \frac{dF}{dr} - \mu^{\eta+3} F = -4\pi\beta\delta(r). \quad (8)$$

The solution of this equation gives the free energy of static charges in a thermal medium interacting through a potential described by equation (5). The equations in terms of screening functions can be obtained by substituting equation (4) in the above equations. In the following we consider these separately for the case of string and Coulomb free energies, by choosing the appropriate values of β, η in equation (5).

2.2 Coulomb Screening

In this section we obtain the r and T dependence of the Coulomb screening function f_c . The temperature dependence appears only through the screening mass μ , as one can see from equation (8). For Coulomb interactions, the two parameters of equation (5) are $\beta = -\alpha$, $\eta = -1$. Substituting the form $F_c(r, T) = -(\alpha/r)f_c(r, T)$ for the Coulomb free energy in equation (8), we get the following equation

$$\frac{1}{r} \frac{d^2 f_c}{dr^2} - \frac{\mu^2}{r} f_c = -4\pi\delta(r). \quad (9)$$

for $f_c(r, T)$. It is easy to see from this equation that $f(r, \mu(T))$ will depend only on the combination μr . The solution is given in terms of the modified Bessel function $K_{1/2}$ and becomes by,

$$\begin{aligned} f_c &= \frac{2^{1/2}}{\Gamma[1/2]} \sqrt{\mu r} K_{\frac{1}{2}}(\mu r) \\ &= e^{-\mu r}, \end{aligned} \quad (10)$$

which is in the familiar Debye screening form. However, in the presence of medium the Coulomb free energy has to satisfy an additional boundary condition when the static charges are infinitely far apart; it must then approach the value $F(r=\infty, T) = -\alpha\mu$ for each pair of opposite color charges [4, 6]. Taking this fact into account, the Coulomb screening function is given by

$$f_c = e^{-\mu r} + \mu r. \quad (11)$$

This form satisfies all the required boundary conditions. The Coulomb free energy in the presence of medium is thus given by

$$F_c(r, T) = -\frac{\alpha}{r} [e^{-\mu r} + \mu r]. \quad (12)$$

2.3 String Screening

Next we will derive the string screening function. As in the Coulomb screening case, the temperature dependence enters in the free energy only through the screening mass μ . For the string interaction, the parameter β of equation (5) is the string tension σ , and $\eta = 1$. We substitute the form $F_s(r, T) = \sigma r f_s(r, T)$ for the string free energy in equation (8), which gives the following Poisson equation for $f_s(r, T)$,

$$\frac{1}{r} \frac{d^2 f_s}{dr^2} + \frac{2}{r^2} \frac{df_s}{dr} - \mu^4 r f_s = 4\pi\delta(r). \quad (13)$$

Here, as in the case of equation (9), the r and μ dependence are such that the solution f_s is a function of the product μr only. The solution of equation (13), which satisfies the boundary condition $f(r, \mu = 0) = 1$, is given by

$$f_s(x) = \frac{1}{x} \left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{x}}{2^{3/4}\Gamma(3/4)} K_{1/4}(x^2) \right], \quad (14)$$

with $x = \mu r$, and where $K_{1/4}$ is the modified Bessel function. The string free energy is now given by

$$F_s(r, T) = \frac{\sigma}{\mu} \left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{x}}{2^{3/4}\Gamma(3/4)} K_{1/4}(x^2) \right]. \quad (15)$$

Because of the medium effects, $F_s(r, T)$ eventually stops rising linearly with r and flattens off for large r , giving

$$F_s(r = \infty, \mu) = \frac{\sigma}{\mu} \frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)}. \quad (16)$$

It is important to note that the treatment of the medium effects in equation (7) is an approximation. In principle, there can be higher order corrections to equation (7) if the medium is not dilute enough. Also one may expect non-trivial confinement effects on equation (7). As a simple ansatz, we here assume that all such effects only change the argument of the modified Bessel function $K_{1/4}$,

$$K_{1/4}(x^2) \rightarrow K_{1/4}(x^2 + \kappa x^4). \quad (17)$$

The resulting higher order corrections affect the solution in the intermediate range of $x = \mu r$. The string free energy in the medium now takes the form,

$$F(r, T) = \frac{\sigma}{\mu} \left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{x}}{2^{3/4}\Gamma(3/4)} K_{1/4}(x^2 + \kappa x^4) \right]. \quad (18)$$

2.4 A Model for Screening of the $\bar{Q}Q$ Free Energy

In the above two subsections we have discussed separately the screening of the two terms contributing to the heavy quark potential given in equation 1. In general, we should now solve the Poisson equation arising in the Debye-Hückel theory for the combined potential. This, however, seems to be difficult to achieve in closed form. At present, we therefore explore a screening ansatz for the heavy quark free energies based on the asymptotic forms discussed above, *i.e.*, we assume that both terms are modified by their appropriate screening functions.

This leads to the modification of the free energy $F(r, T)$ of the form

$$F(r, T) = \sigma r f_s(r, T) - \frac{\alpha}{r} f_c(r, T). \quad (19)$$

This free energy is same as the zero temperature form, modified by the Coulomb and string screening functions, respectively. These screening functions satisfy the boundary conditions

$$\begin{aligned} f_s(r, T) &= f_c(r, T) = 1, & r \rightarrow 0. \\ f_s(r, T) &= f_c(r, T) = 1, & T \rightarrow 0. \end{aligned} \quad (20)$$

The first condition arises because for distances $r \ll 1/T$ the $\bar{Q}Q$ cannot see the medium, the second because there is no medium at $T = 0$.

The total free energy in the medium is then given by the sum of the screened Coulomb and string free energies. Using equation (12) and (18) we get

$$\begin{aligned} F(r, T) &= \frac{\sigma}{\mu} \left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{x}}{2^{3/4}\Gamma(3/4)} K_{1/4}(x^2 + \kappa x^4) \right] \\ &\quad - \frac{\alpha}{r} [e^{-x} + x]. \end{aligned} \quad (21)$$

In the following we will compare this ansatz with lattice results for the free energies of a static quark anti-quark pair in a thermal heat bath of quarks and gluons.

3 Numerical Results

The free energy $F(r, T)$ given in equation (21) contains only two temperature dependent parameters, μ and κ ; we assume the string tension σ and the gauge coupling α to be temperature-independent. Since we are not analyzing the heavy quark free energies at very short distances, where the running of the coupling becomes relevant, we use for α the string model value $\alpha = \pi/12$. To determine the temperature dependence of μ and κ , we fit $F(r, T)$ to the lattice results obtained in 2-flavor QCD [7]. At $r = \infty$, the free energy $F(r, T)$ depends only on μ and not on κ ,

$$F(T) = \frac{\sigma}{\mu(T)} \frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \alpha\mu(T) \quad , \quad (22)$$

with $F(r = \infty, T) = F(T)$. To obtain the temperature dependence of μ , we fit this $F(T)$ to the lattice data for

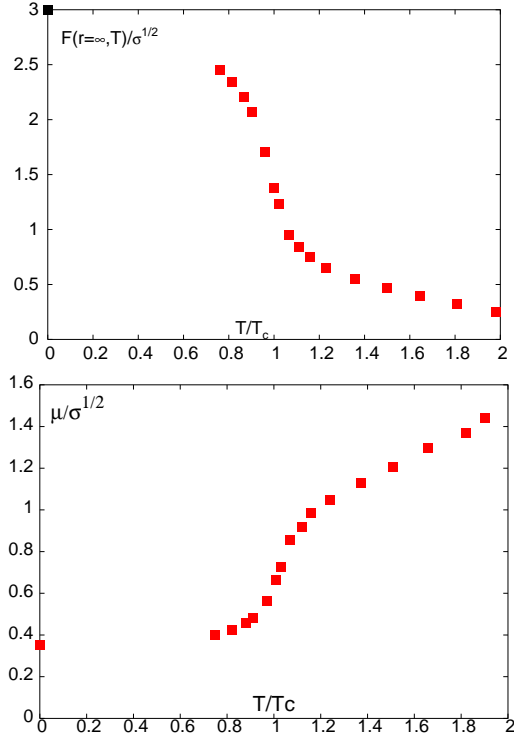


Fig. 1. Results from 2-flavor lattice QCD calculations for the large distance limit of the free energy (upper figure) and the corresponding screening mass (lower figure)

$r = \infty$. For $\mu(T)$, this gives the following form as function of $F(T)$,

$$\mu(T) = -\frac{F(T)}{2\alpha} + \sqrt{\left(F^2(T) + 4\sigma \frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)}\right) / 4\alpha^2} \quad (23)$$

In Fig. 1 we show the lattice results for $F(T)$, obtained from the mentioned lattice calculations in 2-flavor QCD [7], as well as the corresponding fit results for the screening mass $\mu(T)$.

Once we have the temperature dependence of $\mu(T)$, we fit equation (21) to the lattice data to obtain $\kappa(T)$. Our form of the free energy $F(r, T)$ fits these data quite well, as seen in Fig. 2, where we show our fit curves (solid lines) together with the lattice results. The fit results for the temperature dependence of $\kappa(T)$ are also shown in Fig. 2. They indicate that $\kappa(T)$ is finite and nearly constant below $T \lesssim 0.8T_c$, which may reflect the effect of confinement and of string breaking as an additional *non-thermal* screening effect, which we have ignored here. Closer to T_c , $\kappa(T)$ drops sharply and remains small above T_c . In fact, above T_c we obtain good fits also with κ set to zero (as shown by the lower lines for $T=T_c$ and $1.02 T_c$ in Fig. 3). This suggests that our ansatz with two separate screening functions provides a good approximation to the correct screening solution of the Debye-Hückel theory in the QCD plasma phase.

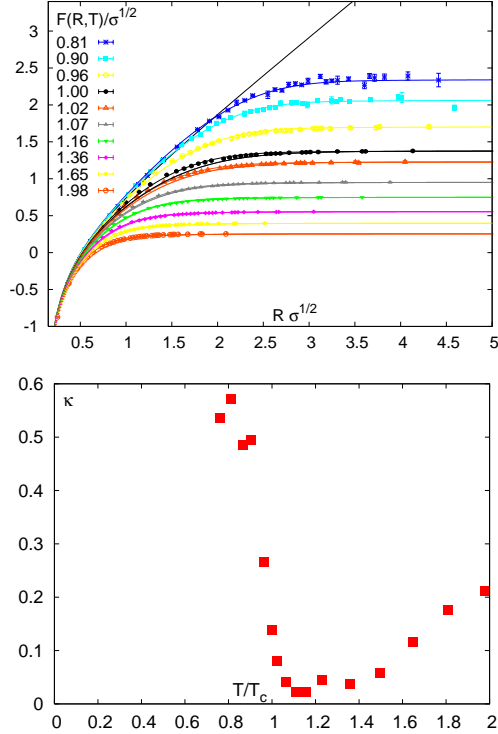


Fig. 2. Fits to the lattice results for the free energy as function of r for different temperatures (upper figure), and for the parameter κ (lower figure).

4 Discussions and Conclusions

Using Debye-Hückel theory, we have constructed a simple functional form for the free energy $F(r, T)$ of a static quark-antiquark pair in a thermal heat bath. For constant string tension and gauge coupling constant, $F(r, T)$ is specified by only two temperature-dependent parameters, μ and κ . We determine their temperature dependence by fitting our form of $F(r, T)$ to the lattice results for the free energy. The resulting $F(r, T)$ fits the lattice data quite well for all r and in a broad range of temperatures from zero to $2 T_c$. Since our $F(r, T)$ is an analytic form, one can now obtain from it the potential energy of a static quark anti-quark system using the thermodynamic relations. This may then be applied to study medium effects on the properties of heavy quarkonia.

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