Heavy quark complex potential in a strongly magnetized hot QGP medium

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We study the effect of strong constant magnetic field, generated in relativistic heavy ion collisions, on the heavy quark complex potential. We work in the strong magnetic field limit with lowest Landau level approximation. We find that the screening of the real part of the potential increases with the increase in the magnetic field. Therefore, we expect less binding of $Q\bar{Q}$ pair in the presence of strong magnetic field. The imaginary part of the potential increases in magnitude with the increase in magnetic field, leading to a increase of the width of the quarkonium state with the magnetic field. All of these effects results in the early dissociation of $Q\bar{Q}$ states in a magnetized hot quark-gluon plasma medium.

Keywords: Heavy quarkonium, Heavy quark-antiquark potential, Dielectric permittivity, Decay width, Debye mass, String tension.

I. INTRODUCTION

Relativistic heavy-ion collisions (HICs) at RHIC and LHC provide a sufficient information regarding the deconfined state of matter known as quark-gluon plasma (QGP). In recent studies [1] it has been realized that noncentral HICs produce a very strong magnetic field in the direction perpendicular to the reaction plane which has a number of interesting phenomenological consequences [1–5]. The estimated strength of the magnetic field is $B = m_\pi^2 = 10^{18}$ Gauss at the RHIC energies and $B = 15m_\pi^2 = 1.5 \times 10^{19}$ Gauss at the LHC energies [1, 6], where m_π is the pion mass. Possibly, such a huge magnetic field might have existed in the early universe as an origin of the present large scale cosmic magnetic field [7, 8].

The medium in the presence of magnetic field requires the modification of the existing theoretical tools that can be used to investigate the various properties of QGP. How does the magnetic field modify the properties the magnetic field modified the properties of strongly interacting matter has been the subject of various theoretical efforts [9]. One could study the various novel phenomena in hot and dense nuclear matter by considering the interaction between the field and the hot medium formed in the later stage of the collisions. These include chiral magnetic effect [1, 10, 11], chiral vortical effect [12, 13], chiral separation effect [1, 10, 12], chiral electric separation effect [14], chiral magnetic wave [15–17], finite-temperature magnetic catalysis [18–20] and inverse magnetic catalysis [21–25], chiral and color-symmetry broken/restoration phases [26–28], soft photon production from the conformal anomaly [29–31] in HICs, dilepton production [2, 32–37], synchrotron radiation and enhancement of elliptic flow of charged particles [32], thermodynamic properties [28, 38] and the perturbation theory in two-loop order [39]. Effects of magnetic field on the bulk properties of strongly interacting matter has also been attempted regarding chiral symmetry breaking [40],transport properties [41, 42], the hadron resonance gas model [43], the Polyakov linear-sigma model [44], bulk properties of Fermi gas [45], refractive indices, decay constants of mesons, thermal mass, and dispersion relations [46–50], phase structure of QCD [22, 28] using different theories of strong interaction. The QCD equation of state (EoS) has also been studied in the pres-

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ence of magnetic field by using LQCD [51, 52] as well as Hard Thermal loop perturbation theory (HTLpt) [53, 54]. Recently, the four-point quark-gluon vertex has been studied by applying HTLpt in the presence of weak magnetic field in Ref. [55].

The PHENIX Collaboration [56] has provided the experimental evidence of photon anisotropy which has posed a challenge for existing theoretical models. Eventually various theoretical calculations are made in the presence of a large anisotropic magnetic field in heavy-ion collisions [29]. All these studies suggest that there is great interest regarding the study of the effects of strong background magnetic fields on different aspects and observables of noncentral HICs. In the strong magnetic field approximation only the lowest Landau levels (LLL), which are the energy levels of a moving charged particles, remains active, so only the LLL dynamics becomes important [57]. Various studies, based on direct numerical investigations using lattice QCD [58–60] and effective theoretical investigations using different methods including, e.g., perturbative QCD studies, model studies, and anti-de Sitter/conformal field theory correspondence studies [47, 61–80] have predicted many interesting phenomena affecting the properties of strongly interacting matter in the presence of strong magnetic backgrounds. It is still not certain though to what extent such phenomena will be detectable in heavy ion experiments.

In this aspect, effects regarding the physics of heavy quark bound states ($Q\bar{Q}$) are of particular interest, since they are more sensitive to the conditions taking place in the early stages. They have large masses and are not affected by the thermal medium. Thus, the heavy quarkonium states are considered as the most powerful probes to study the deconfining properties of the strongly interacting medium. Matsui and Satz [81] predicted a suppression of the J/ψ, being caused by the shortening of the screening length for color interactions in the QGP. As regards the effects more directly related to color interactions, various studies have considered the possible influence of an external magnetic field on the static quark-antiquark potential [82]. So far, different studies have been carried out on the effects of magnetic field for static properties of quarkonia [82–89] and of open heavy flavors [90–94]. The effect of magnetic field on quarkonium production has been discussed in Ref. [85, 90]. Further, the influence of strong magnetic field on the evolution of J/ψ and the magnetic conversion of η_c into J/ψ has been discussed in Ref. [83] and [95] . Since masses of heavy quarks (m_Q) are much larger than QCD scale (λ_{QCD}), the velocity, v of heavy quarks in the bound state is small, and the binding effects in quarkonia at $T = 0$ can be understood in terms of non-relativistic potential models such as Cornell potential which can be derived directly from QCD using the effective field theories (EFTs) [96–98].

In this work we investigate the effects of the magnetic fields on the properties of quarkonium states. Quarkonium states are well understood to be a $Q\bar{Q}$ pair bound by the Cornell potential which is the sum of the Coulombic potential induced by a perturbative one-gluon exchange at short range and linearly rising potential at large distances [99]. In recent years, the quarkonium properties has been investigated by correcting both the coulombic and string terms of the heavy quark potential through the perturbative HTL dielectric permittivity in both the isotropic and anisotropic medium in the static [100–102] as well as moving medium [103]. As the magnetic field is produced at the early stages of the non central HICs, thus it is of great interest to see the effects of magnetic field on the properties of quarkonium states. In the present investigation, we focus on the modification of the heavy quark potential in the presence of strong magnetic field. In order to incorporate the magnetic field effect in the heavy quark potential we first calculate the fermionic contribution to the gluon polarization tensor in the presence of magnetic field and calculate the Debye screening mass. We calculate the gluon self energy in Euclidean space using imaginary time formalism by using the Schwinger propagator [104]. Since gluons do not interact with the external magnetic field so the magnetic field contribution will come from the quark loop only. We have taken only the LLL contribution in our calculation. This is a reasonable appex as quarkonia are produced during initial stages of collision when B is very high. Our aim is to find a field-theory motivated parametrization of the potential that depends both on the temperature and the magnetic field, which will be identified with the Debye screening mass, m_D. Here we follow the Ref. [105] where the potential is modified by bringing together the generalized Gauss law [106] with the characterization of in-medium effects through the perturbative dielectric permittivity. We demonstrate the effect of strong homogeneous magnetic field on the Debye screening and Landau damping induced thermal width obtained from the imaginary part of the quark-antiquark potential. In the medium, both the perturbative (coulombic) and non-perturbative (string) part of the heavy quark potential get modified. As the potential is a complex quantity [100, 103, 107–112] due to which an additional complication will arise. Therefore, a significant description of a relevant physics must illustrate both the effects of Debye screening in the real part of the potential and Landau-damping induced thermal width obtained from the imaginary part of the potential. Recently, heavy quark potential in a static and strong homogeneous magnetic field has been studied in [113]. However, the authors have studied the effect of magnetic field on real part of the potential only and they have neglected the gluonic contribution in the Debye mass whereas we have included the gluonic contribution also.

This paper is structured as follows. In Sec. II we first calculate the gluon self energy using the imaginary time formalism of thermal field theory and then calculate the Debye mass by taking the static limit of self energy. In Sec. III we discuss the heavy quark complex potential in the presence of magnetic field and shows the variation of the real and imaginary parts of the potential with various values of magnetic field at different temperature. In Sec. IV we investigate the decay width and discuss its variation with the magnetic field for both bottomonium and charmonium ground states. Finally, we present our conclusion in Sec. V.

II. GLUON SELF ENERGY IN LLL IN THE PRESENCE OF MAGNETIC FIELD

We consider that a charged particle of charge q_f and mass m_f is moving in a constant magnetic field (B) which is acting along z-direction i.e. $\vec{B} = B\hat{z}$. In this scenario the propagator of the charged particle is a function of both the transverse and longitudinal component of the momentum (w.r.t., B), which is due to the breaking of Lorentz invariance.

The fermion propagator in coordinate space is written as [104]

$$S(y,y') \;=\; \Phi(y,y') \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(y-y')} S(K), \tag{1}$$

where $\Phi(y,y')$ is the phase factor and can be gauged away. In Eq. (1), the Fourier transform of the fermion propagator, i.e., $S(K)$ in Euclidean space is written as [114]:

$$\begin{aligned} S(K) \;=\; -i\int_0^\infty ds \exp\Big(-s(K_\parallel^2 + K_\perp^2 \frac{\tanh(q_f Bs)}{q_f Bs} + m_f^2)\Big)\Big[(m_f - \not{K} + i(K_2\gamma_1 - K_1\gamma_2) \\ \times\; \tanh(q_f Bs))(1 + i\gamma_1\gamma_2 \tanh(q_f Bs)\Big], \end{aligned} \tag{2}$$

where Euclidean 4-vector $K_\mu = (K_4, K_1, K_2, K_3)$ and Dirac matrices $\gamma_\mu = (\gamma_4, \gamma_1, \gamma_2, \gamma_3)$ with $K_4 = -iK_0$, $\gamma_4 = -i\gamma_0$. $K_\parallel = (K_4, K_3)$ and $K_\perp = (K_1, K_2)$ are the 4-momentum component parallel and perpendicular to the magnetic field. It is convenient to write the propagator in the Landau level representation. To do so we rewrite Eq.(2) bringing in the perpendicular component into the square bracket as

$$S(K) \;=\; -i\int_0^\infty \frac{dv}{q_fB}\exp\Big(-\frac{v}{q_fB}(K_\parallel^2+m_f^2)\Big)\Big[(m_f-\not{K})\exp(-\lambda\tanh(v))-i[(m_f-\not{K})\gamma_1\gamma_2 + K_2\gamma_1-K_1\gamma_2]\frac{\partial}{\partial\lambda}\exp(-\lambda\tanh(v))-[K_2\gamma_1-K_1\gamma_2]\gamma_1\gamma_2\frac{\partial^2}{\partial\lambda^2}\exp(-\lambda\tanh(v))\Big], \tag{3}$$

where $v=q_fBs$ and $\lambda=\frac{K_\perp^2}{q_fB}$. Now we use the identity $\tanh(z)=1-\frac{2\exp(-2z)}{1+\exp(-2z)}$ and the relation

$$(1-y)^{-(\alpha+1)}e^{\frac{xy}{y-1}}=\sum_{l=0}^{\infty}L_l^\alpha(x)y^l.$$

where $L_l^\alpha(x)$ is generalised Laguerre polynomial. Proceeding in a similar way as done in Ref. [115] for Minkowski space, the resulting propagator can be written as

$$S(K)=-ie^{-\frac{K_\perp^2}{q_fB}}\sum_{l=0}^{\infty}\frac{(-1)^lD_l(q_fB,K)}{K_\parallel^2+m_f^2+2l|q_fB|}, \tag{4}$$

where the sum is over all the Landau levels ($l=0,1,2..$), and

$$D_l(q_fB,K) \;=\; (m_f-\not{K}_\parallel)\Big((1+i\gamma_1\gamma_2)L_l\Big(\frac{2K_\perp^2}{q_fB}\Big)-(1-i\gamma_1\gamma_2)L_{l-1}\Big(\frac{2K_\perp^2}{q_fB}\Big) + 4\not{K}_\perp L_{l-1}^1\Big(\frac{2K_\perp^2}{q_fB}\Big)\Big), \tag{5}$$

The energy spectrum of a charged fermion in the presence of magnetic field can be written as [116]

$$E_l(K_3)=K_3^2+m_f^2+2l|q_fB|, \tag{6}$$

which can be obtained from Eq. (4) by equating the denominator of the propagator to zero. From Eq. (6), it is clear that the energy of fermion is discrete in the direction perpendicular to B and continuous in the direction parallel to the magnetic field. The discretization of energy levels is known as Landau levels.

In the presence of a very strong magnetic field, i.e., $q_fB\gg T^2$ the higher Landau levels ($l\gg 1$) are at infinity as compared to LLL [34, 117] due to which the LLL dominates and the dimensional reduction takes place. For LLL approximation we use the relations $L_n(x)=L_n^0(x)$ and $L_{-1}^\alpha(x)=0$ (by definition). In this approximation, the fermion propagator (Eq. 4) becomes

$$S(K)=ie^{-\frac{K_\perp^2}{q_fB}}\Big(\frac{\not{K}_\parallel-m_f}{K_\parallel^2+m_f^2}\Big)(1+i\gamma_1\gamma_2), \tag{7}$$

where the factor, $(1+i\gamma_1\gamma_2)$ is the projection operator which appears because the spin of the fermion in the LLL is polarized parallel to the magnetic field hence signifies the spin-polarized nature of LLL [114]. The spin orientation is parallel/anti-parallel to the magnetic field direction for positively/negatively charged fermion. The form of the propagator in Eq. (7) clearly demonstrates the dimensionally reduced character (1+1) of the LLL. It is evident that the dimensional reduction restricts the motion of charged particles perpendicular to the magnetic field.

For evaluating the gluon polarization tensor and Debye screening mass we use the following identities

$$b_\parallel^\mu=(b_4,0,0,b_3); \qquad a_\parallel^2=a_4^2+a_3^2; \qquad b_\perp^\mu=(0,b_1,b_2,0),$$

$$(a \cdot b)_{\parallel} = a_4 b_4 + a_3 b_3; \qquad (a \cdot b)_{\perp} = a_1 b_1 + a_2 b_2,$$

where $\parallel$ and $\perp$ are the parallel and perpendicular components. In a background magnetic field, the only contribution to the gluon self energy comes from the quark loop as shown in (Fig. 1). As gluons do not interact with the magnetic field so their contribution remain same as that at $B = 0$. Therefore, the contribution to the gluon self energy in the presence of magnetic field from quarks,

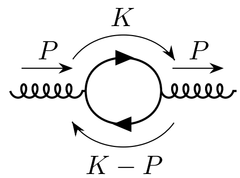

FIG. 1: Gluon self energy in the presence of strong magnetic field

$\delta\Pi_{\mu\nu}(P, B)$ can be written as

$$\delta\Pi_{\mu\nu}(P, B) = \int \frac{d^4K}{(2\pi)^4} Tr[(gt^a\gamma_\mu)S(K)(gt^b\gamma_\nu)S(K-P)], \tag{8}$$

where S(K) is fermion propagator as defined in Eq. (7). As the dimensional reduction separate the parallel and perpendicular component of the momentum in the propagator. So the self energy can also be separated into components. In Eq. (7), the perpendicular part (exponential term) is written as

$$\begin{aligned} I_\perp &= \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dK_x dK_y \exp[\frac{1}{q_fB}(-K_x^2 - (K_x - P_x)^2)] \exp[\frac{1}{q_fB}(-K_y^2 - (K_y - P_y)^2)] \\ &= \sum_f \frac{\pi q_f B}{2(2\pi)^2} \exp\left(\frac{-P_\perp^2}{2q_fB}\right) \end{aligned} \tag{9}$$

Therefore, Eq. (8) becomes

$$\delta\Pi_{\mu\nu}(P, B) = g^2 Tr(t^at^b) I_\perp \int \frac{d^2K_\parallel}{(2\pi)^2} \frac{Tr[\gamma_\mu(\not{K}_\parallel - m_f)(1 + i\gamma_1\gamma_2)\gamma_\nu((\not{K} - \not{P})_\parallel - m_f)(1 + i\gamma_1\gamma_2)]}{(K_\parallel^2 + m_f^2)((K-P)_\parallel^2 + m_f^2)}. \tag{10}$$

To take into account the thermal effects we use imaginary time formalism so that the integration over the temporal component becomes a summation over discrete Matsubara frequencies. Within the HTL approximation the temporal component of self energy can be written as

$$\delta\Pi_{44}(P, B) = g^2 Tr(t^at^b) I_\perp \int \frac{dK_3}{(2\pi)} T \sum_n \frac{4\omega_n^2 + 4K_3^2 - 8K_3^2 - m_f^2}{(\omega_n^2 + K_3^2 + m_f^2)((\omega - \omega_n)^2 + (K_3 - P_3)^2 + m_f^2)} \tag{11}$$

where $\omega_n = (2n+1)\pi T$ is fermionic Matsubara frequency and ω is bosonic Matsubara frequency. For massless fermion ($m_f = 0$) we get

$$\delta\Pi_{44}(B) = 4g^2 Tr(t^at^b) I_\perp \int \frac{dK_3}{(2\pi)} T \sum_n \left(\tilde{\Delta}(K) - 2K_3^2\tilde{\Delta}(K)\tilde{\Delta}(K-P)\right) \tag{12}$$

where $\tilde{\Delta}(K) = \frac{1}{\omega_n^2 + K_3^2}$, $\tilde{\Delta}(K-P) = \frac{1}{(\omega-\omega_n)^2 + (K_3-P_3)^2}$. In order to calculate the Debye screening mass we take the static limit of temporal component of self energy, Π_{44} *viz.* $m_D^2 = -\Pi_{44}(\omega \to 0,$

$\vec{P}=0$). After taking the static limit Eq. (12) becomes

$$\begin{aligned}\delta\Pi_{44}(B)|_{(\omega\to 0,\vec{P}=0)} &= 4g^2Tr(t^at^b)I_\perp\int\frac{dK_3}{(2\pi)}T\sum_n\Big(\tilde{\Delta}(K)-2K_3^2(\tilde{\Delta}(K))^2\Big)\\ &= 4g^2Tr(t^at^b)I_\perp\int\frac{dK_3}{(2\pi)}T\sum_n\Big(\tilde{\Delta}(K)+2K_3^2\frac{\partial}{\partial {K_3}^2}\tilde{\Delta}(K)\Big).\end{aligned}\tag{13}$$

The fermionic frequency sum is given as

$$T\sum_n\tilde{\Delta}(k)=\frac{1-2\tilde{f}(E)}{2E},$$

where $\tilde{f}(E)$ is the Dirac distribution function and $E=K_3$, for massless fermion in the LLL, and

$$2K_3^2T\sum_n\frac{\partial}{\partial K_3}\tilde{\Delta}(k)=-\frac{\beta(\tilde{f}(E))^2e^{\beta E}}{2}-\frac{1-2\tilde{f}(E)}{2E}$$

After using above relations in Eq. (13), we get

$$\delta m_D^2=2g^2\beta I_\perp\int\frac{dK_3}{(2\pi)}\tilde{f}(E)(1-\tilde{f}(E)).\tag{14}$$

By using Eq. (9), the Debye screening mass in the presence of magnetic field becomes

$$\delta m_D^2=\sum_f\frac{|q_f|Bg^2}{2\pi T}\int_0^\infty\frac{dK_3}{2\pi}\tilde{f}(E)(1-\tilde{f}(E)).\tag{15}$$

The Debye screening mass in the presence of magnetic field was earlier derived in Ref. [34, 93]. By taking the gluonic contribution into account the total Debye mass becomes

$$m_D^2=\frac{4\pi\alpha_s(T)T^2N_c}{3}+\sum_f\frac{|q_f|Bg^2}{2\pi T}\int_0^\infty\frac{dK_3}{2\pi}\tilde{f}(E)(1-\tilde{f}(E)).\tag{16}$$

For massless fermions, $\int_0^\infty dK_3\tilde{f}(E)(1-\tilde{f}(E)=T/2$. Therefore, Eq. (16) reduces to

$$m_D^2=\frac{4\pi\alpha_s(T)T^2N_c}{3}+\sum_f\frac{|q_f|B\alpha_s(T)}{2\pi}.\tag{17}$$

where $\alpha_s(T)$ is the one loop coupling constant and which can be written as

$$\alpha_s(T)=\frac{g_s^2(T)}{4\pi}=\frac{6\pi}{(33-2N_f)\ln\left(\frac{2\pi T}{\Lambda_{\overline{\rm MS}}}\right)}.\tag{18}$$

Here we use $N_f=3$ and $\Lambda_{\overline{\rm MS}}=0.176$ GeV [118].

In the left panel, Fig. 2 shows the variation of Debye screening mass for $m_f=0$ with T for different values of magnetic field, i.e. $eB=15\ m_\pi^2$, $30\ m_\pi^2$ and $45\ m_\pi^2$ respectively. In the right panel Figure shows the variation of Debye screening mass with B for different values of temperature, i.e. $T=0.20$ GeV, 0.25 GeV and 0.30 GeV respectively. Here the Debye screening mass (Eq. 17) depends on the two scales, i.e. T and B. We find that the Debye screening mass increases with the increase in temperature and magnetic field in a magnetized hot QCD medium. Therefore, we can say that there is a finite amount of Debye screening with T and B for $m_f=0$.

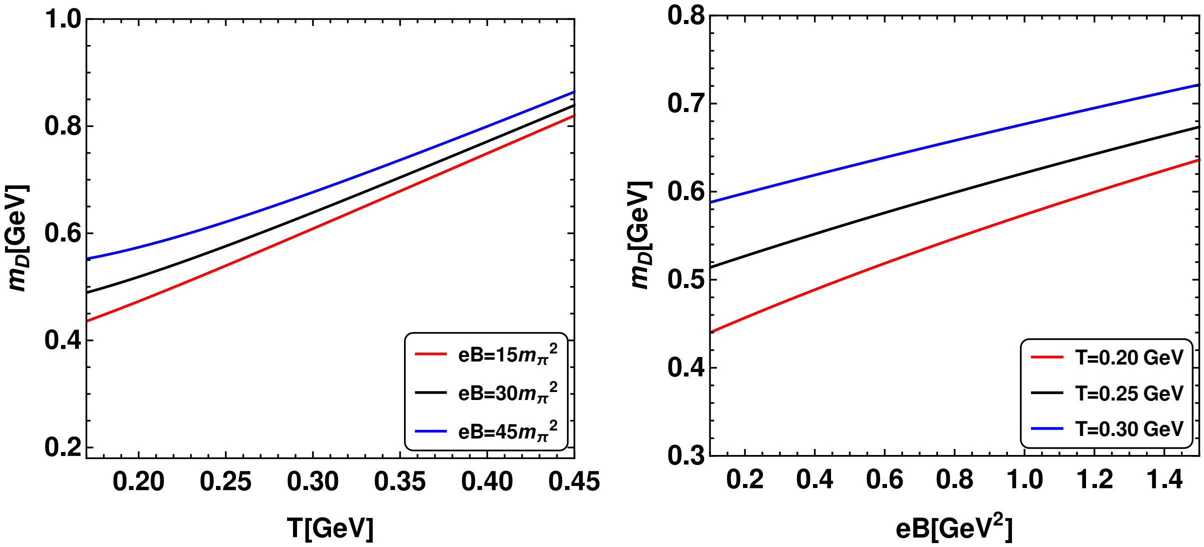

FIG. 2: Left panel: Variation of the Debye screening mass (m_D) with temperature for different values of magnetic field (eB). Right panel: Variation of m_D with magnetic field for different values of T.

III. IN-MEDIUM HEAVY QUARK POTENTIAL IN MAGNETIC FIELD

In this section we study the modification of the Cornell potential in the presence of a hot medium endowed also with a magnetic field. Before that let us first discuss the heavy quark-antiquark potential for vanishing magnetic field. Consider that a test charged particle is placed at origin and an auxilar vector field due to this is written as $\vec{E} = qr^{b-1}\hat{r}$, where "$b$" is a parameter that can have any value. The corresponding potential can be defined by using the relation -$\vec{\nabla}V(r) = \vec{E}(r)$. Therefore, the Gauss law can be defined as [105, 106]

$$\vec{\nabla}.\left(\frac{\vec{E}}{r^{b+1}}\right) = 4\pi q\delta(r). \tag{19}$$

For $b = -1$, Eq. (19) reduces to the Coulomb potential, where $q = \alpha(= \alpha_s C_F = \frac{g_s^2 C_F}{4\pi}; C_F = 4/3)$ and for $b = 1, q = \sigma$ the linearly rising (confining) potential can be obtained. As per the Debye-H$\ddot{u}$ckle framework, the medium gets polarized and leads to a change of the source term on the R.H.S. of the Eq. (19) from $\delta(r)$ to $\delta(r) + \langle\rho(r)\rangle$. Here $\langle\rho(r)\rangle$ is the induced charged density and the Gauss law can be written as

$$\vec{\nabla}.\left(\frac{\vec{E}}{r^{b+1}}\right) = 4\pi q(\delta(r) + \langle\rho(r)\rangle), \tag{20}$$

The induced charge density, $\langle\rho(r)\rangle$ can be written as the difference of the particle and anti-particle charge deviation which in the Boltzmann's distribution can be written as

$$\rho(\vec{r}) = q(n_0 e^{-\beta V(\vec{r})} - n_0) - q(e^{\beta V(\vec{r})} - n_0),$$

where $\beta = 1/T$ and n_0 is the charge density in the absence of test charge. At high temperatures and weak potential the charge density can be approximated as

$$\langle\rho(r)\rangle = -2q\beta n_0 V(r), \tag{21}$$

which is equivalent to the linear response approxomation. After substituting the above equation into Eq. (20), one can obtain [106]

$$-\frac{1}{r^{b+1}}\nabla^2 V(r) + \frac{1+b}{r^{b+2}}\nabla V(r) + 8\pi q n_0 \beta V(r) = 4\pi q \delta(r). \tag{22}$$

For $b = -1, q = \alpha$, one can obtain the coulombic part of the potential as [105]

$$-\nabla^2 V_C(r) + 8\pi\alpha n_0 \beta V_C(r) = 4\pi\alpha\delta(r), \tag{23}$$

and for $b = 1, q = \sigma$, one can find the string part of the potential as

$$-\frac{1}{r}\frac{d^2 V_s(r)}{dr^2} + 8\pi\sigma r n_0 \beta V_s(r) = 4\pi\sigma r\delta(r), \tag{24}$$

In a quantum field theory framework, the medium effects, i.e. the effect of finite temperature and magnetic field can be incorporated by modifying the vacuum potential, $V(r)$ with dielectric permittivity as in Ref. [100]. The in-medium permittivity, $\epsilon(\vec{k}, m_D)$ can be written as

$$\epsilon^{-1}(\vec{k}, m_D) = \frac{k^2}{k^2 + m_D^2} - i\pi T \frac{k m_D^2}{(k^2 + m_D^2)^2}, \tag{25}$$

where m_D is the Debye screening mass in the presence of magnetic field as given in Eq. (17). The Coulomb part of the potential in Fourier space in the presence of medium can be written as [119]

$$k^2 V_C(\vec{k}) = 4\pi \frac{\alpha}{\epsilon(\vec{k}, m_D)} \tag{26}$$

The Fourier transformation of Eq. (26) in coordinate space gives

$$-\nabla^2 V_C(r) + m_D^2 V_C(r) = \alpha(4\pi\delta(r) - iT m_D^2 h(m_D r)) \tag{27}$$

where $h(y) = 2\int_0^\infty dx \frac{x}{(x^2+1)} \frac{\sin(yx)}{yx}$.

Comparing the Eqs. (23) and (27), the charge density (n_0) can be written as $n_0 = \frac{m_D^2}{8\pi\alpha\beta}$. In a similar manner, we have calculated the string part of the potential by assuming the same number density and get the differential equation for $V_S(r)$ as

$$-\frac{1}{r^2}\frac{d^2 V_S(r)}{dr^2} + \mu^4 V_S(r) = \sigma(4\pi\delta(r) - iT m_D^2 h(m_D r)), \tag{28}$$

where $\mu = (\frac{m_D^2 \sigma}{\alpha})^{1/4}$ and σ is the string tension. After using the boundary conditions $m_D \to 0$, $\Re V_C(r) = -\frac{\alpha}{r}$ and $r \to 0$, $\Im V_C(r) = 0$, we obtain both the real and imaginary part of the coulomb potential as

$$\Re V_C(r, T, B) = -\alpha \frac{e^{-m_D r}}{r} - \alpha m_D, \tag{29}$$

and

$$\Im V_C(r, T, B) = -2\alpha T g(m_D r), \tag{30}$$

where $g(y) = \int_0^\infty dx \frac{x}{(x^2+1)^2}\left(1 - \frac{\sin(yx)}{yx}\right)$. Thus, the magnetic field dependence of the potential have arises from the field dependent Debye mass. Since the Debye mass, m_D depends on the magnetic field, hence the potential also depend on the magnetic field.

Similarly, for the string part of the potential we use the following boundary conditions $\mu \to 0, \Re V_S(r) = \sigma r$, $r \to 0, \Im V_S(r) = 0$ and $r \to \infty, \frac{d\Im V_S(r)}{dr} = 0$. After using the boundary conditions we get both the real and imaginary part of string potential as [105]

$$\Re V_S(r,T,B) = -\frac{\Gamma(\frac{1}{4})}{2^{\frac{3}{4}}\sqrt{\pi}}\frac{\sigma}{\mu}D_{-\frac{1}{2}}(\sqrt{2}\mu r) + \frac{\Gamma(\frac{1}{4})}{2\Gamma(\frac{3}{4})}\frac{\sigma}{\mu}, \tag{31}$$

where $D_\nu(x)$ is the parabolic cylinder function, and

$$\Im V_S(r,T,B) = -\frac{\sigma m_D^2 T}{\mu}\phi(\mu r), \tag{32}$$

where

$$\begin{aligned} \phi(\mu r) \;&=\; D_{-\frac{1}{2}}(\sqrt{2}\mu r)\int_0^r dx \Re D_{-\frac{1}{2}}(i\sqrt{2}\mu x)x^2 g(m_D x) + \Re D_{-\frac{1}{2}}(i\sqrt{2}\mu r)\int_\infty^r dx D_{-\frac{1}{2}}(\sqrt{2}\mu x)x^2 g(m_D x) \\ &-\; D_{-\frac{1}{2}}(0)\int_0^\infty dx D_{-\frac{1}{2}}(\sqrt{2}\mu x)x^2 g(m_D x), \end{aligned}$$

The total real part of the potential after combining both the coulombic and string parts in a magnetized medium can be written as

$$\begin{aligned} \Re V(r,T,B) \;&=\; \Re V_C(r,T,B) + \Re V_S(r,T,B) \\ &=\; -\alpha\frac{e^{-m_D r}}{r} - \alpha m_D - \frac{\Gamma(\frac{1}{4})}{2^{\frac{3}{4}}\sqrt{\pi}}\frac{\sigma}{\mu}D_{-\frac{1}{2}}(\sqrt{2}\mu r) + \frac{\Gamma(\frac{1}{4})}{2\Gamma(\frac{3}{4})}\frac{\sigma}{\mu}. \end{aligned} \tag{33}$$

Similarly, the total imaginary part of the potential by combining both the coulombic and string parts in a magnetized medium becomes

$$\begin{aligned} \Im V(r,T,B) \;&=\; \Im V_C(r,T,B) + \Im V_S(r,T,B) \\ &=\; -2\alpha T g(m_D r) - \frac{\sigma m_D^2 T}{\mu}\phi(\mu r), \end{aligned} \tag{34}$$

Figure 3 shows the variation of real part of the potential with the separation distance (r) between the Q$\bar{Q}$ pair for different values of magnetic field ($eB = 15m_\pi^2,\ 30m_\pi^2,\ 45m_\pi^2$) at $T = 200$ MeV (left) and $T = 250$ MeV (right). We use the value of the string tension $\sigma = 0.174$ GeV^2 from Ref. [105] . From the figure we find that the screening increases with the increase in magnetic field. The screening is more at higher temperature ($T = 250$ MeV) as compared to lower temperature ($T = 200$ MeV) because at higher temperatures the quarkonium state is loosely bound as compared to the lower temperatures and gets easily dissociated. Alternatively, we can say that with the increase in temperature gluonic contribution becomes more which results in more screening.

Figure 4 shows the variation of the imaginary part of the potential with the separation distance (r) for various values of magnetic field ($eB = 15m_\pi^2,\ 30m_\pi^2,\ 45m_\pi^2$) at temperature $T = 200$ MeV and $T = 250$ MeV. We find that the imaginary part of the potential increases in magnitude with the increase in magnetic field and hence it contributes more to the Landau damping induced thermal width obtained from the imaginary part of the potential. The increase in the magnitude of imaginary part of the potential is more at higher temperature ($T = 250$ MeV) as compared to lower temperature ($T = 200$ MeV).

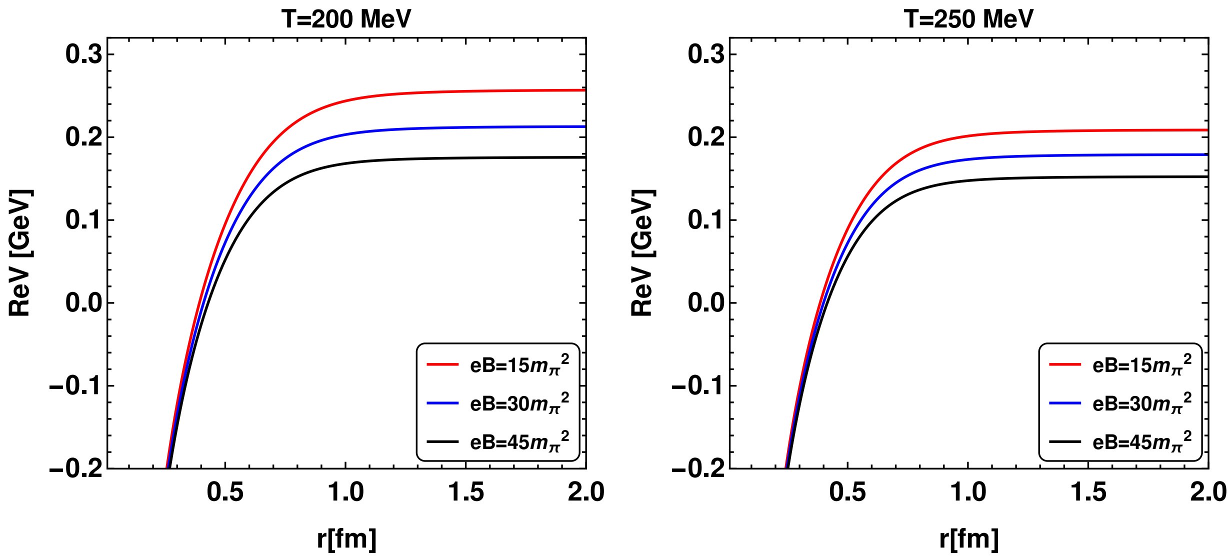

FIG. 3: Variation of the real part of potential with separation distance r between $Q\bar{Q}$ for various values of magnetic field at $T = 200$ MeV (left) and $T = 250$ MeV (right).

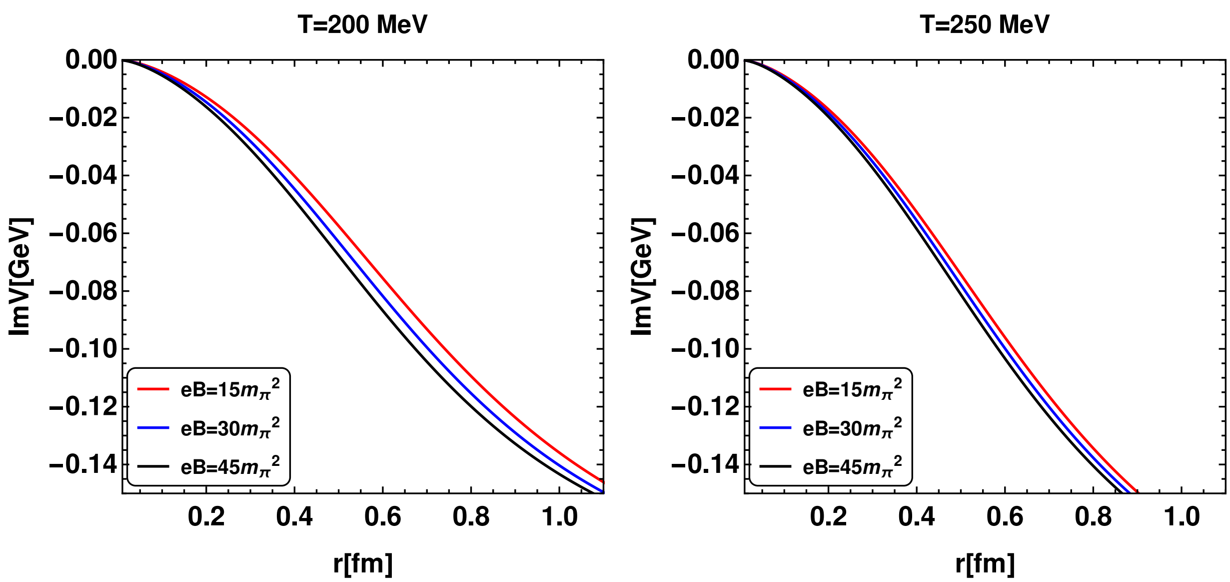

FIG. 4: Variation of Imaginary part of the potential with separation distance r for various values of magnetic field at $T = 200$ MeV (left) and $T = 250$ MeV (right).

IV. DECAY WIDTH

The decay width (Γ) can be calculated from the imaginary part of the potential. The following formula gives a good approximation to the decay width of $Q\bar{Q}$ states [100, 103, 120]

$$\Gamma = -\int d^3\mathbf{r}\,|\psi(\mathbf{r})|^2\,\Im V(\mathbf{r}, T, B) \tag{35}$$

where $\psi(\mathbf{r})$ is the Coulombic wave function for the ground state and is given by

$$\psi(\mathbf{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}, \tag{36}$$

where $a_0 = 2/(m_Q \alpha)$ is the Bohr radius of the heavy quarkonium system. We use the Coulomb-like wave functions to determine the width since the leading contribution to the potential for the deeply bound quarkonium states in a plasma is Coulombic. After substituting the expression for

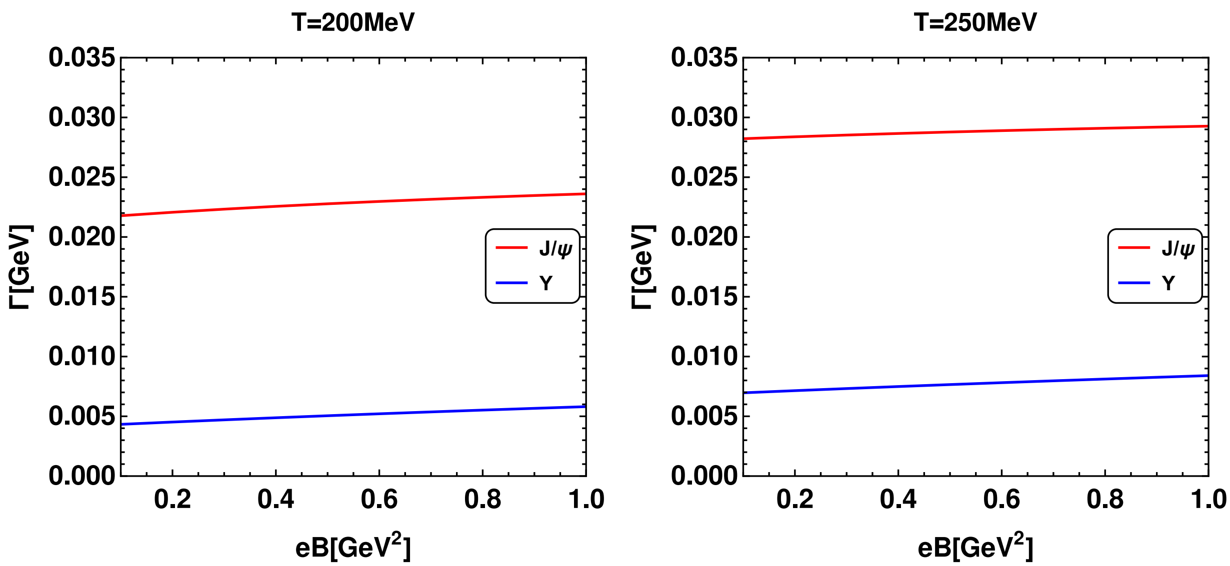

FIG. 5: Variation of decay width with the magnetic field for J/ψ and Υ at $T = 200$ MeV (left) and $T = 250$ MeV (right)

.

imaginary part of the potential in Eq. (35), we show numerically the variation of the total decay width [Eq. (35)] with magnetic field for the ground states of charmonium and bottomonium in Fig. 5 at $T = 200$ MeV (left panel) and $T = 250$ MeV (right panel). Here we take charmonium and bottomonium masses as $m_c = 1.275$ GeV and $m_b = 4.66$ GeV respectively from [121]. We find that the thermal width increases with the increase in magnetic field. The width for the Υ is much smaller than the J/ψ because the bottomonium states are smaller in size and larger in masses than the charmonium states and hence get dissociated at higher temperature. The width at higher temperature ($T = 250$ MeV) is more as compared to lower temperature ($T = 200$ MeV) for both J/ψ and Υ. Alternatively, we can say that the Γ increases with the increase in magnetic field which results in the early dissociation of quarkonium states.

Figure. 6 shows the variation of decay width with temperature at $eB = 30m_\pi^2$ (left) and $eB = 0$ (right). We find that width increases with the increase in temperature for both J/ψ and Υ states. The width is larger in the presence of magnetic field ($eB = 30m_\pi^2$) than in the absence of magnetic field ($eB = 0$). The magnetic field effects become less at high temperature as compared to low temperature because the gluons will dominate at higher temperature. Also we are considering the strong field LLL approximation i.e. $eB \gg T^2$. This approximation may not hold good at high temperature.

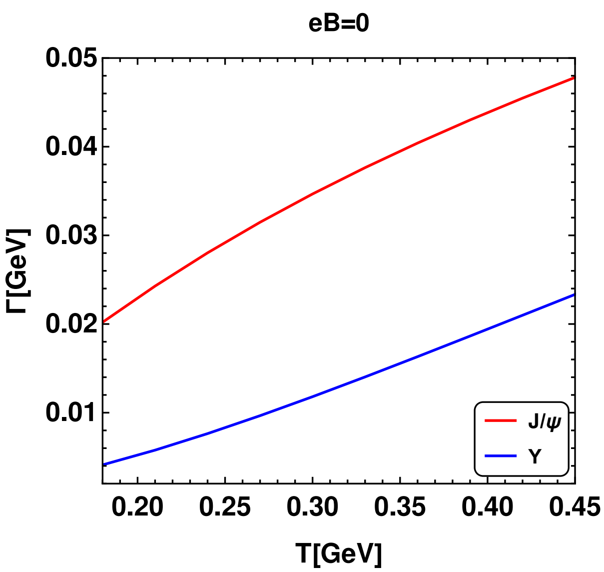

FIG. 6: Variation of decay width with temperature for J/ψ and Υ at $eB = 0$ (left) and $eB = 30\ m_\pi^2$ (right)
.

V. CONCLUSION

In this work we have studied the effect of strong magnetic field on the heavy quark complex potential in a thermal QGP medium. In order to incorporate the magnetic field effect in the heavy quark potential we first calculated the gluon self energy in Euclidean space in the imaginary time formalism by using the Schwinger propagator in the presence of magnetic field and calculated the Debye screening mass (m_D). We have shown the variation of Debye mass with magnetic field and temperature for massless fermions. We found that Debye screening increases in a hot magnetized medium. We have taken only the LLL contribution in our calculation. This is a reasonable appex as quarkonia are produced during initial stages of collision when the magnetic field is very high because higher Landau levels are at infinity due to which the LLL dominates and the dimensional reduction takes place. Further, we study the magnetic field effects on the heavy quark complex potential. The heavy quark complex potential is obtained by bringing together the generalized Gauss law with the characterization of in-medium effects, i.e., finite temperature and magnetic field, through the perturbative dielectric permittivity. We have shown the effect of magnetic field on the Debye screening and Landau damping induced thermal width obtained from the imaginary part of the quark-antiquark potential. In order to see the magnetic field effect on the screening we first show the effect of magnetic field on the real part of the potential. We found that the real part of the potential decreases with increase in magnetic field and becomes more screened. The screening also increases with the increase in temperature. Since the $Q\bar{Q}$ potential is effectively more screened in the presence of magnetic field which results in the earlier dissociation of quarkonium states in a strongly magnetized hot QGP medium.

We have also shown the effect of magnetic field on the imaginary part of the potential and hence on the thermal width. The imaginary part of the potential increases in magnitude with the increase in magnetic field and temperature. As a result, the width of the quarkonium states (J/ψ and Υ) get more broadened with the increase in magnetic field and results in the earlier dissociation of quarkonium states in the presence of magnetic field. The width for Υ is much smaller than the J/ψ because bottomonium states are tighter than the charmonium state and hence get dissociated at a higher temperature. The magnetic field effects become very less at high temperature this may be because of the LLL approximation, i.e., $eB \gg T^2$ and this approximation may not be a good approximation at high temperature. Combining both the effects of screening and the broadening due to damping, we expect a lesser binding of a $Q\bar{Q}$ pair in a strongly magnetized hot QGP medium.

Clearly, the present investigation is limited to very high magnetic field compared to temperature where only lowest Landau level contributes to the dielectric function. For moderate magnetic field one has to include the effects from higher Landau levels. Such an investigation is in progress and will be represented elsewhere.

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