Basic Analog Circuits: DC analysis

CSE562M: Analog Integrated Circuits

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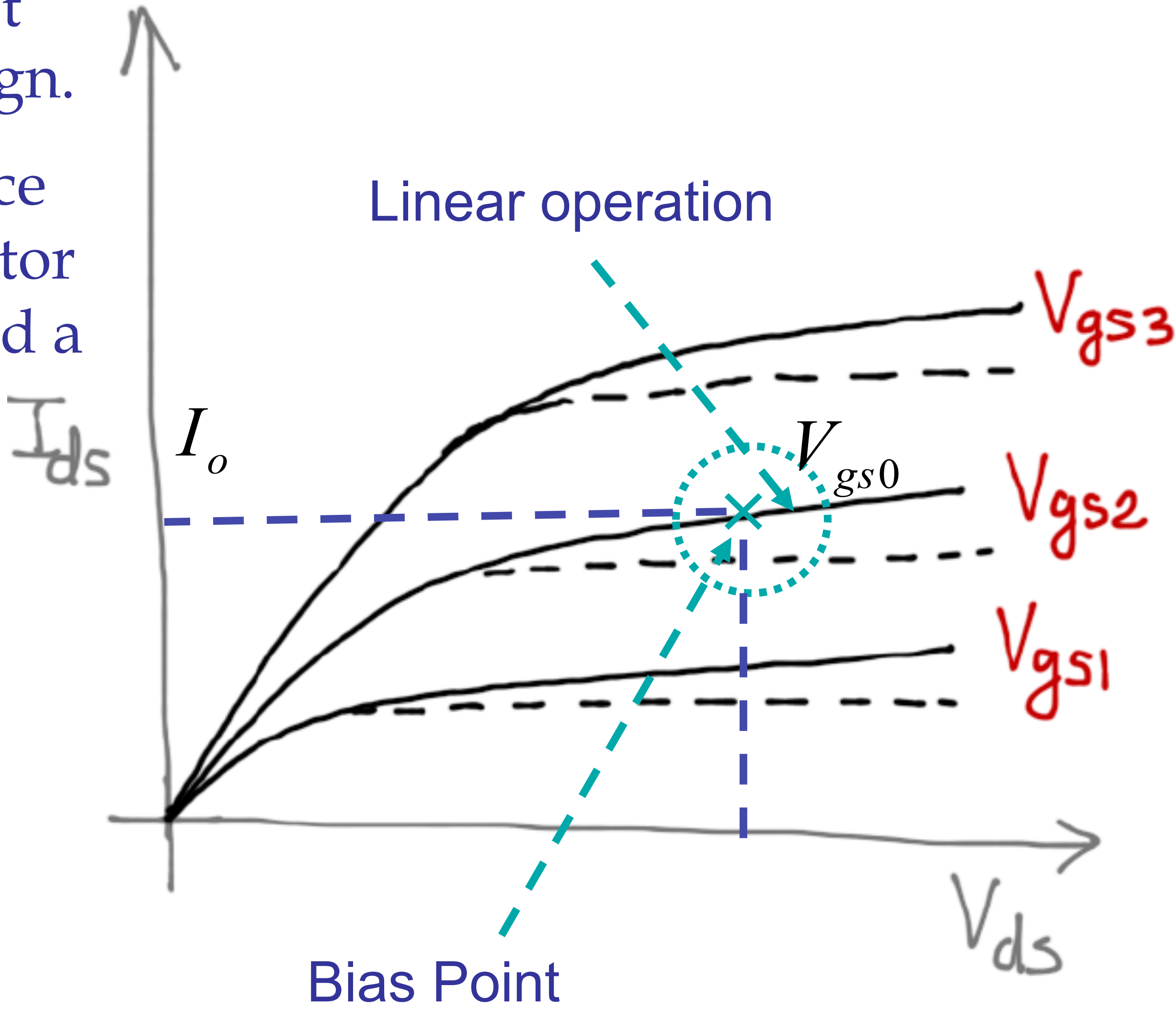
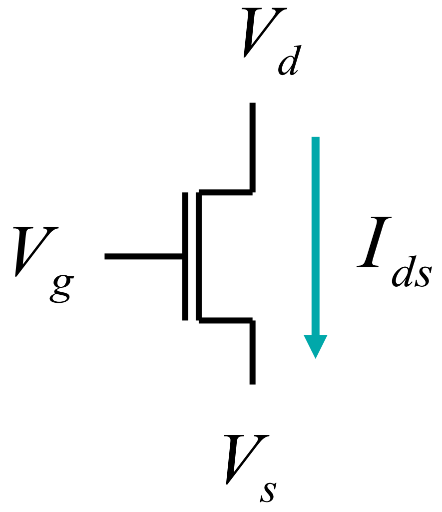
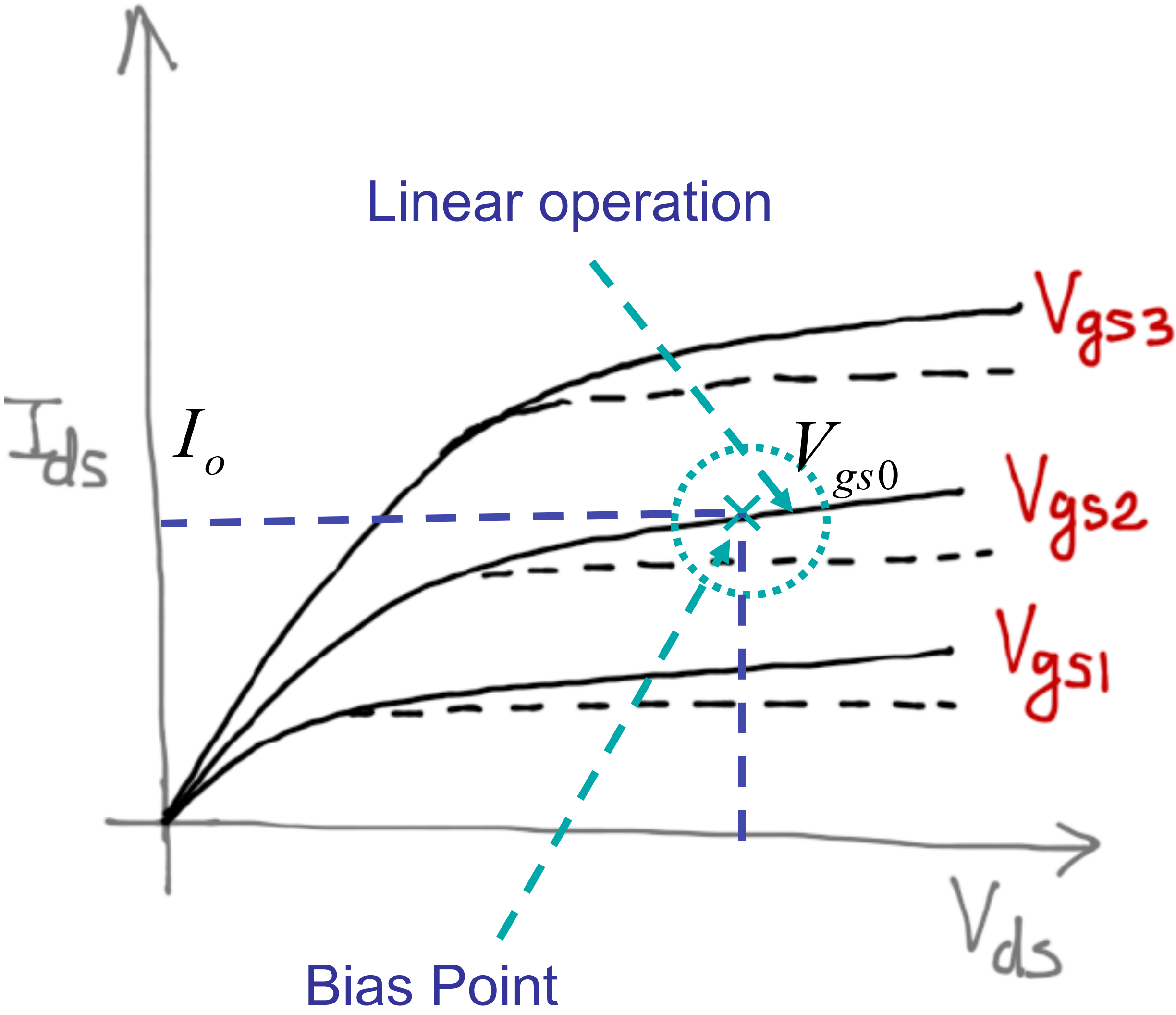
Bias point and Small-signal Models

- Large signal models are not useful in linear circuit design.
- Small signal models: Reduce the operation of the transistor about a small region around a bias point.
- Bias point of a transistor is determined by its drain current and the gate-to-source voltage.



Small-signal Models





Change in drain current when gate-to-source voltage changes

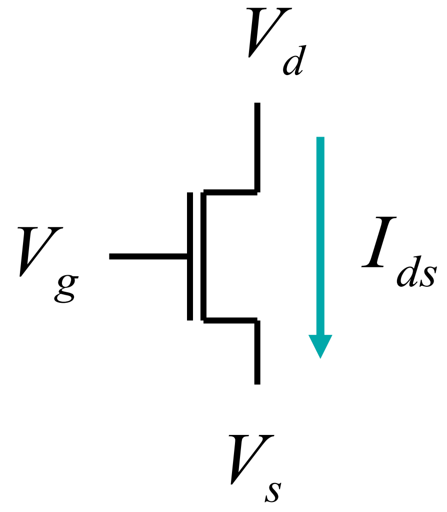
$$\Delta I_{ds} = g_m \Delta V_{gs}$$

Change in drain current when drain-to-source voltage changes

$$\Delta I_{ds} = g_d \Delta V_{ds}$$

$$\boxed{g_m >> g_d}$$

Summary of Small-signal Models



$$\Delta I_{ds} = g_m \Delta V_{gs}$$

Change in drain current when gate-to-source voltage changes

$$\Delta I_{ds} = g_d \Delta V_{ds}$$

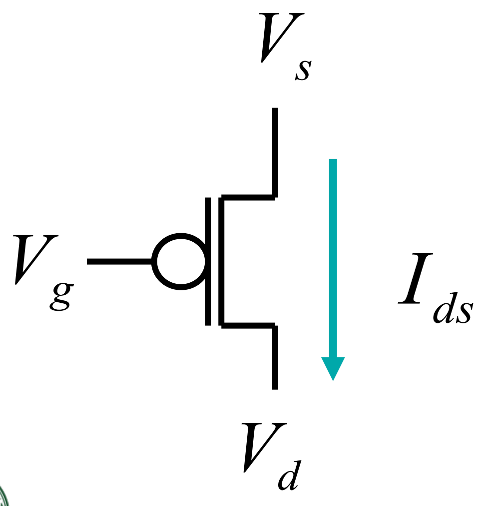
Change in drain current when drain-to-source voltage changes

By superposition principle

$$\Delta I_{ds} = g_m \Delta V_{gs} + g_d \Delta V_{ds}$$

$$\Delta I_{ds} = g_m (\Delta V_g - \Delta V_s) + g_d (\Delta V_d - \Delta V_s)$$

$$\boxed{\Delta I_{ds} = g_m \Delta V_g + g_d \Delta V_d - (g_m + g_d) \Delta V_s}$$

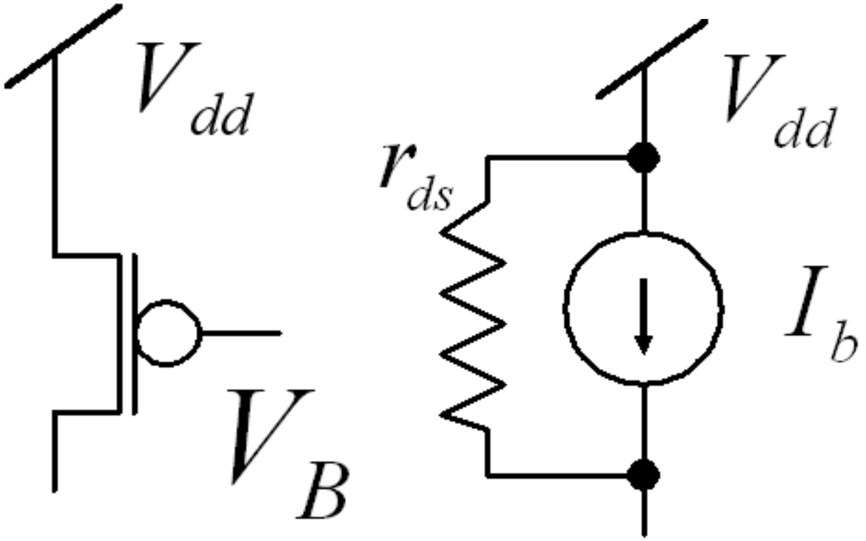


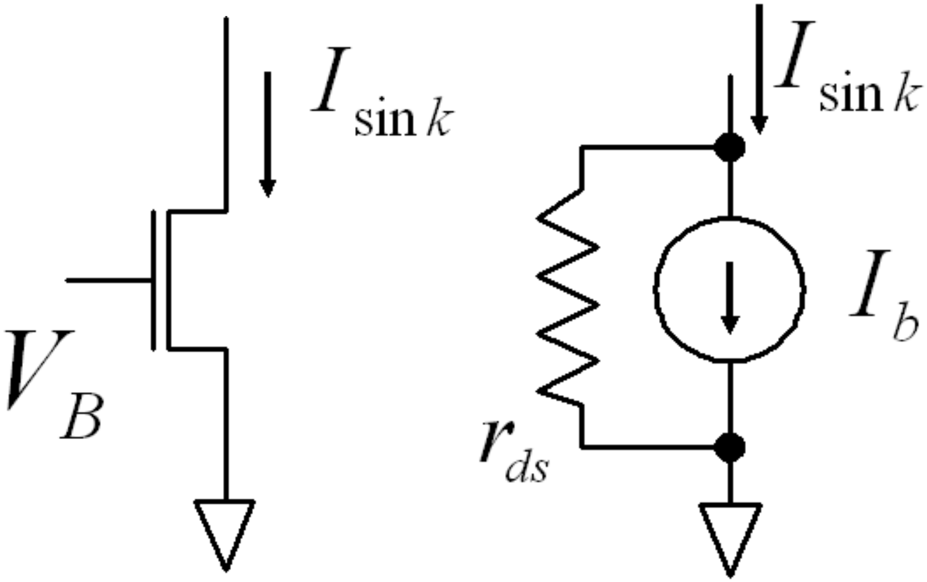
$$\boxed{\Delta I_{ds} = -g_m \Delta V_g - g_d \Delta V_d + (g_m + g_d) \Delta V_s}$$

Current sources and current sinks

Ideal current sources and sinks have infinite output/input impedance.

Only in saturation/active region, nmos and pmos transistors act as a reasonable current sink or source.





$$Z_{out} = r_{ds} = 1/\, g_{dp}$$

$$Z_{in} = r_{ds} = 1/\, g_{dn}$$

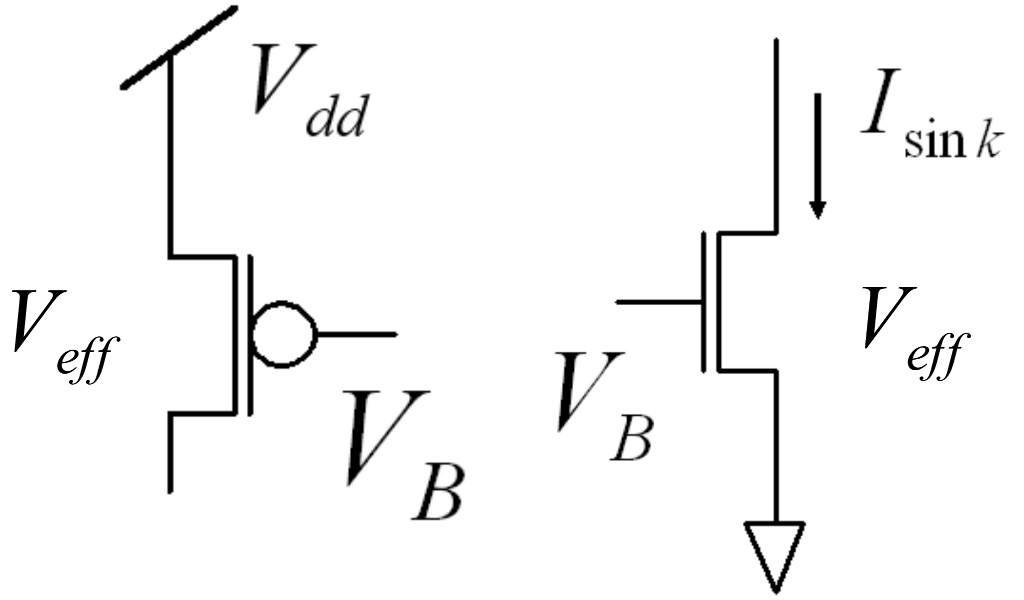
I_b Is determined by the gate-to-source voltage Vb, which is constant.

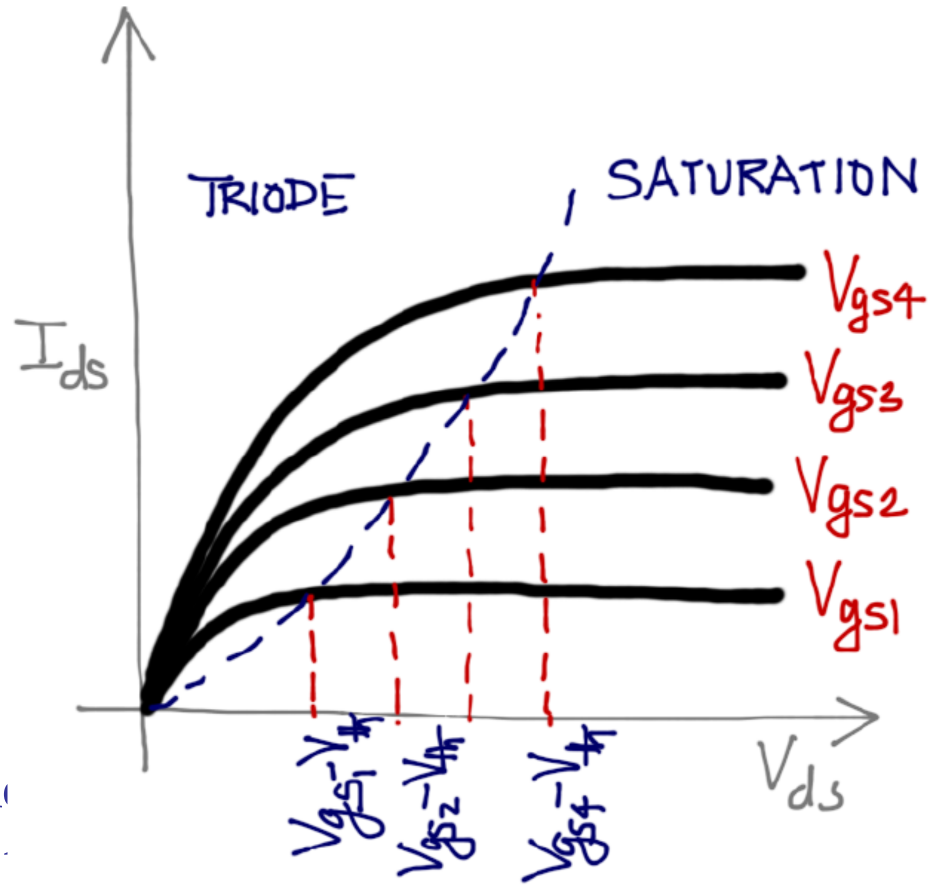
Worksheet

Output voltage range

I_b Is determined by the gate-to-source voltage V_b, which is constant.

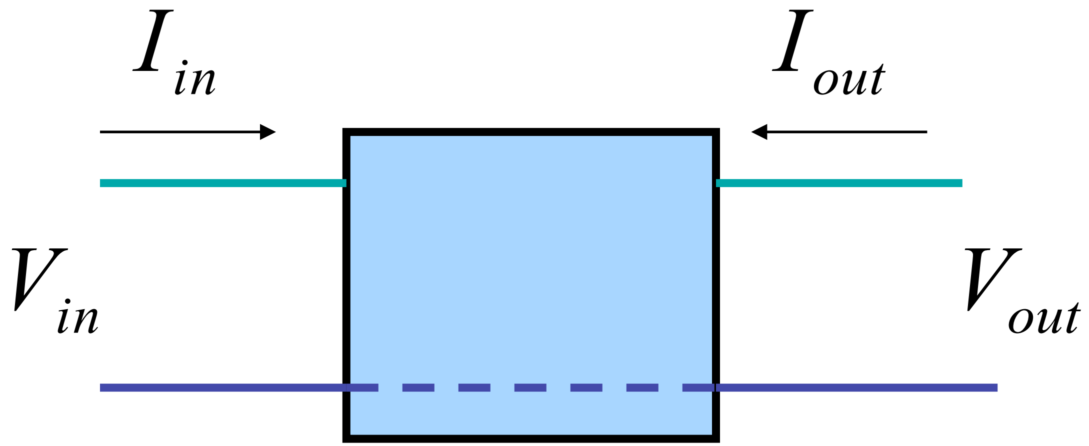
Current can be varied by changing the gate-to-source voltage.





Transistors are in saturation, therefore th
is a saturation voltage drop across drain
source terminal (Important !!!)

Input and Output Impedance



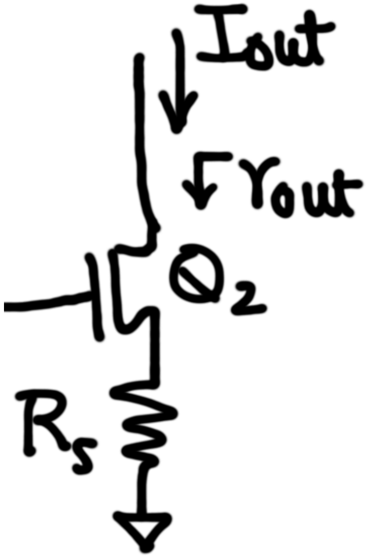
$$Z_{in} = \left.\frac{\Delta V_{in}}{\Delta I_{in}}\right|_{V_{out=constant}} \qquad Z_{out} = \left.\frac{\Delta V_{out}}{\Delta I_{out}}\right|_{V_{in=constant}}$$

Lower the input impedance, the higher will be the input current .

Higher the output impedance implies the output current remains insensitive to changes in output voltage.

Impedance Calculations

Source degeneration: when load is connected to the source of MOS transistors



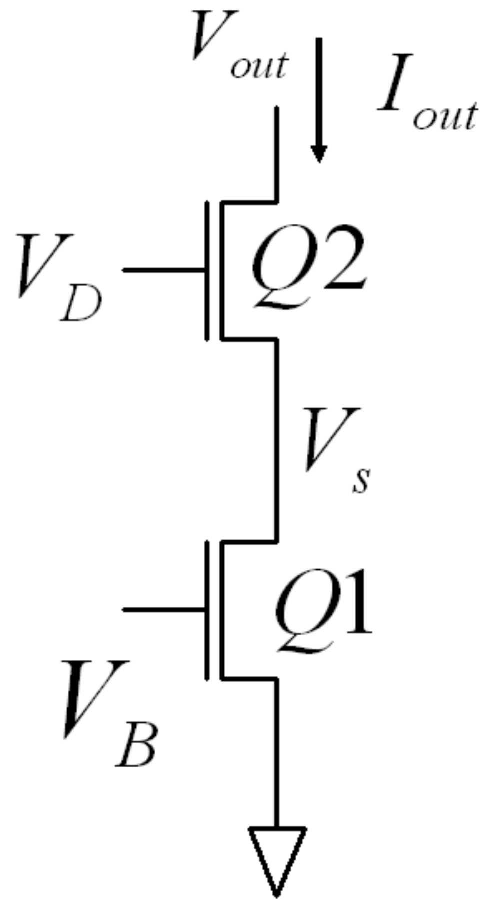
$$r_{out} = \frac{V_{out}}{I_{out}} = r_{ds2}[1 + R_s g_{m2}]$$

Output impedance increases multiplicatively with source resistance.

Worksheet

Cascoding

If we can increase the output impedance or if we can keep the drain voltage as constant as possible – use cascoding.



Cascoded current sink

Compute the output impedance of this source ?

$$Q2: \Delta I_{out} = g_{d2}\Delta V_{out} - (g_m + g_{d2})\Delta V_s$$

$$Q1: \Delta I_{out} = g_{d1}\Delta V_s$$

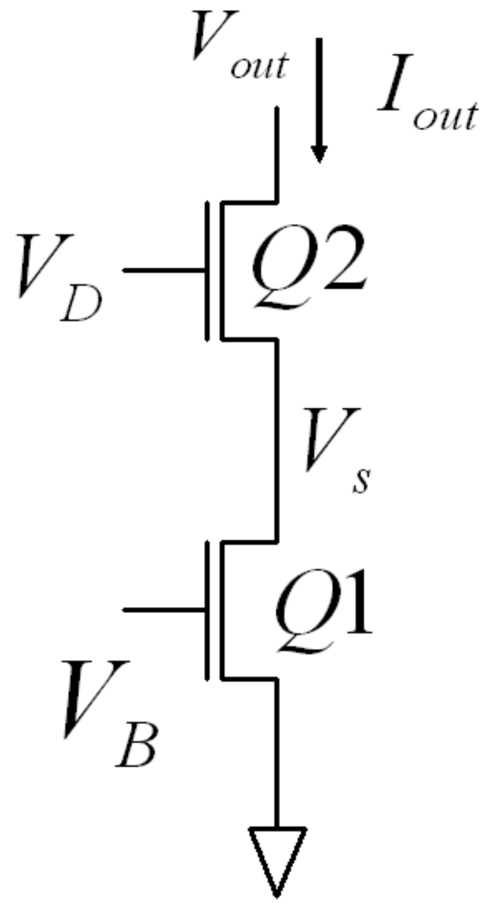
$$\Delta I_{out} = g_{d2}\Delta V_{out} - \frac{g_m}{g_{d1}}\Delta I_{out}$$

$$\boxed{g_m >> g_d}$$

$$\frac{\Delta V_{out}}{\Delta I_{out}} = \frac{g_m}{g_{d1}g_{d2}} = g_m r_{ds1} r_{ds2} \approx 10G\Omega$$

What is the disadvantage?

Driven by higher voltages.



Both Q1 and Q2 have to be maintained in saturation.

$$V_{out} \geq 2V_{eff}$$

$$V_{out} \geq V_D - V_{th}$$

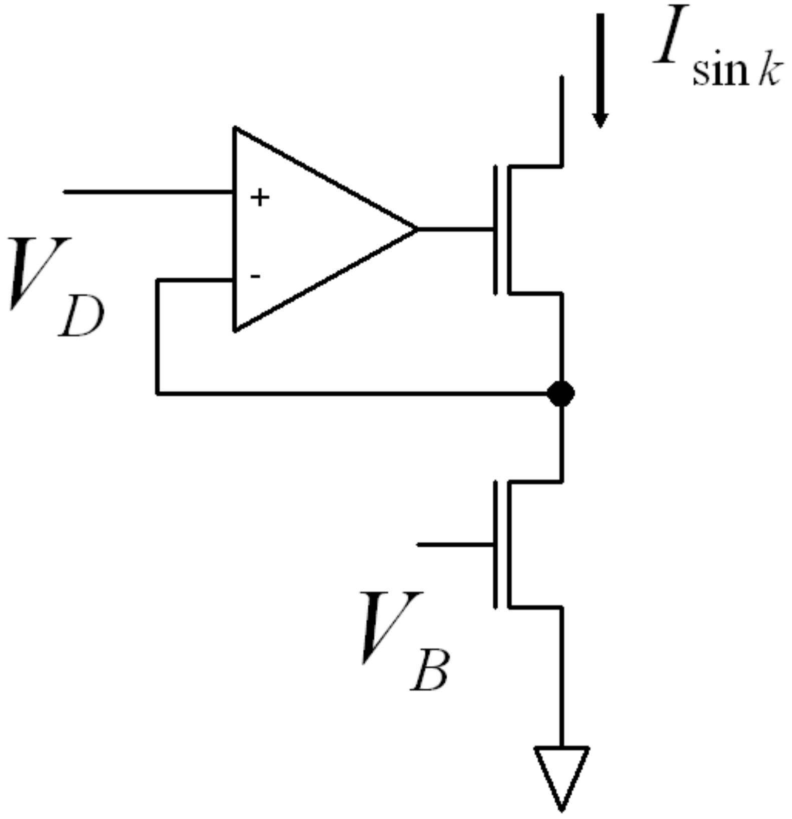
$$V_D \geq V_{eff} + V_{th}$$

Each transistor Q1 and Q2 has an effective voltage drop across the drain-to-source terminal. Also saturation is determined by V_D and V_B.

Choose the tighter constraint. $V_{out} > 2_{Veff} = 400mV$.

How to improve the current sink ?

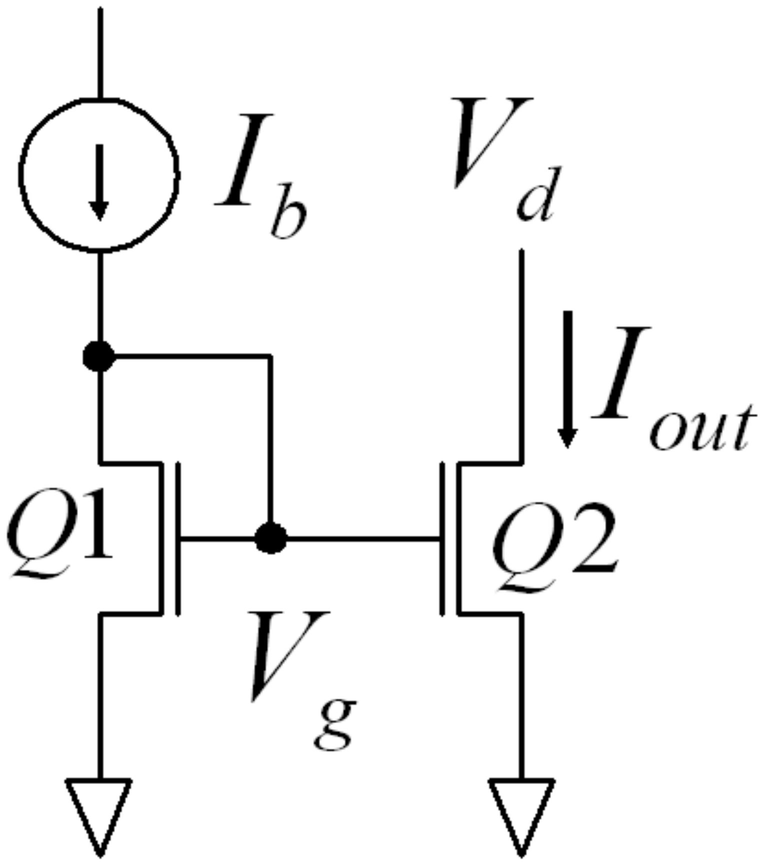
Assuming the opamp is ideal,



What is it's output impedance of this current sink ?

Worksheet

Current Mirrors



Aim: To accurately copy the current I_b

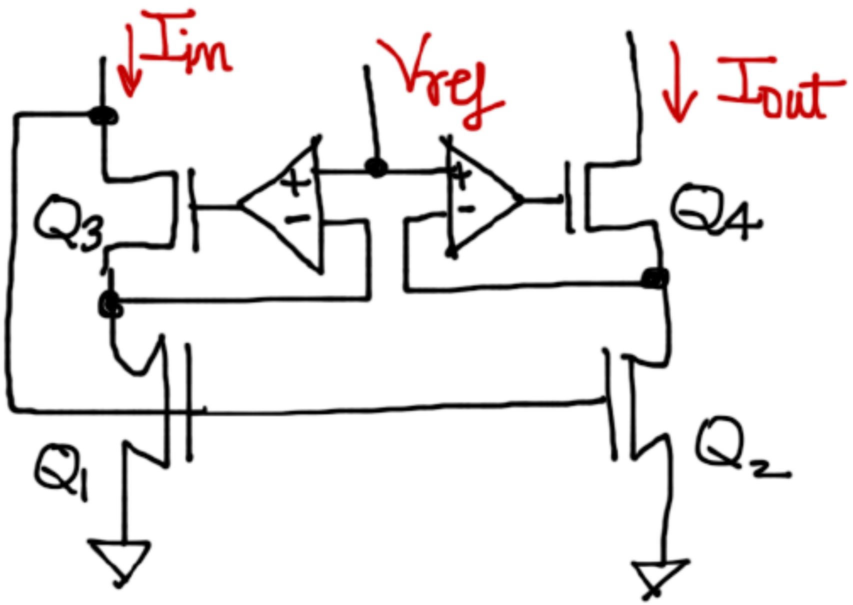
Factors that affect the accuracy:

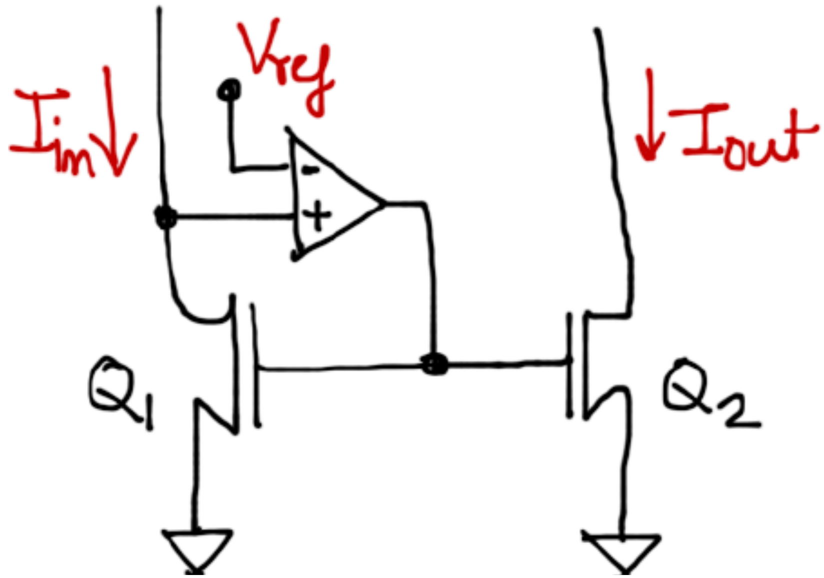
1. Drain voltage difference between Q1 and Q2.
2. Q2 should be a good current sink
3. Difference in size of the transistors.
4. Difference in threshold voltage.

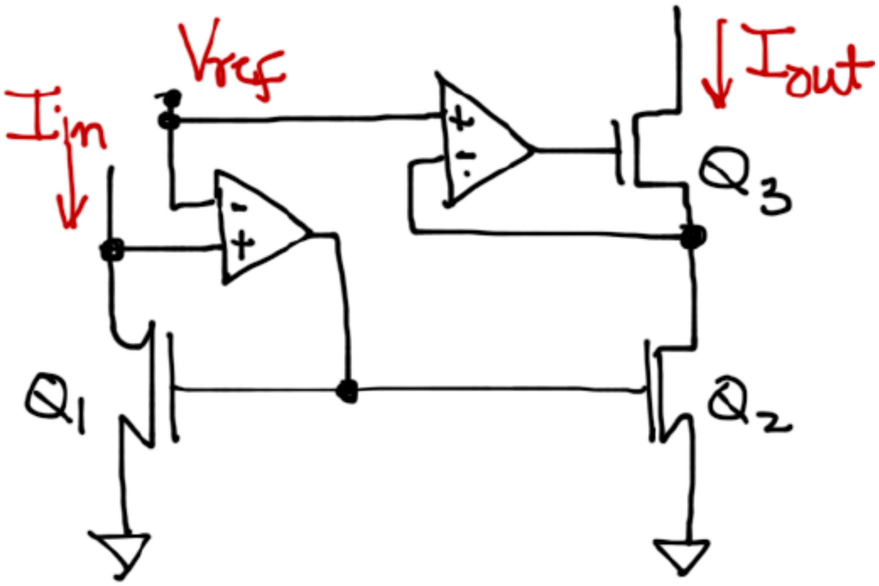
Matching determined by layout and fabrication process (very important !!!)

Worksheet

More Current Mirrors

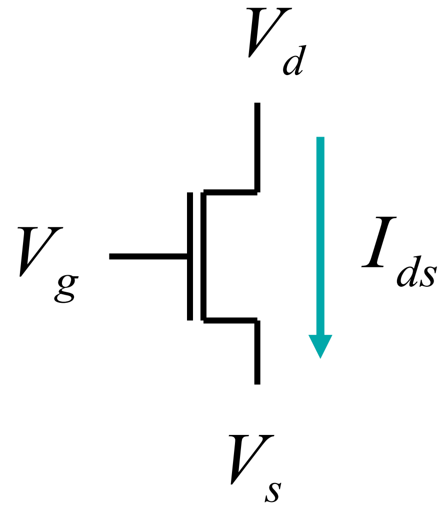






Worksheet

Summary of Small-signal Models



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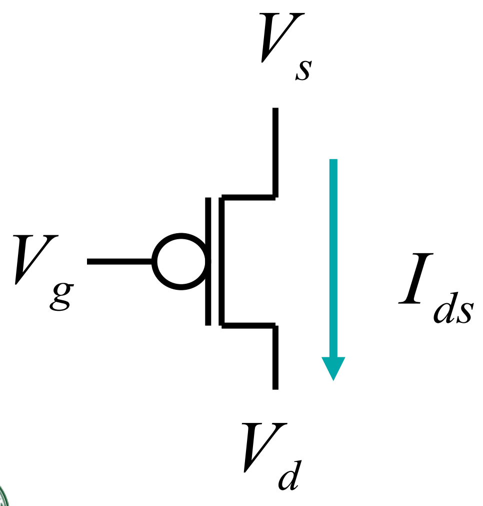
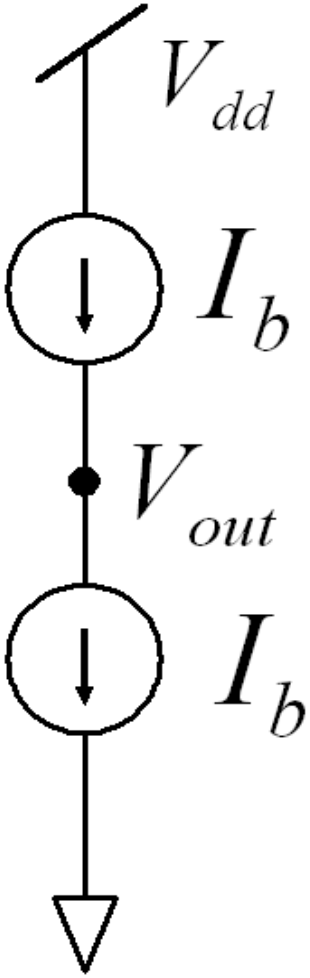
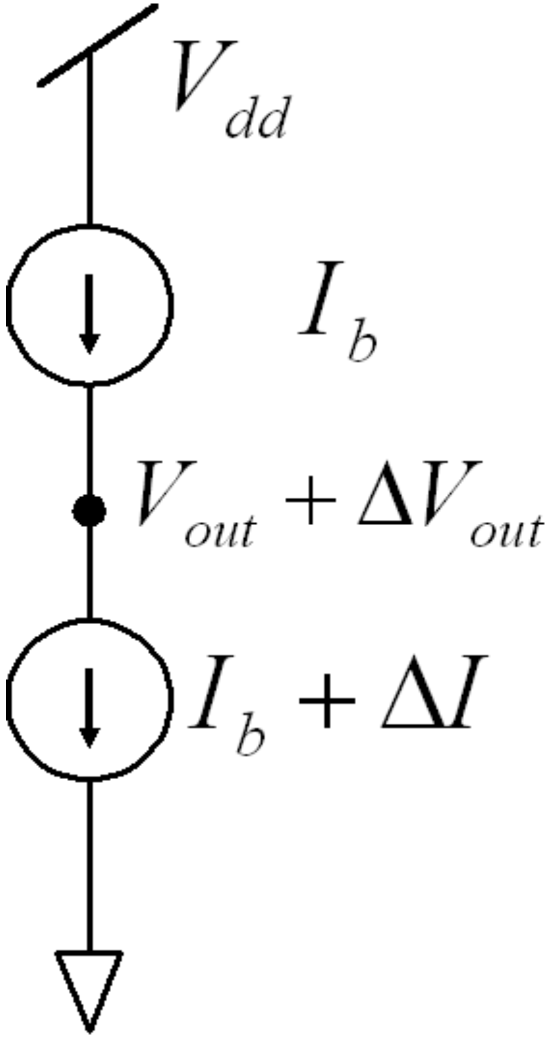


$$\boxed{\Delta I_{ds} = -g_m \Delta V_g - g_d \Delta V_d + (g_m + g_d) \Delta V_s}$$

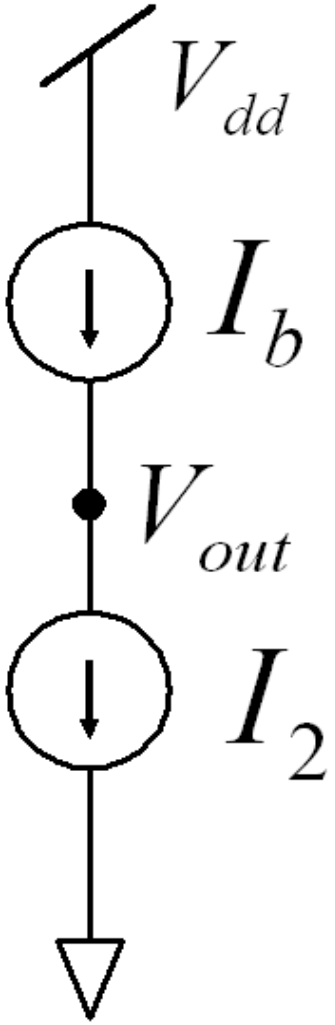
$$\boxed{g_m >> g_d}$$

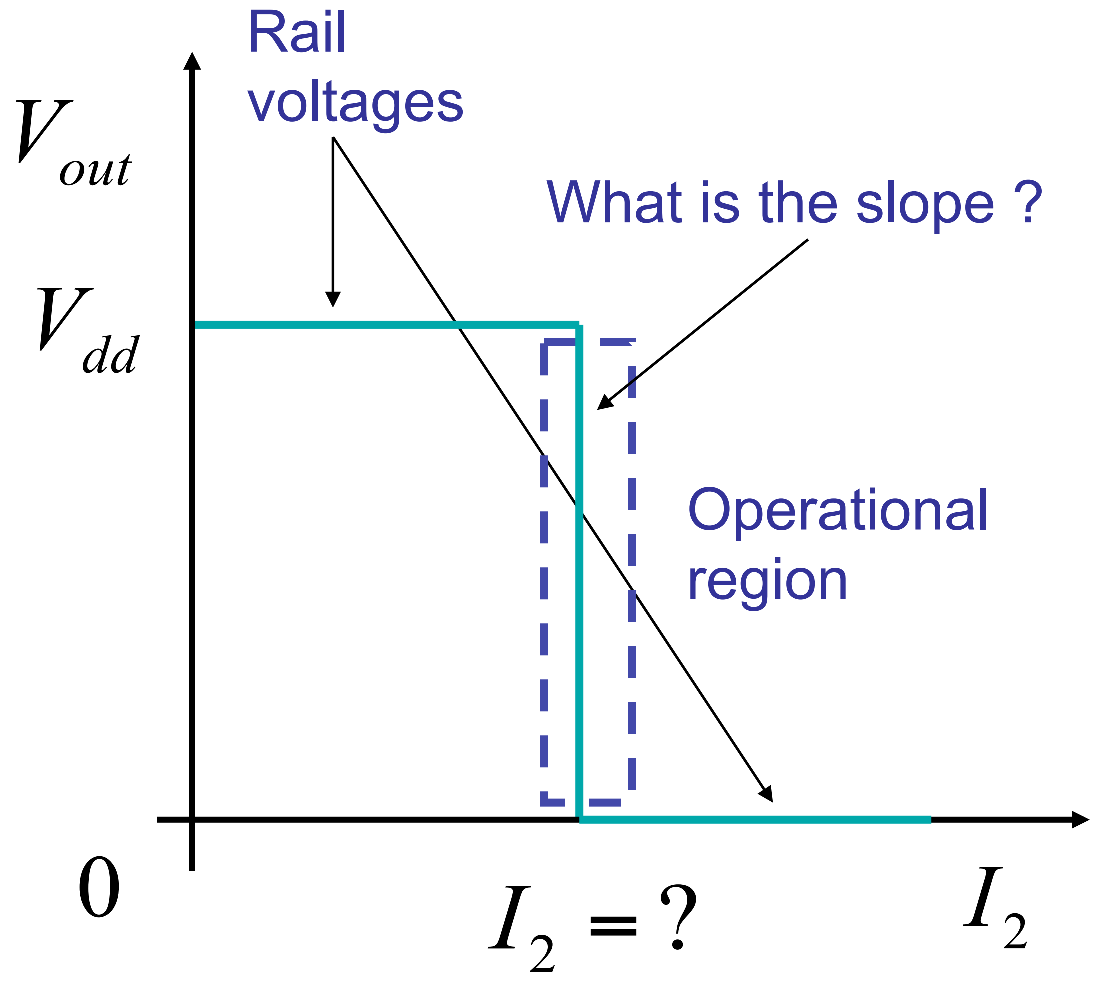
Amplifier Basics





Amplifier Concepts

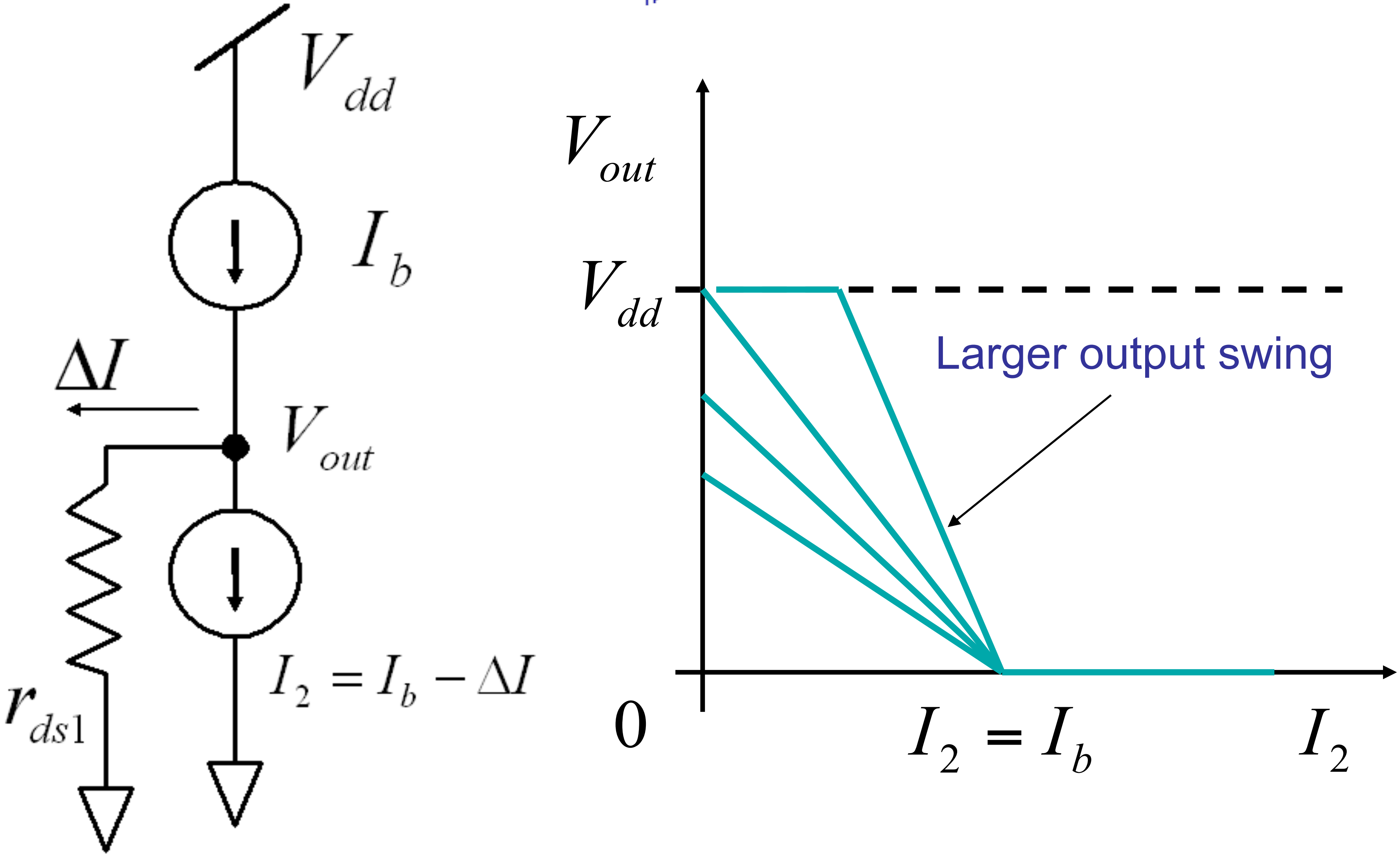




When I_2 is varied how does V_{out} change ?

Worksheet

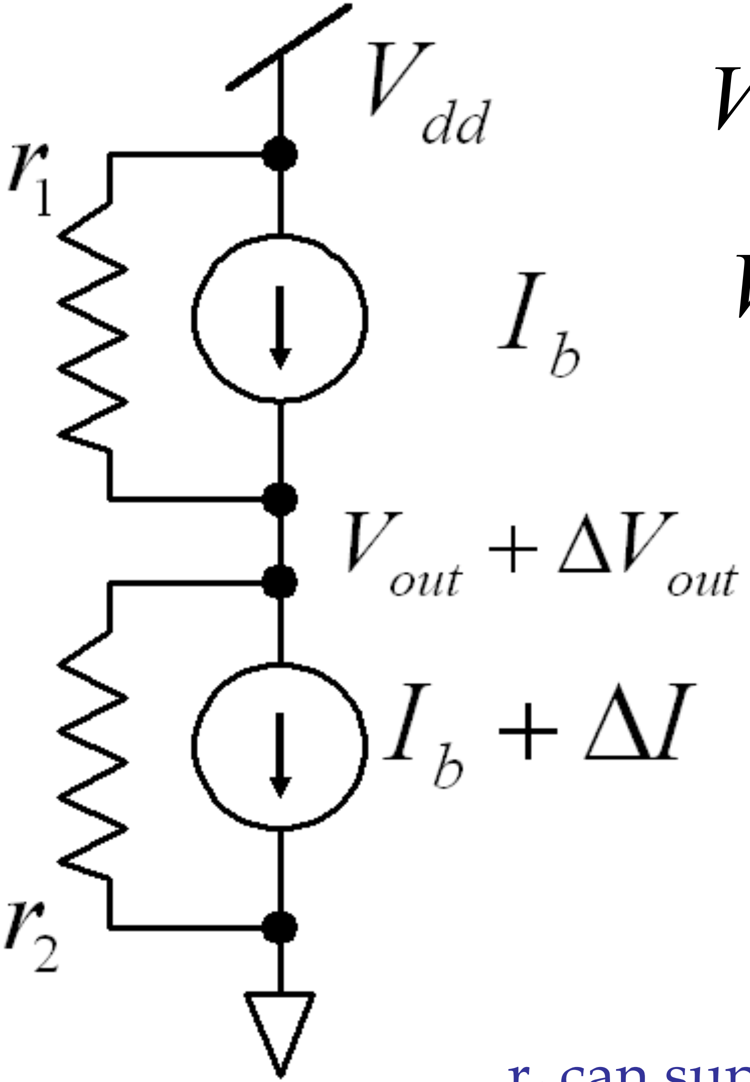
What happens when resistance increases ?

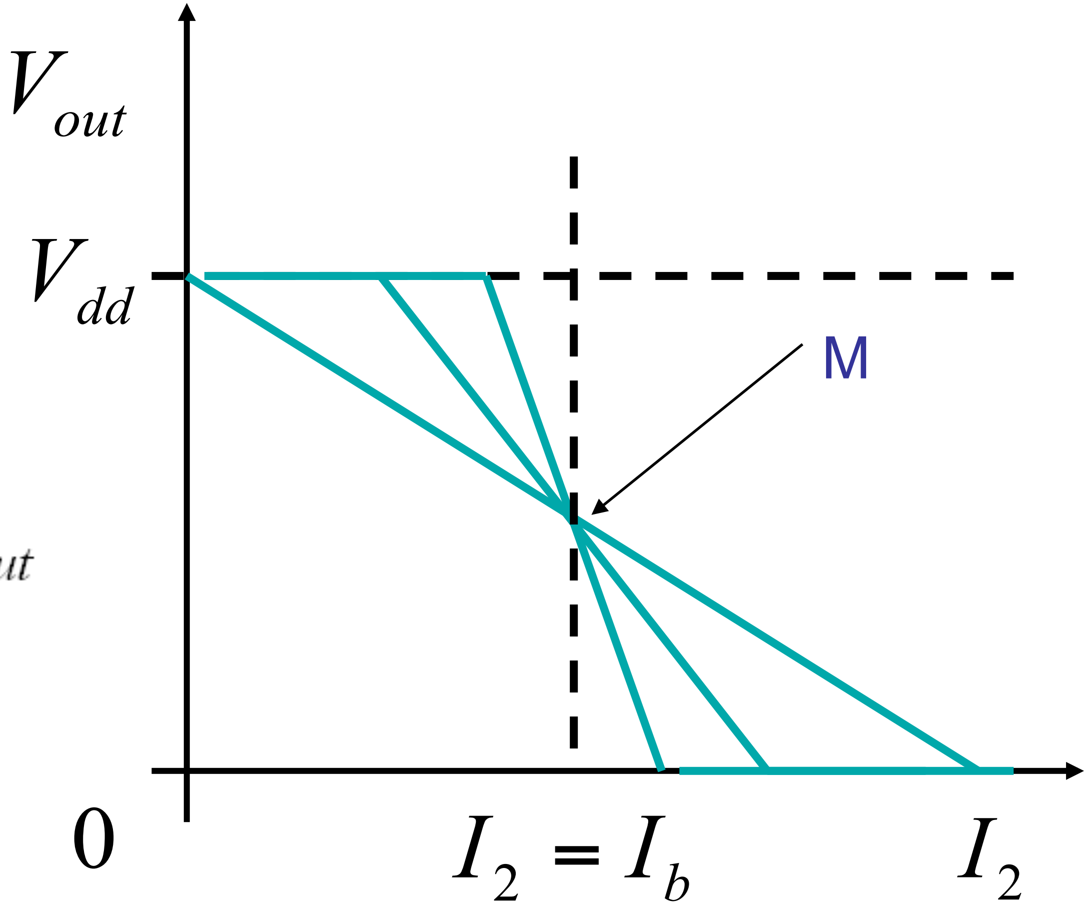


Output swing increases when the load increases

Amplifier Concepts

If r_1 and r_2 are equal what is the voltage at M ?

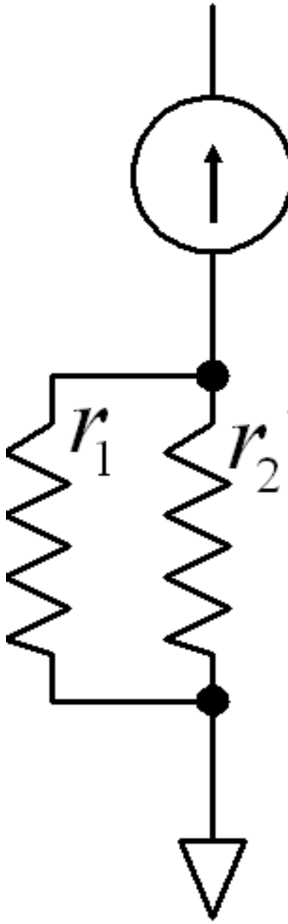




r_1 can supply current from V_{dd}

Worksheet

Amplifier Small-signal Model



ΔI

$$\Delta V_{out} = -\Delta I \boxed{(r_1 \parallel r_2)} = \frac{-\Delta I}{g_1 + g_2}$$

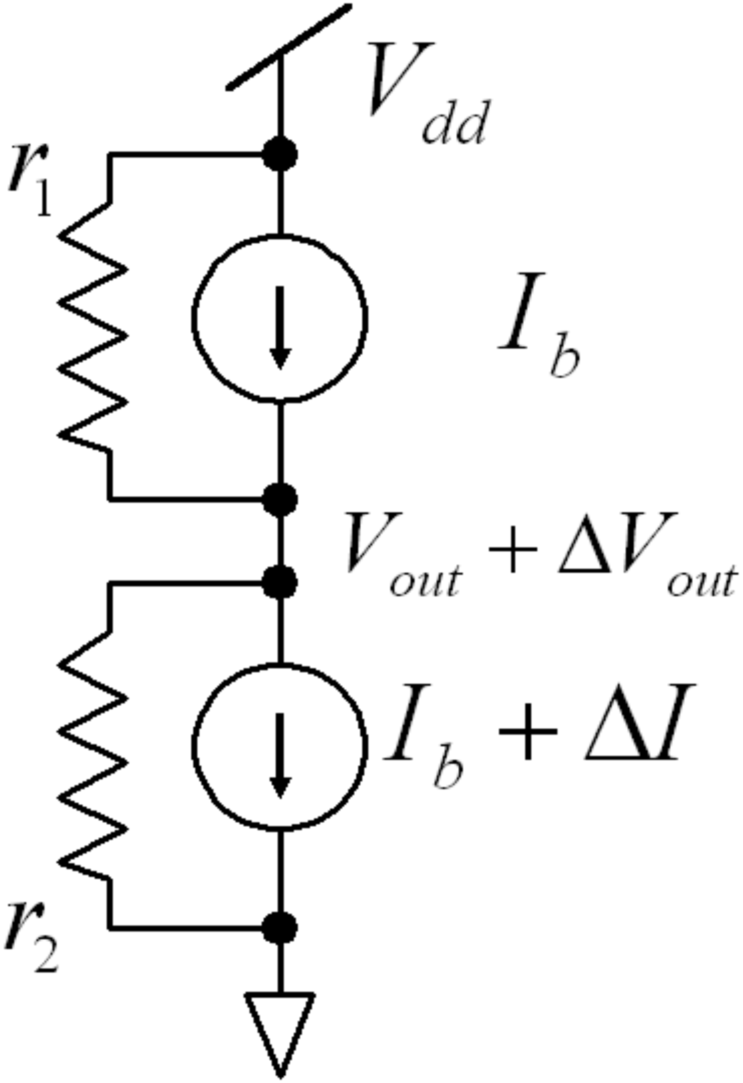
Load of the amplifier

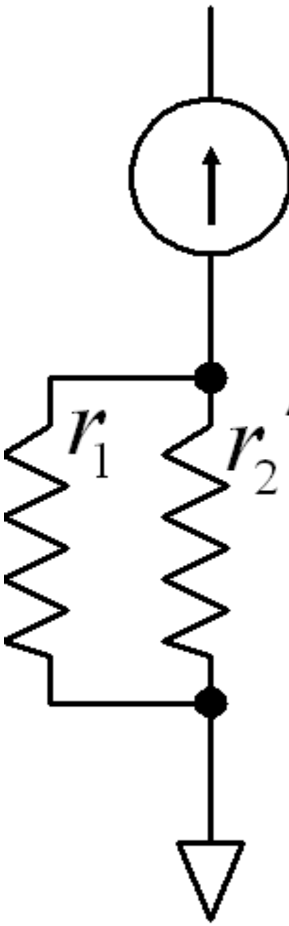
Larger the load, greater is the output slope.

How do we control ΔI ?

- Use a MOS transistor and control one of the three terminals.

Amplifier Basics





$$\Delta V_{out} = -\Delta I\,(r_1 \parallel r_2)$$

$$= \frac{-\Delta I}{g_1 + g_2}$$

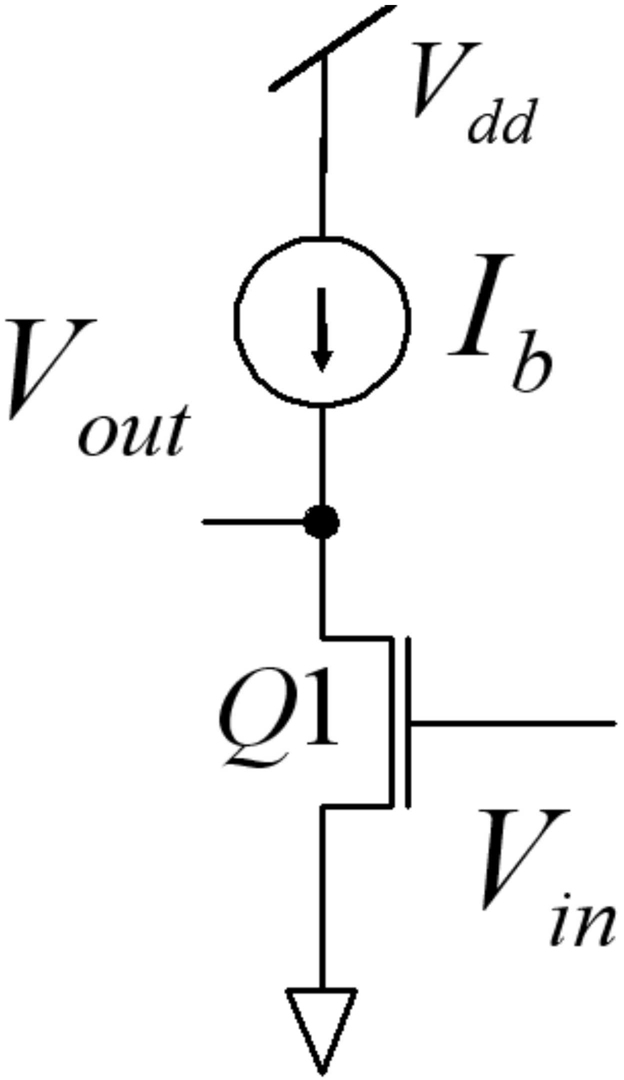
Single Stage Amplifier

Common source amplifier – Gate of the MOS transistor is the input, drain of the transistor is the output and source is the reference potential for input and output.

Common gate amplifier – Source of the MOS transistor is the input, drain is the output and gate is the reference for input and output.

Common drain amplifier – Gate of the MOS transistor is the input, source is the output and drain is the reference for input and output.

Common-source Amplifier



Small signal gain

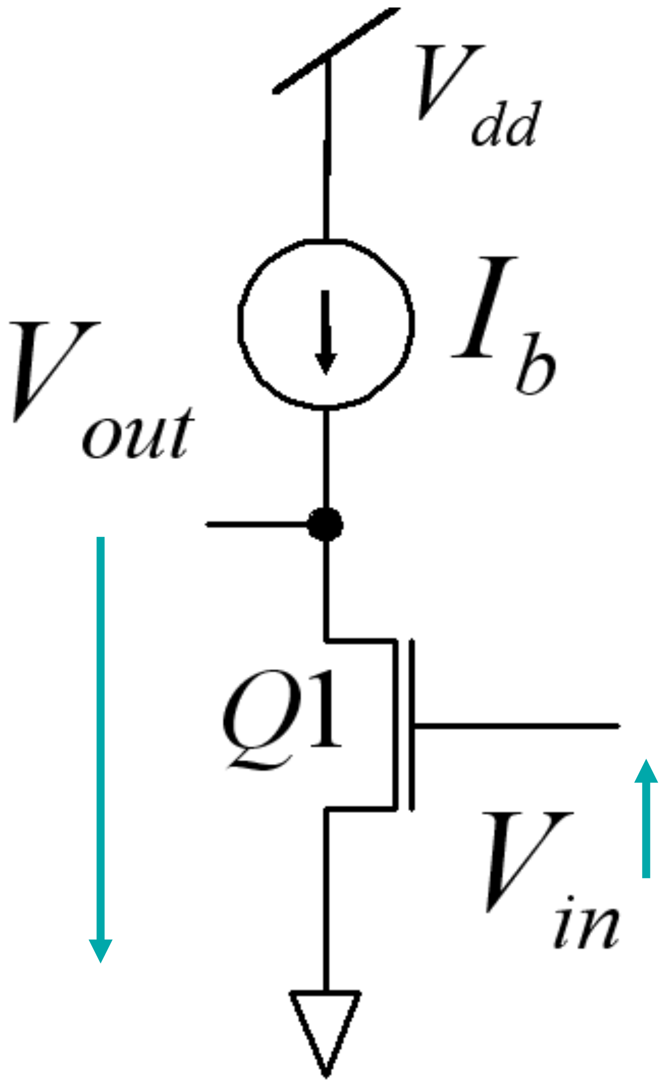
$$A_v \approx -g_m r_{ds}$$

First solve for gain by writing small signal equations

$$g_m \Delta V_{in} + g_d \Delta V_{out} = 0$$

$$\frac{\Delta V_{out}}{\Delta V_{in}} = -\frac{g_m}{g_d}$$

Conceptual Operation



Initially Q1 was conducting current I_b.

Suppose the input voltage increases by a small amount.

The drain current through Q1 will increase.

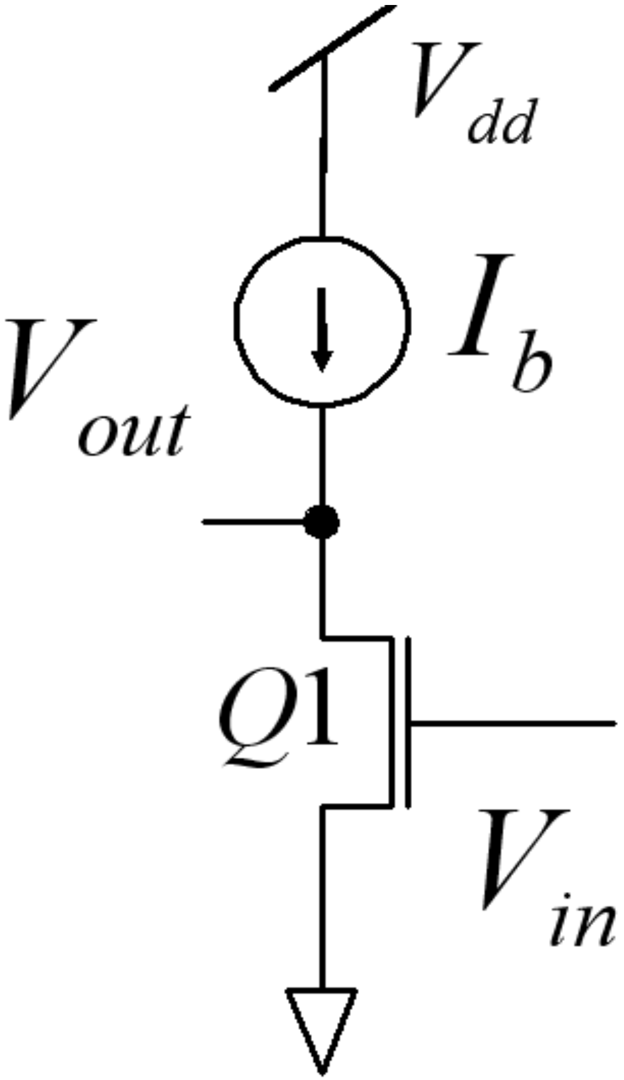
This increased current will pull the voltage V_{out} down.

Question is how much is V_{out} pulled down ?

V_{out} has to decrease such that it can compensate the current change due to V_{in}.

Because V_{in} is more sensitive than V_{out}, V_{out} has to decrease by a large amount to compensate for change in V_{in}.

Common-source Amplifier Specification



Gain $A_v = -g_m r_{ds}$

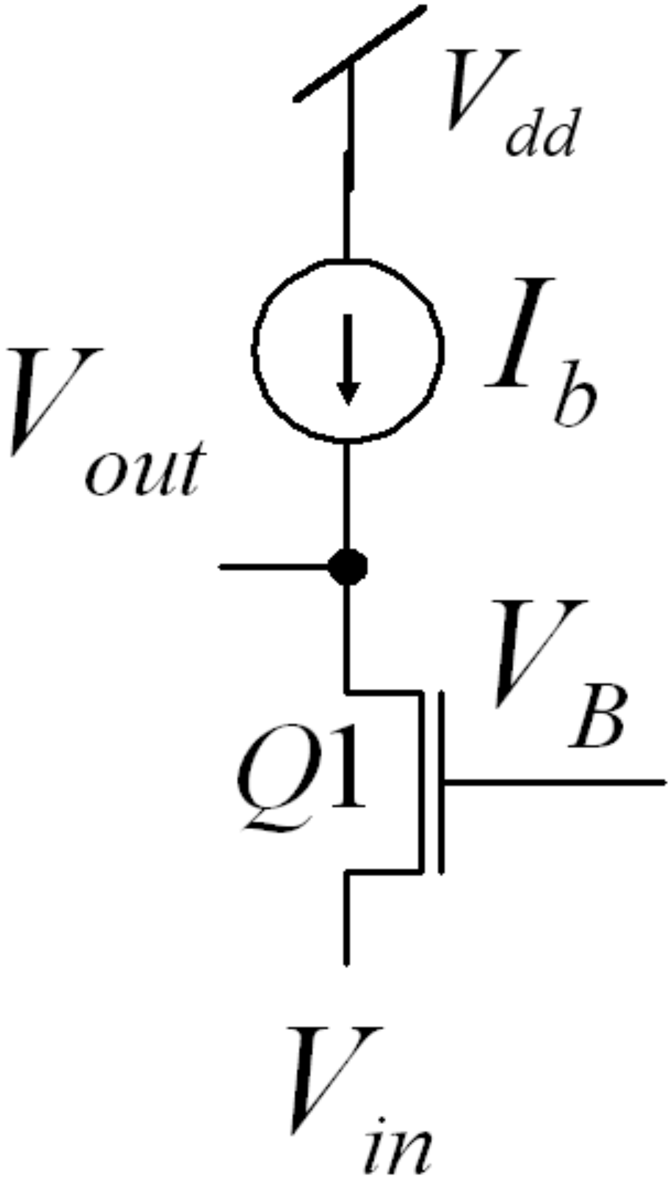
Input impedance $Z_{in} = \infty$

Output impedance $Z_{out} = r_{ds}$

Output voltage swing $V_{dd} > V_{out} \geq V_{eff}$

Power dissipation $P_d = V_{dd} I_b$

Common-gate Amplifier



Gain $A_v = 1 + g_m r_{ds}$

Input impedance $Z_{in} = 1/g_m$

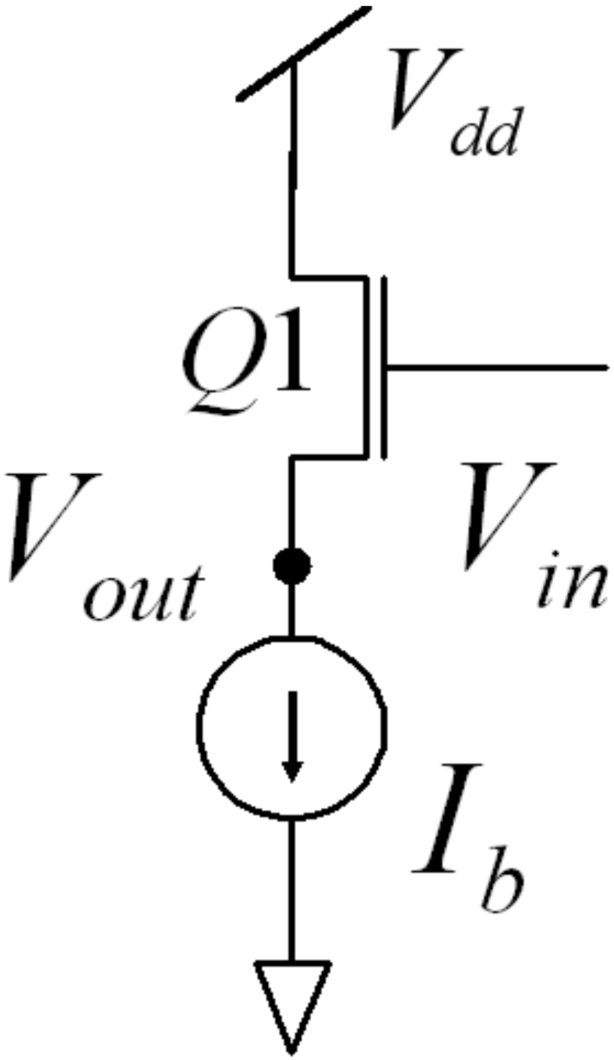
Output impedance $Z_{out} = r_{ds}$

Output voltage swing $V_{dd} > V_{out} \geq V_{eff}$

Power dissipation $P_d = V_{dd} I_b$

Worksheet

Common-drain Amplifier



Gain $$A_v = \frac{g_m}{g_m + g_d}$$

Input impedance $$Z_{in} = \infty$$

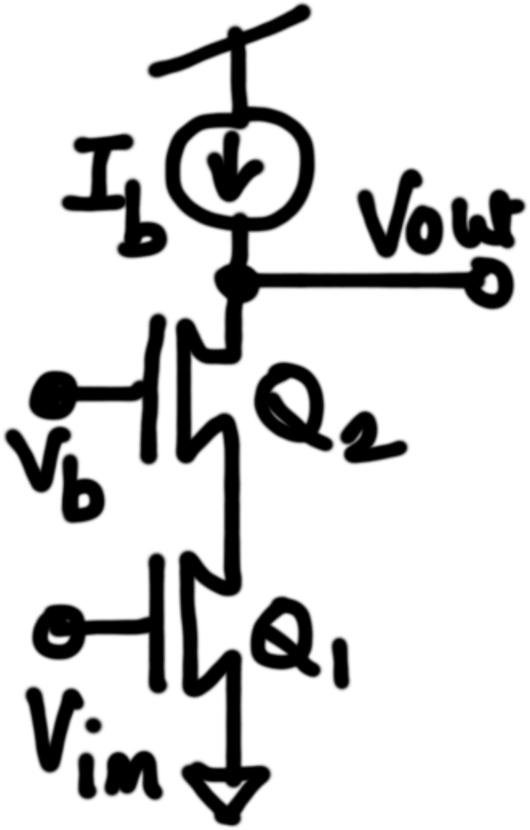
Output impedance $$Z_{out} = \frac{1}{g_m + g_d}$$

Output voltage swing $$V_{dd} - V_{th} > V_{out} \geq 0$$

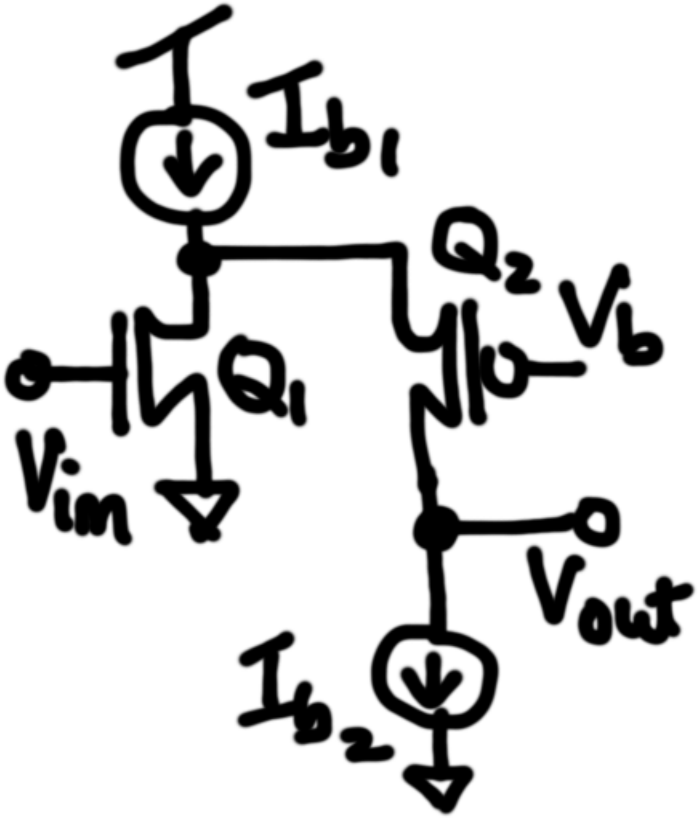
Power dissipation $$P_d = V_{dd} I_b$$

Worksheet

More Cascoded Amplifiers



Telescopic Cascoded gain stage



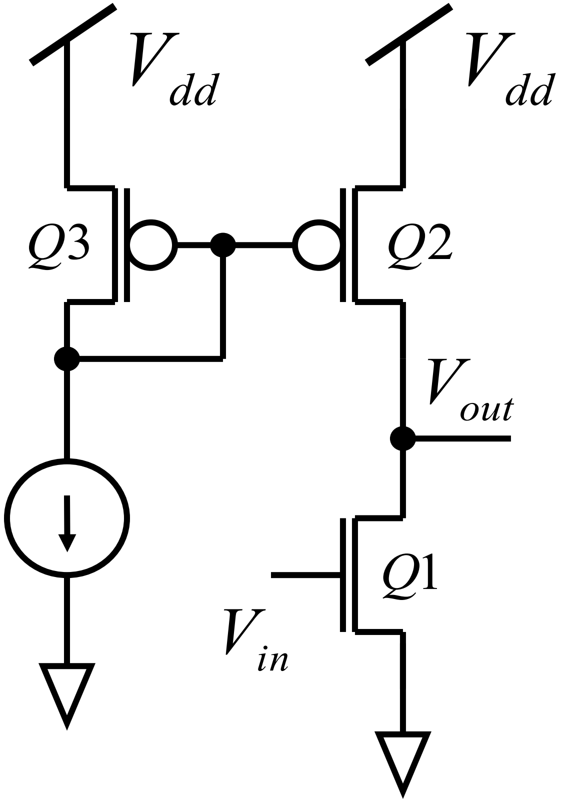
Folded Cascoded gain stage

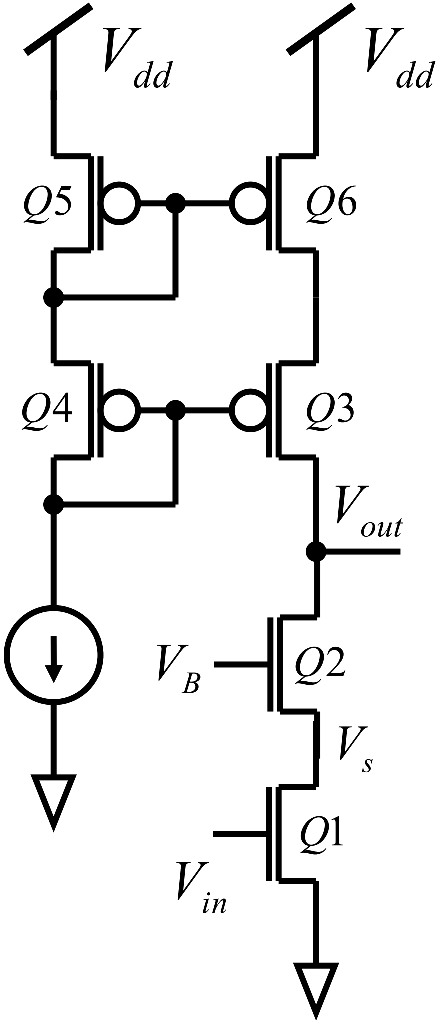
$$A_v = -\left(\frac{g_m}{g_{ds}}\right)^2$$

Assuming that the current source is a cascoded current source.

Worksheet

Single to Multi-stage Amplifier





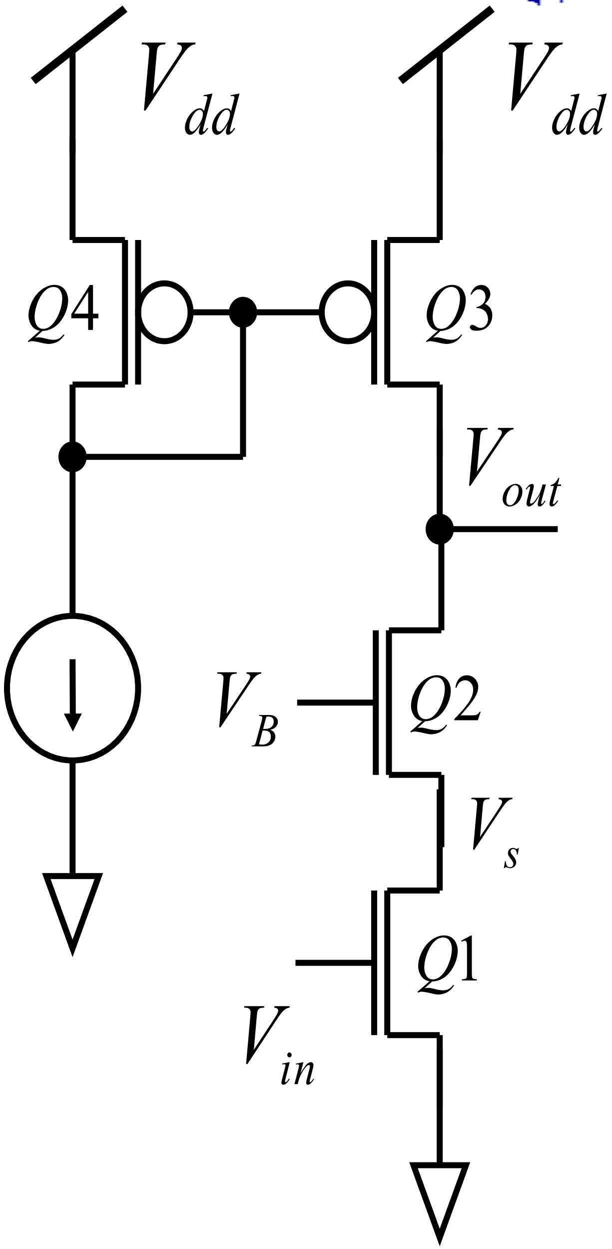
Gain

$$A_v = \frac{\Delta V_{out}}{\Delta V_{in}} = -g_{m1}(r_{ds1} \parallel r_{ds2})$$

$$A_v = -g_{m1}(g_{m2} r_{ds1} r_{ds2} \parallel g_{m3} r_{ds3} r_{ds6})$$

Worksheet

Cascoding – common mistake !



Why will the gain reduce ?

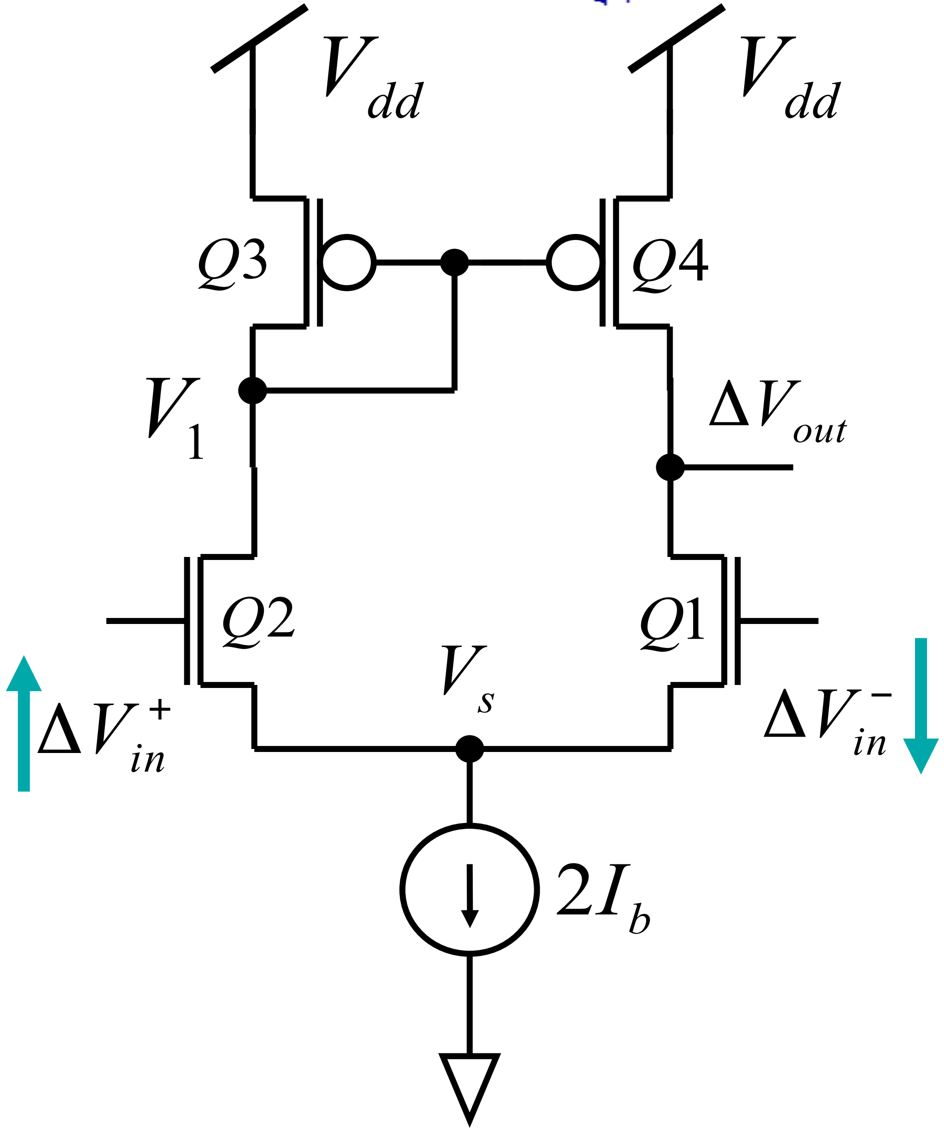
$$A_v = -\left(\frac{g_m}{g_{ds3}}\right)$$

Output voltage range to maintain Q1 and Q2 in saturation

$$V_{dd} - V_{eff} > V_{out} \geq 2V_{eff}$$

Worksheet

Single-ended Differential Amplifier



The gate of Q2 increases

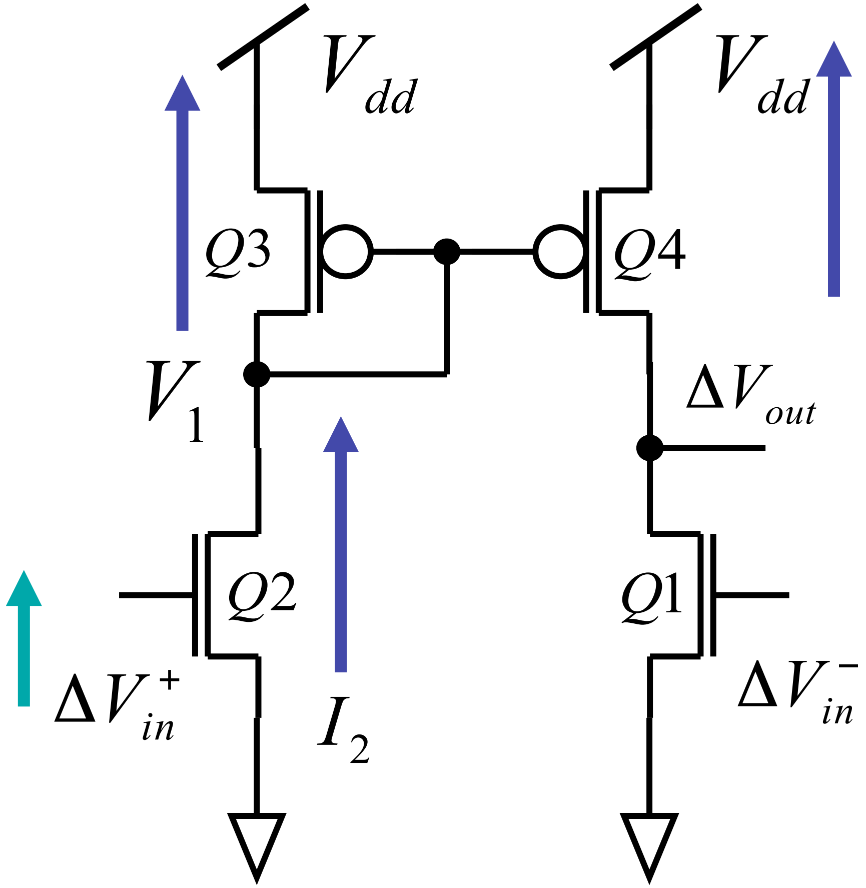
Two things can happen: either Vs can change or V1 can change.

But at the same time gate of Q1 decreases.

Since Q2 and Q1 are matched, Vs will not change (approximation).

Therefore Vs can be replaced by a constant potential for differential operation

Conceptual Operation



The gate of Q2 increases

The drain current through Q2 increases by $g_m \Delta V_{in}$.

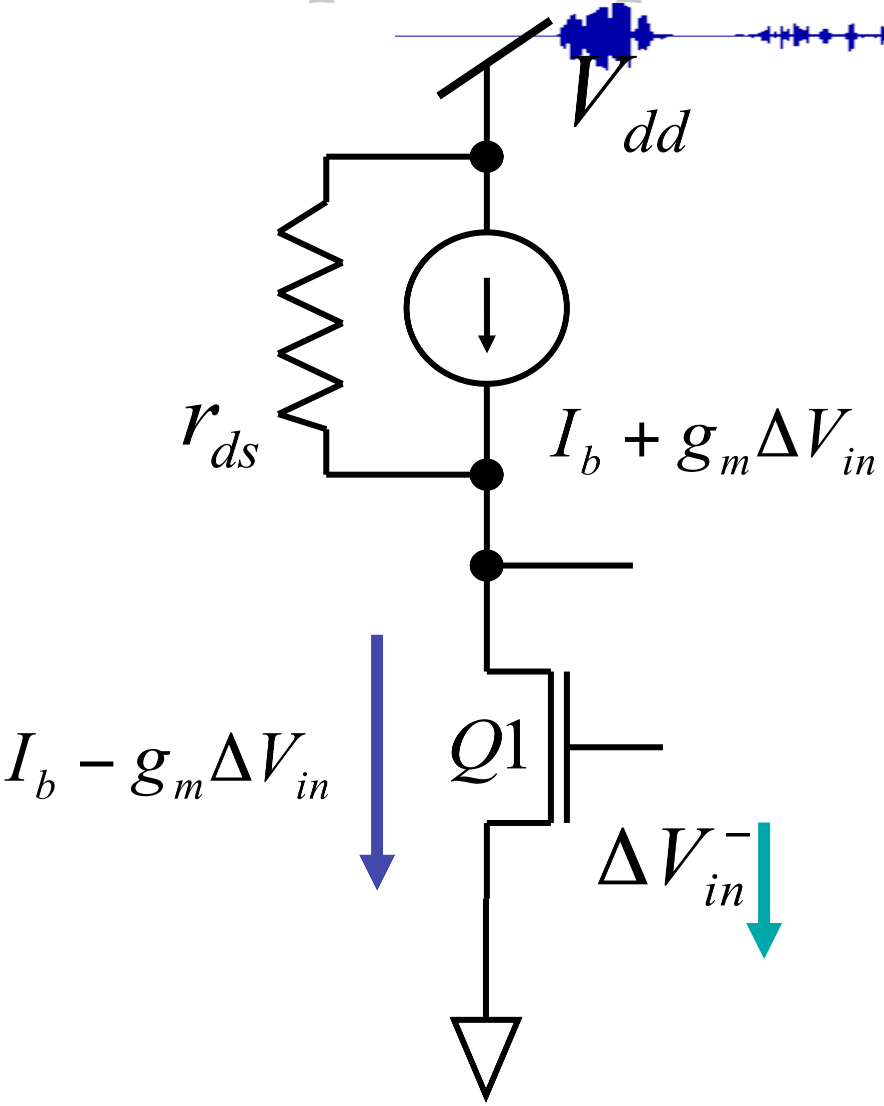
Because Q3 and Q4 are mirrors, the current through Q4 will increase.

Remember in a mirror Q4 acts as a current source.

The current through Q4 is controlled by gate of Q2

When the gate voltage of Q2 increases by ΔV_{in} the current through Q4 increases by $g_m \Delta V_{in}$.

Conceptual Operation

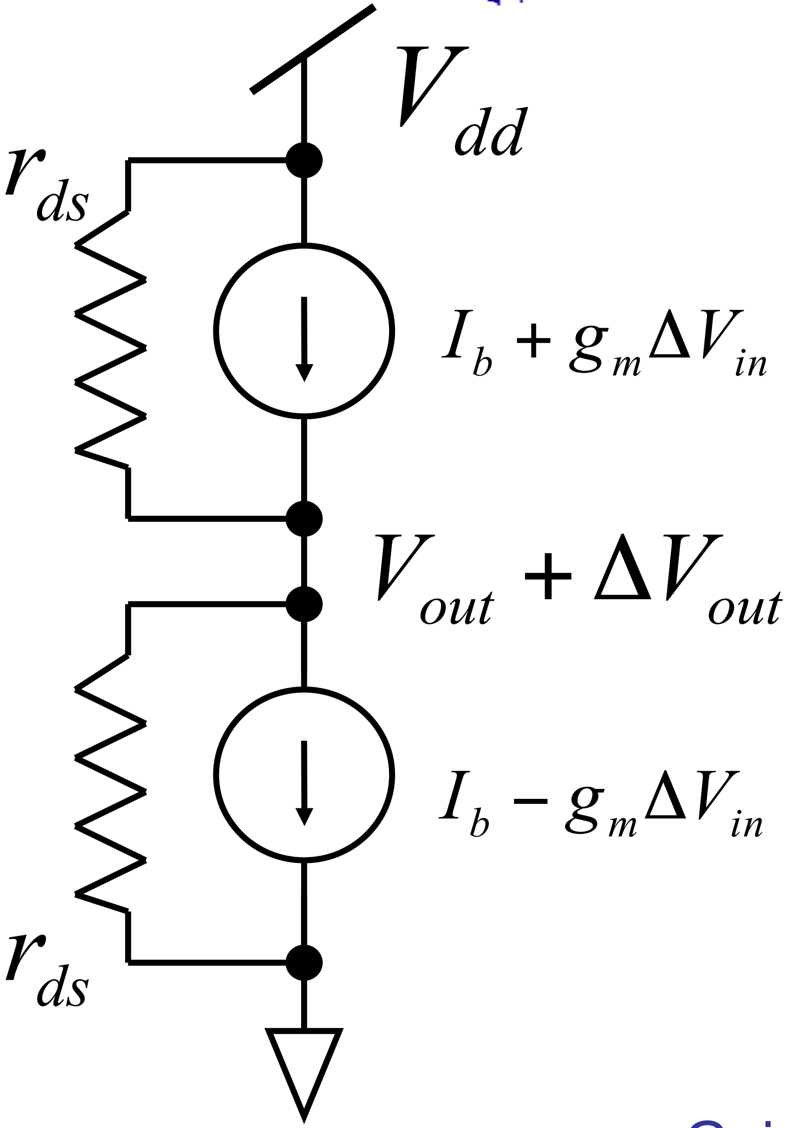


Replace Q4 by an equivalent current source

Now consider the case when the gate of Q1 decreases.

The drain current through Q1 decreases by $g_m \Delta V_{in}$

Conceptual Operation



Difference in current $2 g_m \Delta V_{in}$

This extra current will flow through the load connected at V_{out}.

Total load at Vout is $r_{ds} \parallel r_{ds}$

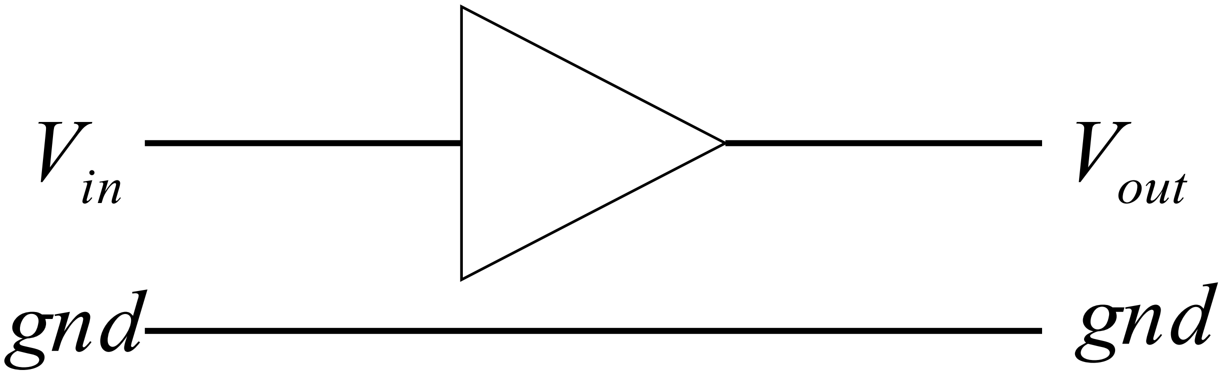
Therefore output voltage change is

$$\Delta V_{out} = g_m r_{ds} \Delta V_{in}$$

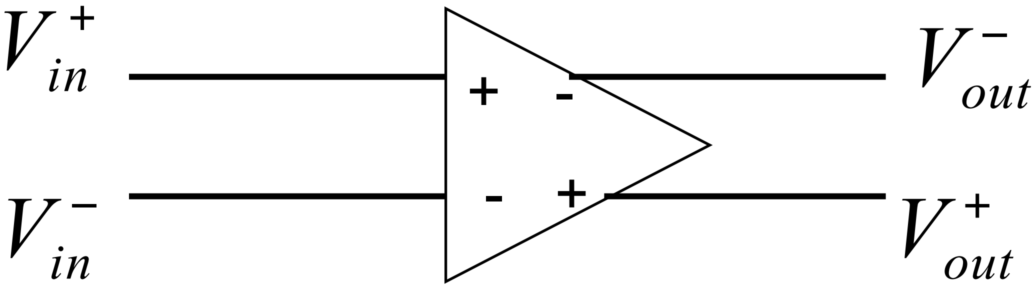
$$\frac{\Delta V_{out}}{\Delta V_{in}} = g_m r_{ds}$$

Gain is given by $A_v = \dfrac{\Delta V_{out}}{2 \Delta V_{in}} = \dfrac{1}{2} g_m r_{ds} = \dfrac{g_m}{2 g_d}$

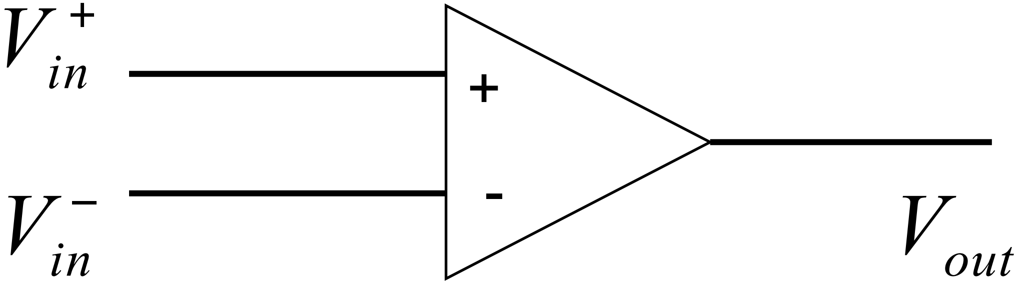
Single and differential amplifiers



$$A_v = \frac{\Delta V_{out}}{\Delta V_{in}}$$

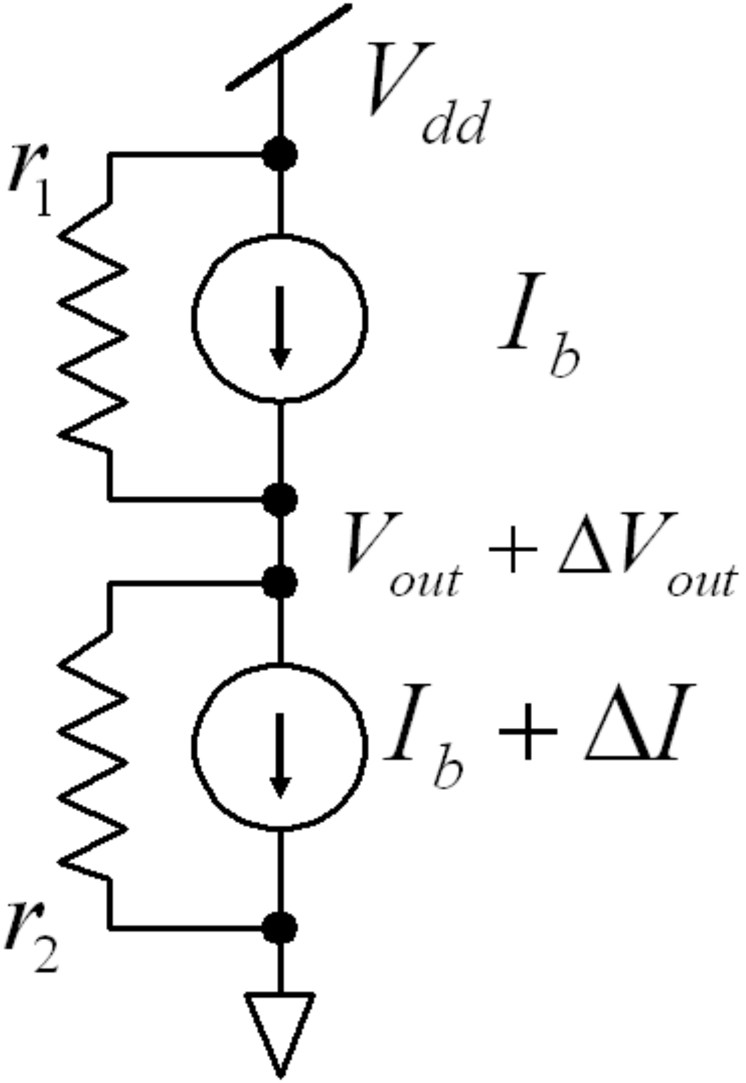


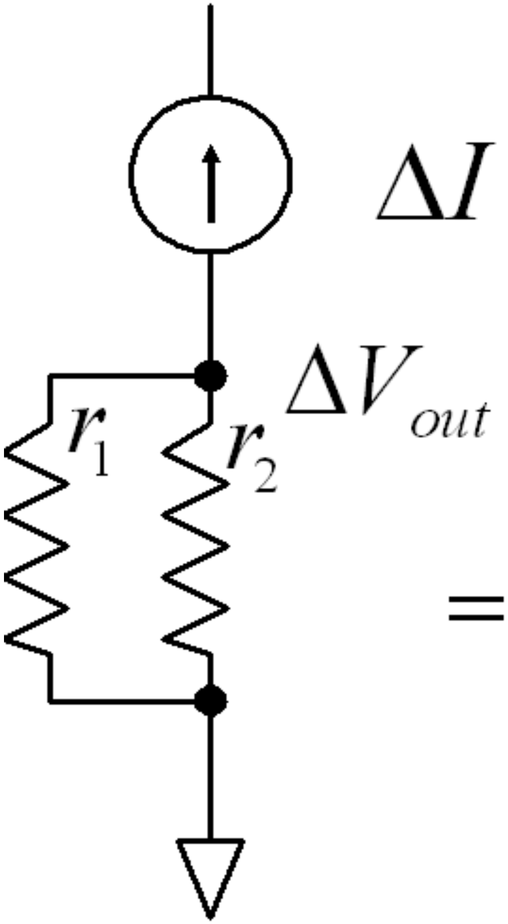
$$A_v = \frac{\Delta V_{out}^{+} - \Delta V_{out}^{-}}{\Delta V_{in}^{+} - \Delta V_{in}^{-}}$$



$$A_v = \frac{\Delta V_{out}}{\Delta V_{in}^{+} - \Delta V_{in}^{-}}$$

Amplifier Basics





$$\Delta V_{out} = -\Delta I \left(r_1 \parallel r_2 \right) = \frac{-\Delta I}{g_1 + g_2}$$

Analysis of Differential Amplifier

- Identify all the current mirrors.

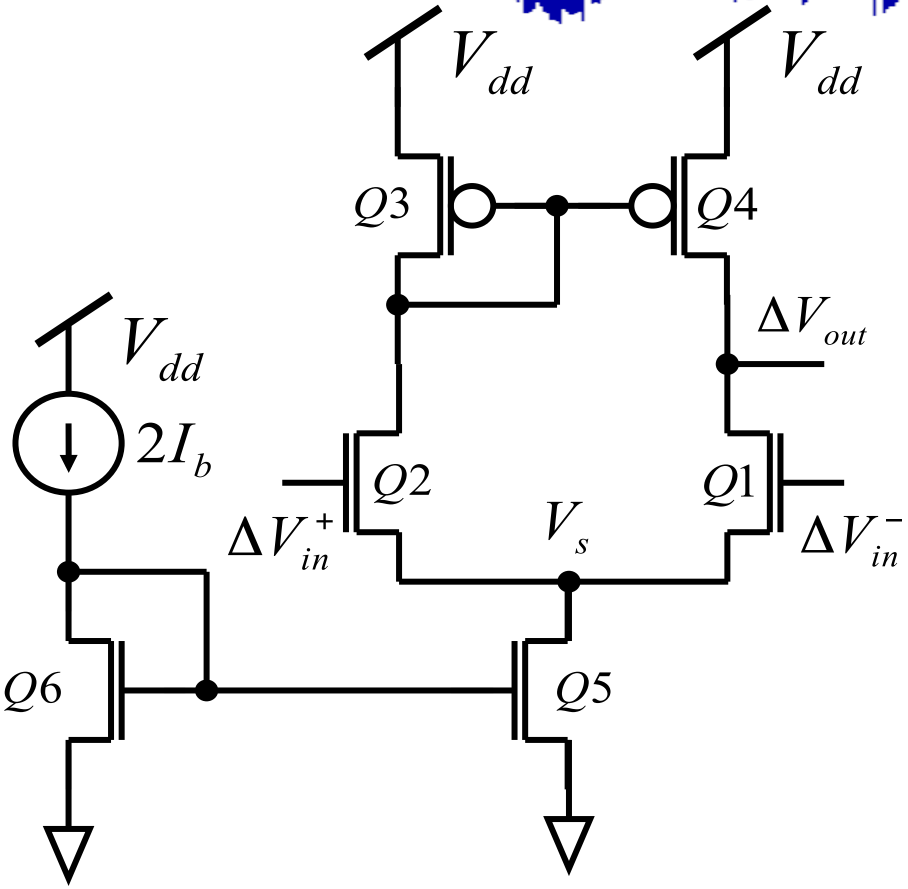
- Replace current mirrors by non-ideal current sources/ sink.

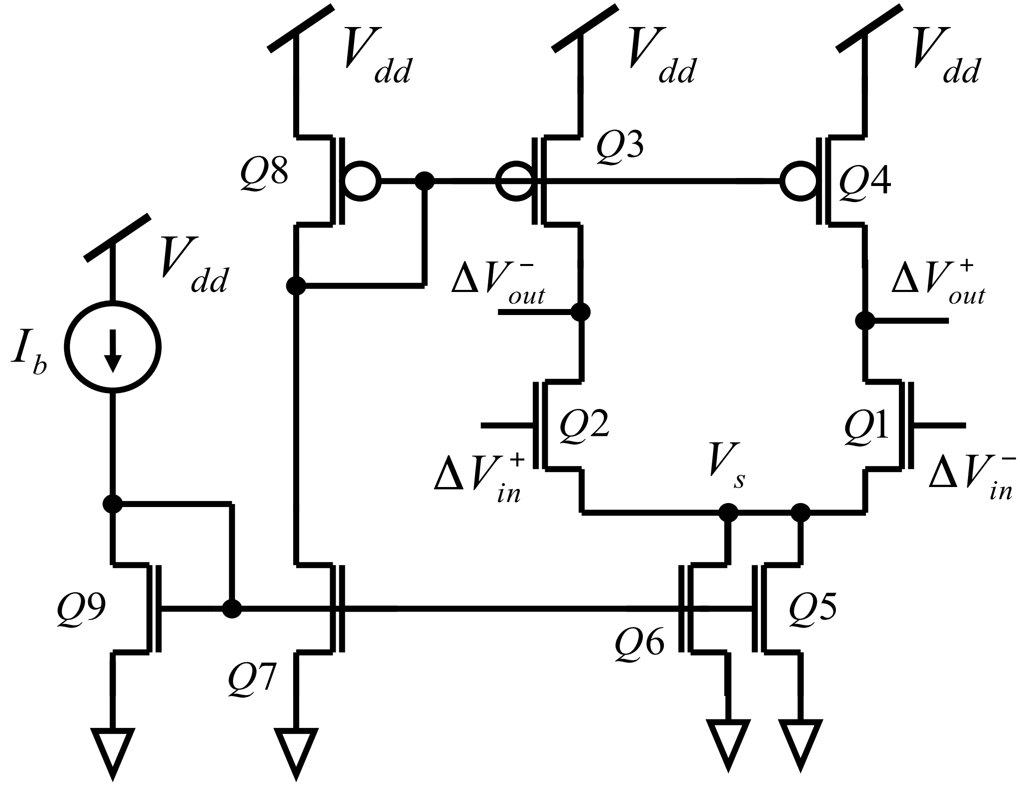
-Identify any point of symmetry and then replace it by gnd.

- Identify the input stage of the amplifier which will determine the incremental change in current.

- Calculate the impedance seen at the output terminal of the amplifier.

Single to Multi-stage Amplifiers



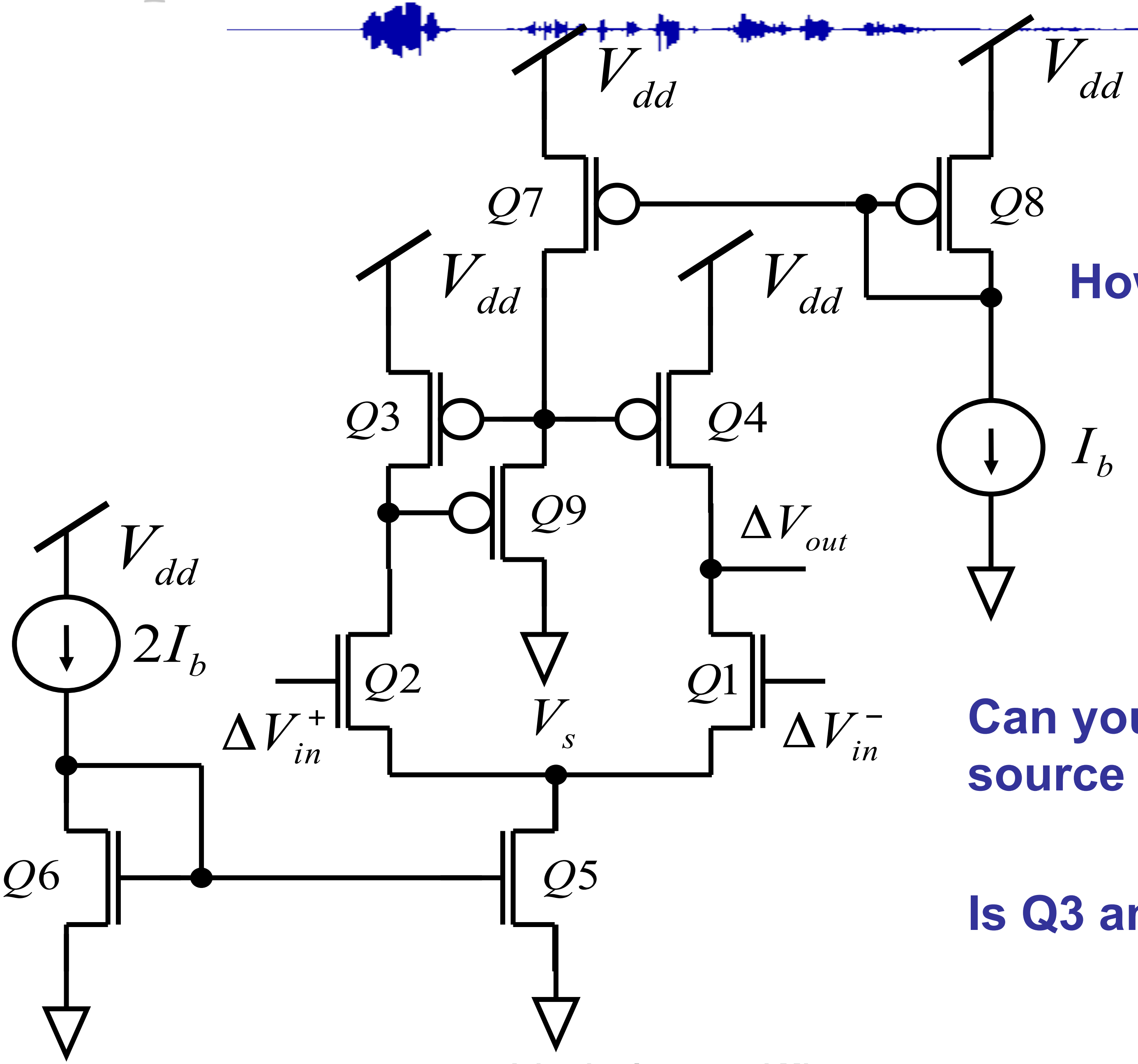


Gain

$$A_v = \frac{\Delta V_{out}}{\Delta V_{in}^{+} - \Delta V_{in}^{-}} = -g_{m1}(r_{ds1} \parallel r_{ds2})$$

$$A_v = \frac{\Delta V_{out}^{+} - \Delta V_{out}^{-}}{\Delta V_{in}^{+} - \Delta V_{in}^{-}} = -g_{m1}(r_{ds4} \parallel r_{ds1})$$

Example 1



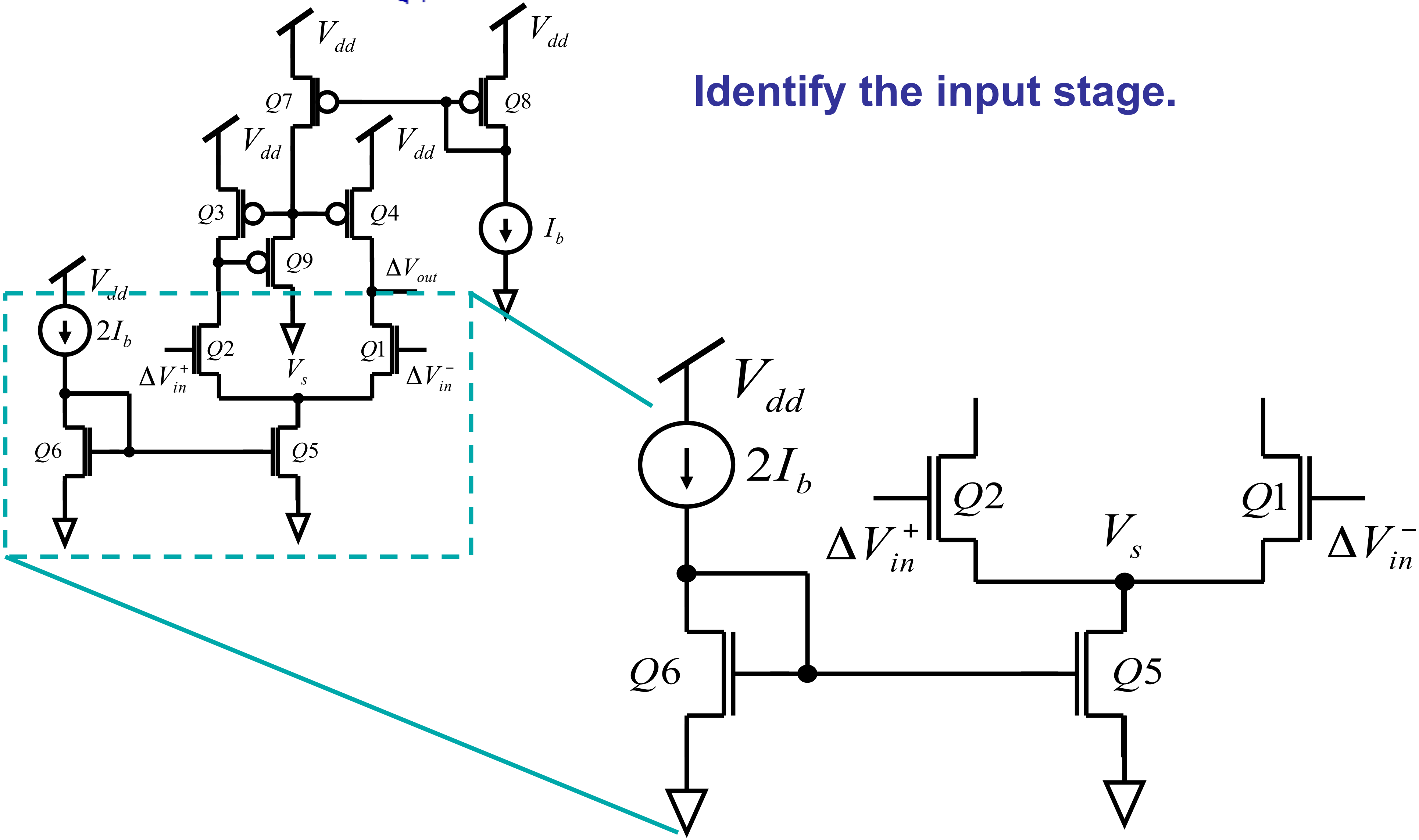
How many mirrors ?

Can you identify the source follower ?

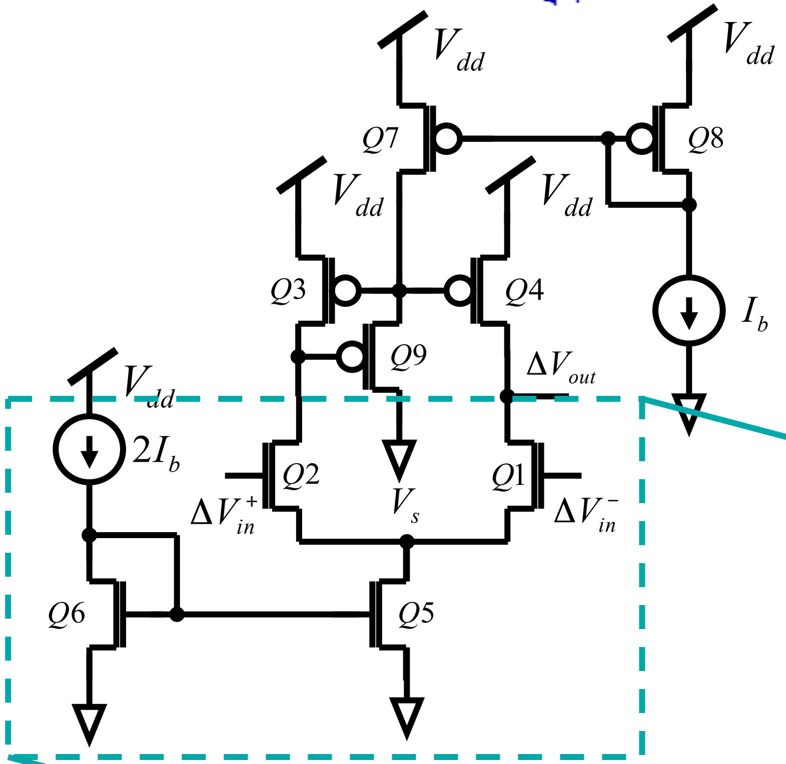
Is Q3 and Q4 mirrors ?

Example (cont.)

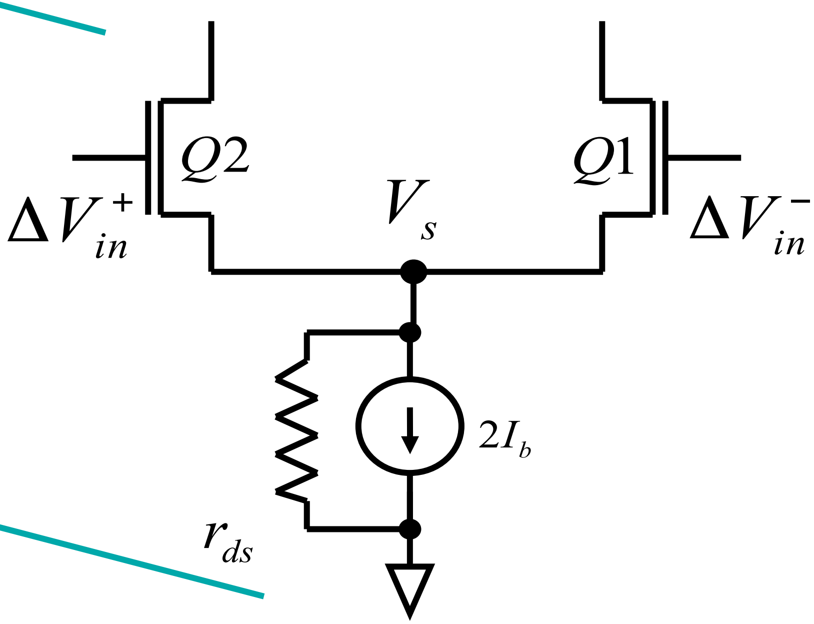
Identify the input stage.



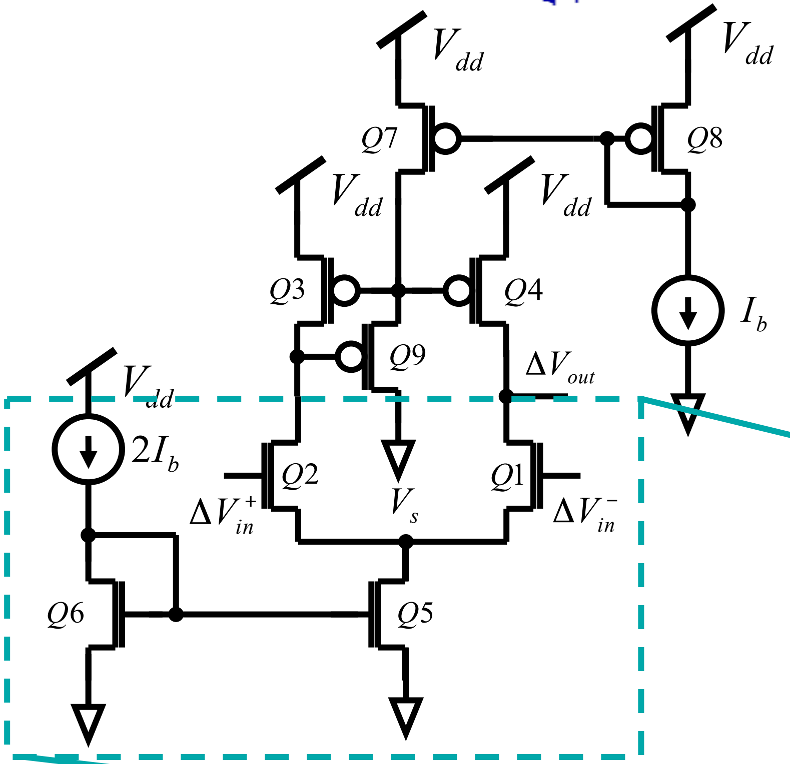
Example (cont.)



Replace the mirror by its equivalent circuit.



Example (cont.)

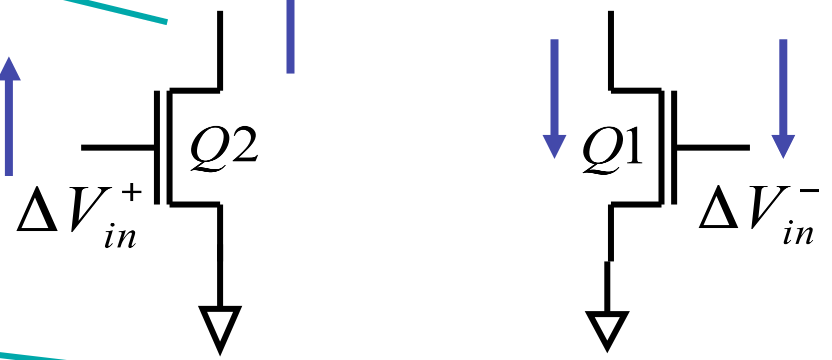


Identify the point of symmetry Vs and then split the input stage about the point of symmetry.

Differential change in current through Q2 and Q1 for a differential input change.

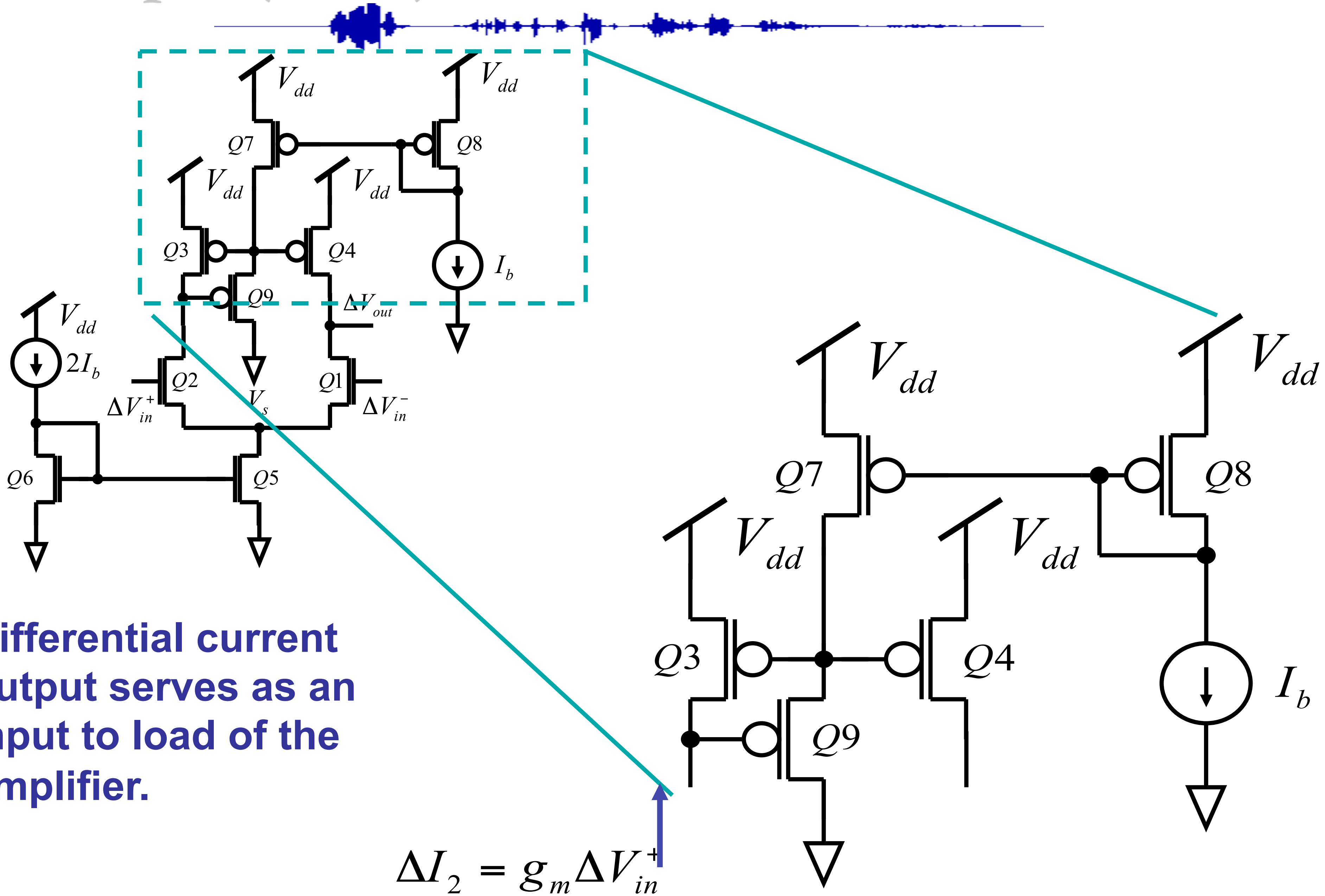
$$\Delta I_2 = g_m \Delta V_{in}^{+}$$

$$\Delta I_1 = g_m \Delta V_{in}^{-}$$



REMEMBER: Symmetry can not be used for analysis in all conditions. Review previous notes to understand why symmetry can be used.

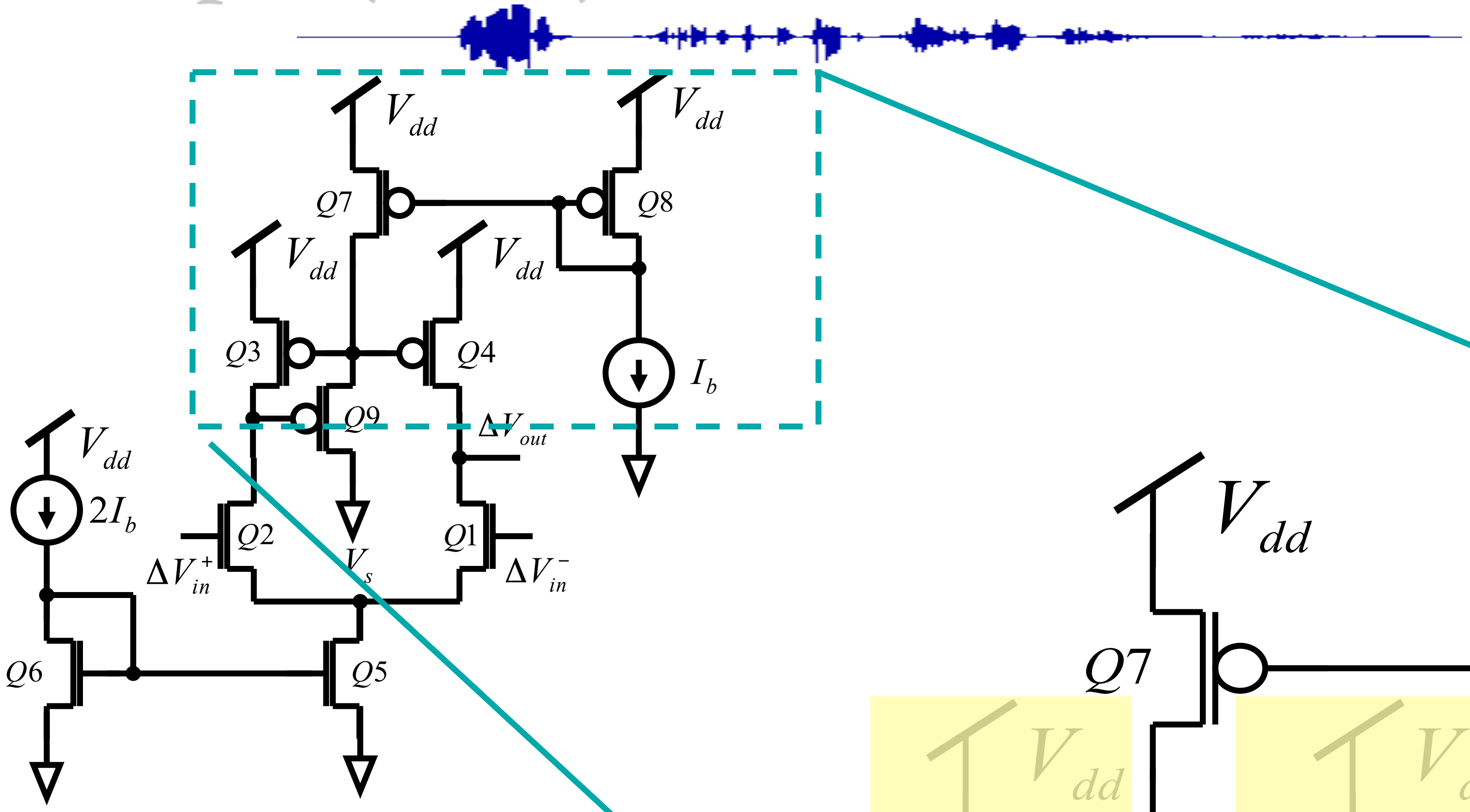
Example (cont.)

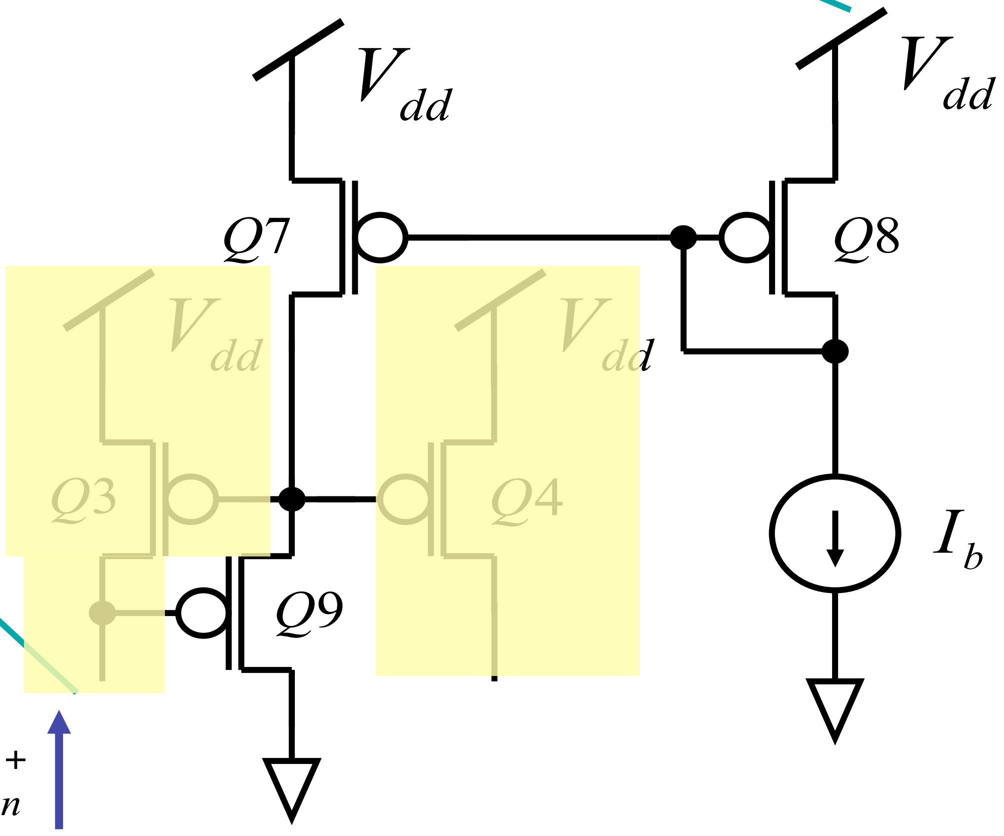


Differential current output serves as an input to load of the amplifier.

$$\Delta I_2 = g_m \Delta V_{in}^+$$

Example (cont.)

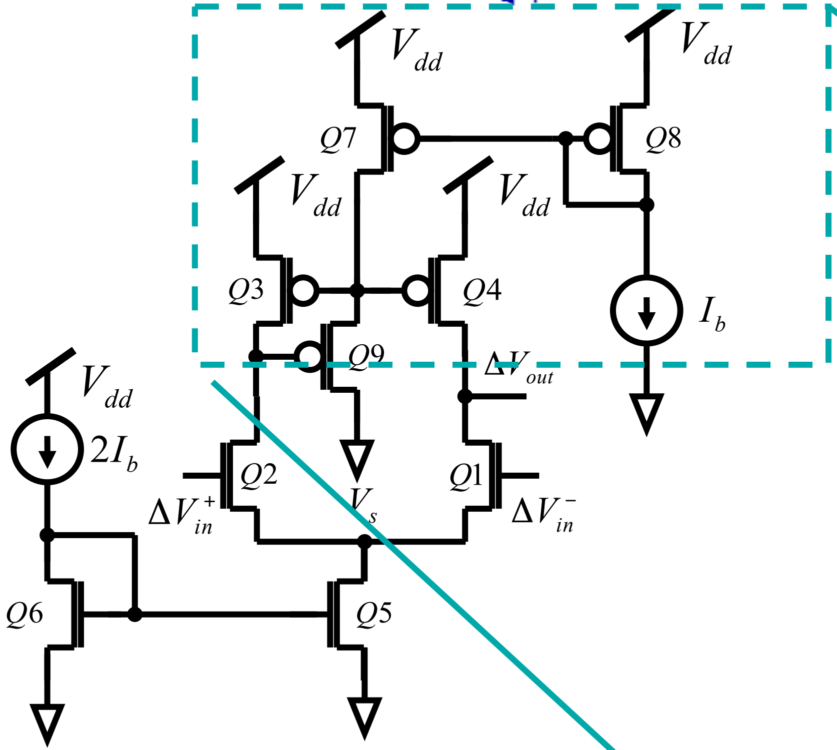




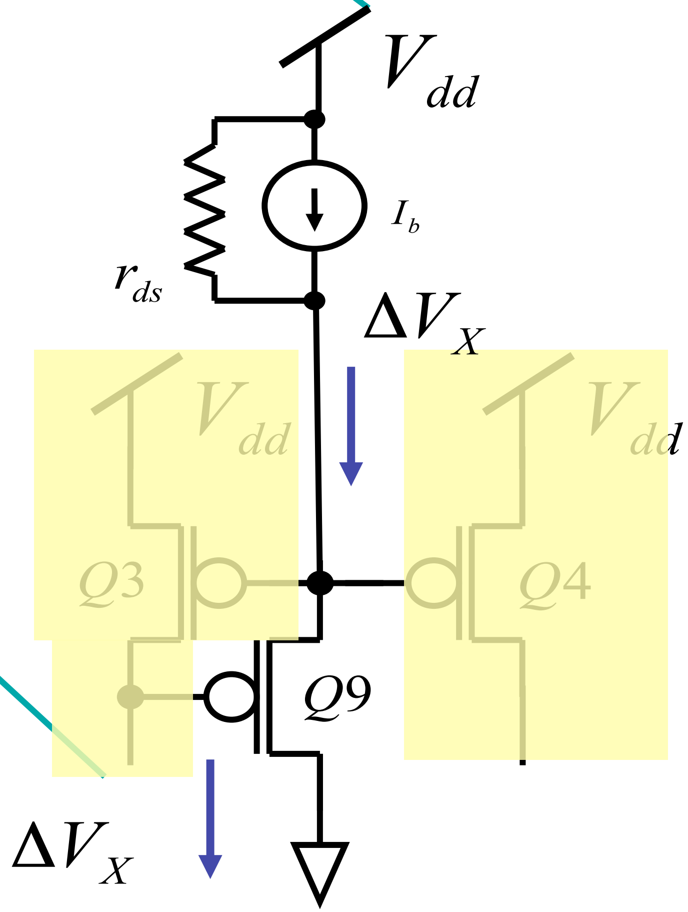
What is the circuit formed by Q7, Q8 and Q9 ?

$$\Delta I_2 = g_m \Delta V_{in}^{+}$$

Example (cont.)



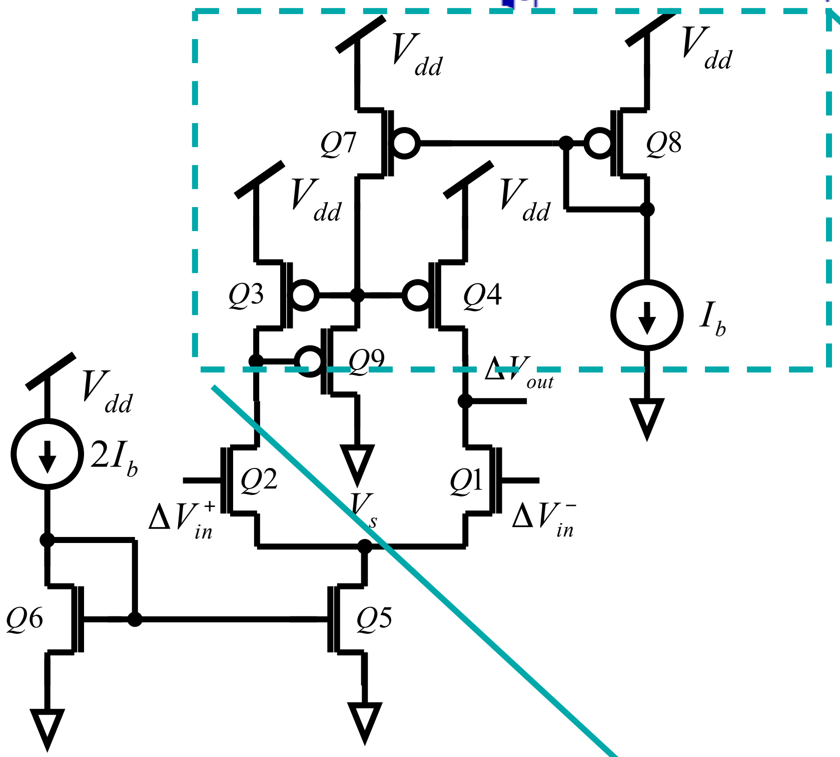
Write down small signal equations to convince yourself.



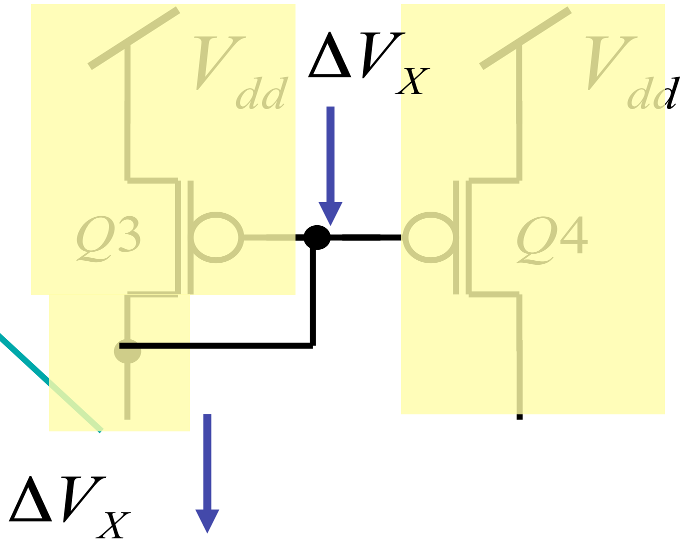
Simplify by replacing the current mirror.

Q9 acts as a source follower.

Example (cont.)

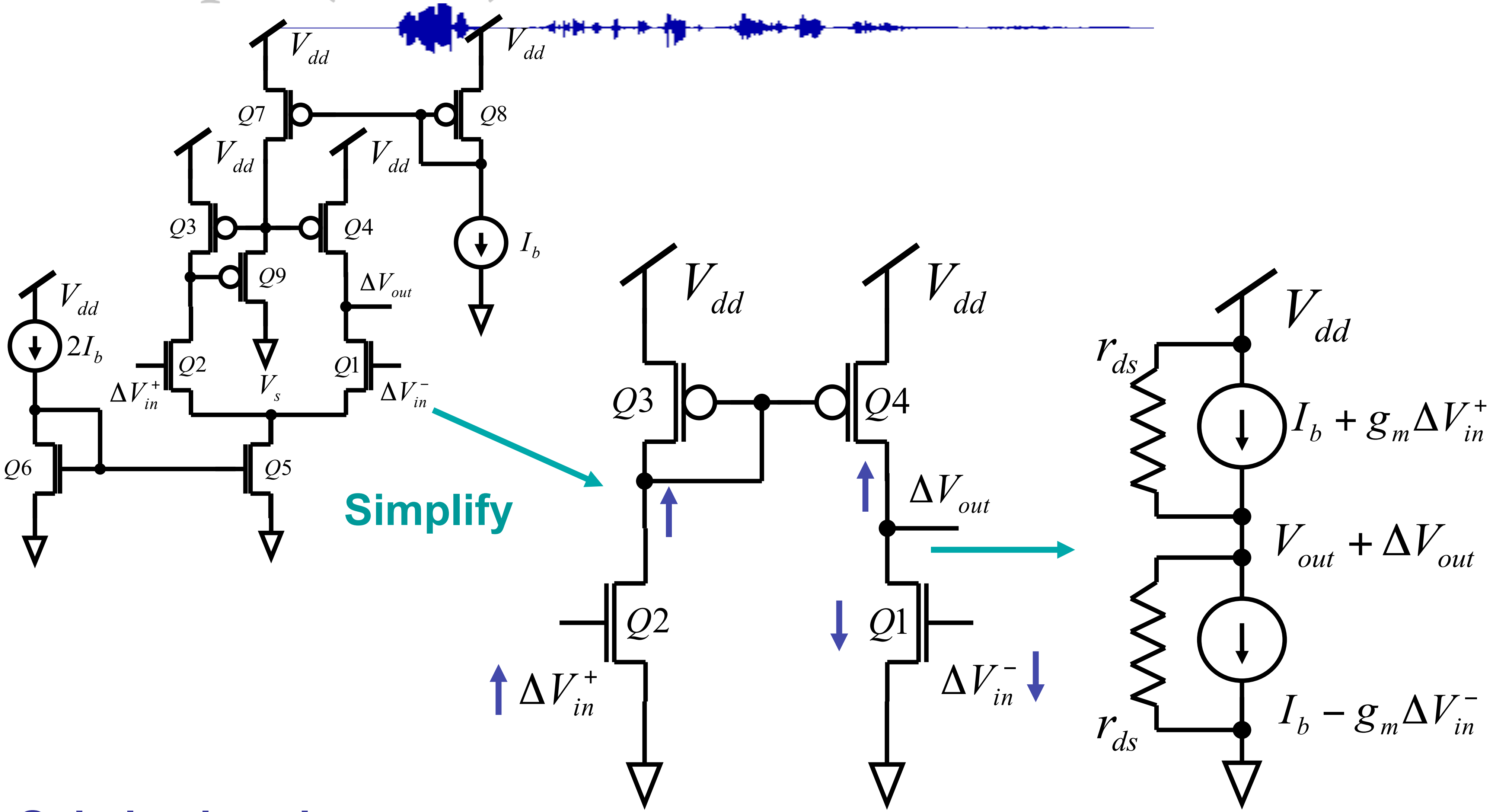


REMEMBER: Input and output are still isolated from each other. (Advantage of this circuit).



For a source follower the input and output can be FUNCTIONALLY connected together

Example (cont.)

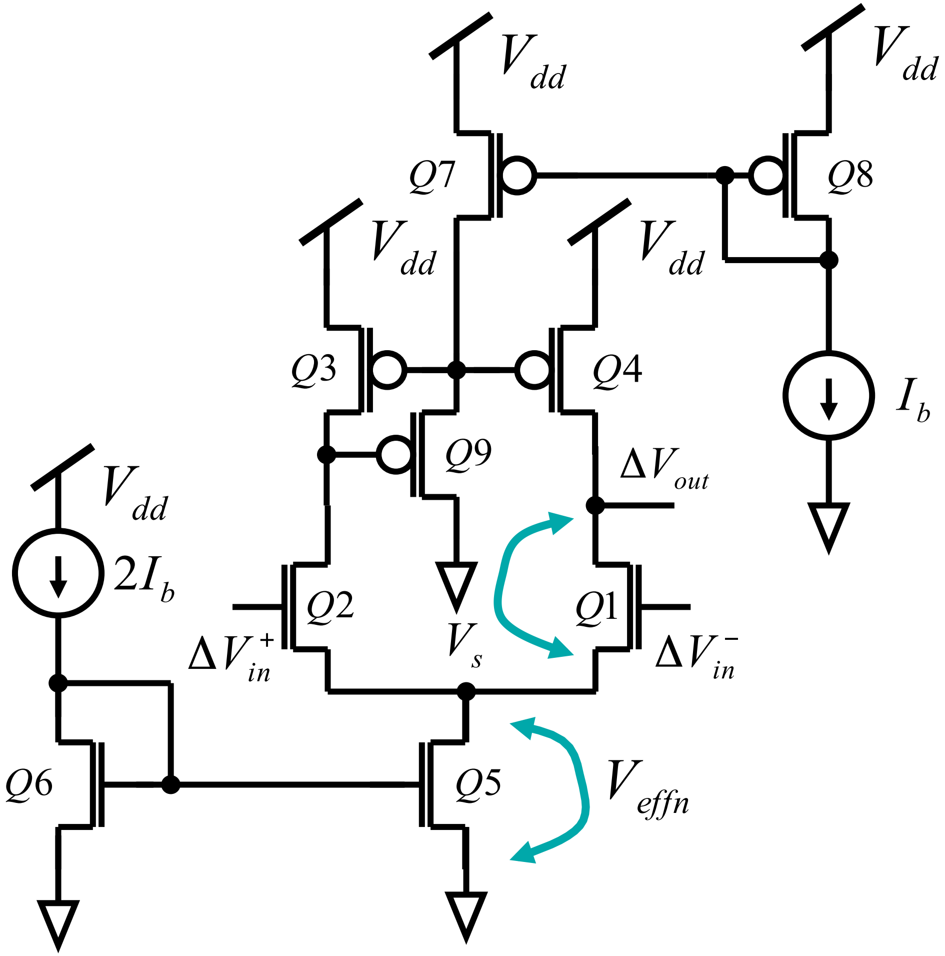


Gain is given by

$$A_v = \frac{\Delta V_{out}}{\Delta V_{in}^{+} - \Delta V_{in}^{-}} = \tfrac{1}{2} g_m r_{ds}$$

Equivalent circuit

Output Voltage Range and Power Dissipation



First calculate effective voltage (overdrive voltage) for each transistor.

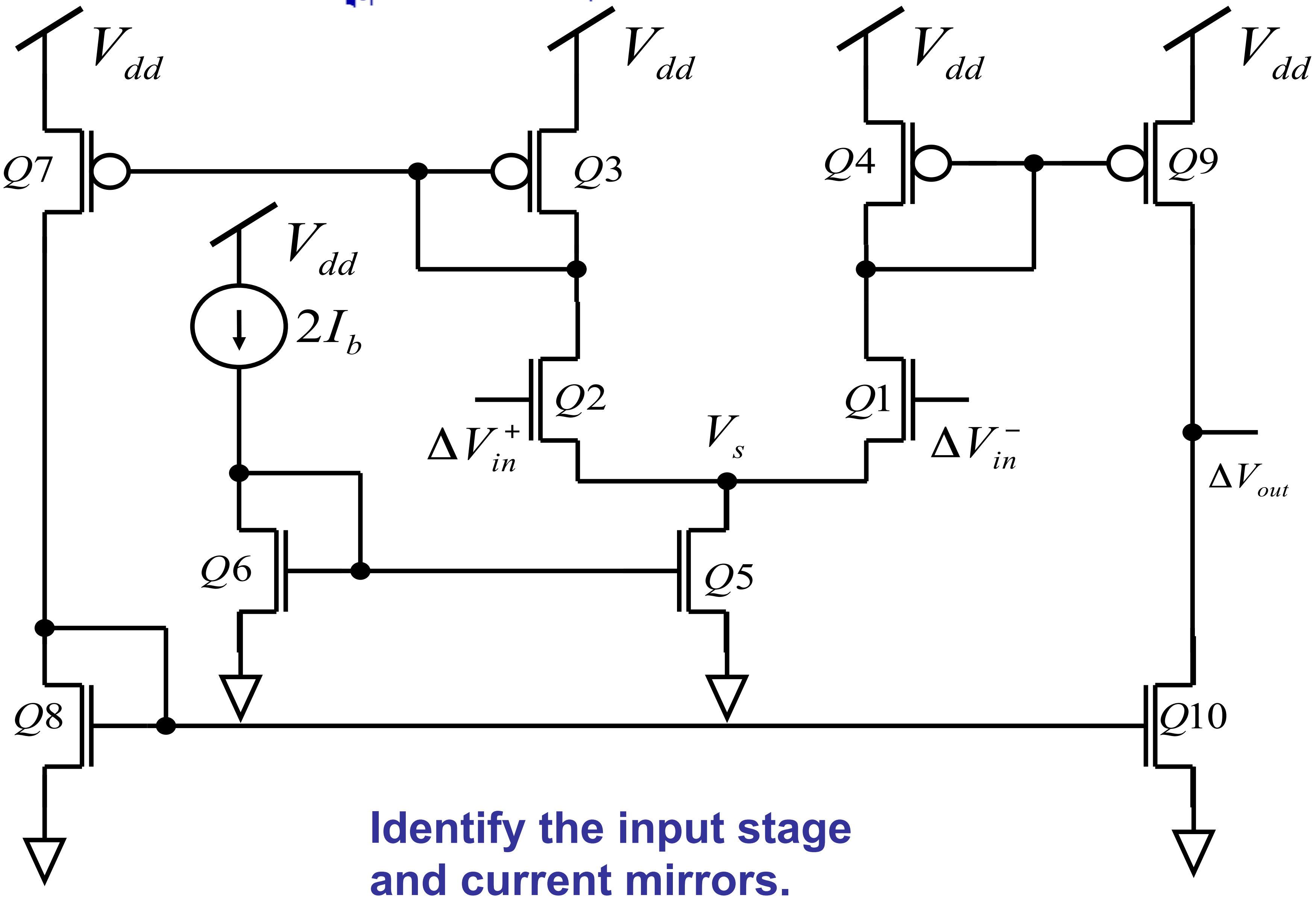
$$\boxed{V_{effn} = V_{gs} - V_{th} = \sqrt{\frac{2I_{ds}}{\mu_n C_{ox} \frac{W}{L}}}}$$

$$V_{dd} - V_{eff\,4} \geq V_{out} \geq V_{eff\,5} + V_{eff\,1}$$

$$P_d \approx 6 V_{dd} I_b \quad watts$$

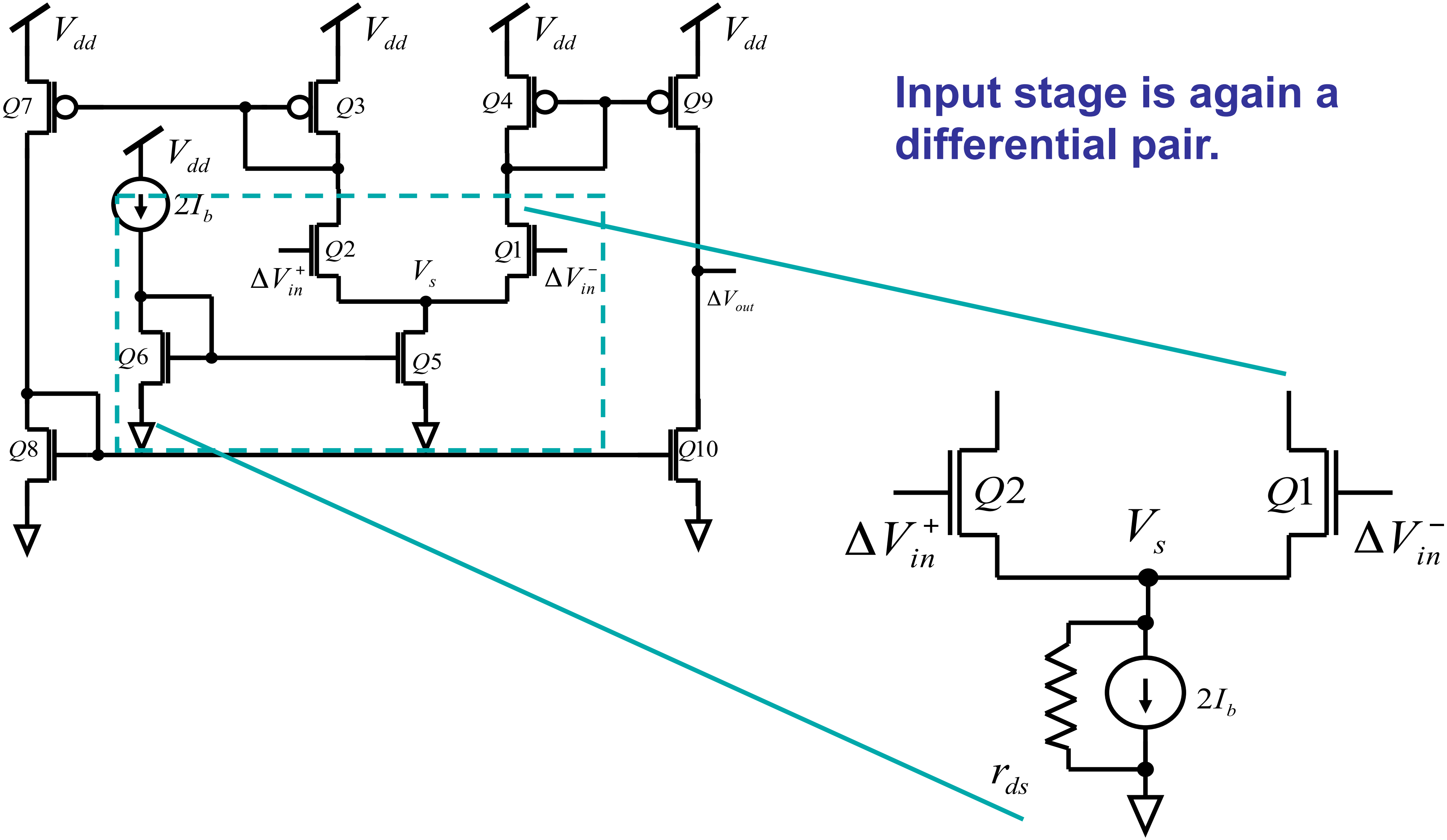
Output voltage swing is 0.8V – 2.6 V for a supply voltage of 3V

Large Output Swing Amplifier



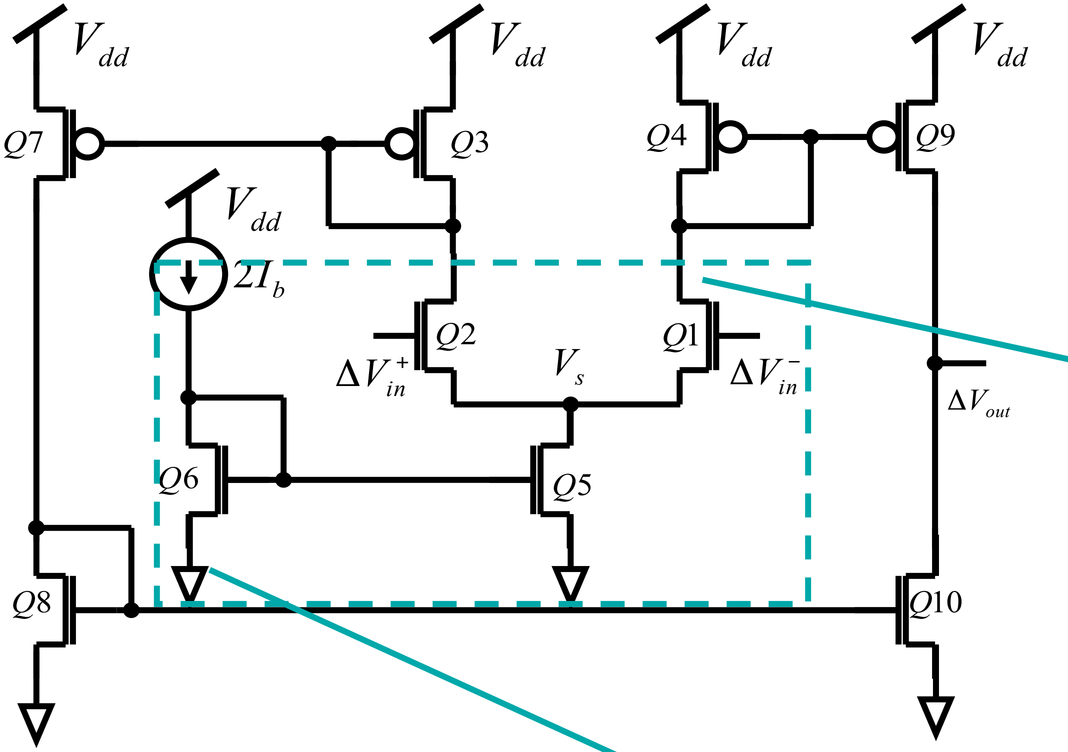
Identify the input stage and current mirrors.

Large Output Swing Amplifier



Input stage is again a differential pair.

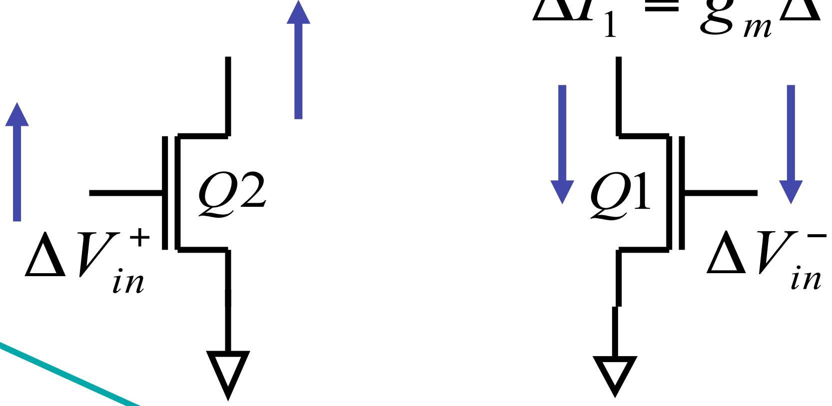
Large Output Swing Amplifier



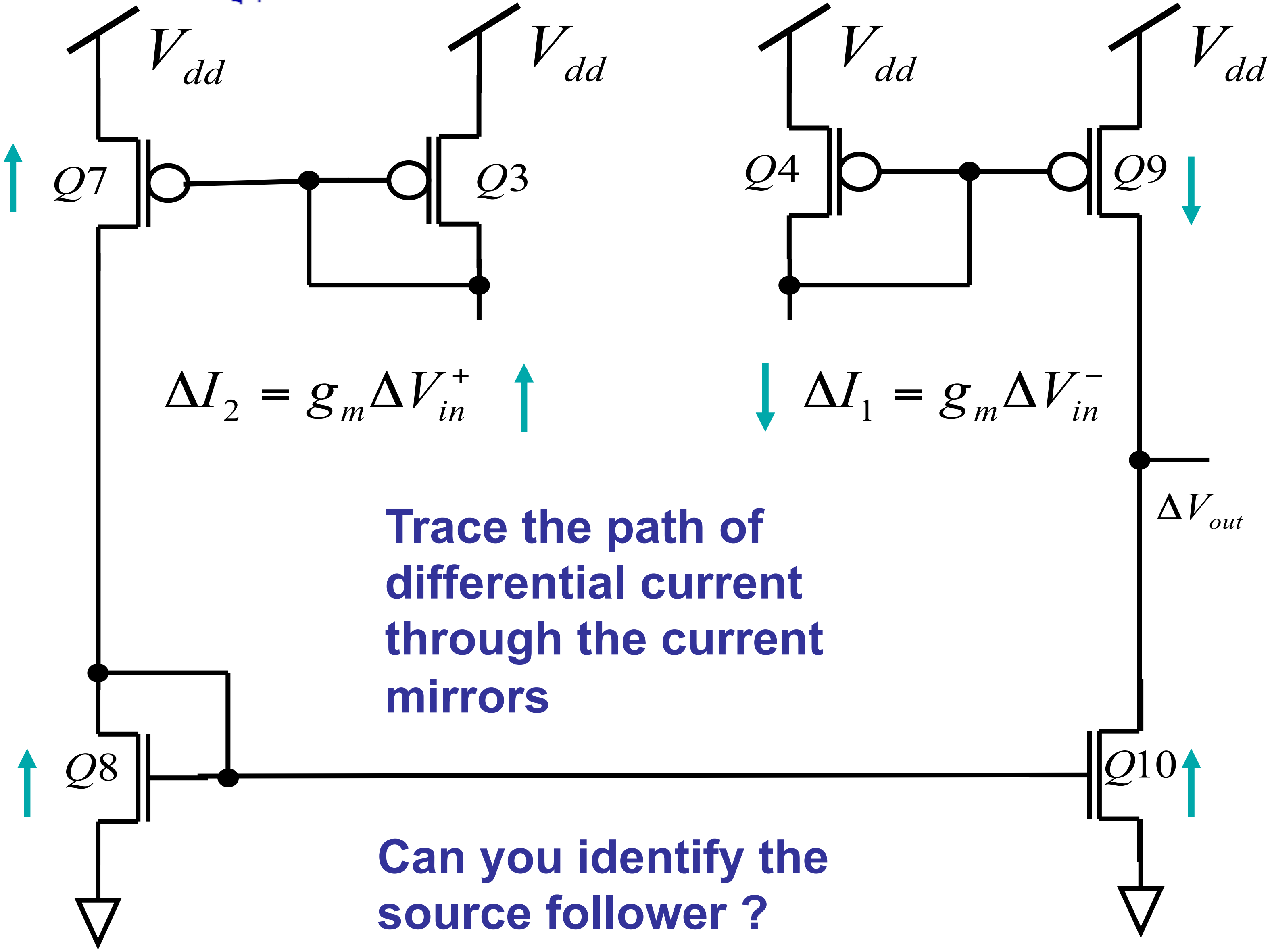
Use symmetry and note the differential output current for differential input voltage.

$$\Delta I_2 = g_m \Delta V_{in}^+$$

$$\Delta I_1 = g_m \Delta V_{in}^-$$



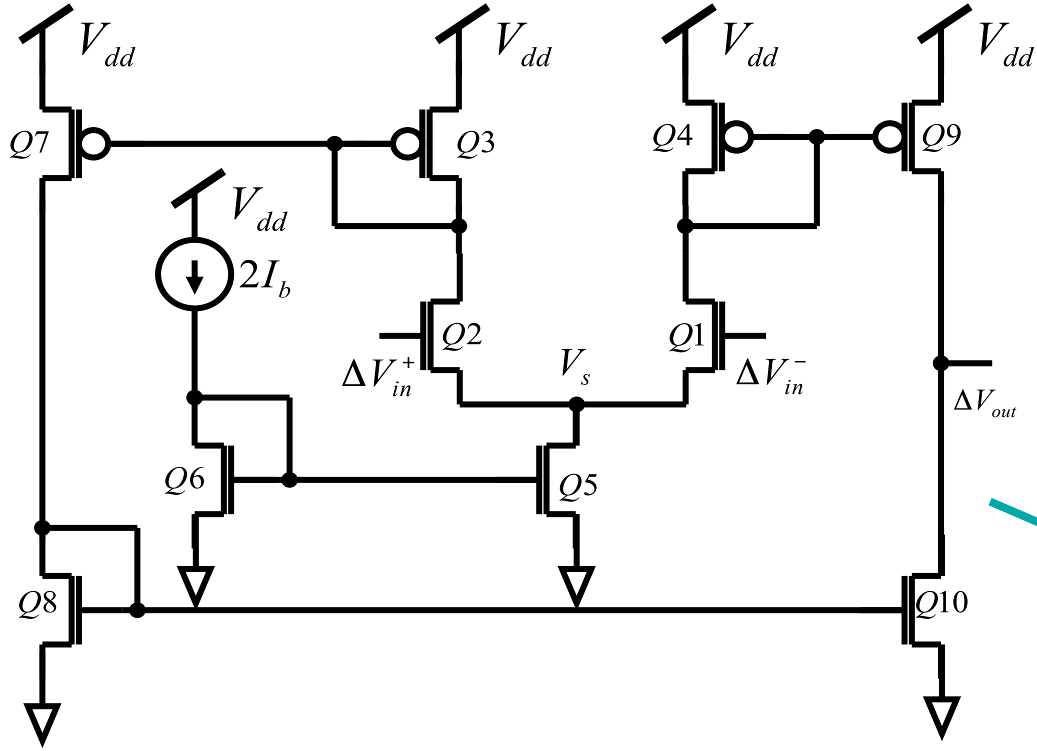
Large Output Swing Amplifier



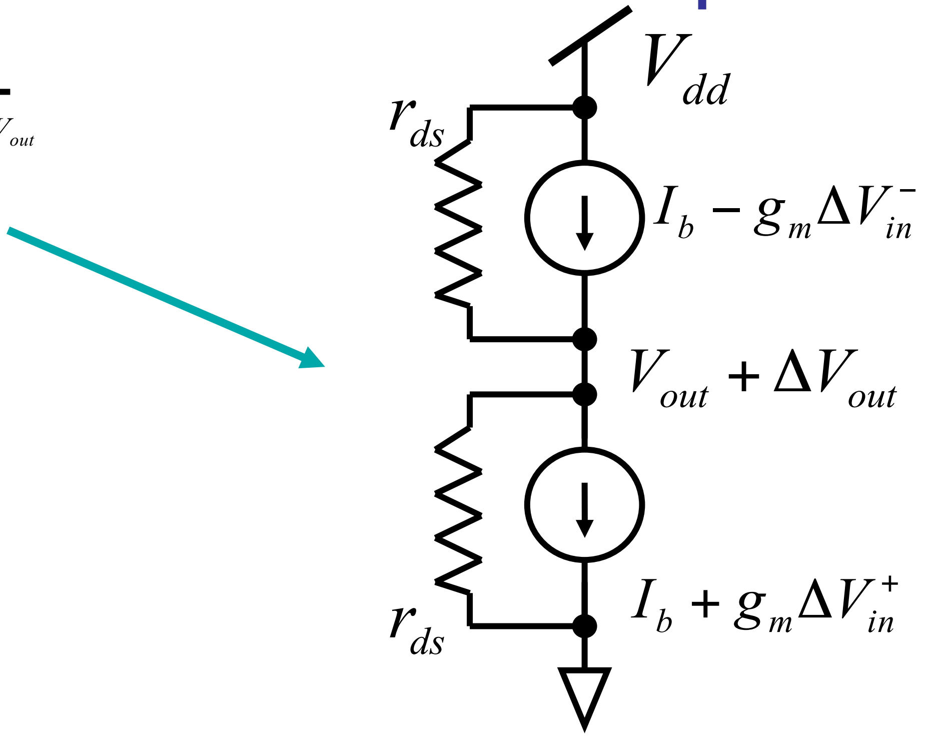
Trace the path of differential current through the current mirrors

Can you identify the source follower ?

Large Output Swing Amplifier



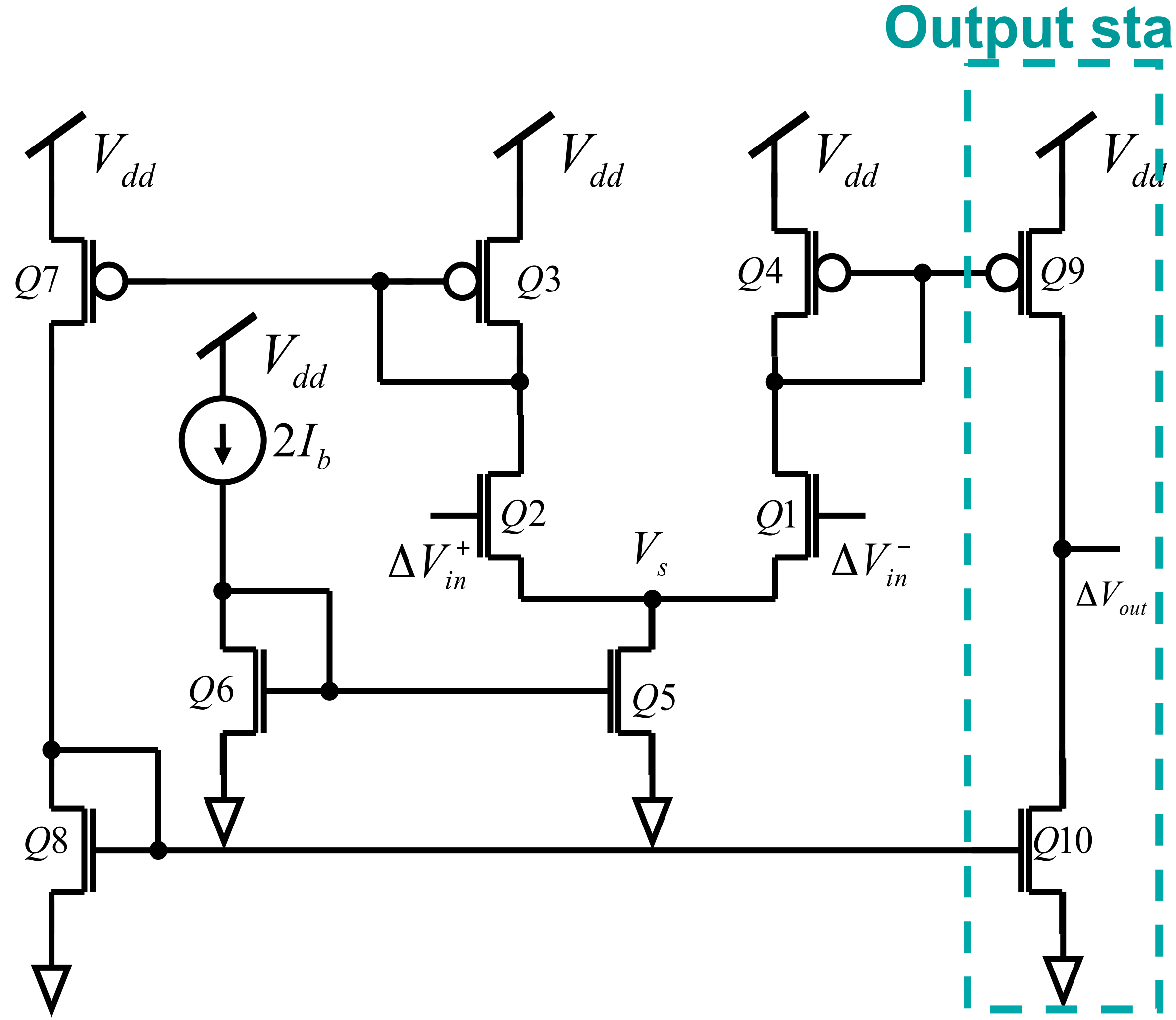
The equivalent circuit is again two current comparators with a load connected to the output.



Gain is given by

$$A_v = \frac{\Delta V_{out}}{\Delta V_{in}^{+} - \Delta V_{in}^{-}} = \tfrac{1}{2} g_m r_{ds}$$

Output Voltage Swing and Power Dissipation

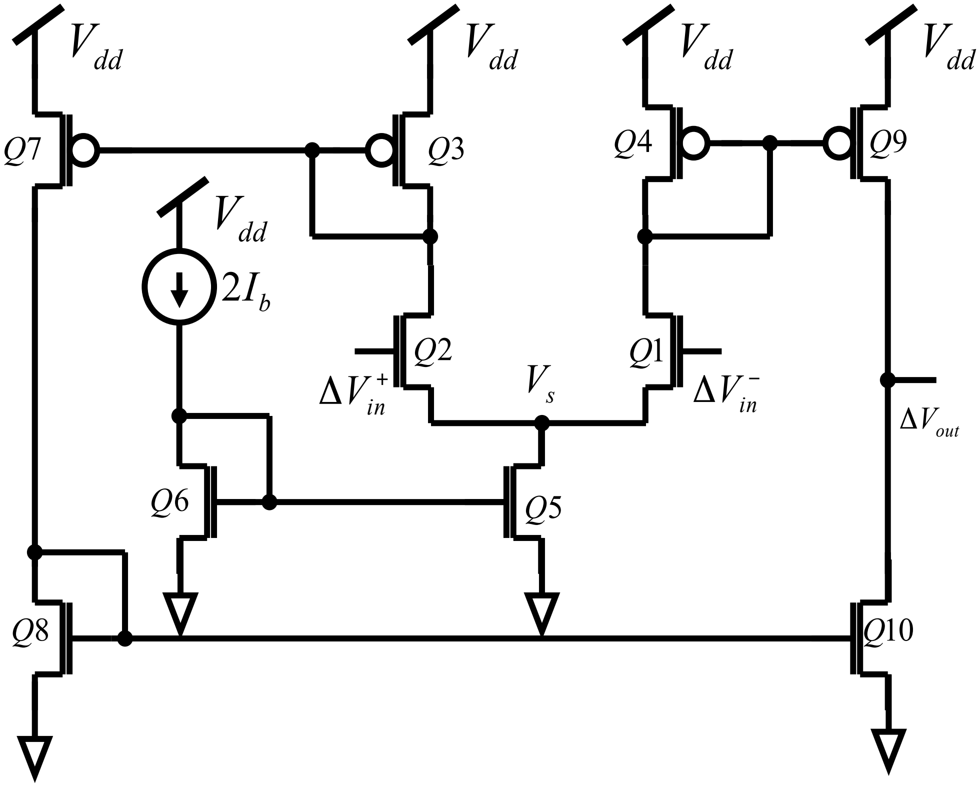


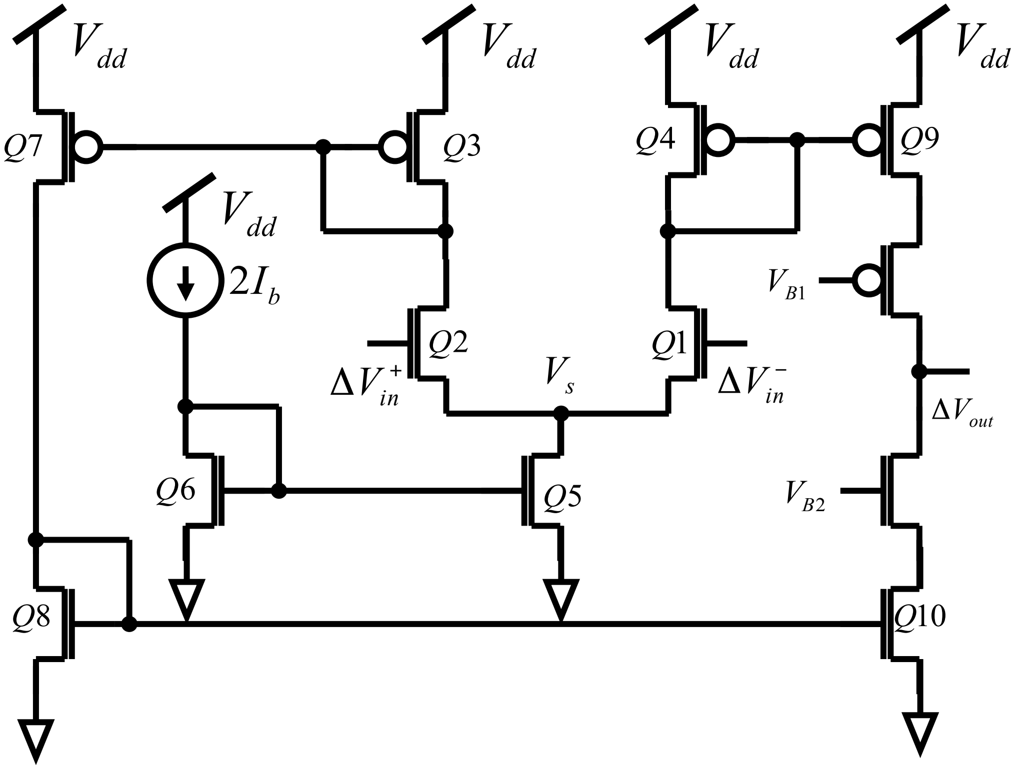
$$V_{dd} - V_{eff\,9} \geq V_{out} \geq V_{eff\,5}$$

Output voltage swing has increased by 0.4V.

Power dissipation = $6V_{dd}I_b$

Single to Multi-stage Amplifier



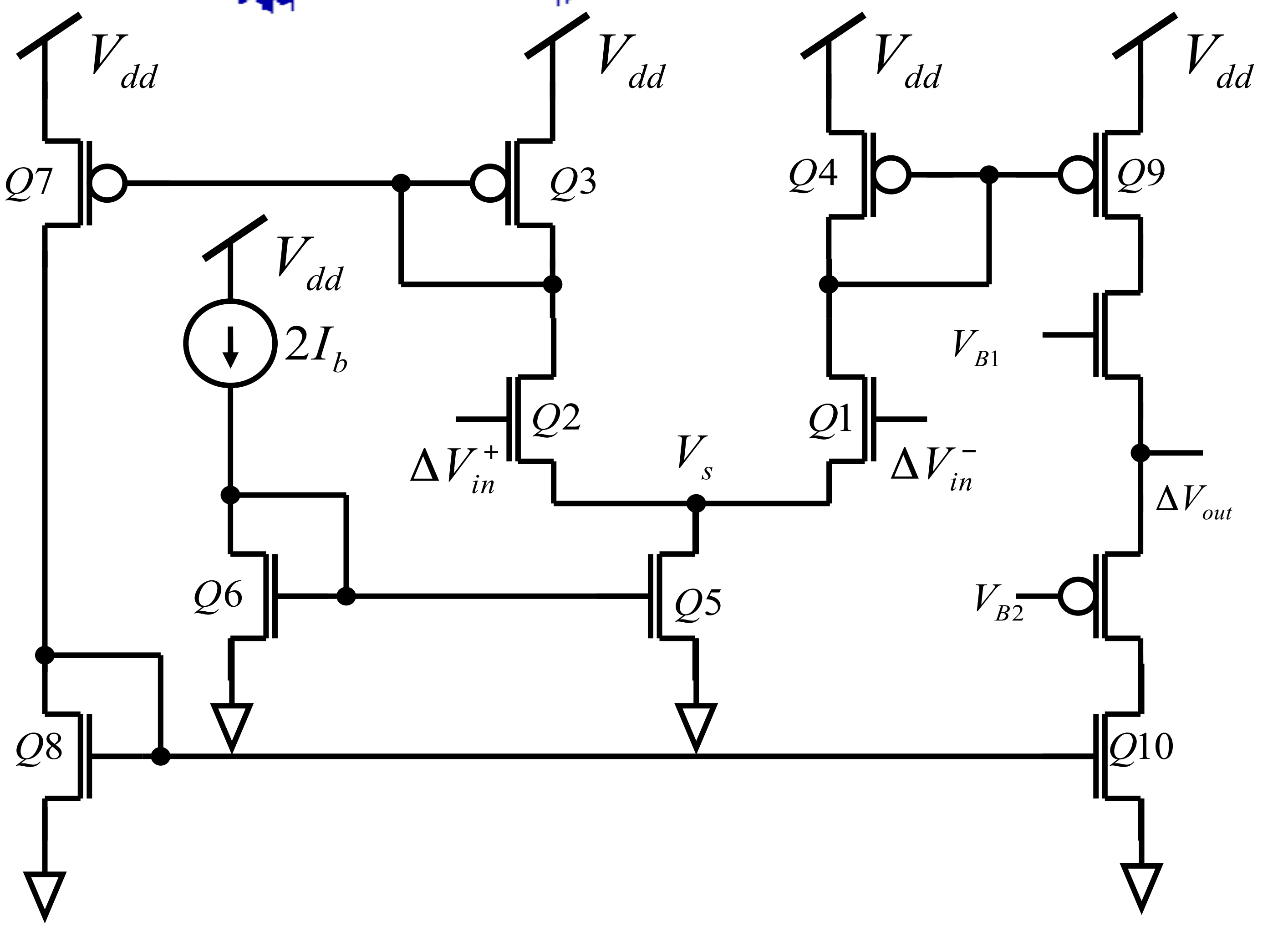


Gain

$$A_v = \frac{\Delta V_{out}}{\Delta V_{in}^{+} - \Delta V_{in}^{-}} = -g_{m1}(r_{ds1} \parallel r_{ds2})$$

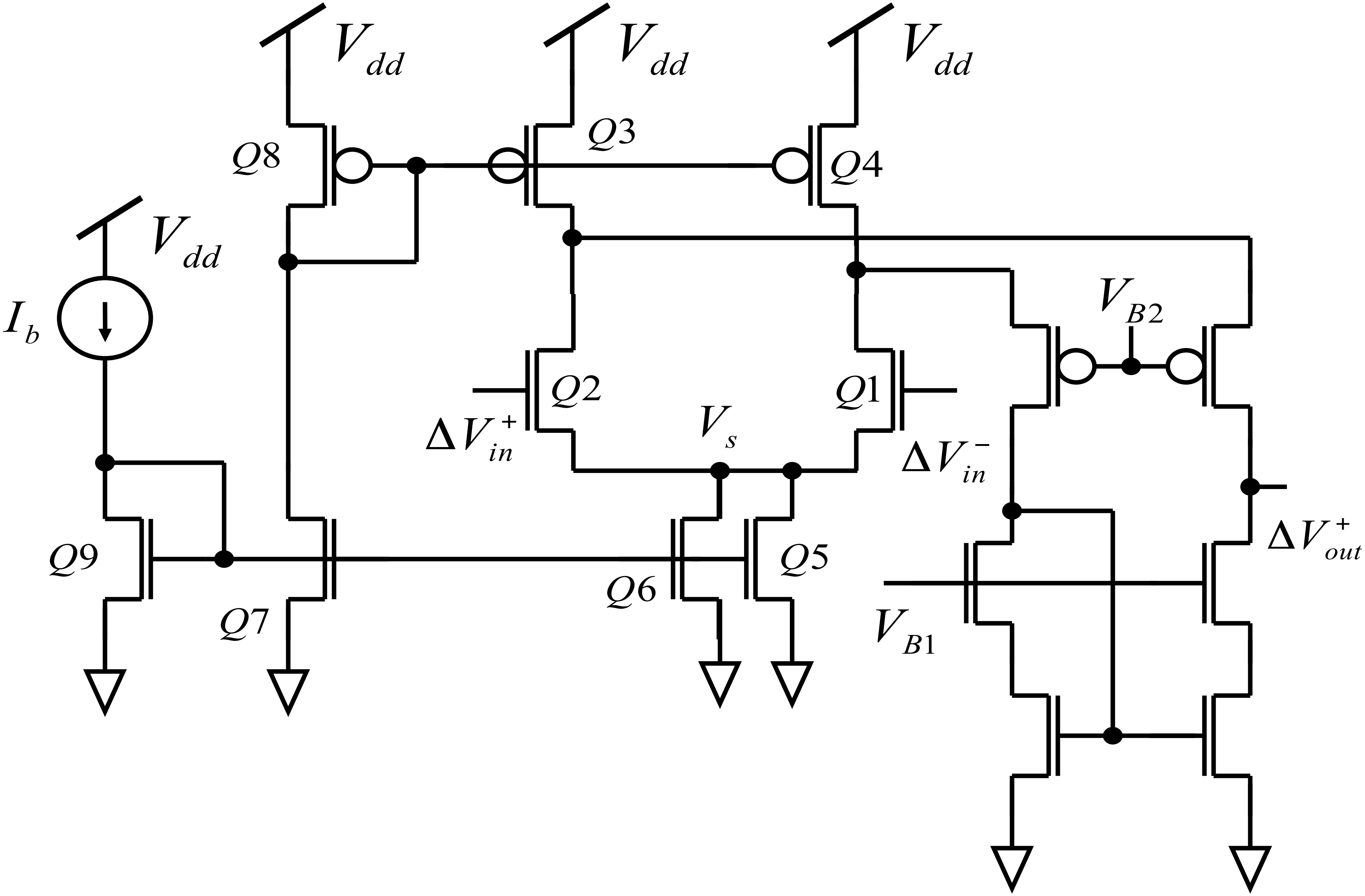
What is the gain of this circuit ?

Design Twist

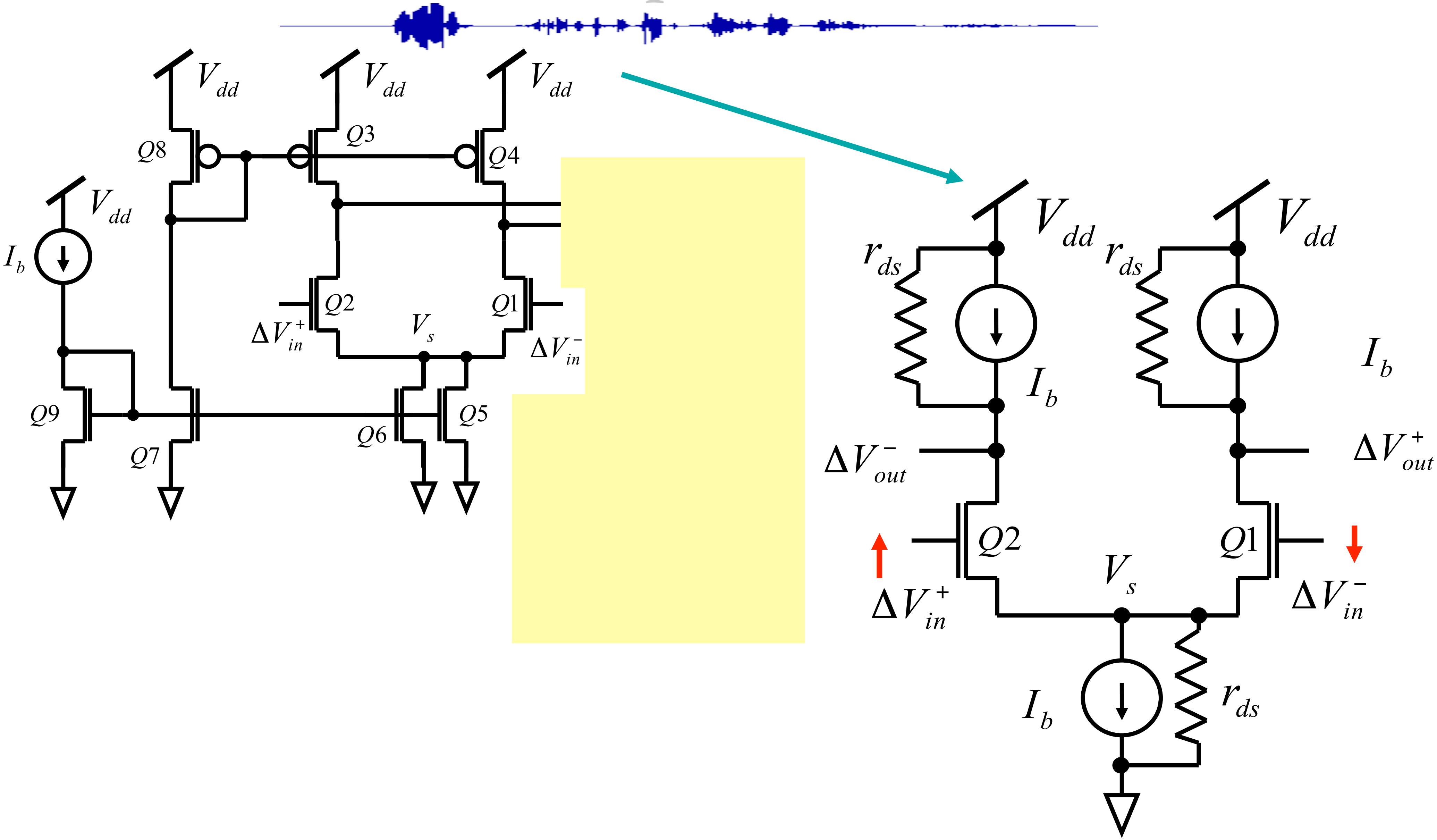


What is the gain of this circuit ?

Folded Cascoded Amplifier

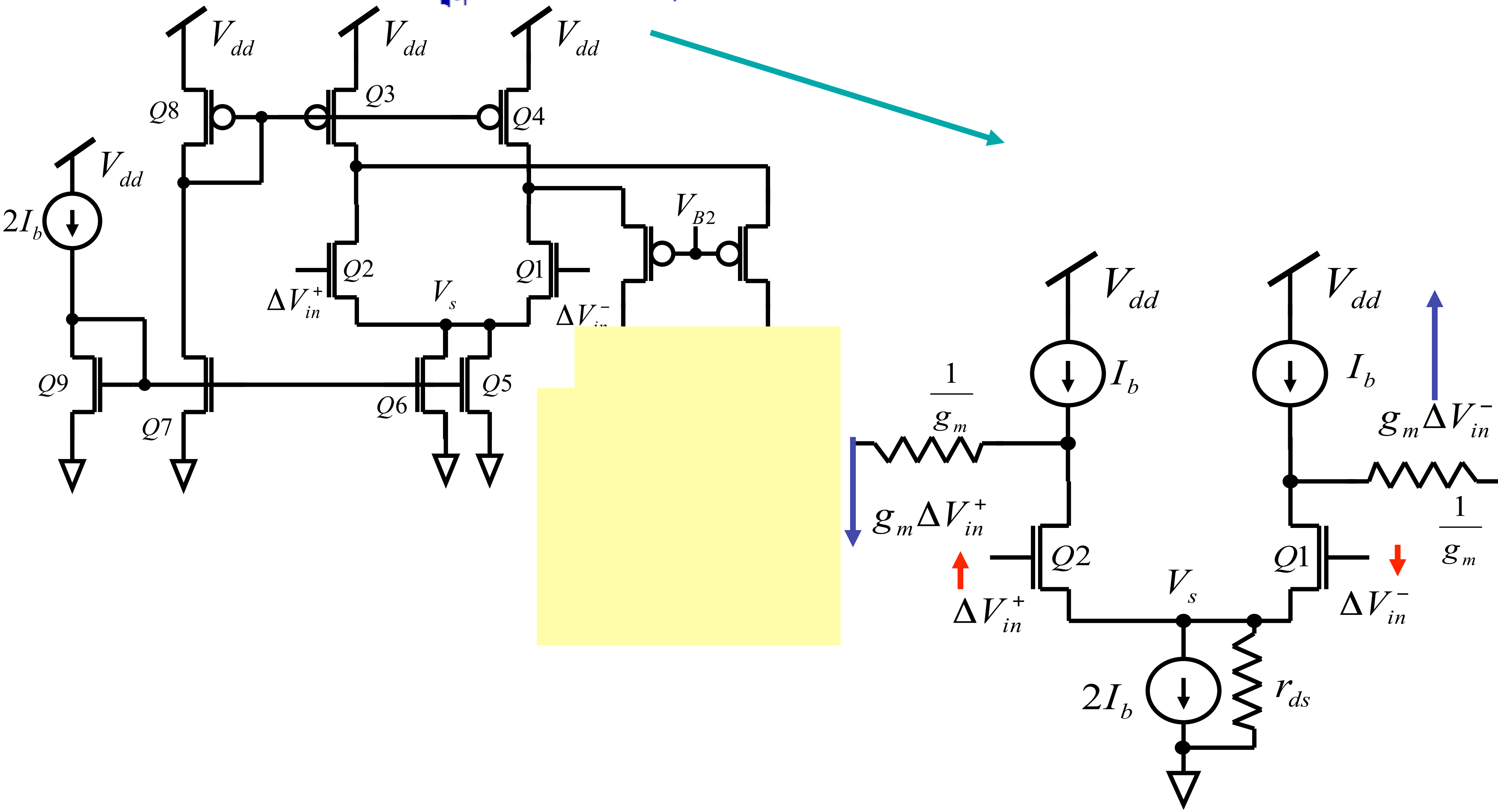


Folded Cascoded Amplifier

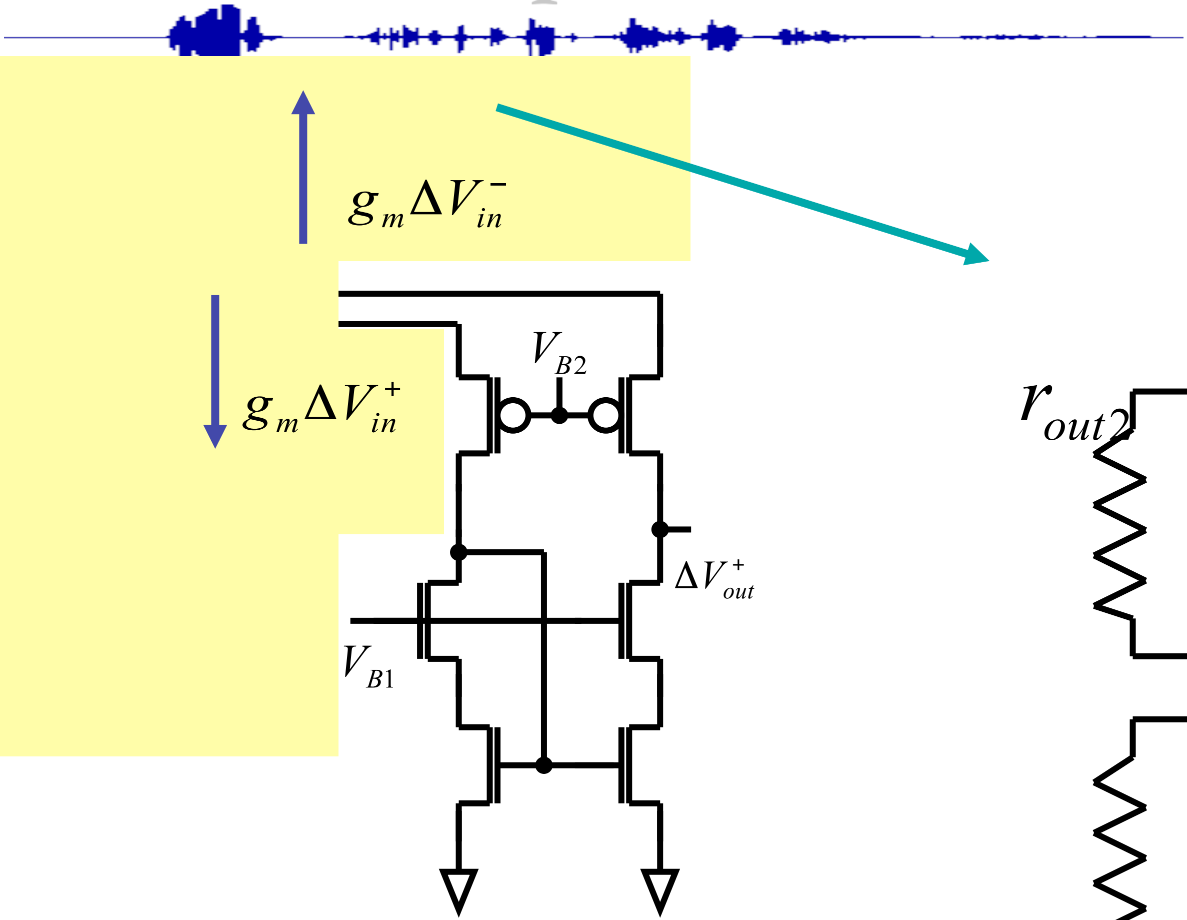


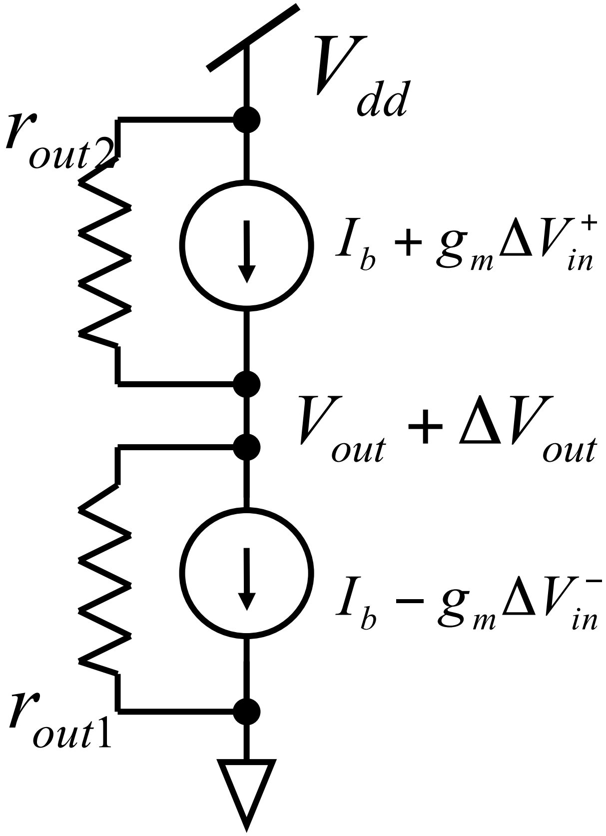
Worksheet

Folded Cascoded Amplifier



Folded Cascoded Amplifier



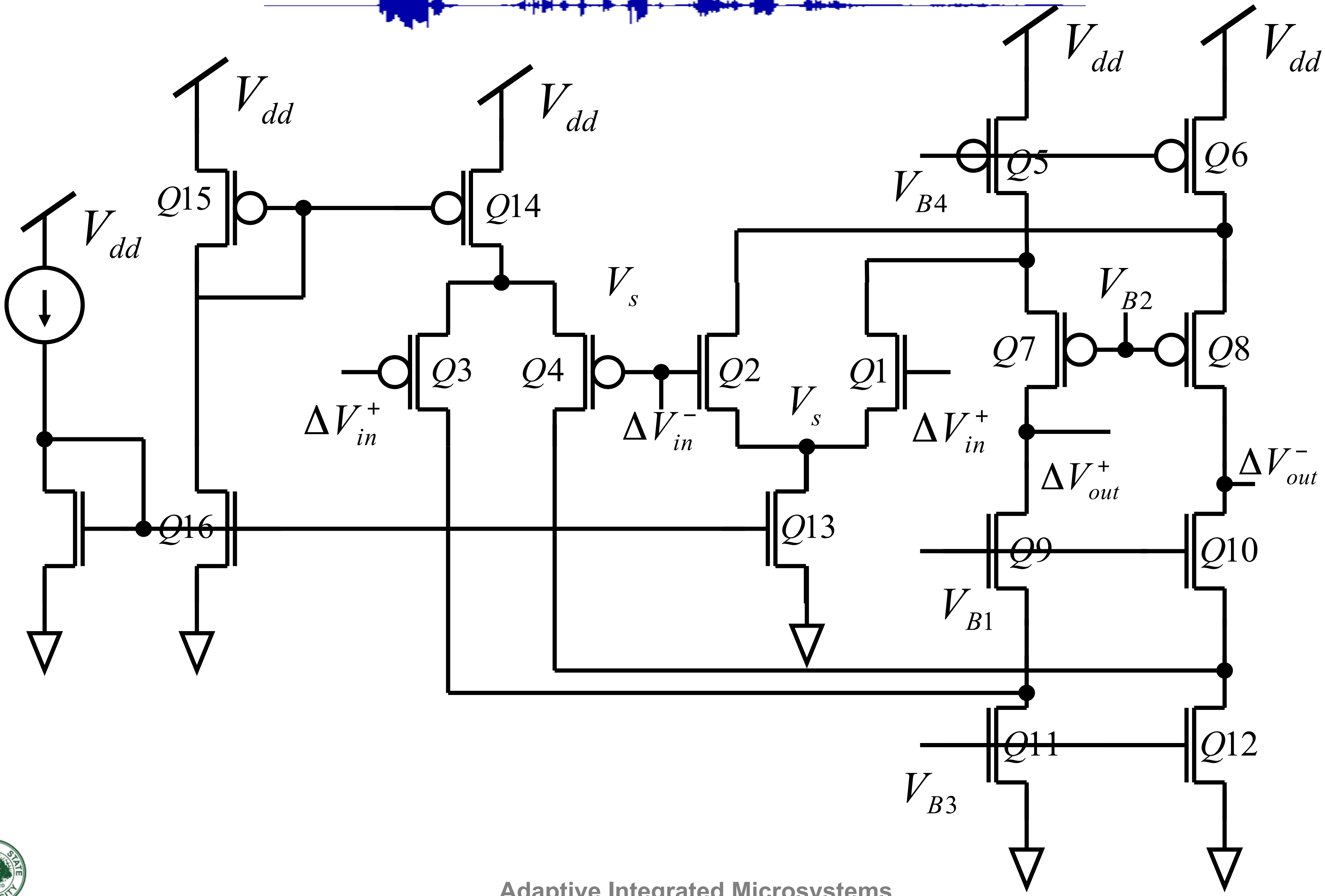


$$r_{out1} \approx (g_m r_{ds}) r_{ds}$$

$$r_{out2} \approx (g_m \frac{r_{ds}}{2}) r_{ds}$$

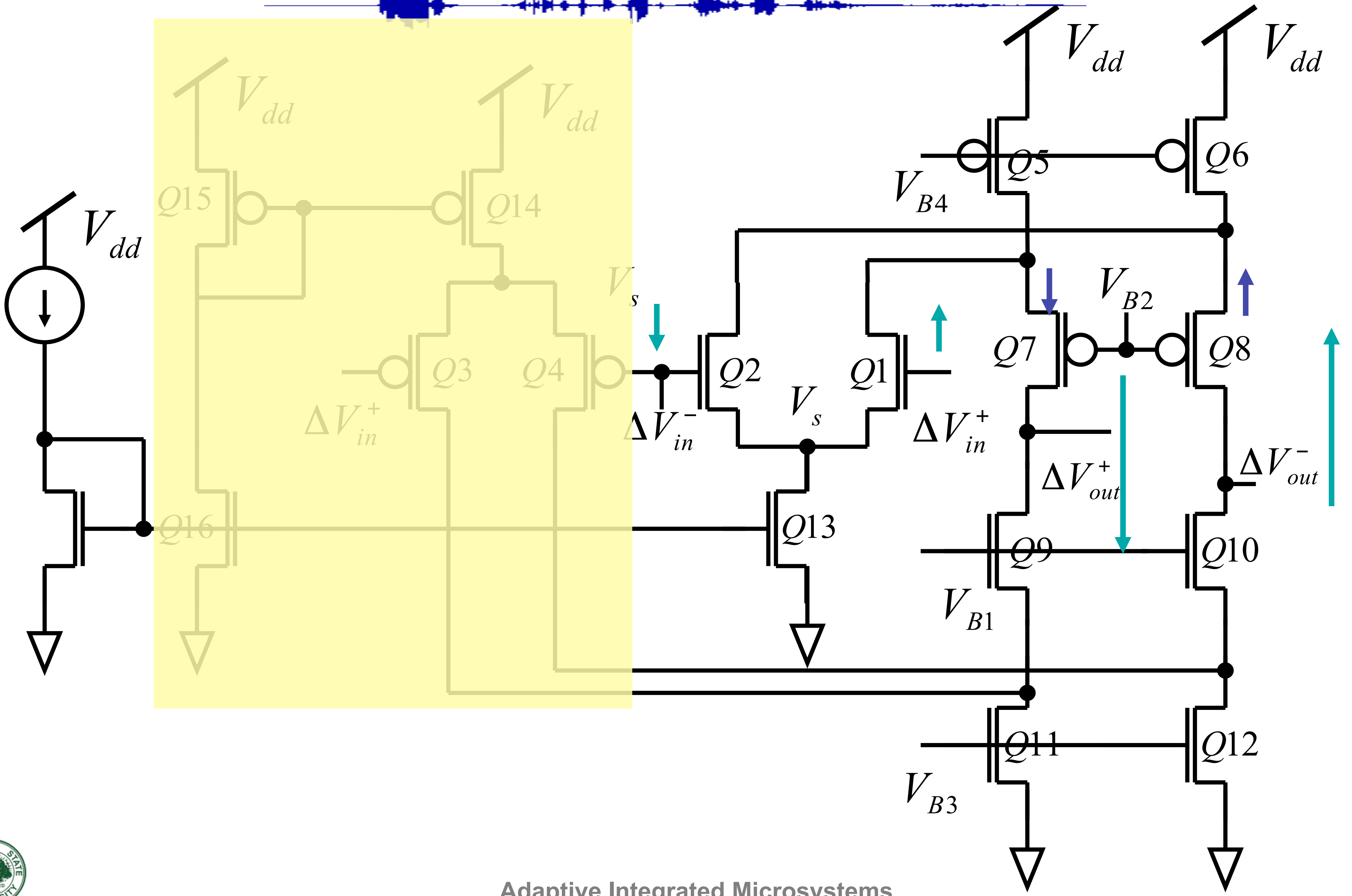
$$A_v = g_m (r_{out1} \parallel r_{out2})$$

Improving the Input Swing of the Amplifier

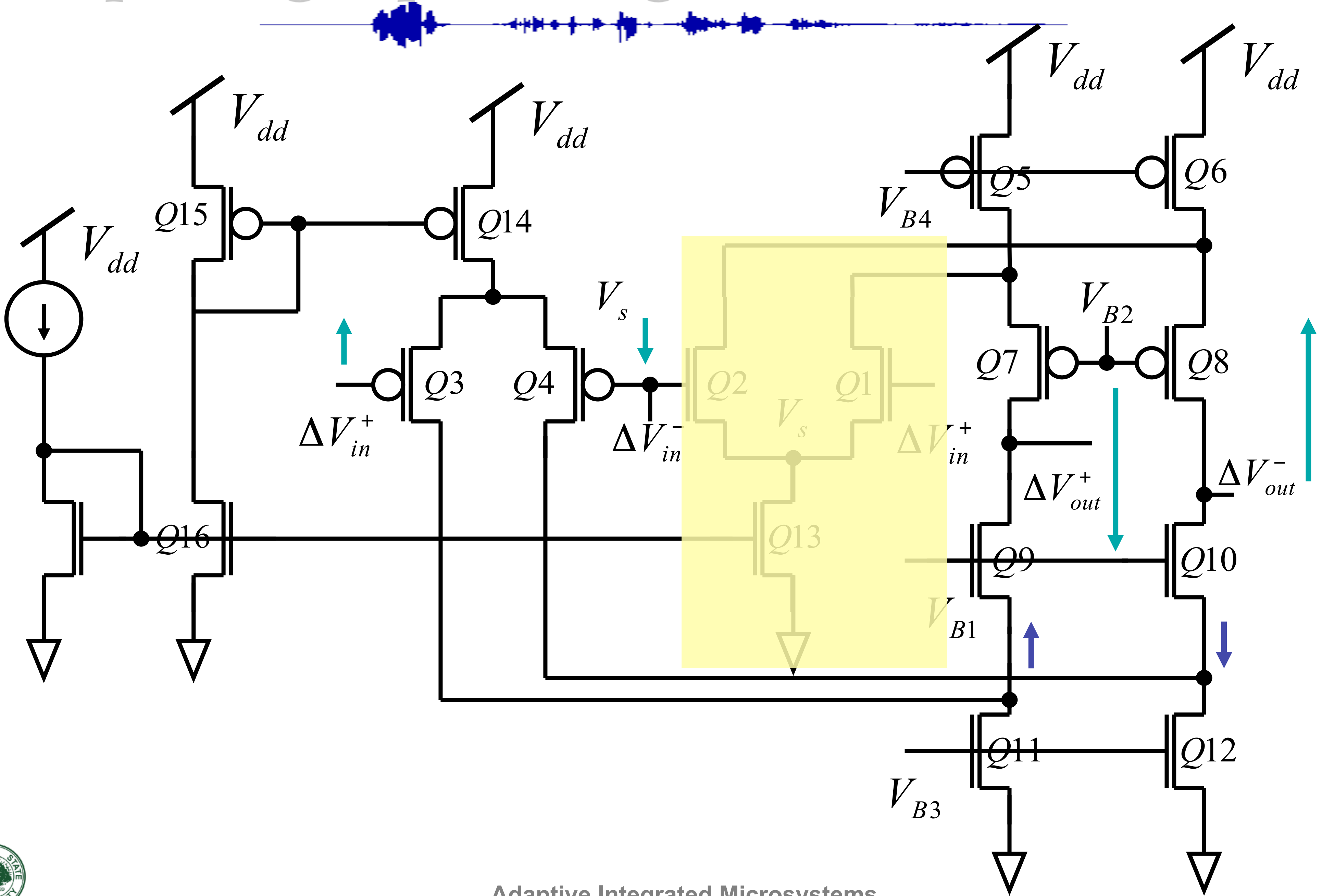


Worksheet

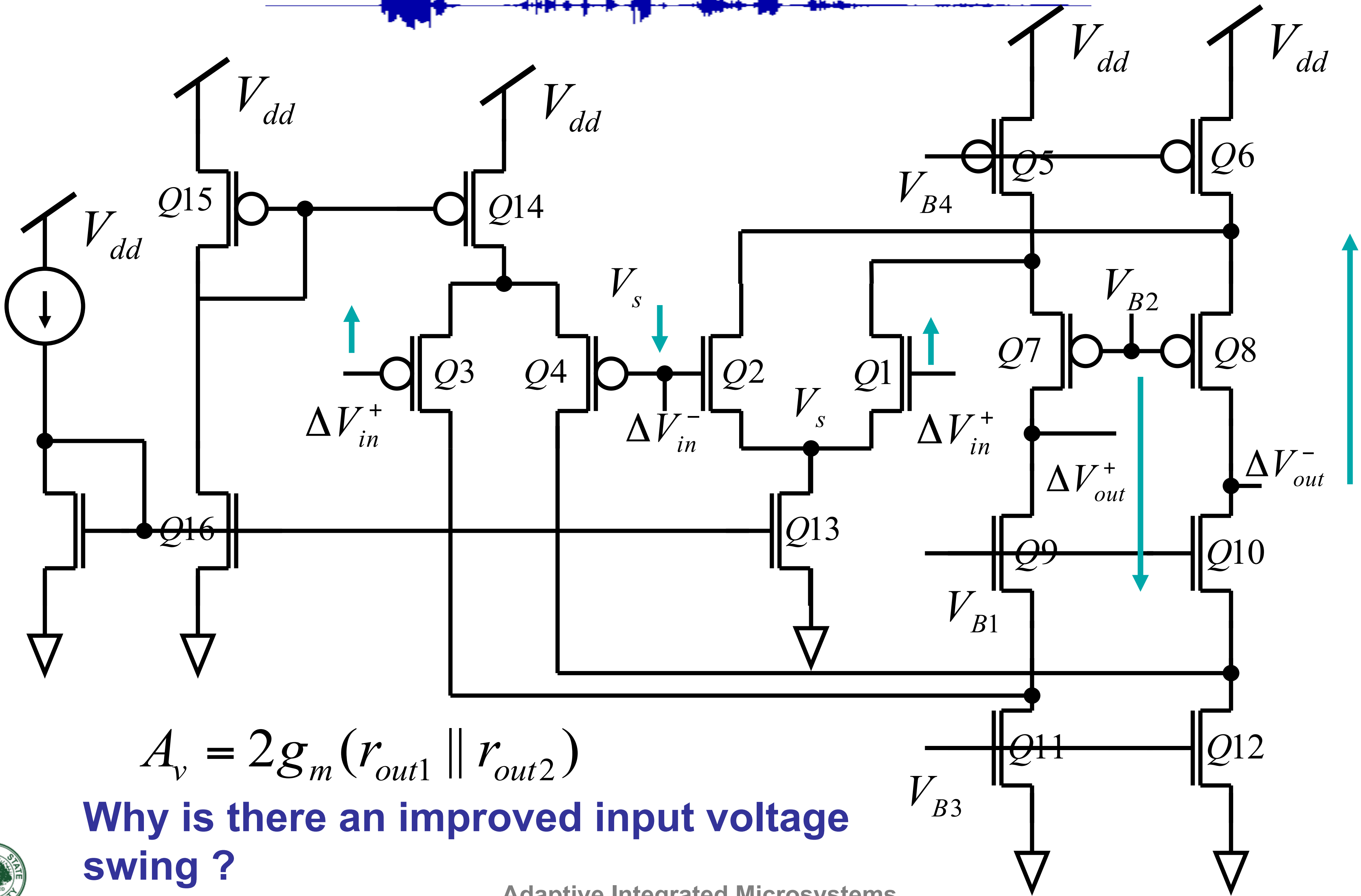
Improving the input swing



Improving Input Swing



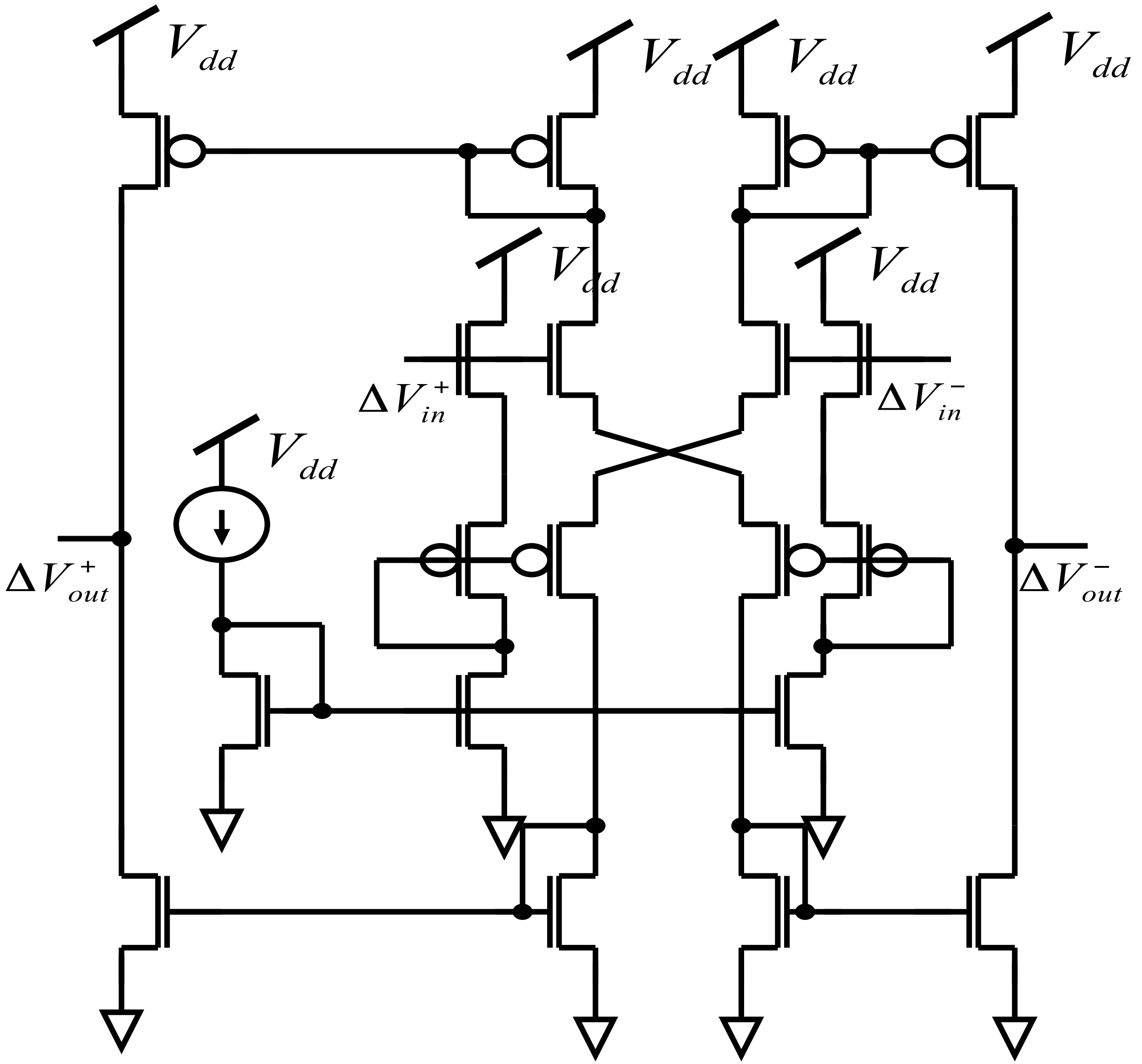
Improving Input Swing



$$A_v = 2g_m(r_{out1} \parallel r_{out2})$$

Why is there an improved input voltage swing ?

Calculate the gain of this circuit



Worksheet