Noise Analysis

CSE562: Analog Integrated Circuits

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What is noise ?

- Any input that is not relevant to the signals of interest.

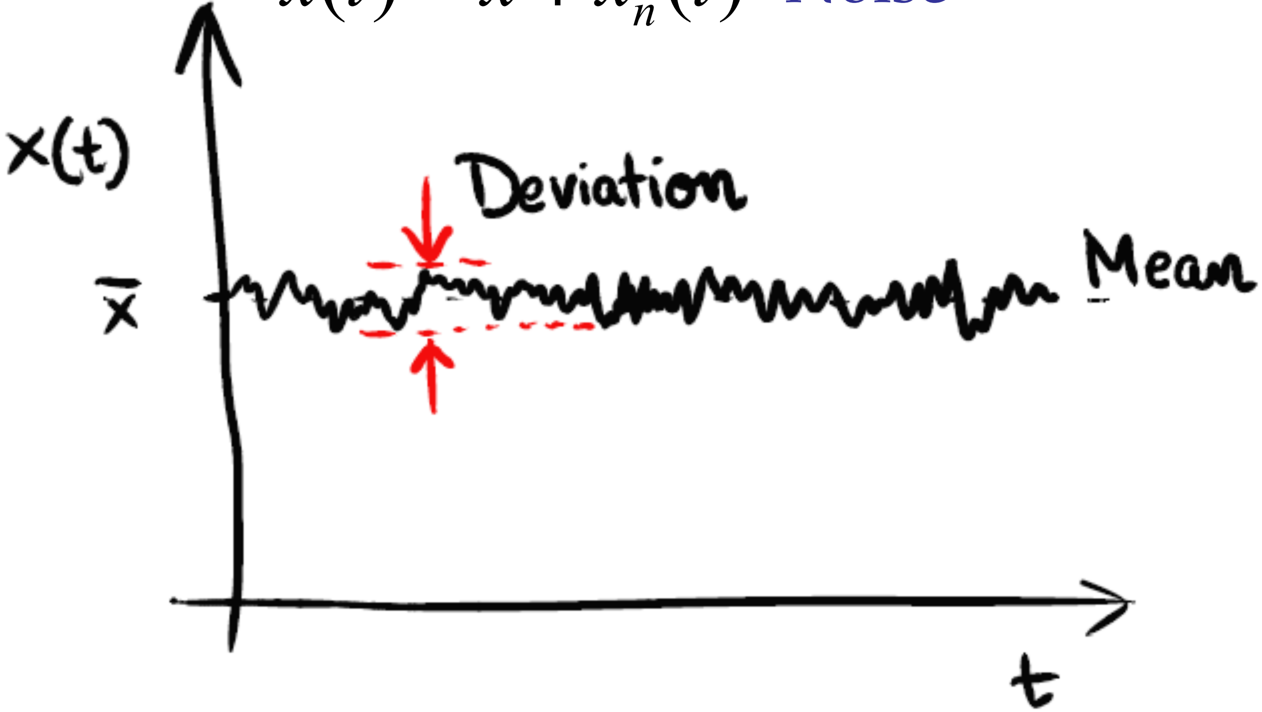
<u>Sources of noise</u>

- **External sources**
  - Coupling from outside of the system of interest. E.g. substrate coupling, interconnect noise, power supply noise.
  - Can be eliminated by careful circuit design, shielding and layout.
- **Fundamental sources**
  - Noise generated from within the device.
  - Fundamental and microscopic in nature and in principle can never be eliminated by can be <u>reduced</u>.

Characterization of noise

- Stochastic signal - follows an underlying probability distribution.
- Stationary – Statistical property of the signal remains constant with respect to time.

$$x(t) = \bar{x} + x_n(t)$$ Noise



- Mean

$$E[x_n(t)] = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} x_n(t)\,dt = 0$$

- Variance – energy in the error signal/noise

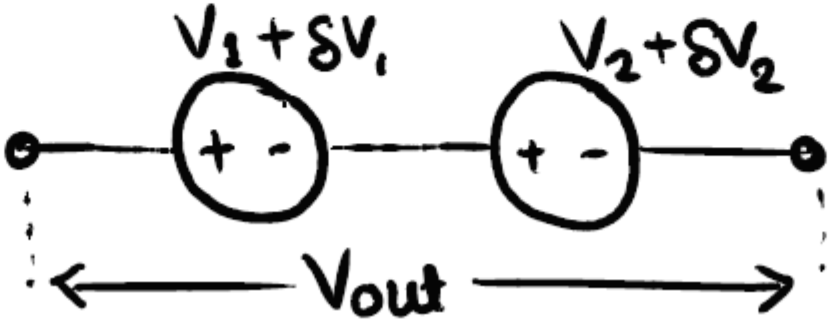
$$\bar{x}_n^2 = E\left[x_n^2(t)\right]$$

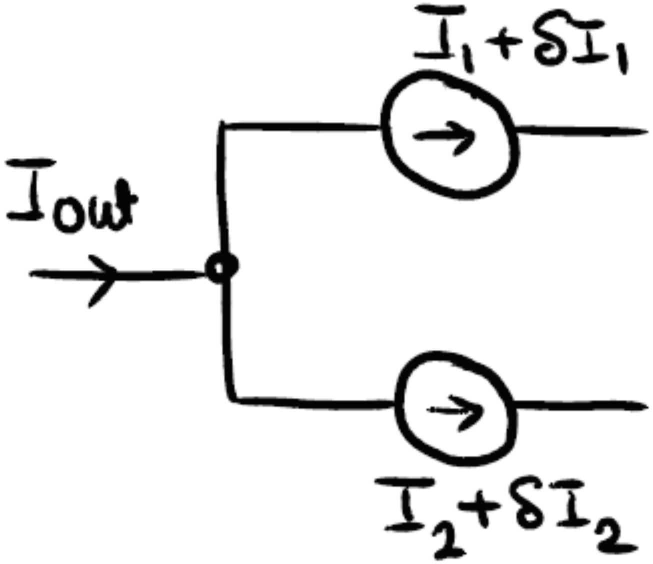
- Autocorrelation

$$R(\tau) = E\left[x_n(t)\,x_n(t+\tau)\right]$$

Combination of noise sources

- Unrelated noise sources are statistically independent.





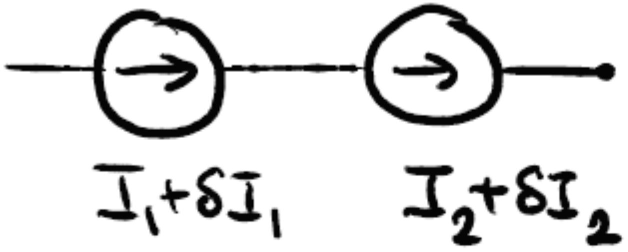
$$V_{out} = V_1 + V_2$$

$$\overline{V}^2_{out} = \overline{V}^2_{n1} + \overline{V}^2_{n2}$$

$$I_{out} = I_1 + I_2$$

$$\overline{I}^2_{out} = \overline{I}^2_{n1} + \overline{I}^2_{n2}$$

- What about this circuit ?



Frequency domain characterization

- Distribution of noise power over frequency/spectrum.
- Characterized by noise power spectral density.

$$\overline{e}_n^2 = \lim_{\tau\to\infty}\frac{1}{\tau}\int_{-\tau}^{\tau} e_n^2(t)dt = \lim_{\tau\to\infty}\frac{1}{\tau}\int_{-\infty}^{\infty} e_n^2(t)W^2(t,\tau)dt$$

- Window function

- Parseval's theorem

$$\overline{e}_n^2 = \lim_{\tau\to\infty}\frac{1}{2\pi\tau}\int_{-\infty}^{\infty}\left|E(\omega,\tau)\right|^2 d\omega$$

$$\overline{e}_n^2 = \frac{1}{2\pi}\int_{0}^{\infty}\lim_{\tau\to\infty}\frac{1}{\tau}2\left|E(\omega,\tau)\right|^2 d\omega$$

$$\overline{e}_n^2 = \frac{1}{2\pi}\int_{0}^{\infty}S_E(\omega)d\omega$$

One sided power spectrum

White noise

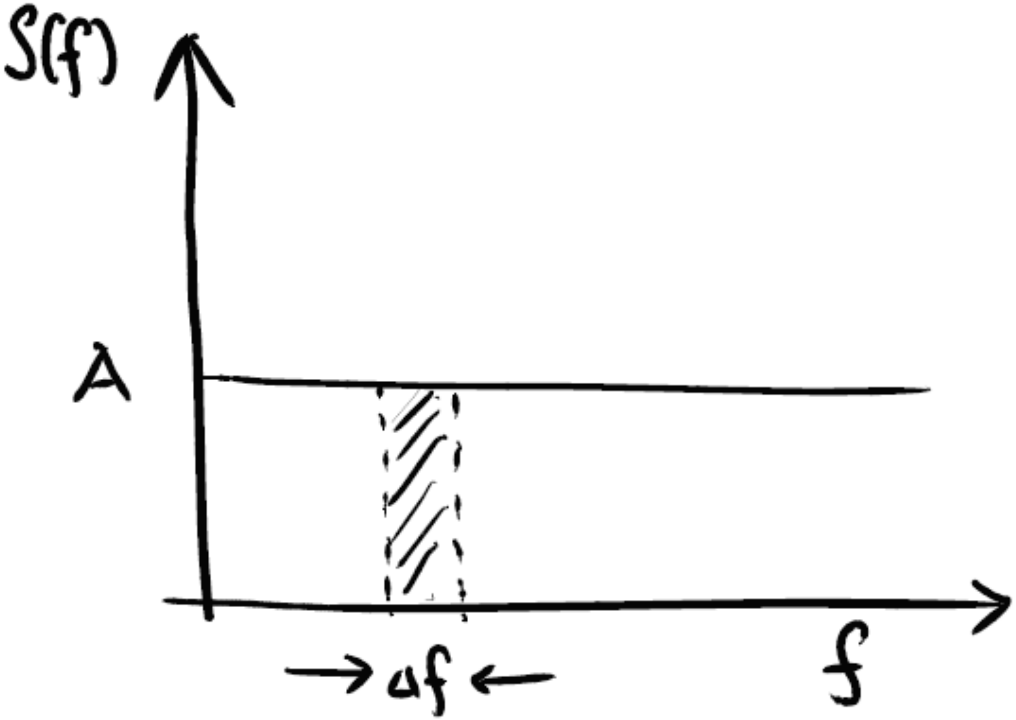
- **Shot noise :** Generated by a source of carriers. Noise due to variability in generation and arrival time of carrier at the source.
  - Requires a current flow.
  - Noise in diodes, BJTs, vacuum diodes, MOSFET biased in sub-threshold.

- **Thermal noise:** Noise due to variability in carrier transport within a transport channel. Due to thermal fluctuations.
  - Requires no current flow.
  - Proportional to temperature.
  - Noise in resistors, MOSFET (above-threshold) and JFETs.

White noise: Frequency domain

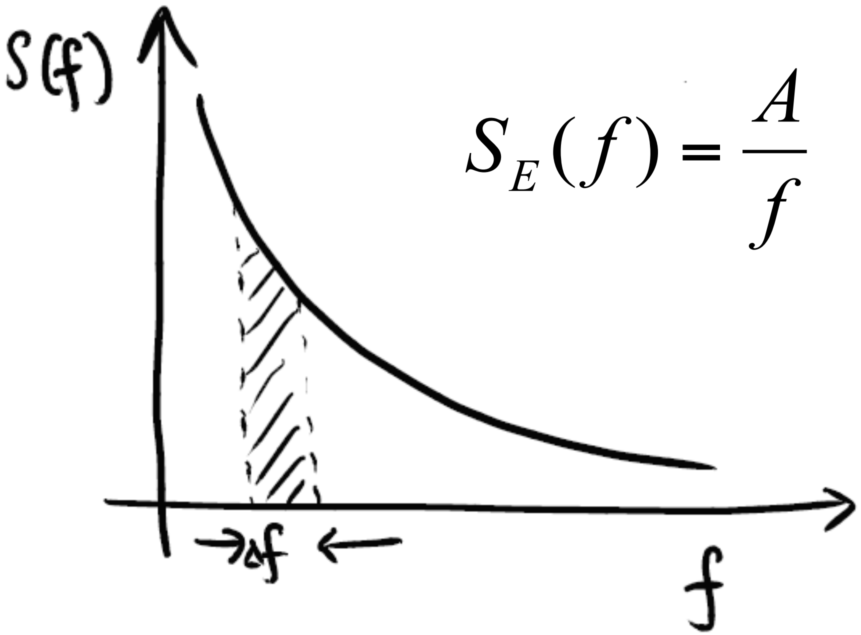
- White noise: power of noise constant within any frequency band.
- Statistically the easiest noise to work with.

$$S_E(f) = A$$

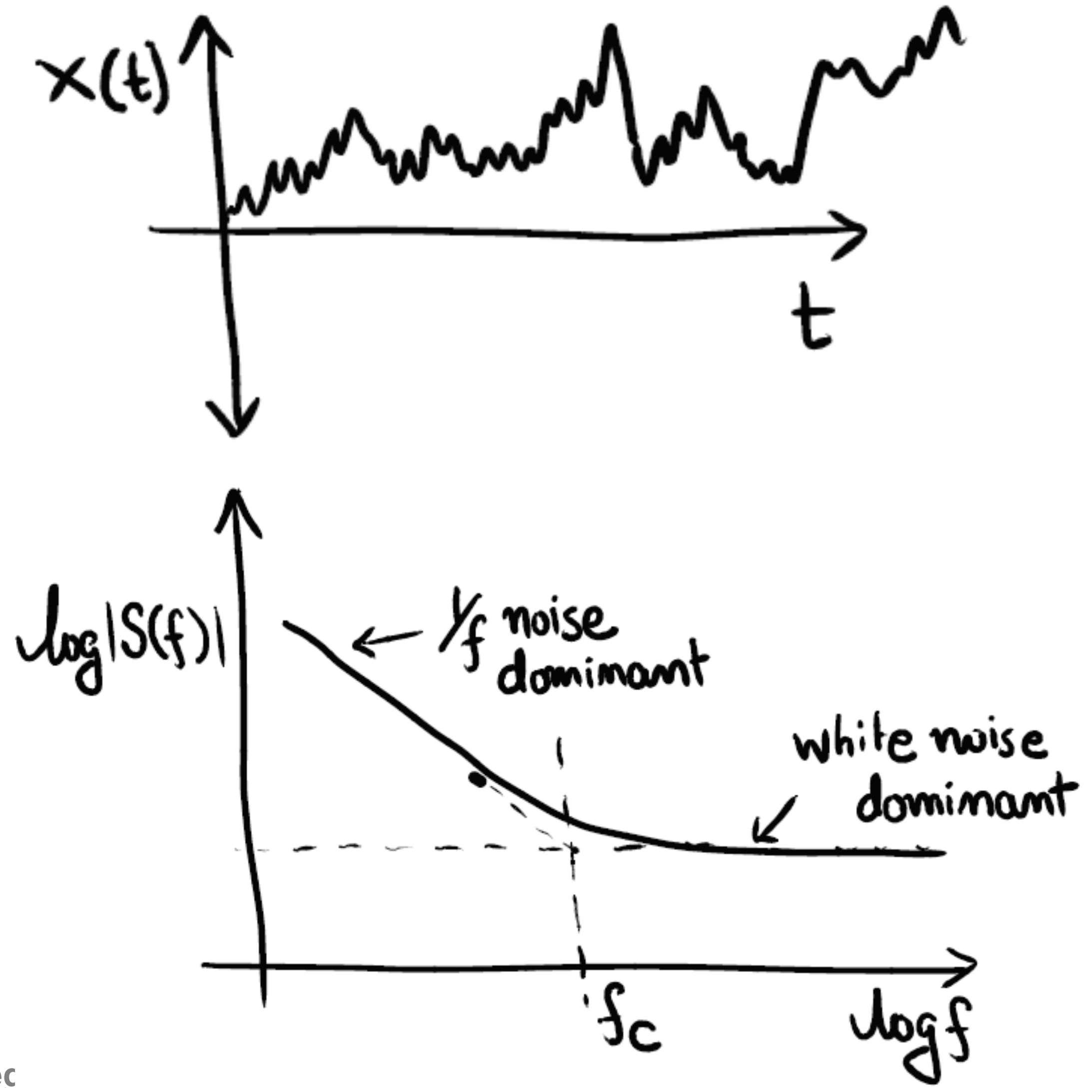


Flicker noise: Frequency domain

- Flicker noise or 1/f noise : noise power concentrated at lower frequencies

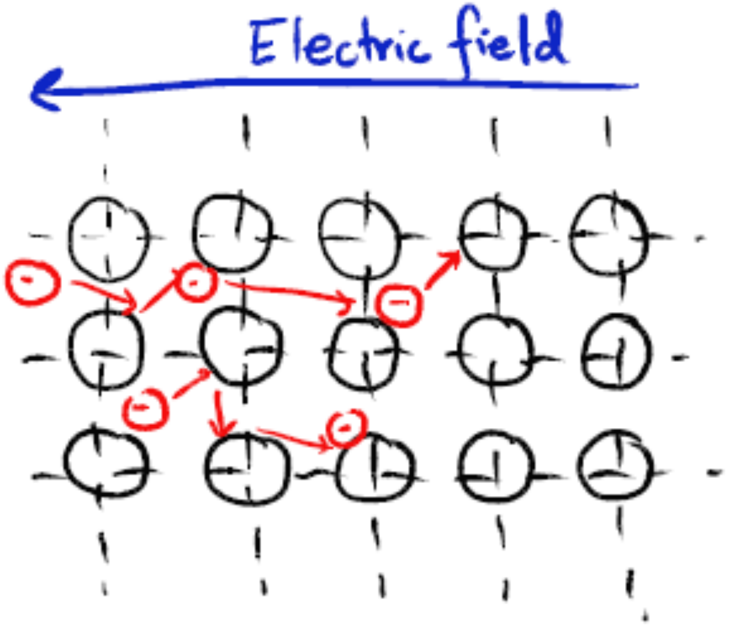


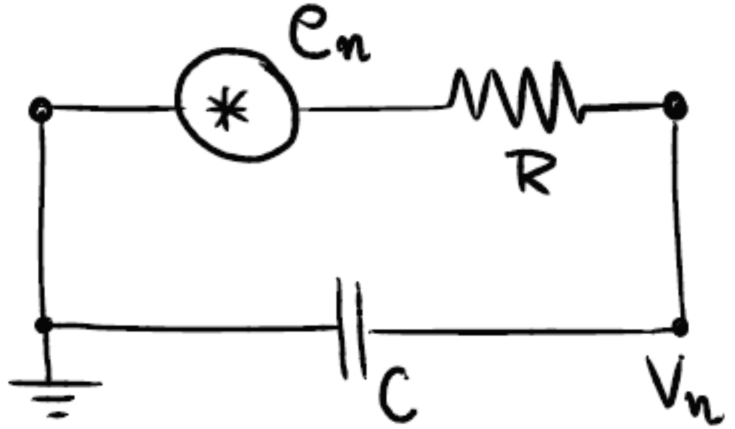
$$S_E(f) = \frac{A}{f}$$



- Most system has both noise components.
- Corner frequency (f_c): where white noise and flicker noise are equal.

Derivation of thermal noise in resistors





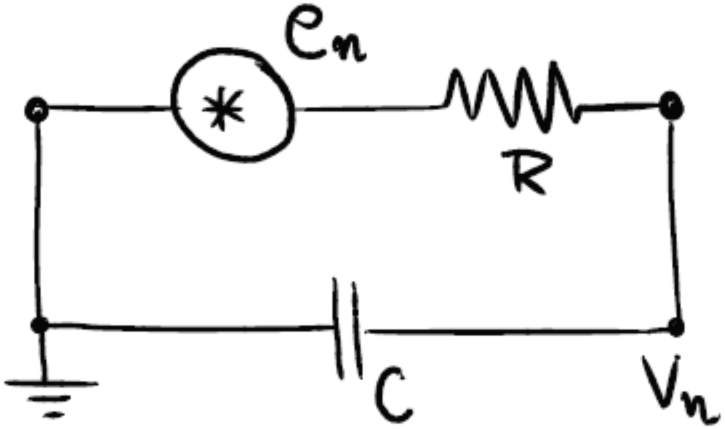
$$v_n(t) + i_n(t)R = e_n(t)$$

$$v_n(t,\tau) + i_n(t,\tau)R = e_n(t,\tau)$$

$$|V_n(\omega,\tau)|^2 = \frac{|E_n(\omega,\tau)|^2}{\left(1+\omega^2 R^2 C^2\right)}$$

- Thermally excited electrons collide with the lattice or potential barriers and change directions.

Derivation of thermal noise in resistors



$$\bar{v}_n^2 = \frac{1}{2\pi}\int_0^\infty \lim_{T\to\infty}\frac{1}{T}2\left|V(\omega,\tau)\right|^2 d\omega$$

$$\bar{v}_n^2 = \frac{1}{2\pi}\int_0^\infty \frac{1}{\left(1+\omega^2R^2C^2\right)}\lim_{T\to\infty}\frac{1}{T}2\left|E(\omega,\tau)\right|^2 d\omega$$

$$\bar{v}_n^2 = \frac{A}{2\pi}\int_0^\infty \frac{1}{\left(1+\omega^2R^2C^2\right)}d\omega = \frac{A}{4RC}$$

Thermal equilibrium: Average potential energy equals thermal energy

$$\boxed{\frac{1}{2}C\bar{v}_n^2 = \frac{1}{2}KT}$$

$$\bar{e}_n^2 = \frac{1}{2\pi}A\Delta\omega \qquad \bar{e}_n^2 = \frac{2}{\pi}C\bar{v}_n^2R\Delta\omega$$

$$\boxed{\bar{e}_n^2 = 4KTR\Delta f} \qquad \boxed{\bar{i}_n^2 = 4KTG\Delta f}$$

Thermal noise: Limits

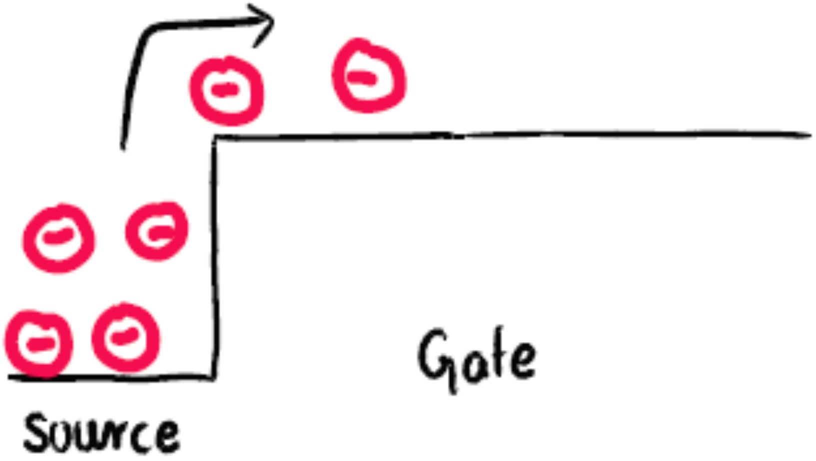
$$\boxed{\bar{e}_n^{\,2} = 4KTR\Delta f}$$

- When is the white noise formula valid ?
- Note that the formula implies infinite energy if integrated over all the frequency bins.
- Include quantum limits – modified version of Plank's law

$$\frac{1}{2}C\bar{v}_n^{\,2} = \frac{1}{2}\frac{hf}{e^{hf/KT}-1} + \frac{1}{4}hf$$

- At TeraHz frequencies *hf* comparable to the thermal energy *KT*.
- At photonic frequencies the uncertainty/noise comes from Heisenberg's uncertainty principle.

Derivation of shot noise



- Current $$\bar{i} = \frac{e}{\Delta t}\bar{n}$$

$$\bar{i}^2 = \left(\frac{e}{\Delta t}\right)^2 \bar{n}^2 = \left(\frac{e}{\Delta t}\right)^2 \bar{n} = \frac{e}{\Delta t}\bar{i}$$

- Randomness in the electrons that jump a potential barrier.
- Can be modeled as a Poisson process – number of electrons jumping the barrier.

$$P(n) = \frac{\lambda^n}{n!}e^{-\lambda}$$

$$\bar{n} = \lambda; \bar{n}^2 = \lambda$$

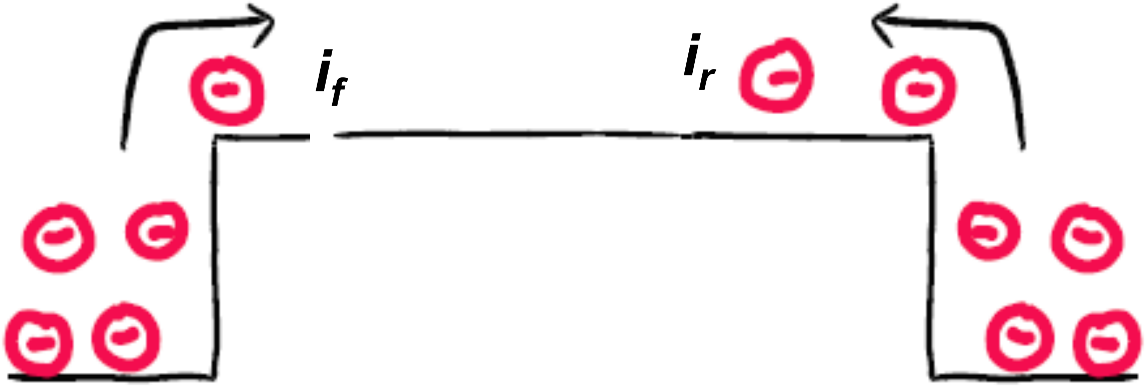
- Double sided spectral density.

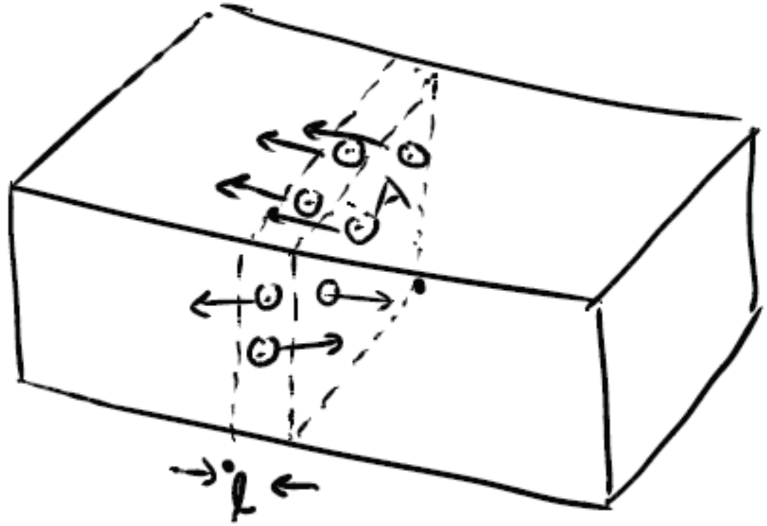
$$\bar{i}^2 = e\bar{i}\Delta f$$

- Single sided spectral density.

$$\bar{i}_n^2 = 2e\bar{i}\Delta f$$

Thermal noise from Shot noise





- Total current $\quad i = i_f - i_r$
- Total noise $\quad \bar{i}_n^2 = \bar{i}_{nf}^2 + \bar{i}_{nr}^2 \qquad \bar{i}_n^2 = 2ei_f\Delta f + 2ei_r\Delta f$
- Electrons diffuse $\quad \bar{i}_{f,r} = enD\dfrac{A}{l} \qquad \bar{i}_n^2 = 4e\dfrac{eDnA}{l}\Delta f$
- Einstein's relationship

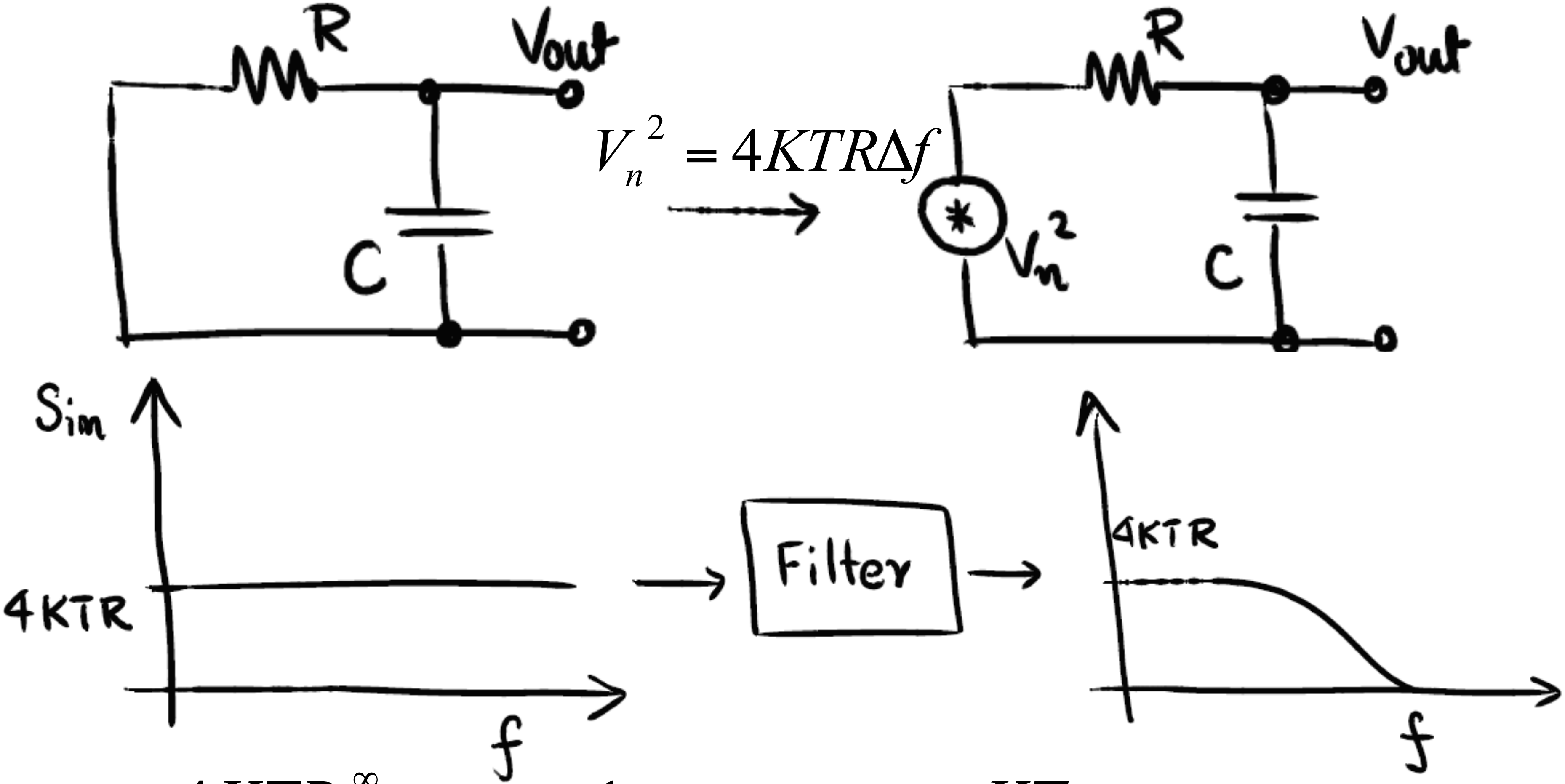
$$D = \left(\frac{KT}{e}\right)\mu$$

$$\boxed{\bar{i}_n^2 = 4KT\frac{e\mu nA}{l}\Delta f = 4\frac{KT}{R}\Delta f}$$

Noise on a capacitor

- An ideal capacitor itself is noiseless.

$$V_n^2 = 4KTR\Delta f$$

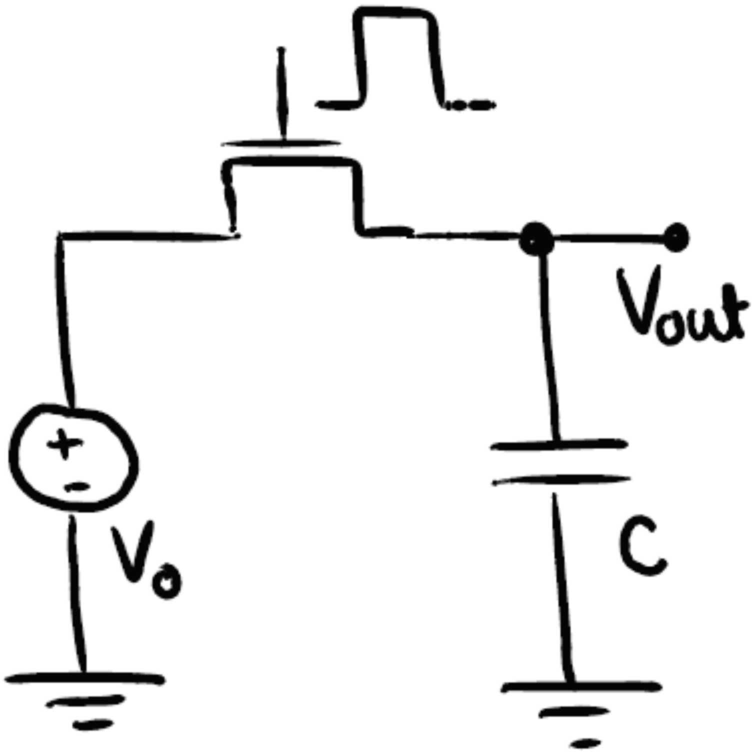


$$\overline{v}_{out}^2 = \frac{4KTR}{2\pi}\int_0^\infty \frac{1}{\left(1+4\pi^2 f^2 R^2 C^2\right)} df = \frac{KT}{C}$$

- Independent of the resistor

Noise on a capacitor

- For a 1pF capacitor charging and discharging during every cycle will contribute to a noise voltage of ±64μV.
- Large value of capacitor is required for reducing the effect of noise.
- For white noise use averaging – sample the voltage multiple times.



$$V_{sum} = V_{out1} + V_{out2} + \ldots + V_{outN}$$

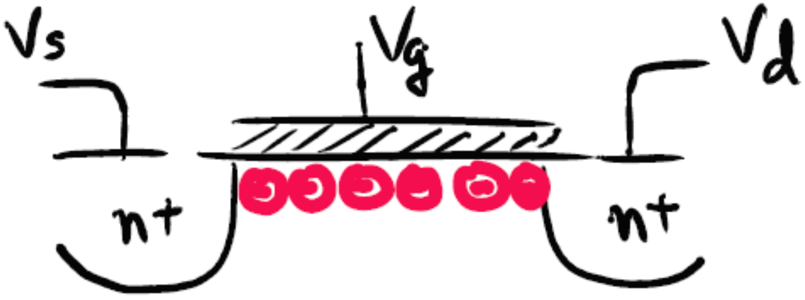
$$\overline{V}_{out1} = \overline{V}_{out2} = \ldots = \overline{V}_{outN} = V_{out}$$

$$Signal\ Power = N^2 V_{out}^2$$

$$Noise\ Power = N\frac{KT}{C}$$

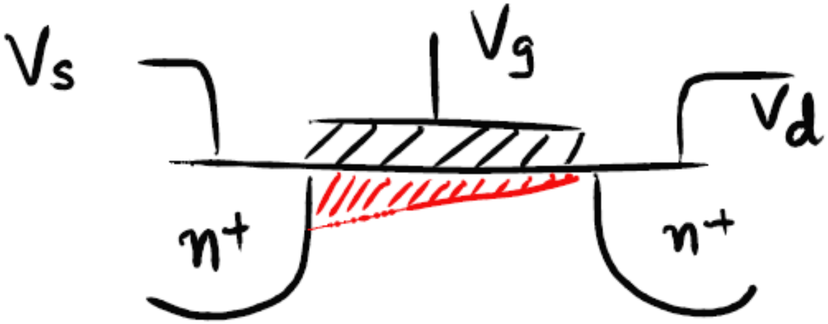
$$SNR \propto N$$

Noise in MOSFET



- Above threshold: Each element of the channel acts as resistor.

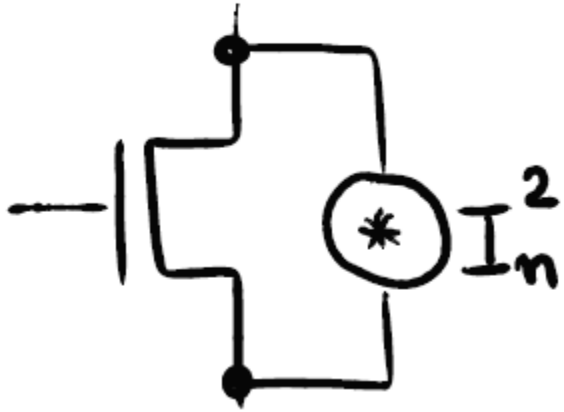
$$R_{ON} = \frac{1}{\frac{\partial I_{ds}}{\partial V_{ds}}} = \frac{1}{\mu C_{ox} \frac{W}{L} (V_{gs} - V_{th})} \approx \frac{1}{g_m}$$

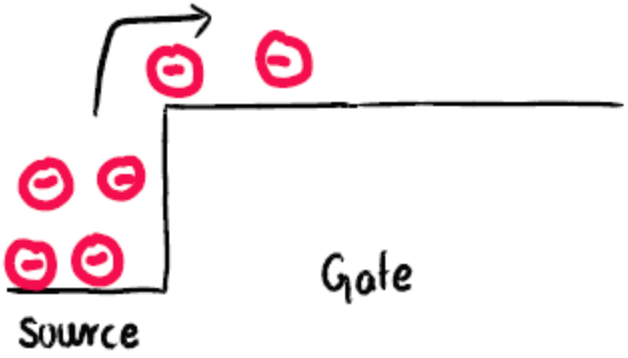


- However the channel width is graded

$$\boxed{\overline{i}_n^{\,2} = 4KT\gamma g_m \Delta f}$$

$$\gamma \approx 2/3$$

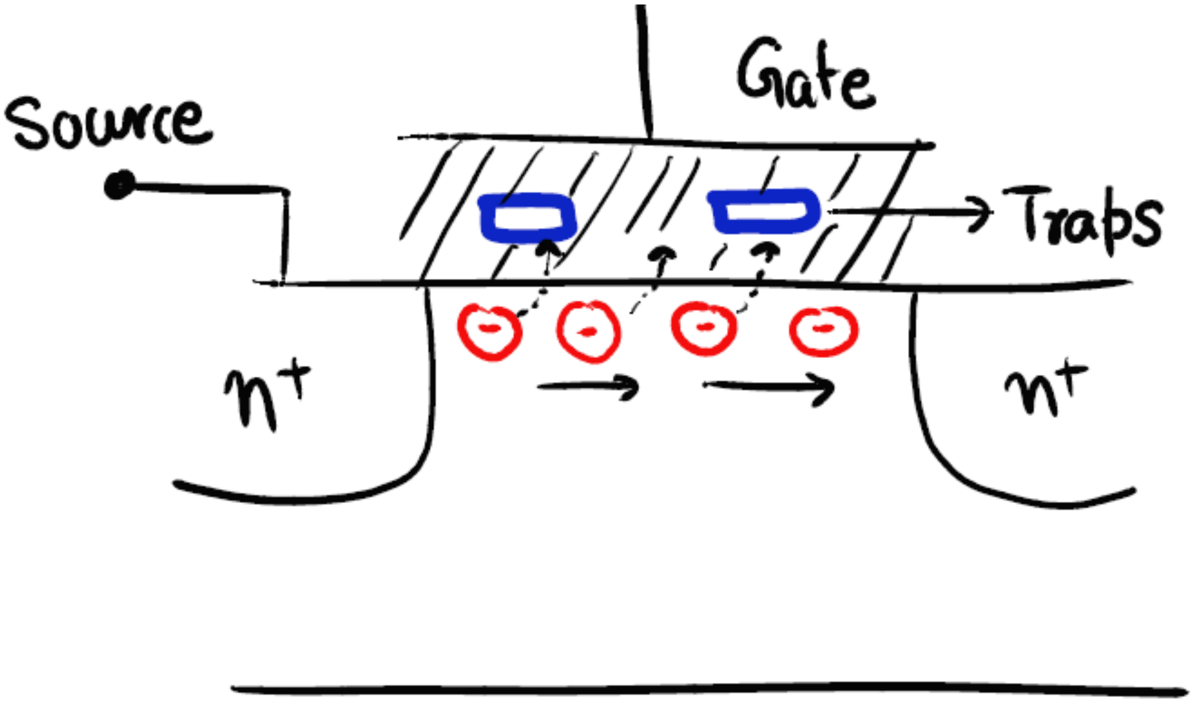




- Sub-threshold: The source and drain act as a random source.

$$\overline{i}_n^{\,2} = 2e\overline{i}\Delta f$$

Flicker noise in MOSFET



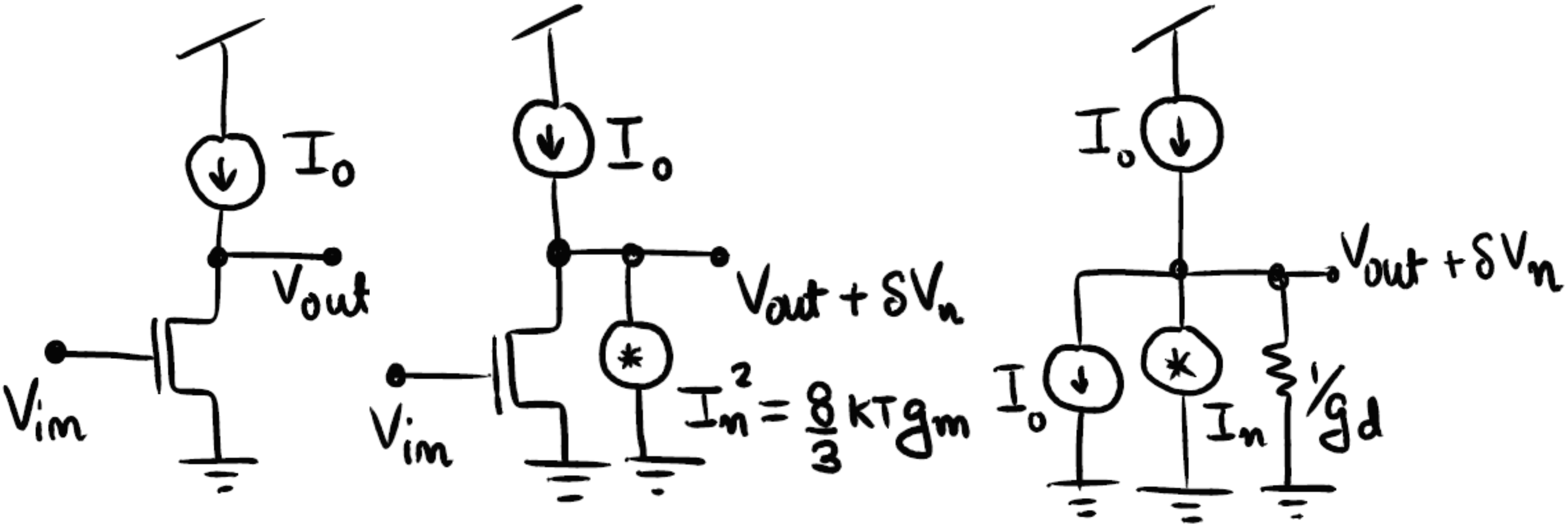
- Due to presence of dangling bonds and surface imperfections.
- The dangling bonds create traps that capture electrons/holes and then later release them back into the channel.

- Trapping and de-trapping is a slow process. Trapping charge leads to change in threshold voltage – changes the conductance and current. PMOS has lower flicker noise.

$$S_{Id}^2(f) = \frac{KF\ I_d^{AF}}{f^{EF} C_{ox} W L_{eff}}$$

Noise analysis

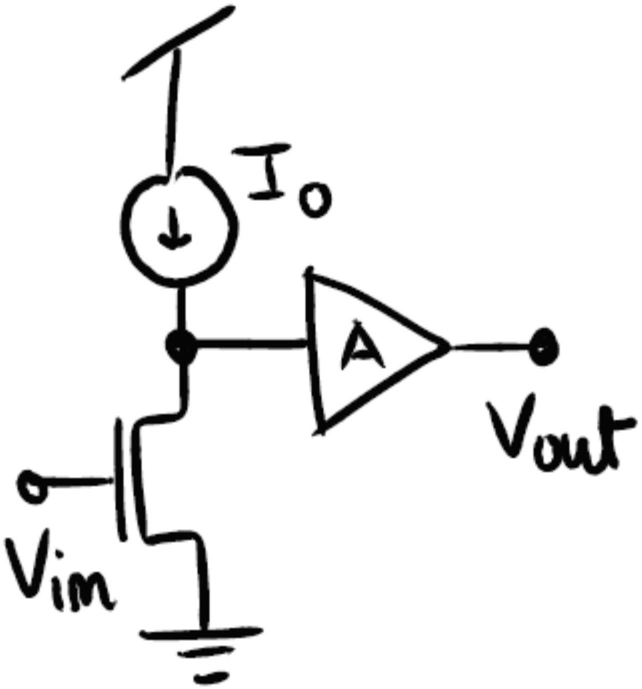
- Thermal and flicker noise can be analyzed in a similar manner.
- Use the amplifier equivalent circuit we discussed before.

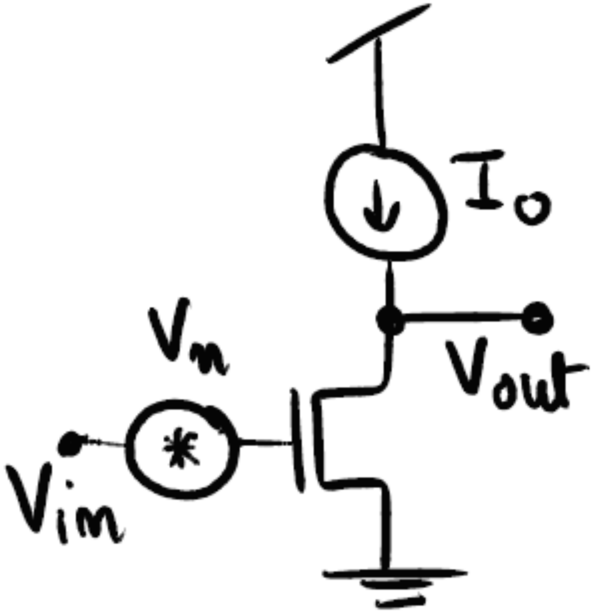


$$\overline{V}_{out} = \frac{1}{g_d} I_n \qquad \overline{V}_{out}^{\,2} = \frac{1}{g_d^2} \overline{I}_n^{\,2} \qquad \overline{V}_{out}^{\,2} = \frac{8}{3} KT \frac{g_m}{g_d^2} \Delta f$$

Input referred noise

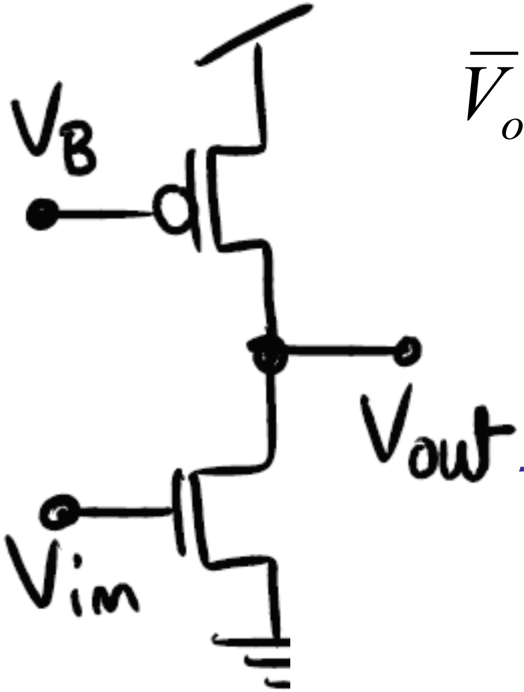
- Analyzing only the output noise leads to a biased perception.
- Standard: Refer the output noise to the input of the amplifier.





$$\overline{V}_{out}^{2} = \frac{8}{3} KTA^{2} \frac{g_m}{g_d^{2}} \Delta f \qquad \overline{V}_{in}^{2} = \frac{\overline{V}_{out}^{2}}{(Gain)^{2}} \qquad \overline{V}_{in}^{2} = \frac{8}{3} KT \frac{1}{g_m} \Delta f$$

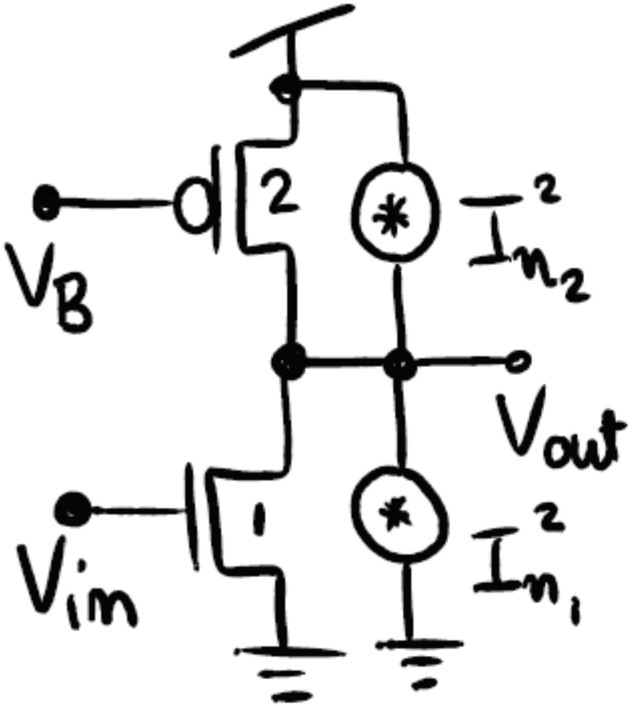
Common source amplifier

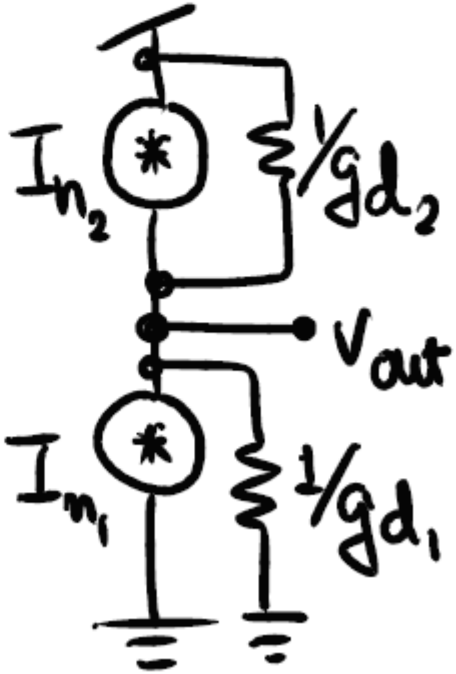


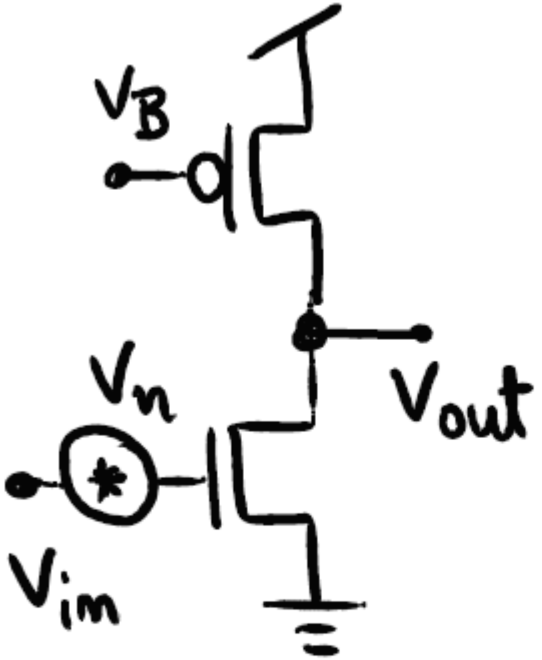
$$\overline{V}_{out}^{2} = \frac{8}{3}KT\frac{g_{m1}}{(g_{d1}+g_{d2})^2}\Delta f + \frac{8}{3}KT\frac{g_{m2}}{(g_{d1}+g_{d2})^2}\Delta f$$

$$\overline{V}_{in}^{2} = \frac{8}{3}\frac{KT}{g_{m1}}\left(1+\frac{g_{m2}}{g_{m1}}\right)\Delta f$$

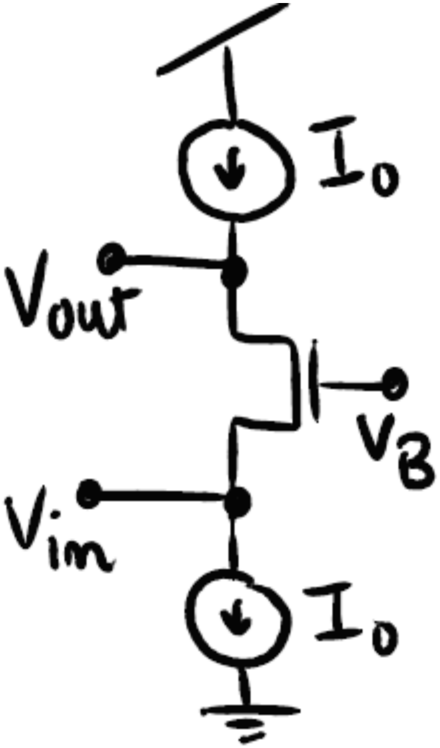
- For a fixed amplifier gain, low noise design means reduce the transconductance of current source.

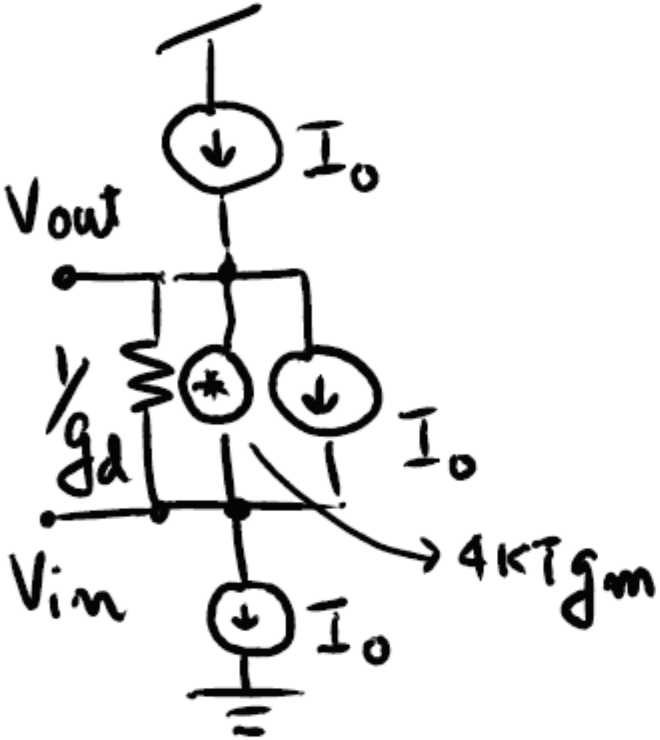


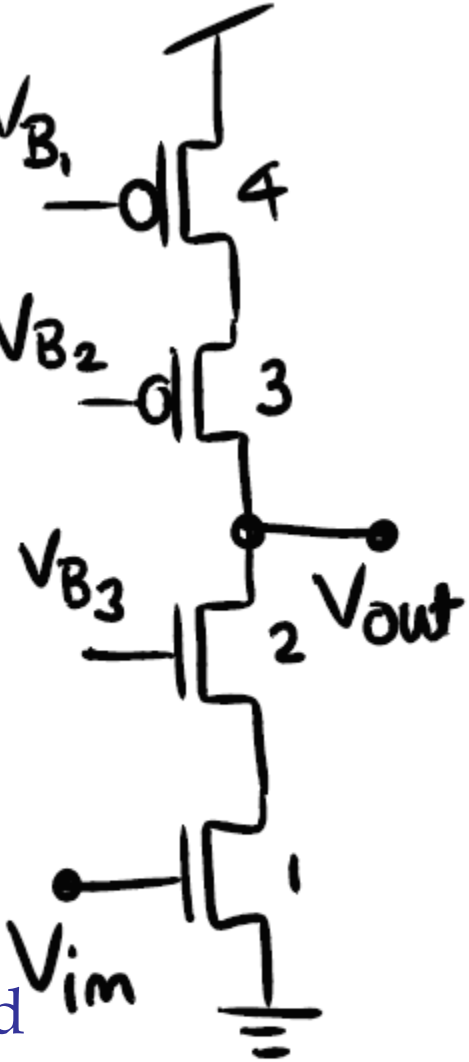




Common gate and cascoded amplifier

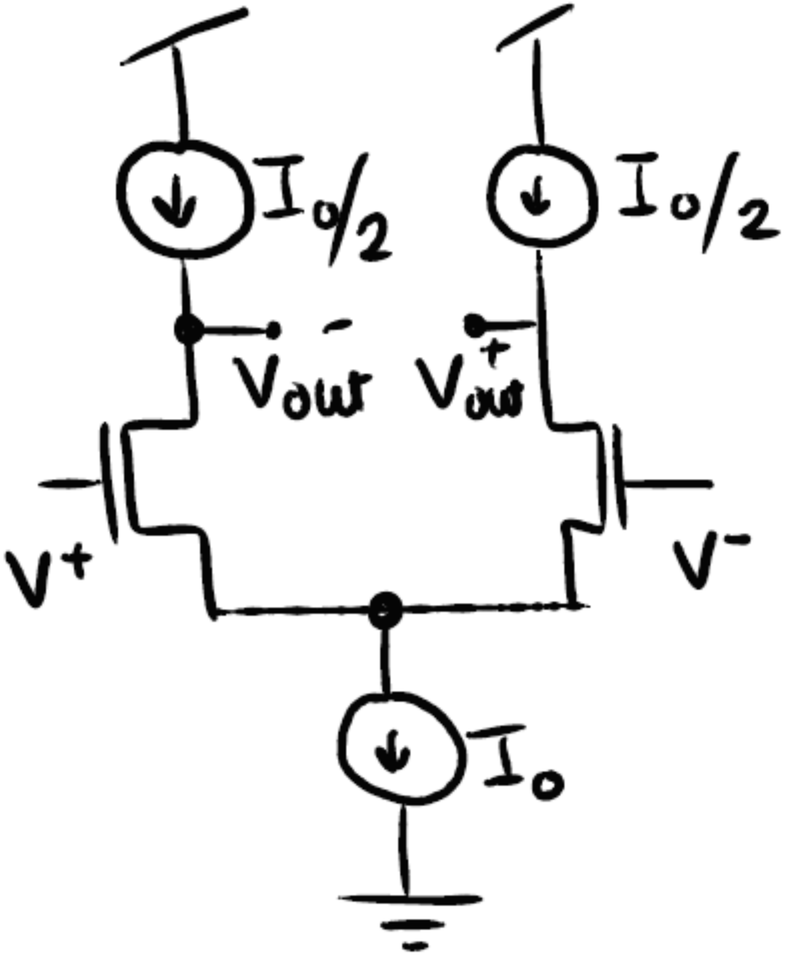


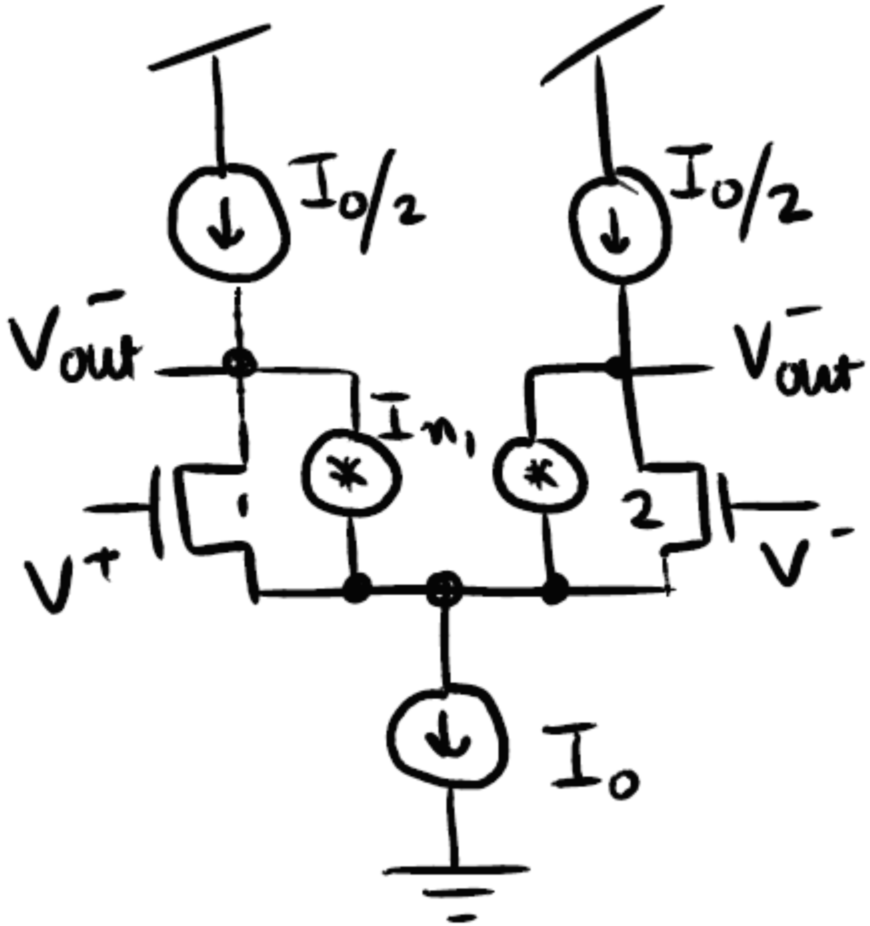




- Common source amplifier – no noise current gain.
- What is the effect of transistors 2 and 3 in cascoded amplifier – does it add to the amplifier noise ?

Differential amplifier



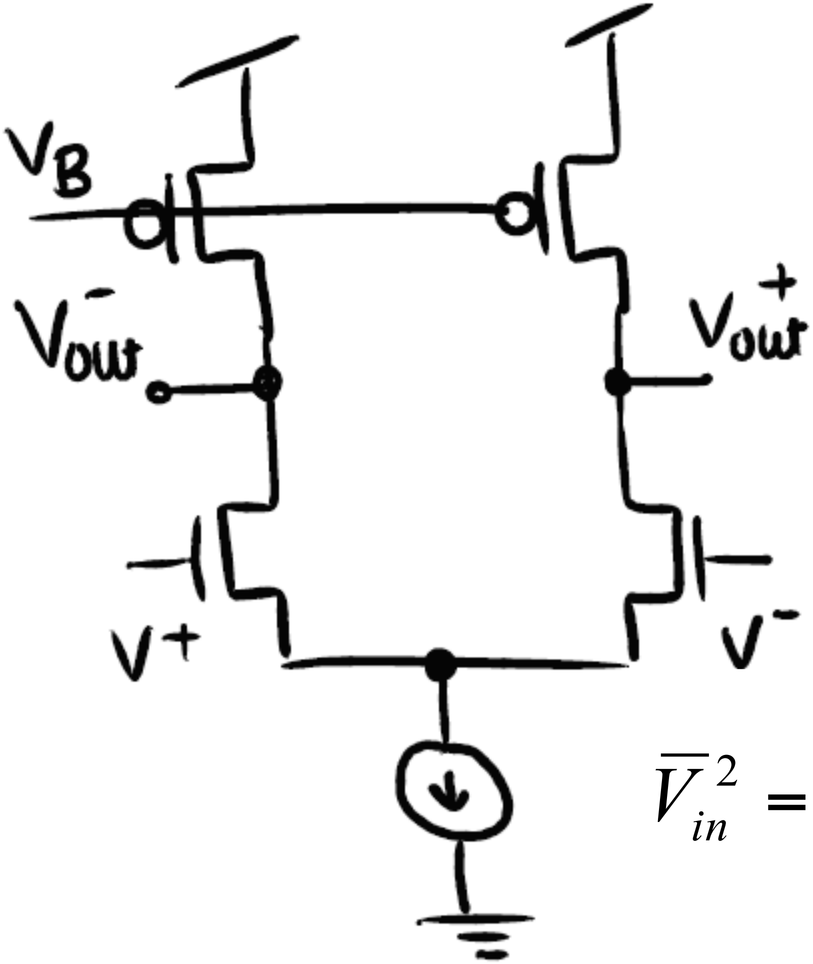


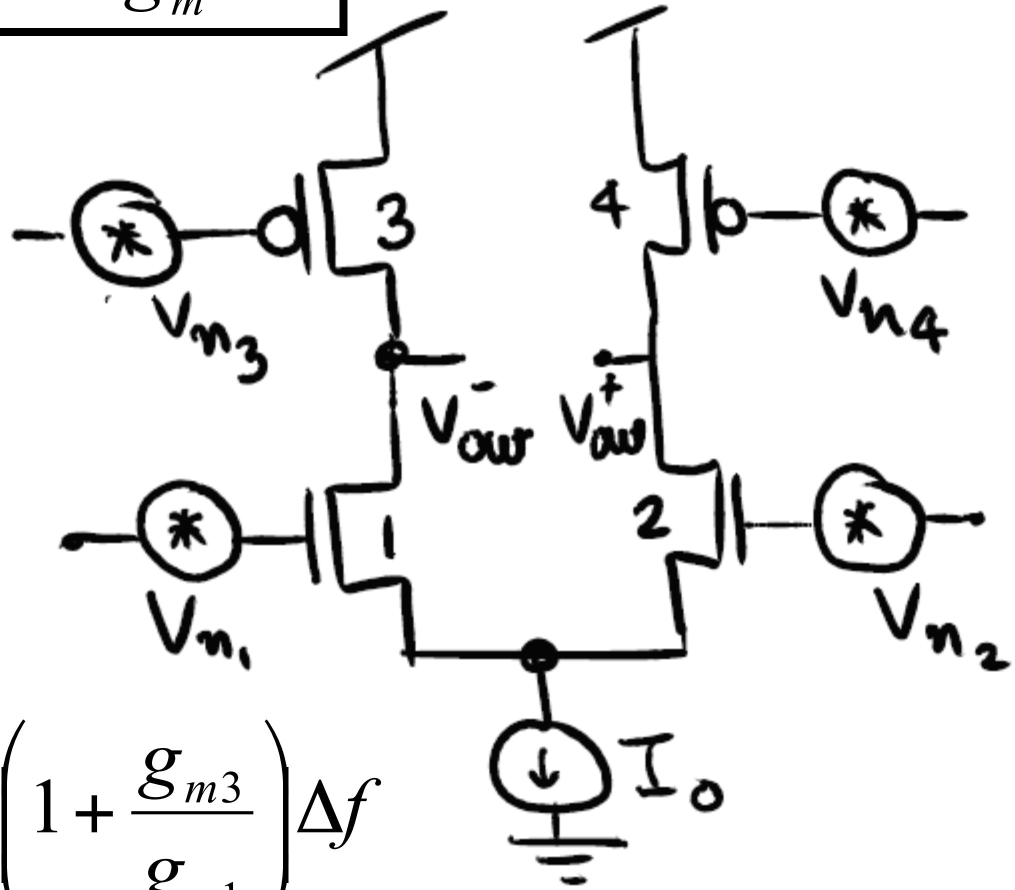
- Use symmetry to create two common source amplifiers.
- Noise in differential pair – twice the noise as a single stage amplifier – why then choose a differential topology ?

Noise in Differential Amplifier

- Easier to refer the noise current as a gate voltage noise.

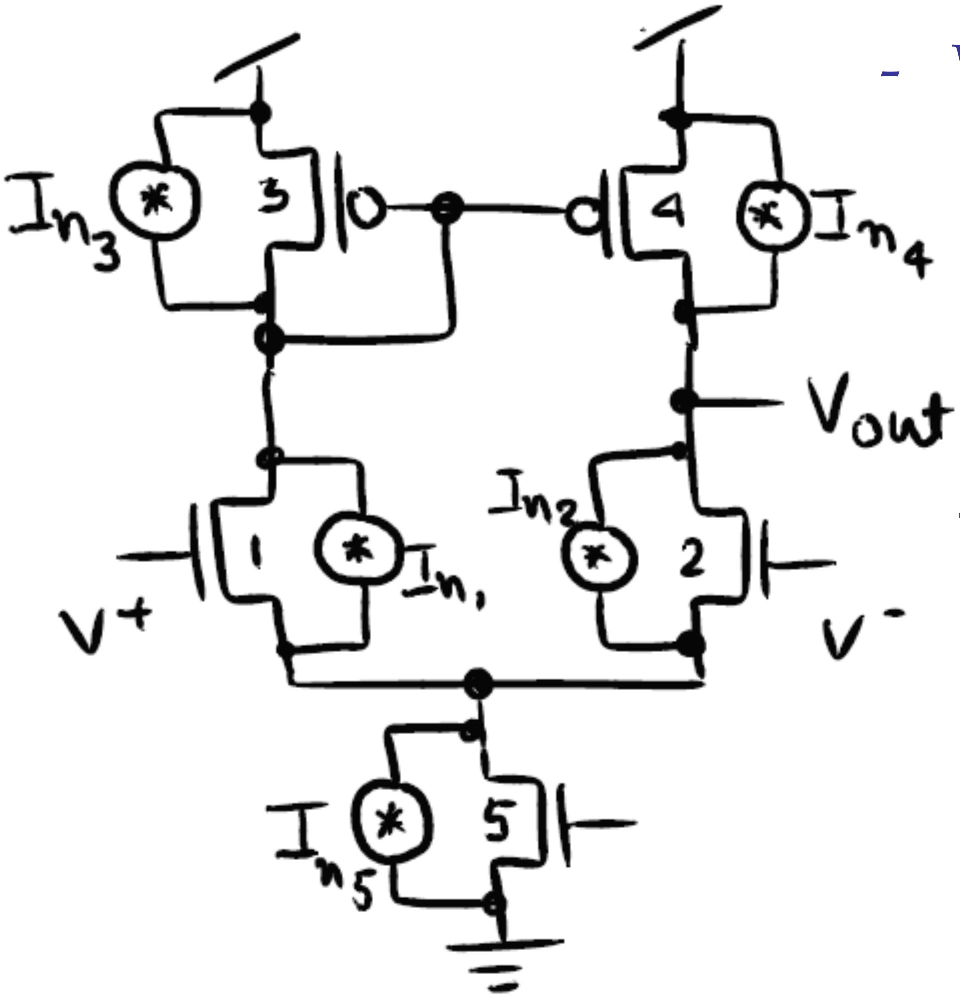
$$\overline{i_n^2} = \frac{8}{3} KTg_m \Delta f \qquad \overline{v_n^2} = \frac{8KT}{3g_m} \Delta f$$





$$\overline{V_{in}^2} = \frac{16}{3} \frac{KT}{g_{m1}} \left(1 + \frac{g_{m3}}{g_{m1}} \right) \Delta f$$

Noise in Differential Amplifier



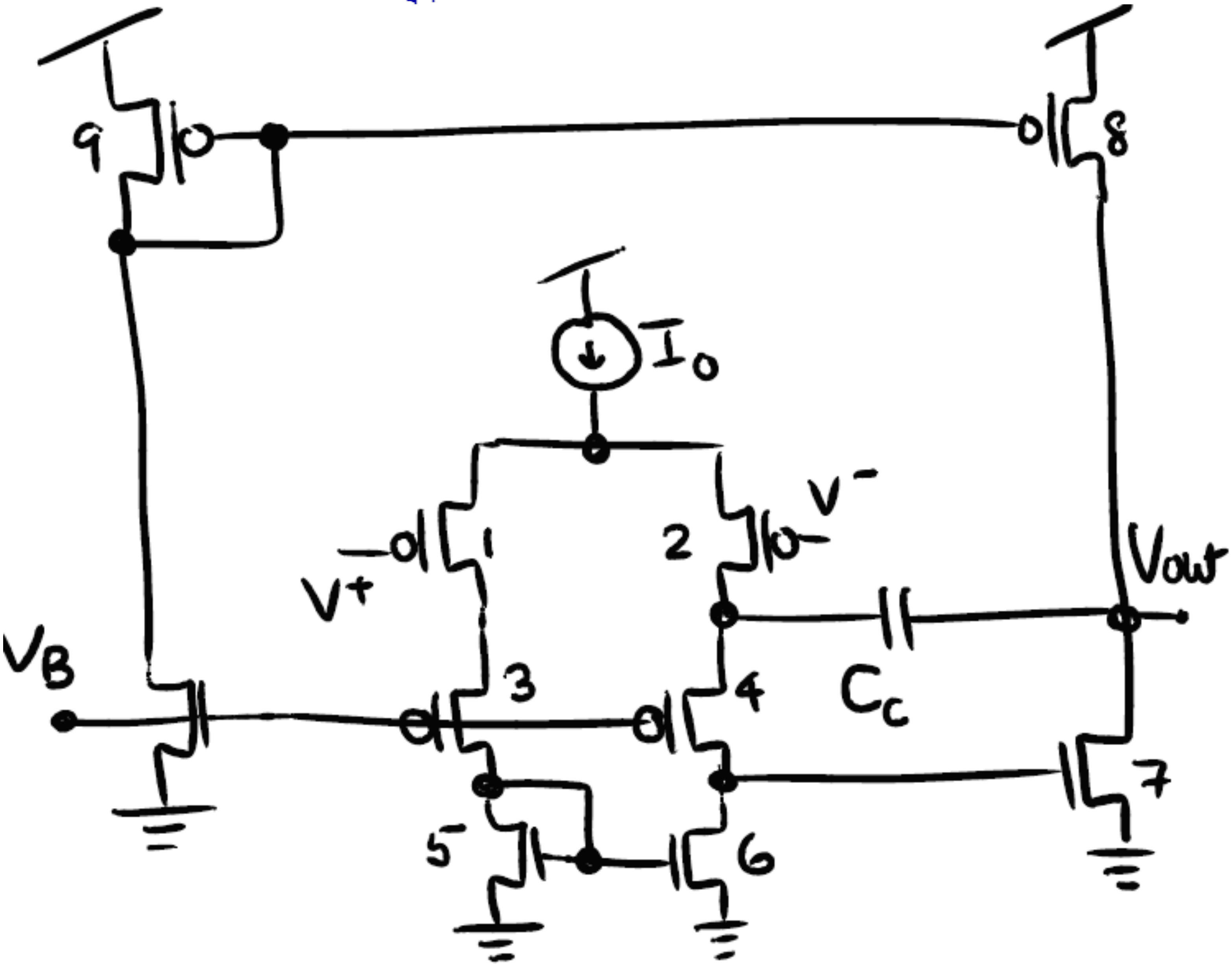
- What about the noise in transistor 5 ?

$$\overline{V}_{in}^{2} = \frac{16}{3}\frac{KT}{g_{m1}}\left(1 + \frac{g_{m3}}{g_{m1}}\right)\Delta f$$

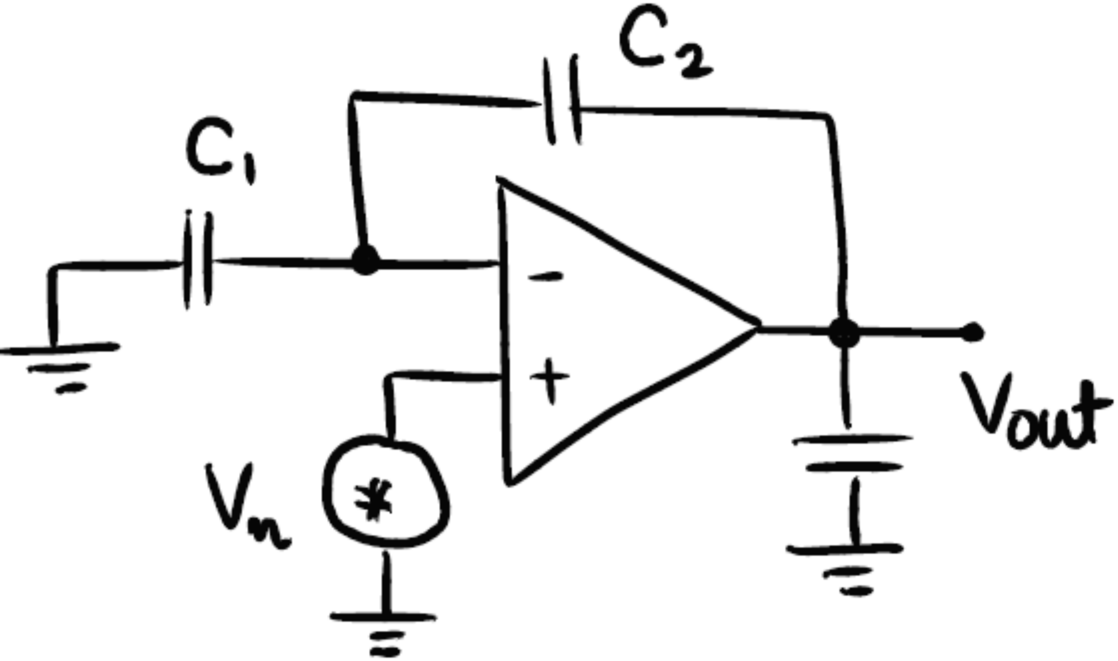
<u>Noise Analysis Principles</u>

- Put noise voltage sources at the gates of the transistors.
- Calculate the effective gain from the voltage sources to the output using superposition.
- Divide by the total gain of the system.

Low noise amplifier



Noise analysis in a feedback network



- Input referred amplifier noise

$$\overline{V_n}^2 = \frac{16}{3}\frac{KT}{g_{m1}}\left(1+\frac{g_{m3}}{g_{m1}}\right)\Delta f$$

- Amplifier noise is filtered by the feedback system formed by the amplifier, C_1 and C_2.

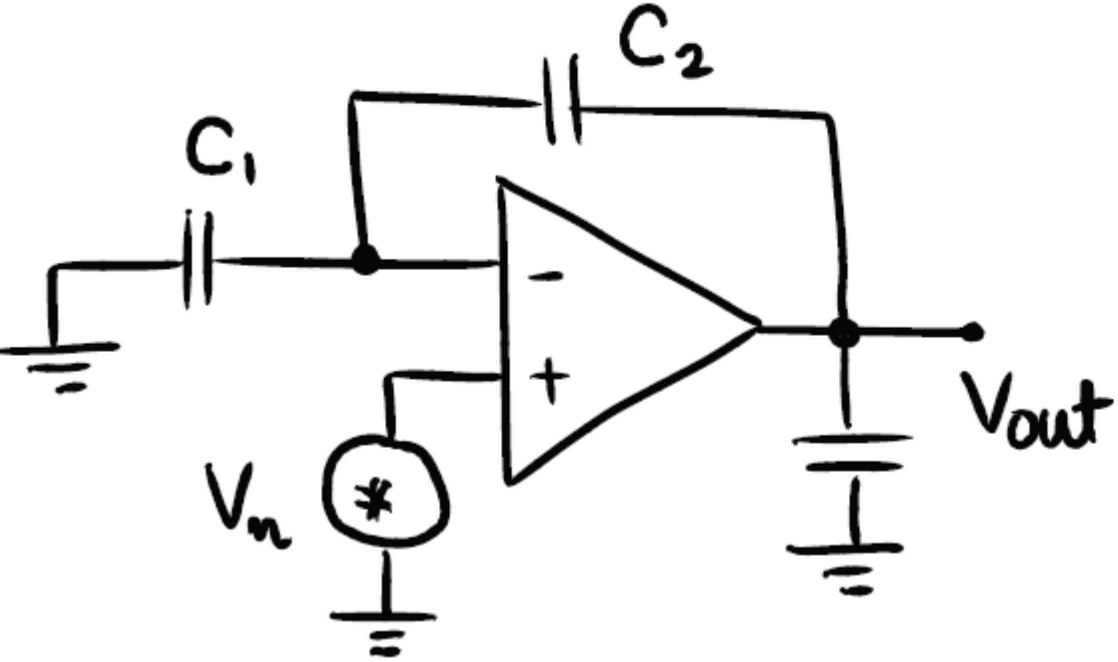
- Open loop amplifier – single pole

$$H_{amp}(s) = \frac{g_m}{2g_d}\frac{1}{(1+\frac{sC_L}{2g_d})}\left(V_{in}^{+} - V_{in}^{-}\right)$$

- Feedback amplifier with capacitor feedback.

$$H(s) = \frac{(1+C_1/C_2)}{1+s\left[C_L/(1+C_1/C_2)g_m\right]}$$

Noise analysis in a feedback network



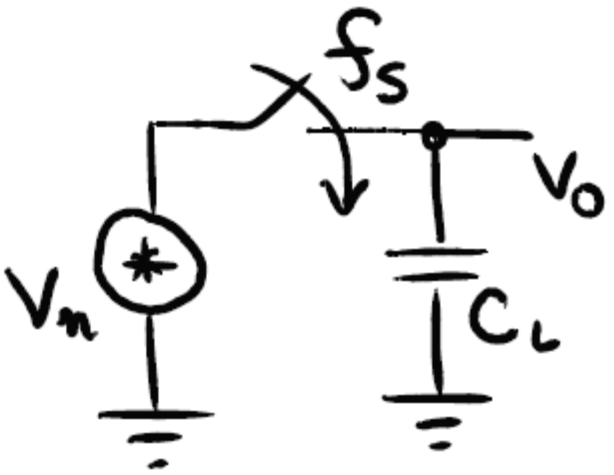
- Input referred amplifier noise

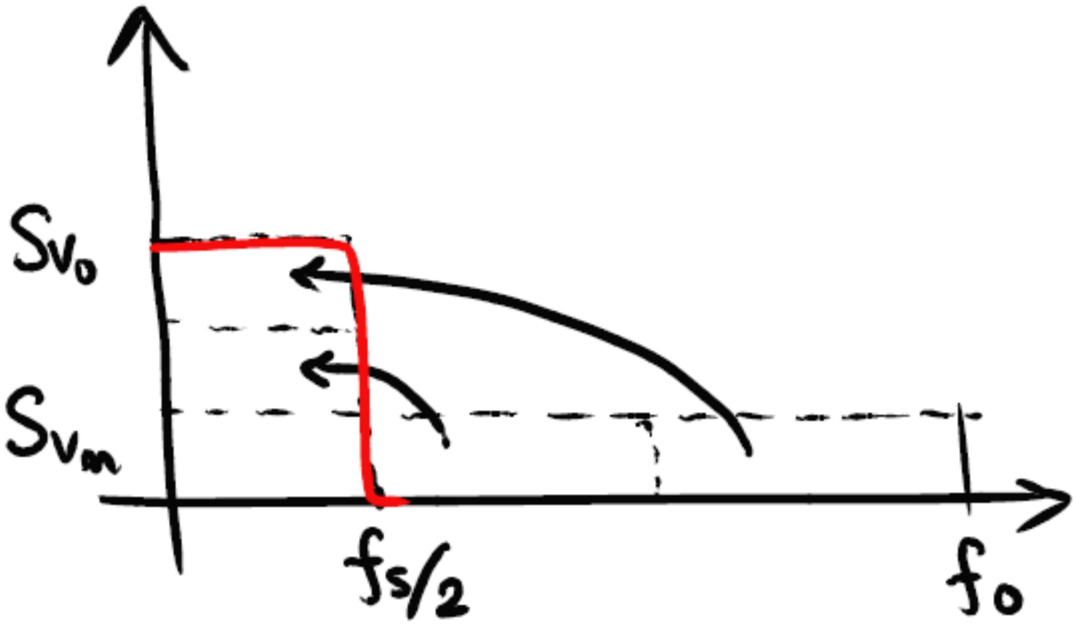
$$\overline{V}_n^{\,2} = \frac{16}{3}\frac{KT}{g_{m1}}\left(1+\frac{g_{m3}}{g_{m1}}\right)\Delta f$$

- Output noise due to input referred amplifier noise.

$$\overline{V}_{out1}^{\,2} = \int_0^{\infty} \left|H(f)\right|^2 \frac{16KT}{3g_{m1}}\left(1+\frac{g_{m2}}{g_{m1}}\right)df = \frac{4KT}{3C_L}\left(1+\frac{C_1}{C_2}\right)\left(1+\frac{g_{m2}}{g_{m1}}\right)$$

Sampled White Noise

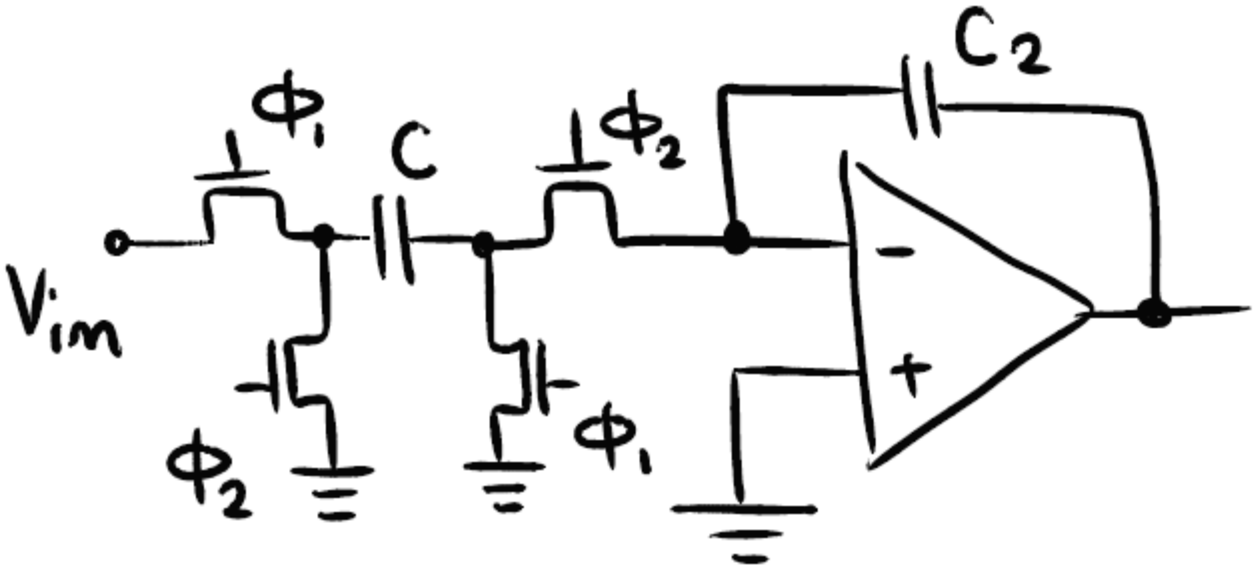




- After sampling the energy should still be conserved.
- Magnitude of sampled noise is larger but bandwidth smaller.

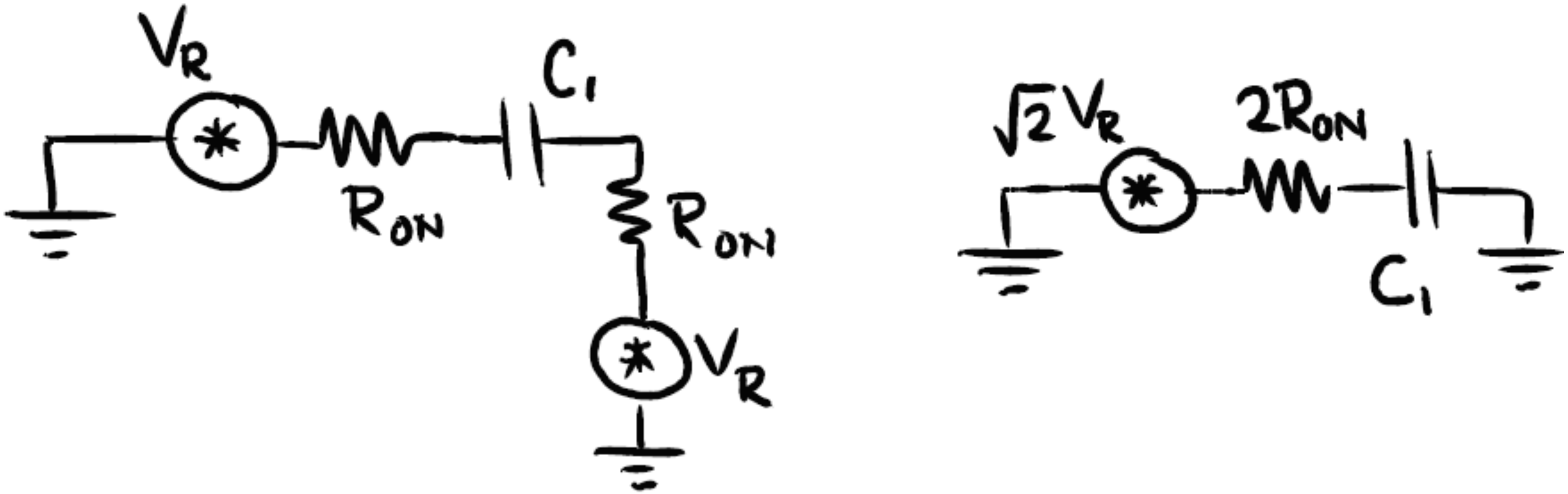
$$S_{v_n} f_0 = S_{v_0} \frac{f_s}{2}$$

Noise Analysis of Switched Capacitor Circuits



- Fundamental noise in switched capacitor due to sampling noise on the capacitors.
- Noise due to the amplifier also contributes to the output noise.
- Fundamental noise typically smaller than other interference – clock feedthrough, channel charge injection (discussed later).

Phase I



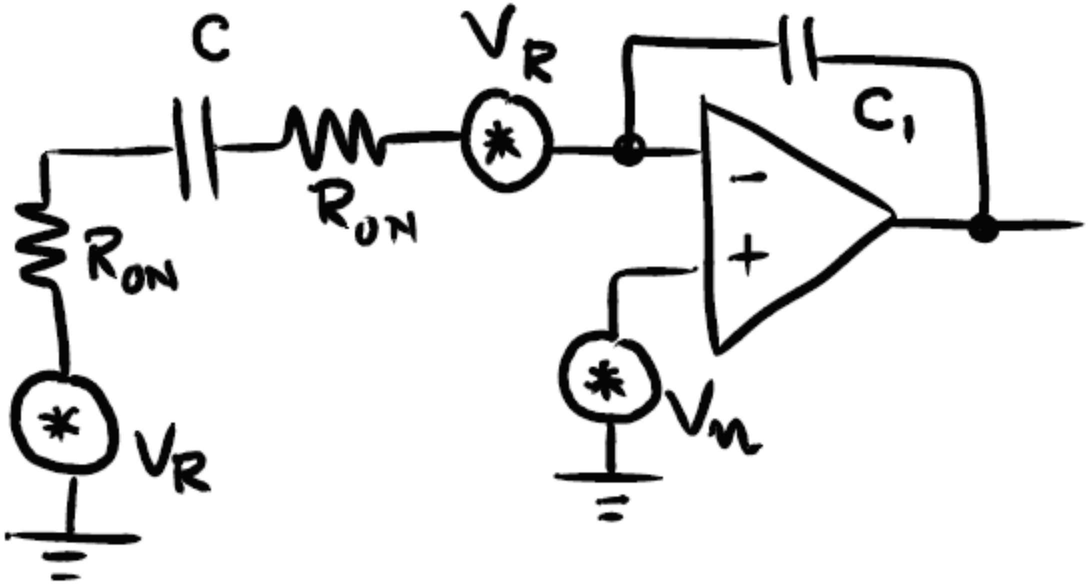
- Two switches form two noise sources which is integrated on the capacitor.

$$S_{C1}(f) = \frac{8KTR_{ON}}{1+(4\pi f R_{ON}C_1)^2}$$

$$\overline{V}_{C1}^{\,2} = \int_0^\infty S_{C1}(f)df = \int_0^\infty \frac{8KTR_{ON}}{1+(4\pi f R_{ON}C_1)^2}df = \frac{8KTR_{ON}}{8R_{ON}C_1} = \frac{KT}{C_1}$$

- The noise sampled in phase I remains stored on the capacitor.

Phase II noise



- Switch noise

$$S_{sw}(f) = 8KTR_{ON}$$

- Amplifier noise

$$S_{amp}(s) = \frac{16}{3}\frac{KT}{g_{m1}}\left(1 + \frac{g_{m3}}{g_{m1}}\right)$$

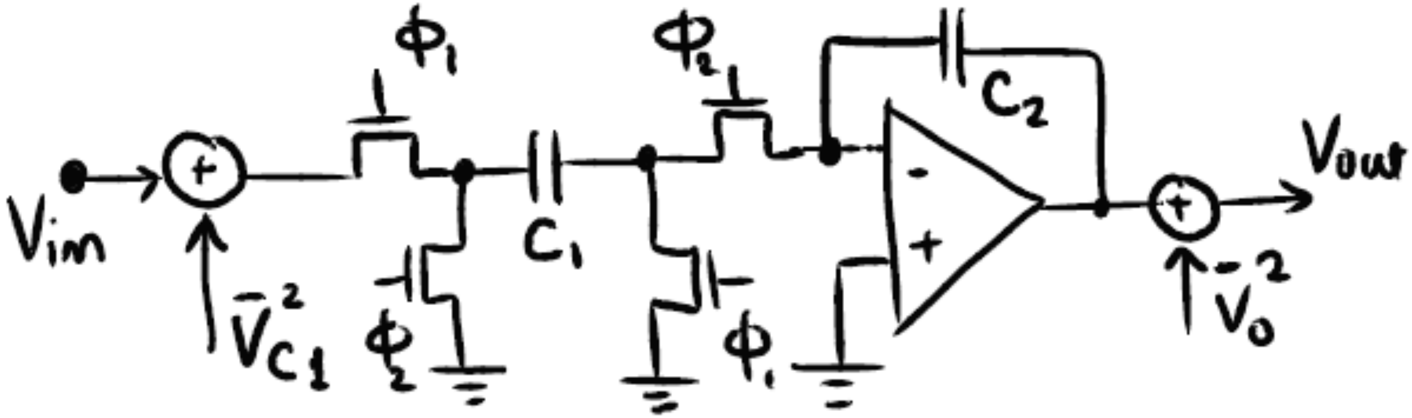
- Filter transfer function

$$H(s) = \frac{1}{(1 + s[2R_{ON} + 1/g_{m1}]C_1)}$$

- Integrate all the noise sources over the entire spectrum

$$\overline{V}^2_{sw,amp} = \int_0^\infty |H(f)|^2 A df = \frac{A}{4(2R_{ON} + 1/g_{m1})C_1}$$

Output Noise in SC integrator



- Transfer function of the integrator

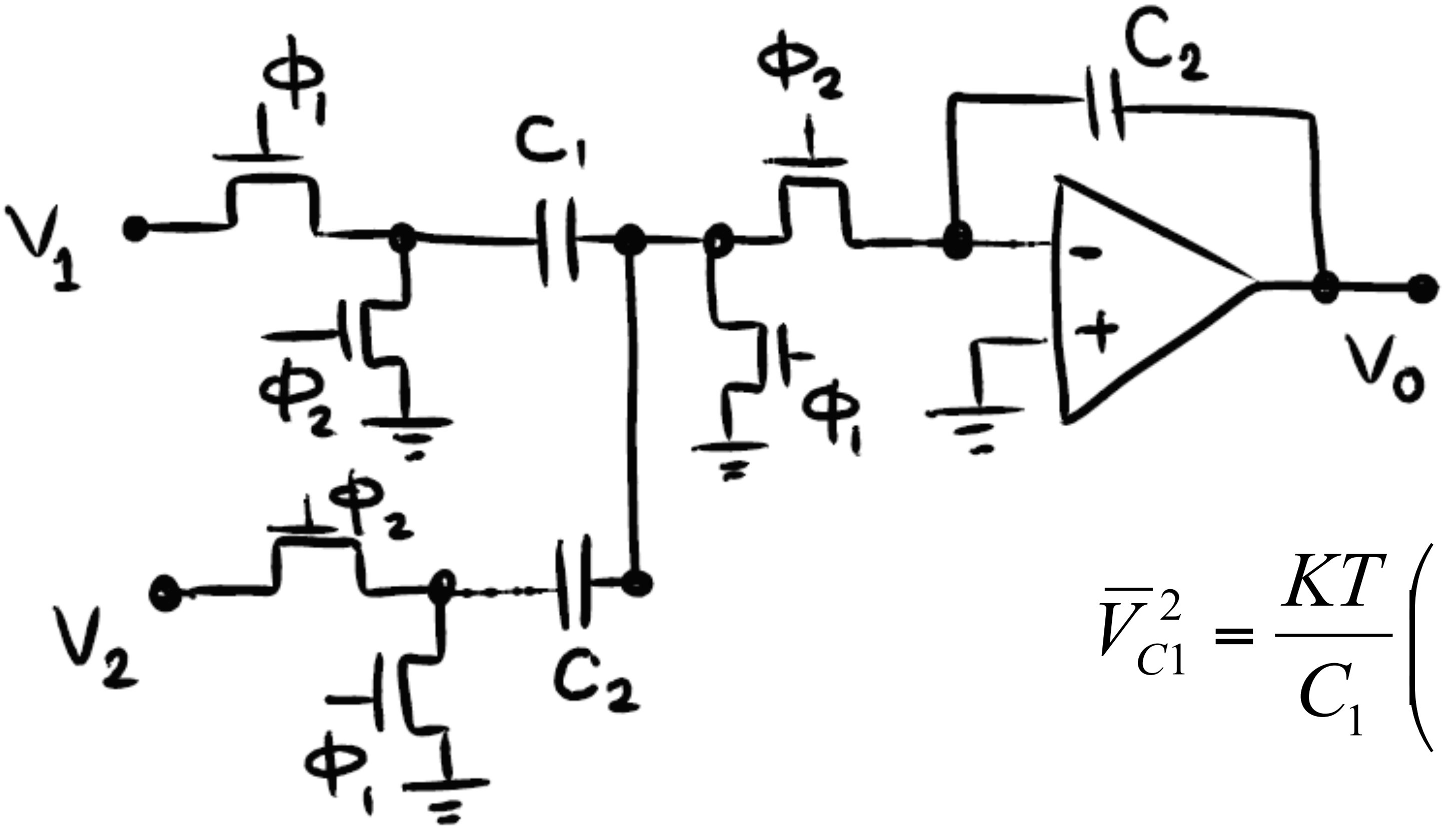
$$\overline{V}_{\text{int}}^{\,2} = \int_0^{f_B} \left(|H(f)|^2 S_{c1}(f) + S_0(f) \right) df$$

- Output noise due to the amplifier.

$$\overline{V}_0^{\,2} = \frac{4KT}{3C_L}\left(1+\frac{C_1}{C_2}\right)\left(1+\frac{g_{m2}}{g_{m1}}\right)$$

- How to get back the power spectrum ? Remember all the noise is sampled and the measurement bandwidth is f_B.

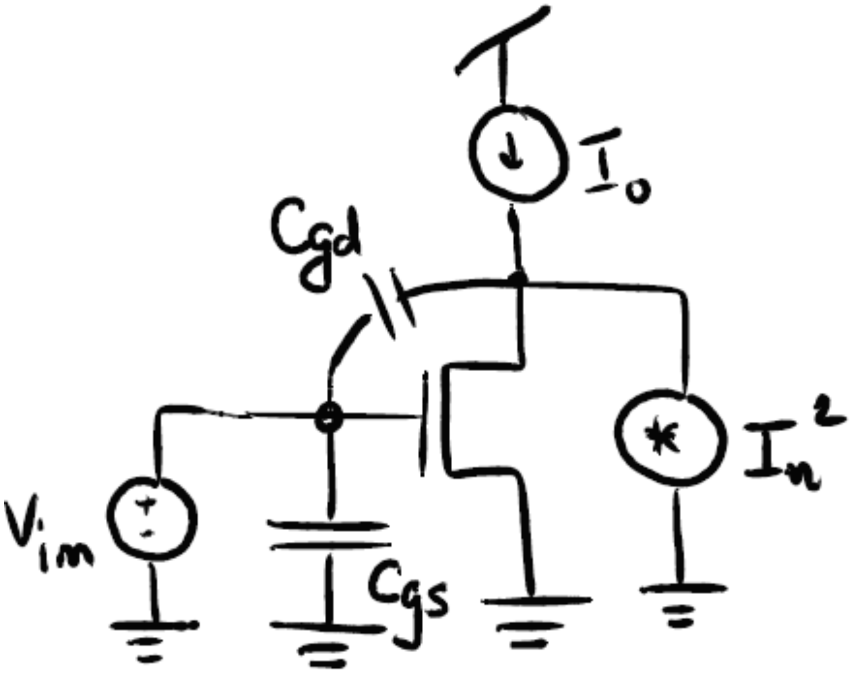
Noise in SC circuits

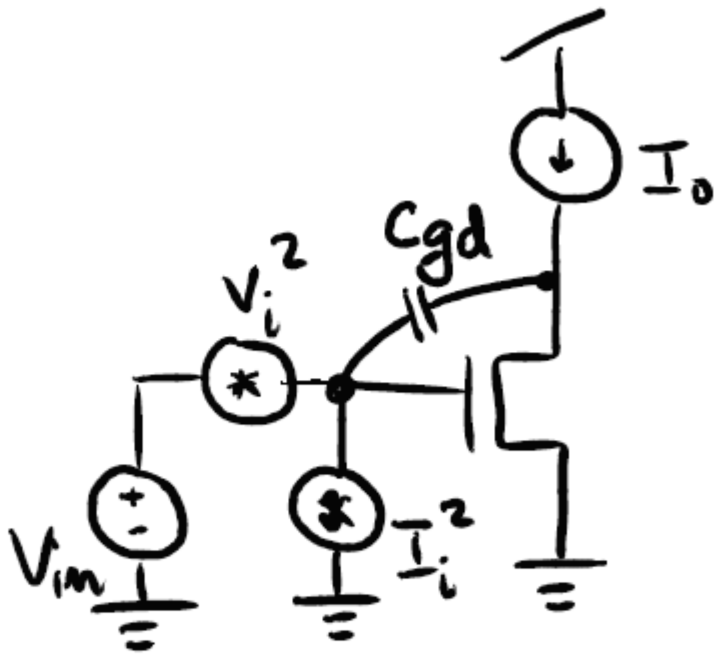


$$\overline{V}_{C1}^{2} = \frac{KT}{C_1}\left(1 + \frac{C_{1a}}{C_1} + \ldots\right)$$

- Each additional input capacitor adds to the total noise.
- Separate capacitors isolates V_1 and V_2.

Input referred High-frequency Noise Models





- At low frequencies, the gate is an insulator – any gate referred noise can be transformed into output noise through the transconductance.
- At high-frequencies, C_{gs} becomes a short-circuit, so only an input-referred voltage source wont work.
- High-frequency noise models – uses an input referred noise current.

External Noise

- Till now noise was considered to be random with known statistics.
- Localized inside circuit components.
- External Noise – not random but not correlated with the signal.
- Quasi-random – baseline fluctuations superimposed on desired signal.

Problems due to external noise

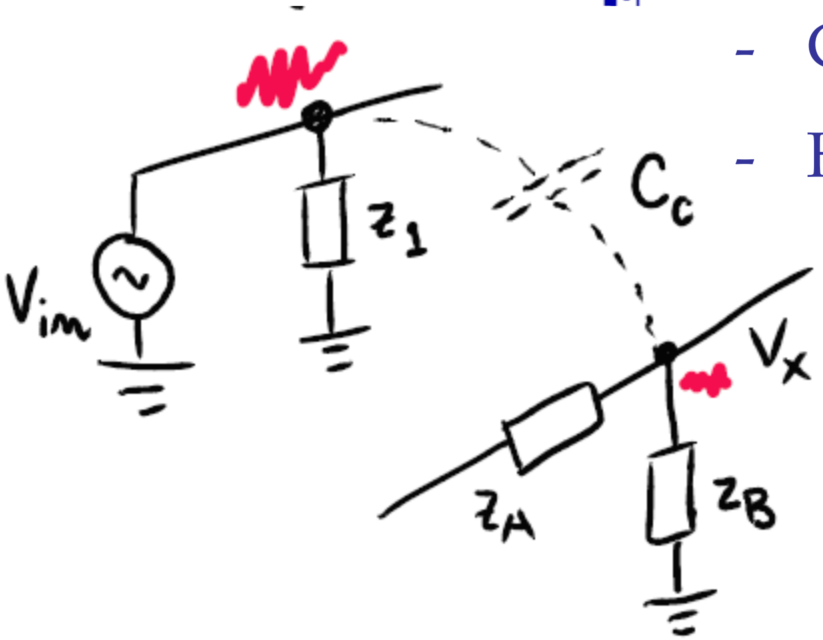
- Increasing detection threshold – degradation of resolution.
- Important to distinguish between – pickup of spurious signals from local or remote sources, or circuit provides feedback path that causes sustained oscillation – output reaching the input.

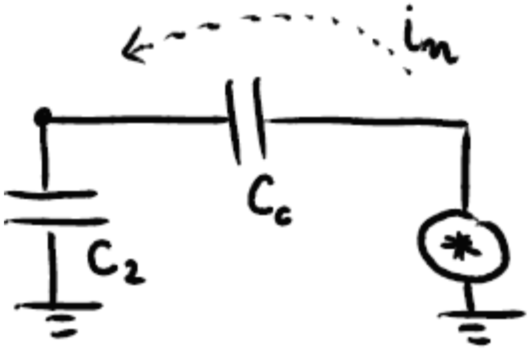
Interference Pickup

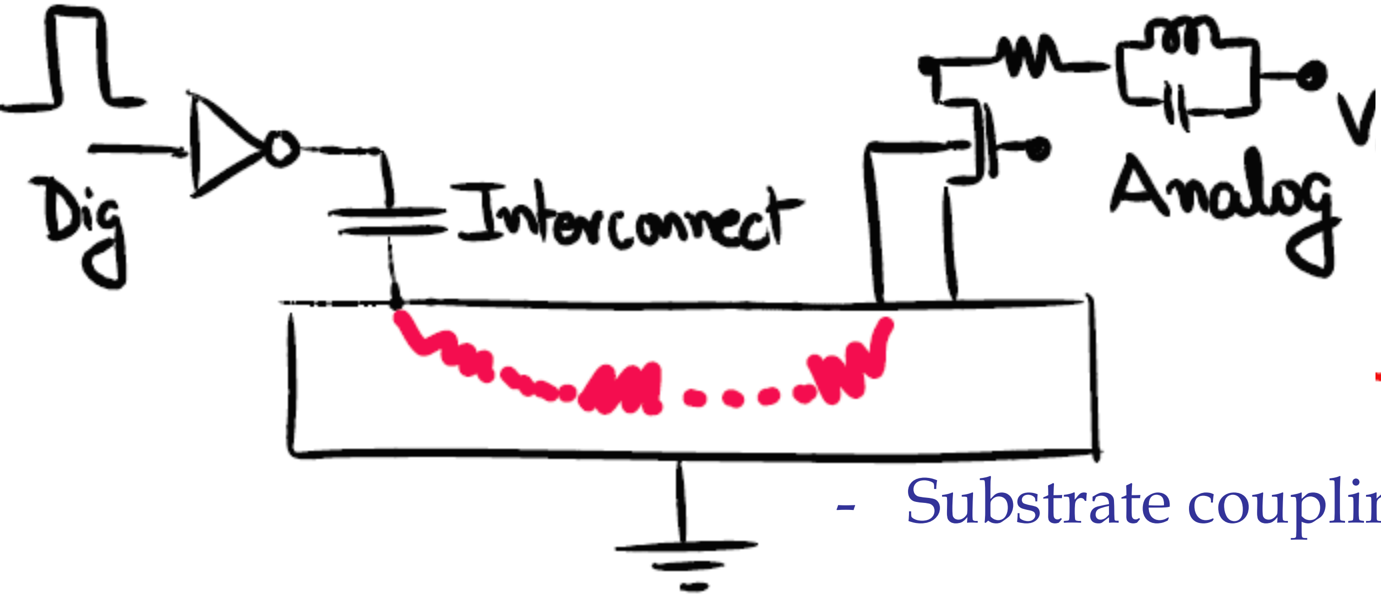
- **Light pickup**: pn junctions act as a photo-diode – pickup from the ambient light sources. Light pick-up in photomultiplier tubes, semiconductor detectors
- Interference is correlated with power line frequency– twice the line frequency – light intensity proportional to voltage square.
- Solution: switch off light, light shield
- **Vibration pickup**: Two capacitor plates at different potentials are susceptible to noise due to displacement currents. Signature: Look for harmonics in the signals at specific frequencies.
- Solution: Anti-vibration shield, coaxial shield to maintain zero potential difference or provide a path for the displacement current to easily flow away from the circuit.
- **RF pickup**: Everything is an antenna – goal is to make the circuit of interest a bad antenna.

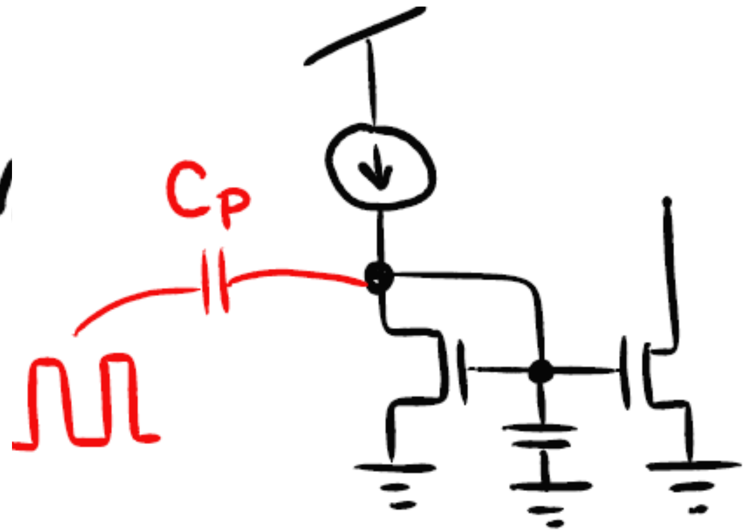
Electrical Interference

- Coupling through parasitic capacitance.
- High-impedance nodes more susceptible



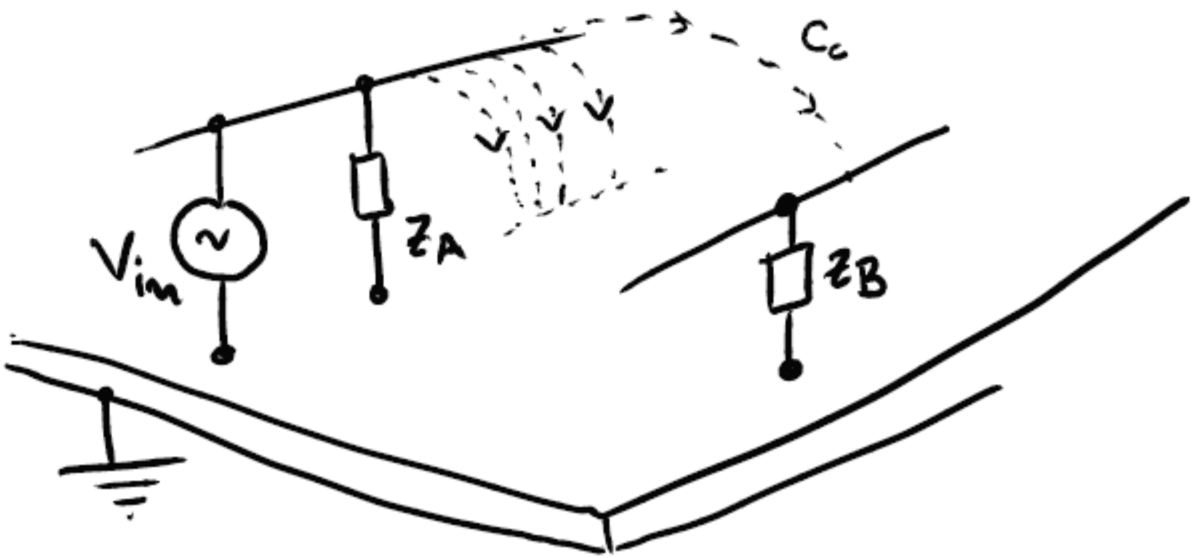


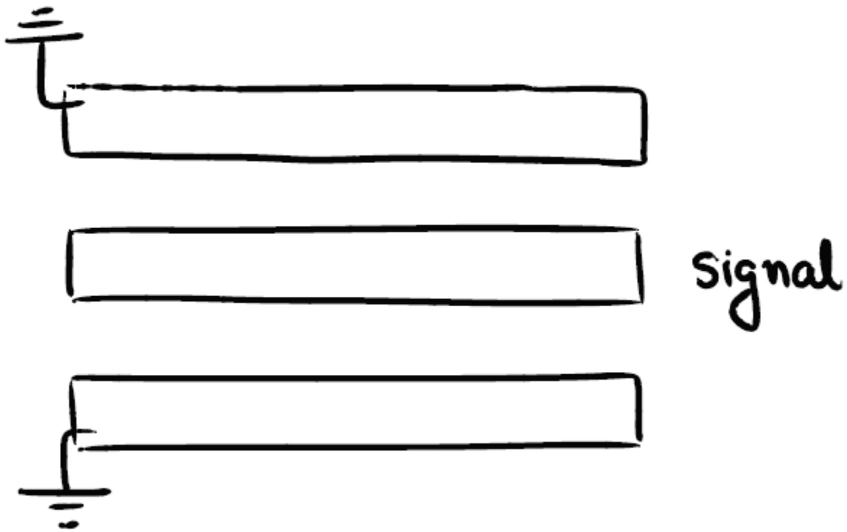


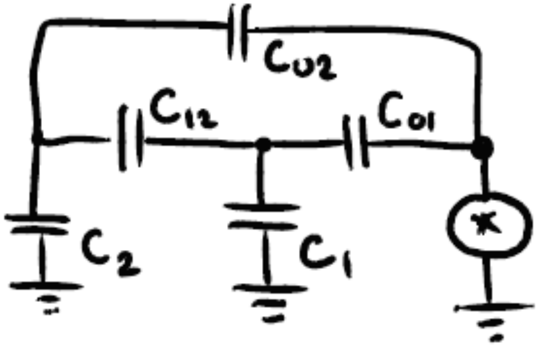


- Substrate coupling

Shielding

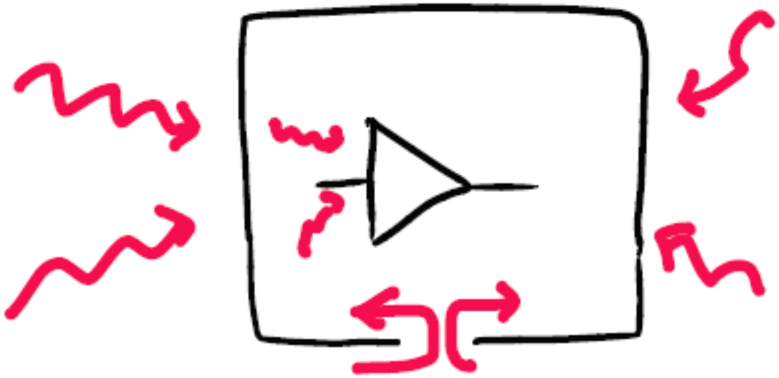




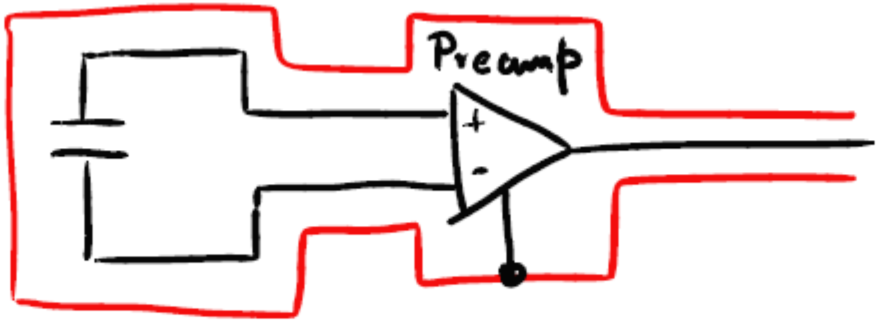


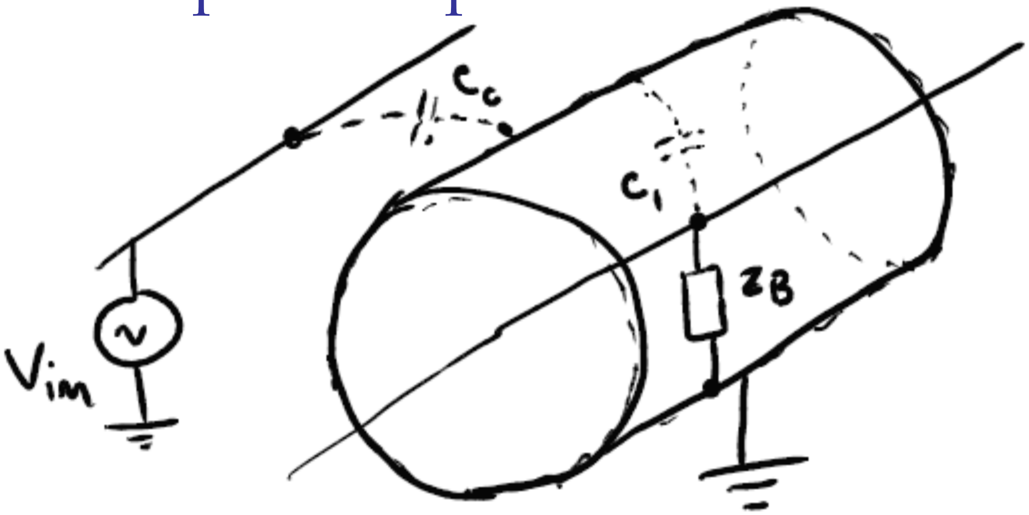
- Introduce a low-impedance node in between the interference and signal node.
- Use ground planes to capture fringing electric fields.

Faraday Cages

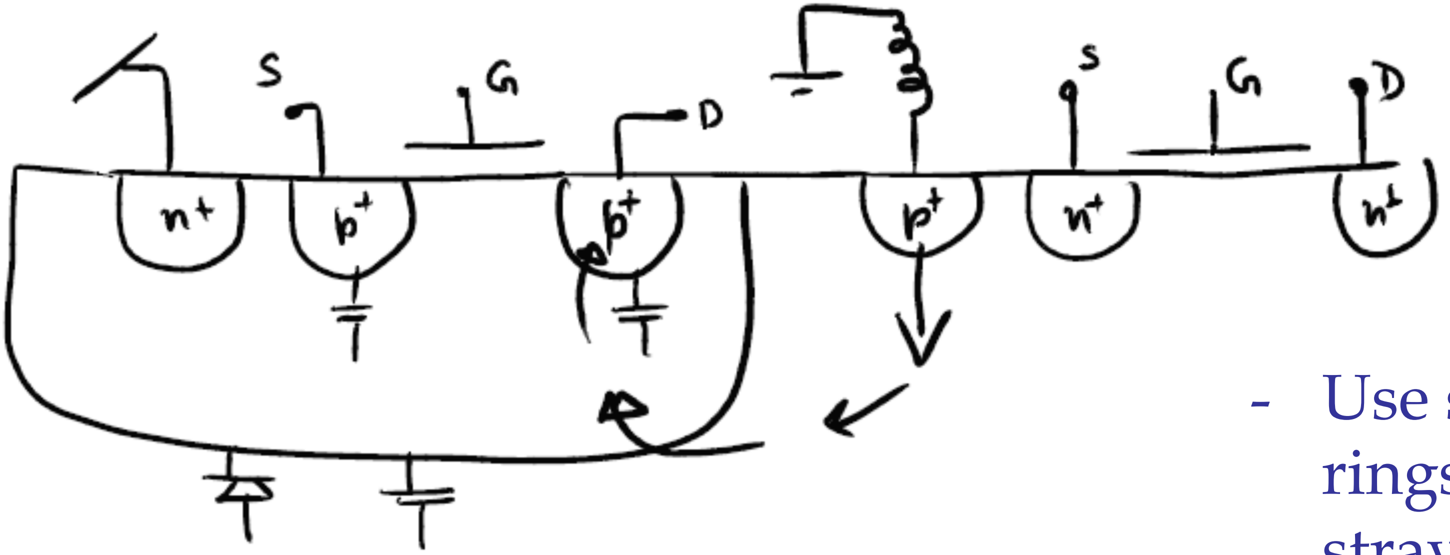


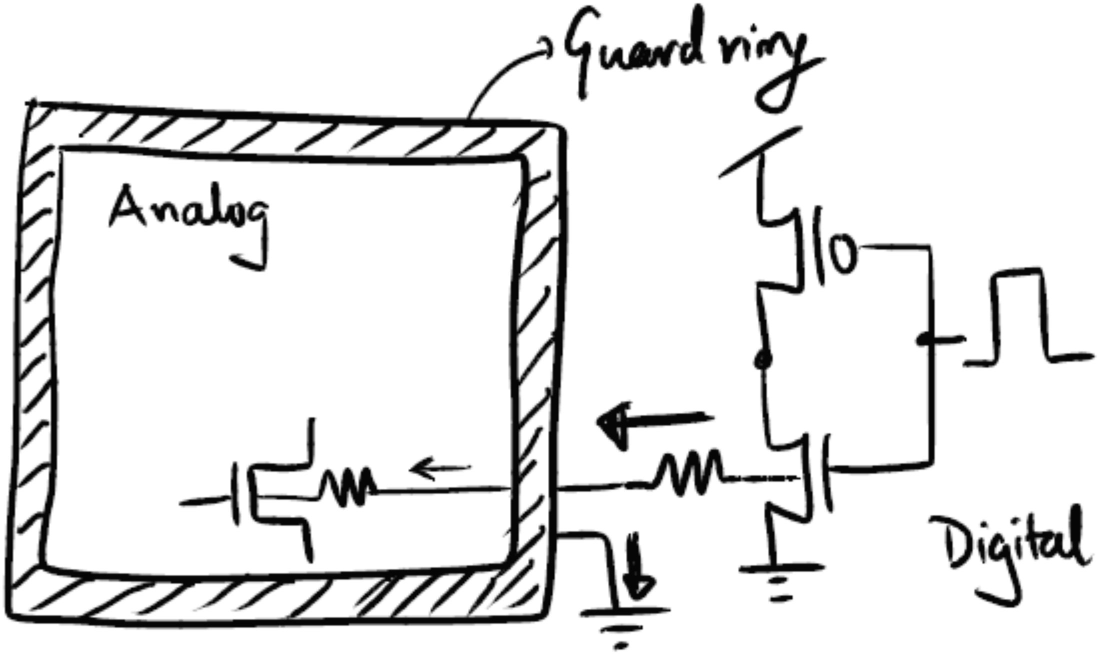
- Completely surround the circuit by a thick conductor.
- Thickness greater than the skin depth.
- Holes need to be smaller than 1/10th the wavelength.
- Ground the shield to provide a low impedance path.





Reducing effect of substrate coupling

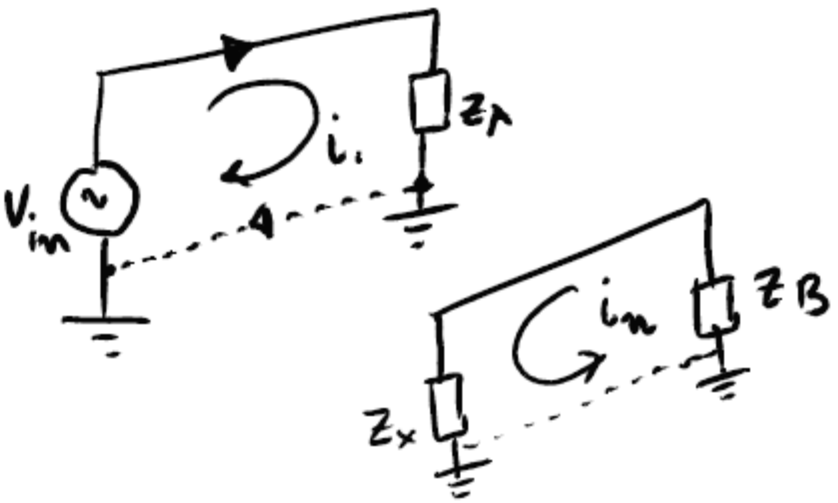




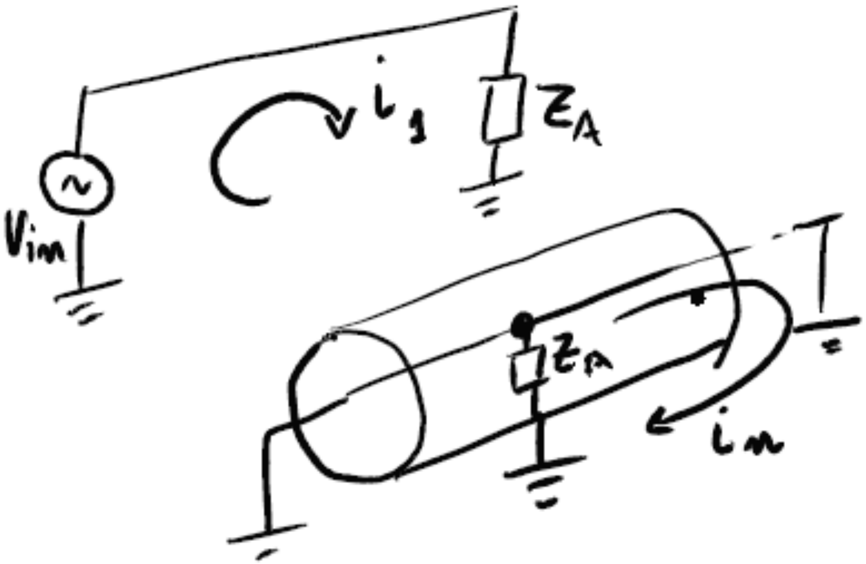
- Use substrate guard rings to capture any stray substrate current.
- Make sure to have a lot of substrate connections. It is difficult to control current that leaks into the substrate.

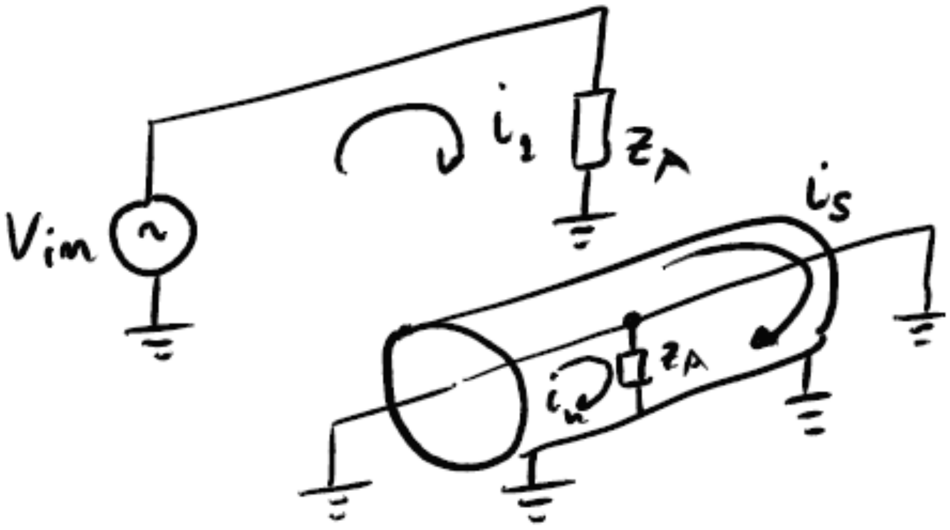
tems

Magnetic pickup

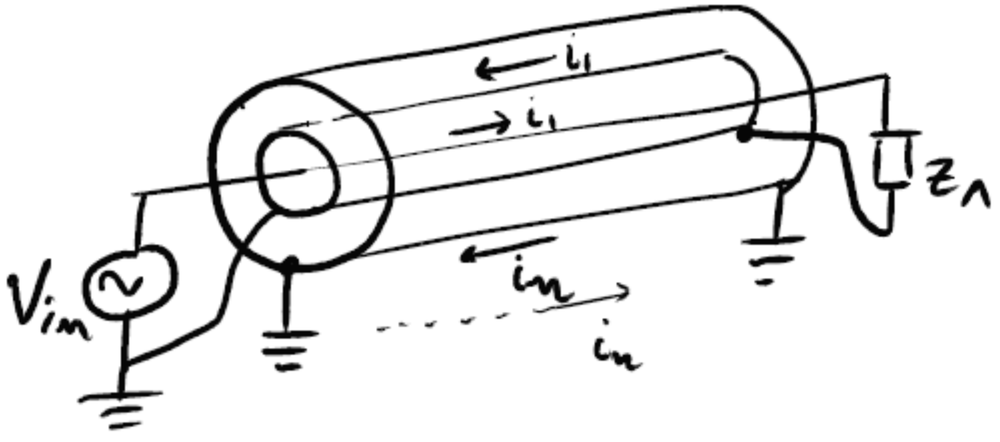


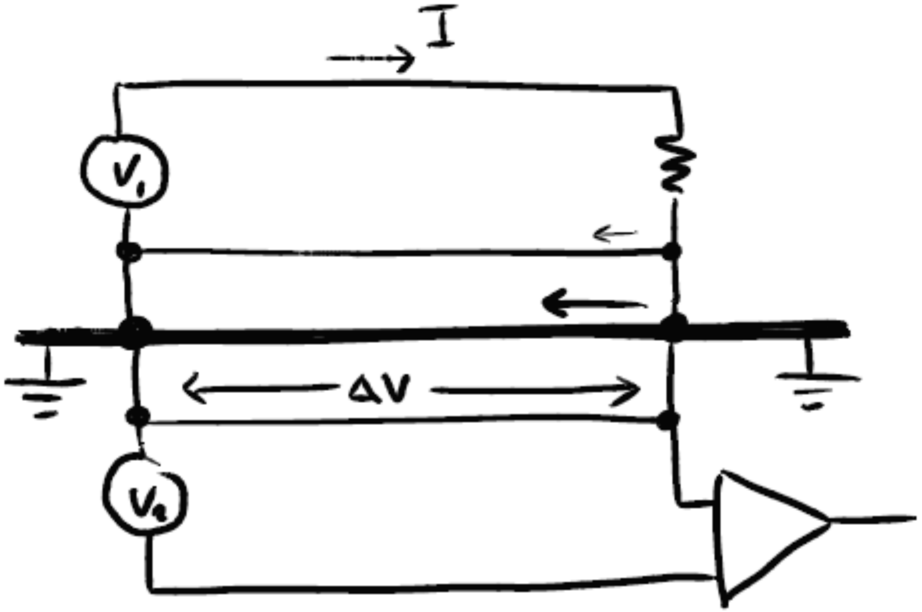
- Faraday cage might not be useful for reducing magnetic interference.
- Ground shield at two end points.
- Return current path cancels the effect of interfering current.

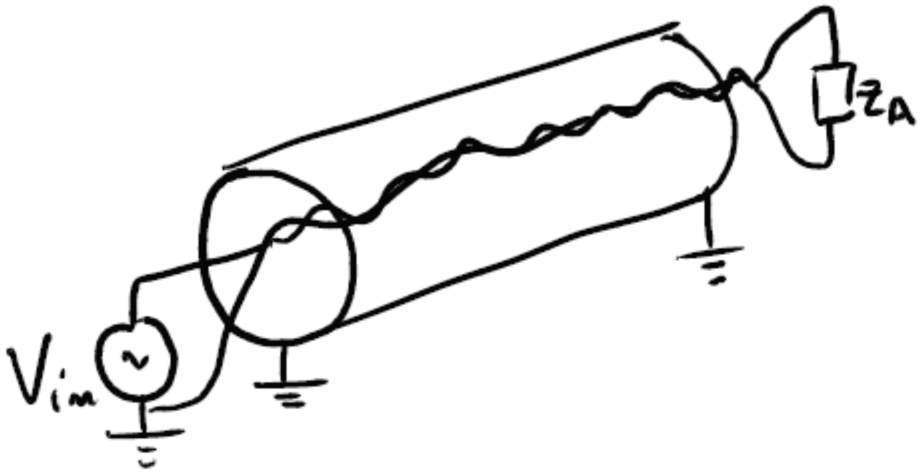




Magnetic pickup due to ground loops



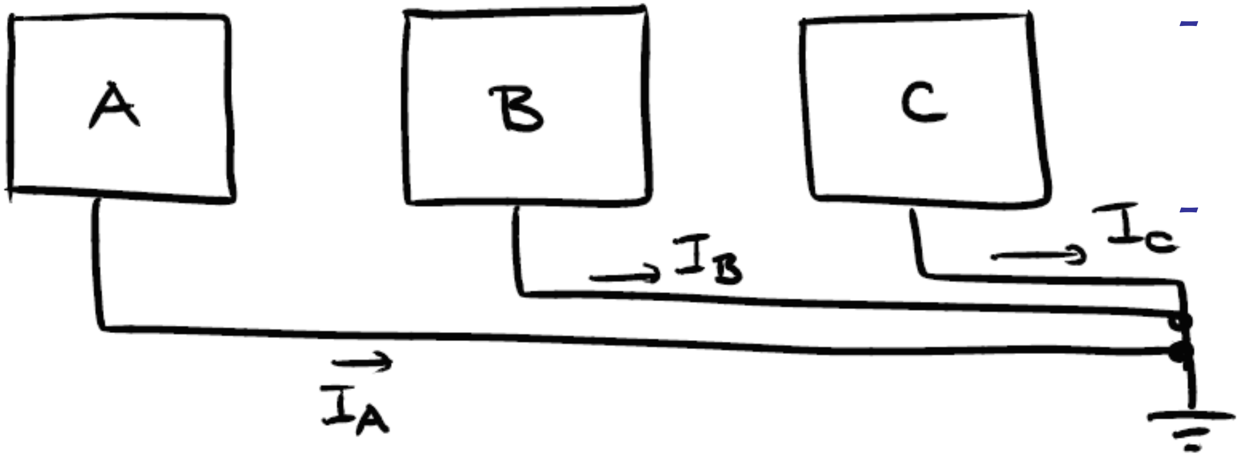


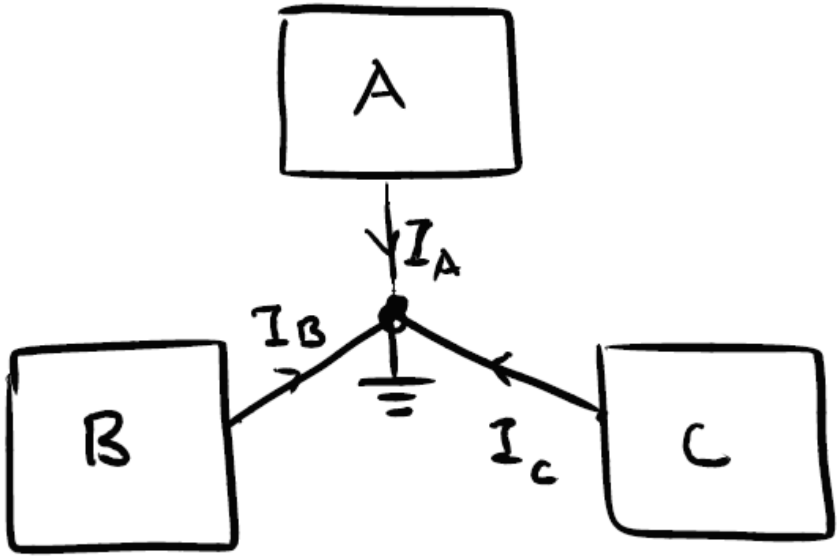


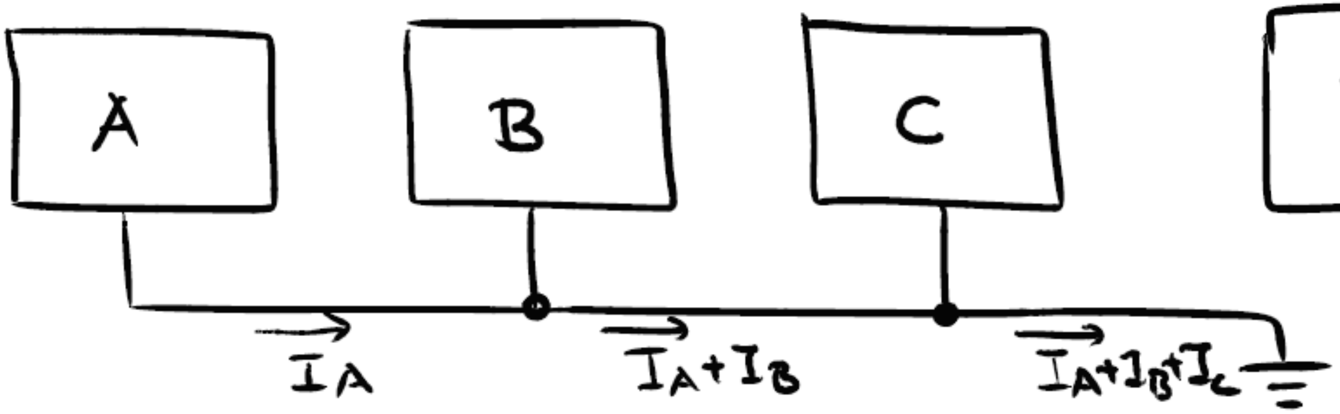
- Provide a separate return current path and not through the ground.
- Use a twisted cable to reduce the magnetic pickup by reducing the effective cross-sectional area.

Ground Noise

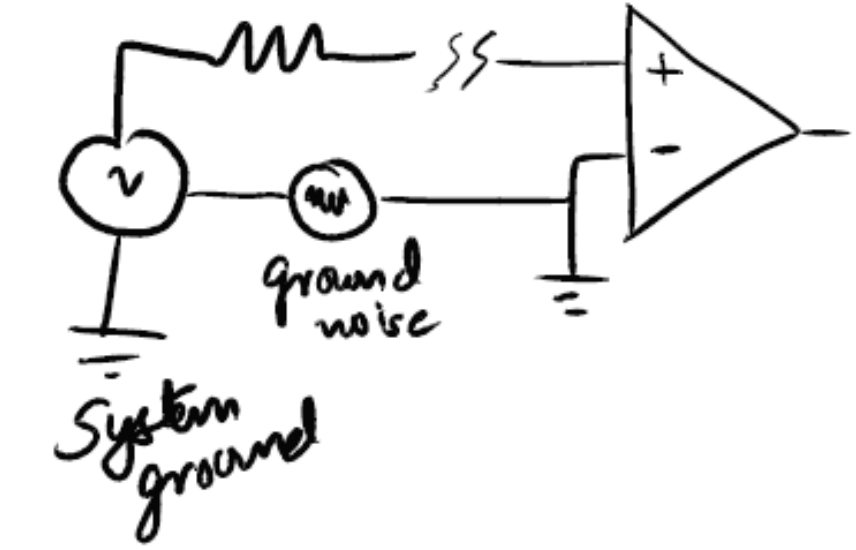
- Ground is never perfect - has a finite resistance.
- Use symmetrical ground connection when possible.



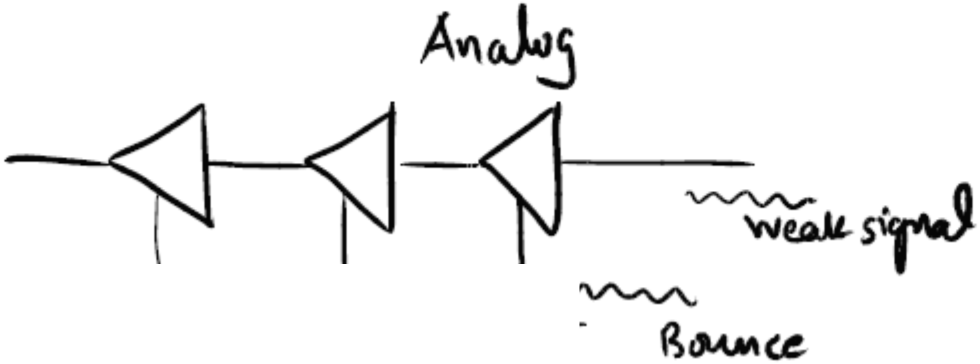


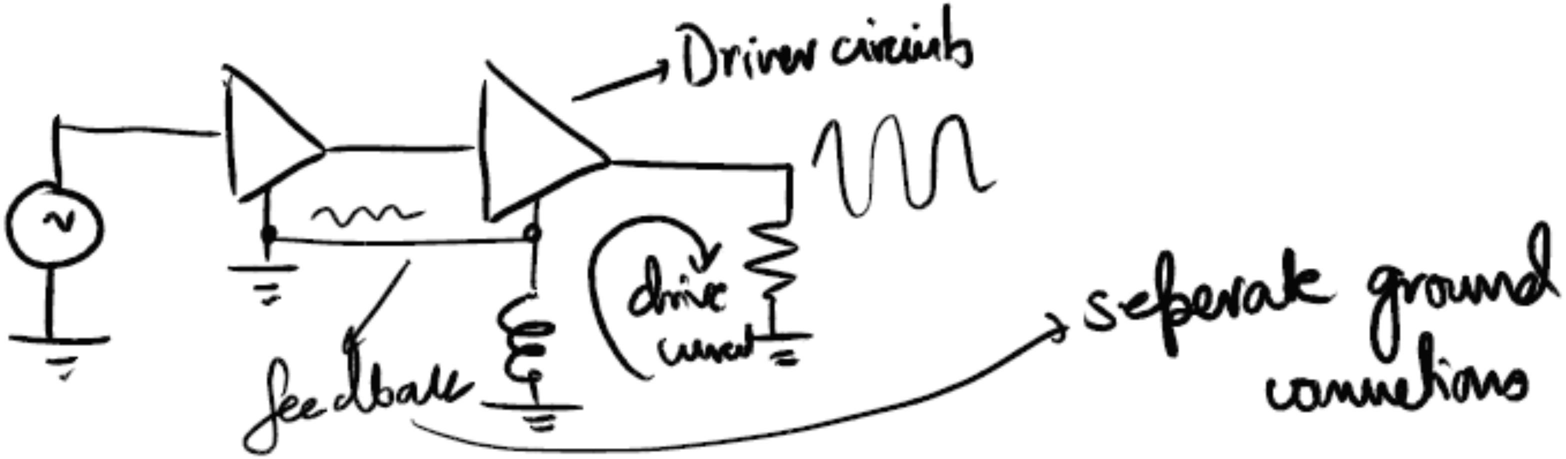


Effect of Ground Noise

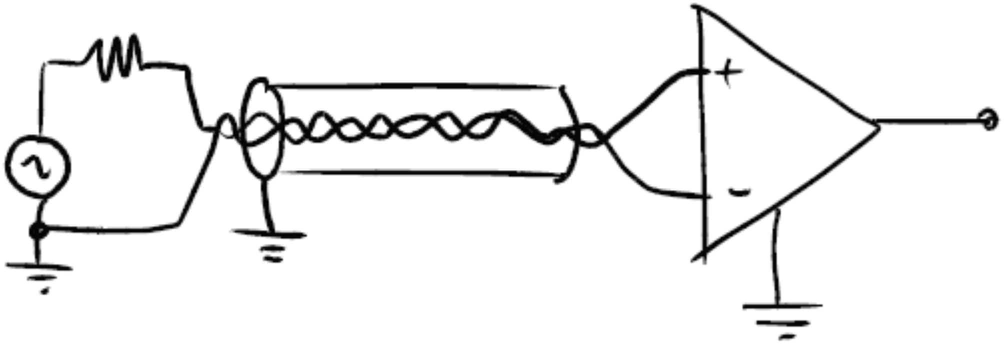


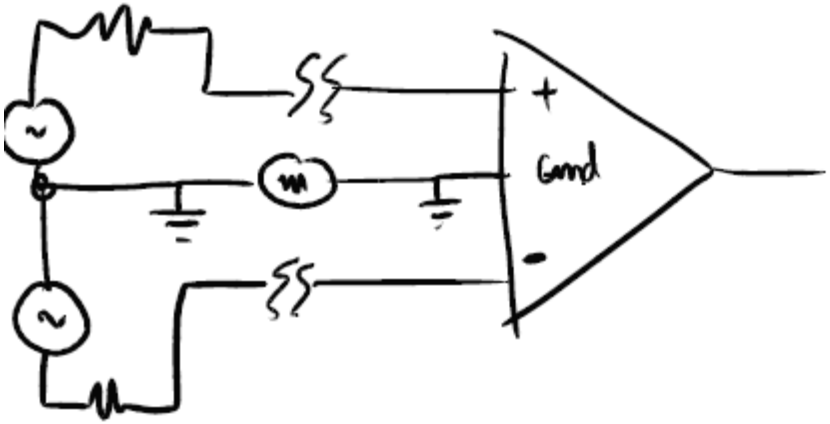
- Reduces signal-to-noise ratio.
- Could induce oscillations due to parasitic feedback.

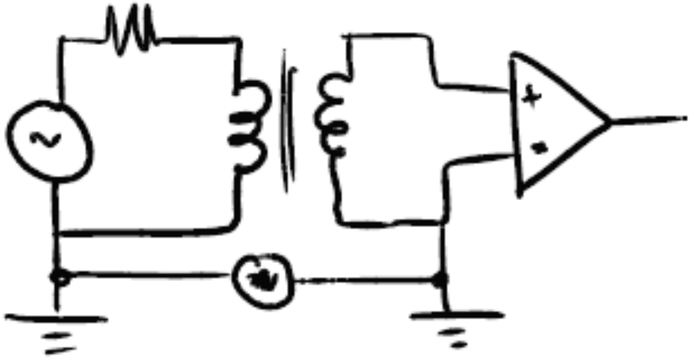




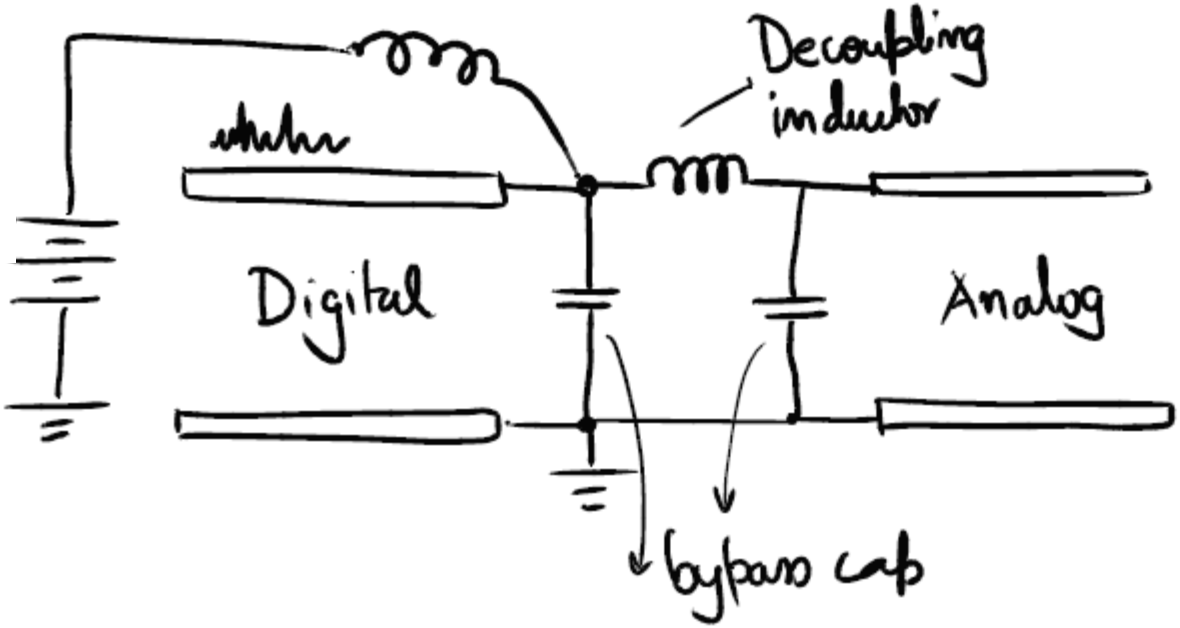
Reducing effect of ground noise



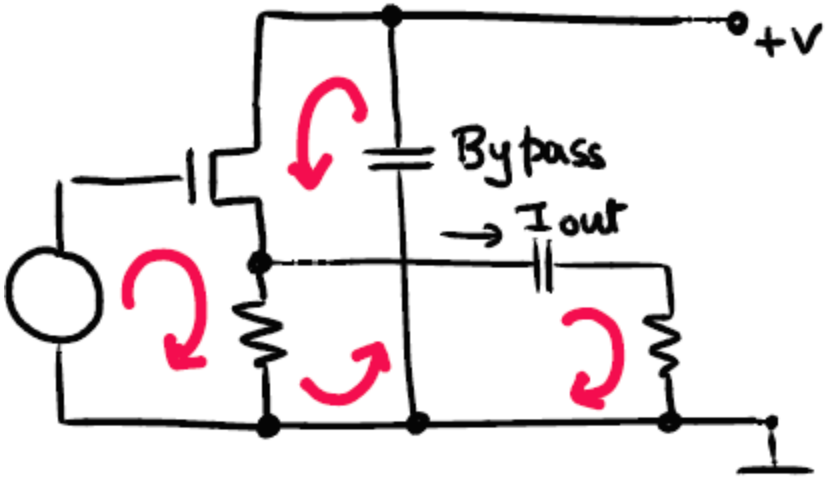




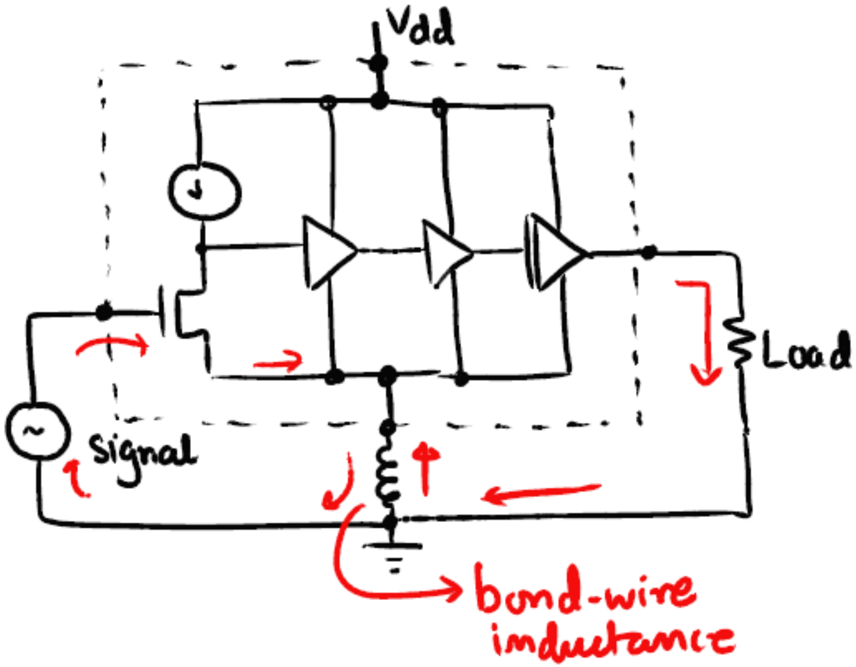
- Provide a separate return path using bypass capacitors.
- Isolate and decouple the ground and supplies.
- Use differential topologies.

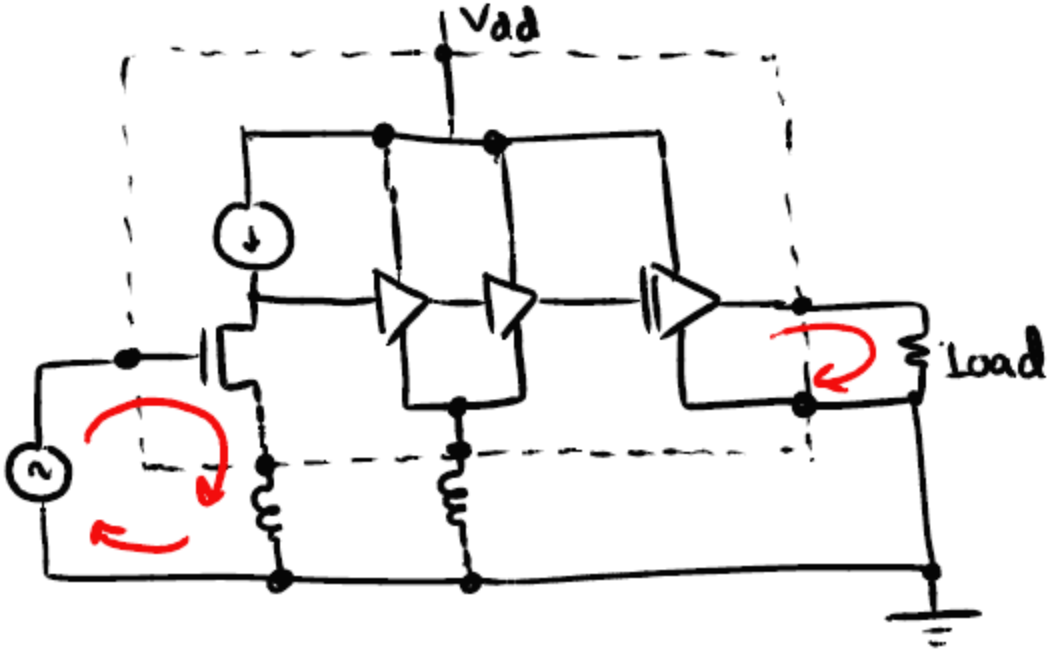


Reducing effect of ground noise

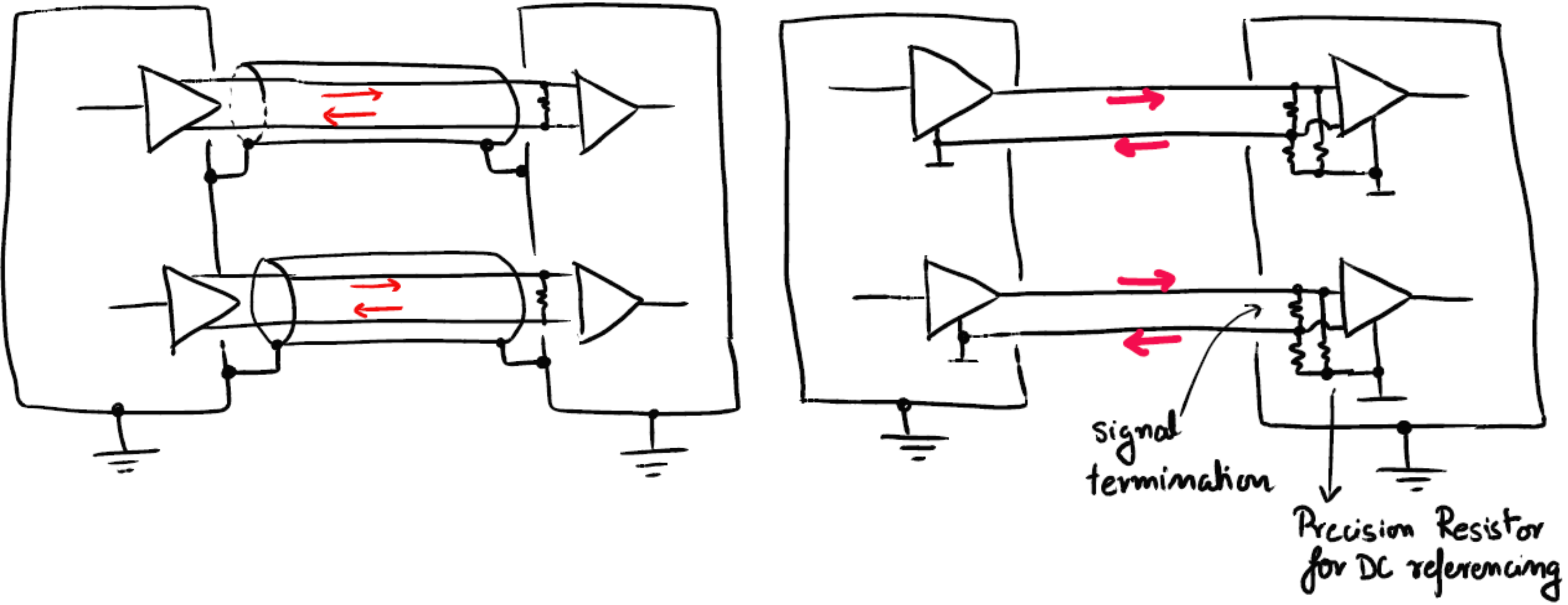


- Closer the current return path less is the noise pickup.
- Avoid signal return path through global ground.





Signal connections



- Signal transmission using a triax.
- Isolate the signal return path using a coax and shield from chassis.