ADONIS' ULTIMATE TEST REVISION PACKAGE FOR SUCCESS THROUGH SUFFERING, PAIN, HARD TIMES, DEPRESSION, LACK OF PRODUCTIVITY, SADNESS, TRIALS AND TRIBULATIONS (STA112)

Welcome to the Probability Party!

Your Adventure Starts Here!

Imagine you're at a superhero party where you get to predict fun things—like whether your favorite hero will win a battle or if it'll rain during your picnic! Probability is all about figuring out how likely something is to happen. It's like a magical tool that uses simple math and cool pictures to help us guess outcomes. This guide is designed to be short, exciting, and easy to follow, even if you're new to this. So, grab your cape, and let's jump into this adventure together!

What Is Probability, Really?

The Big Idea Made Simple!

Probability is a way to measure how likely something is to happen. Think of it like a score between 0 and 1: - A probability of **0** means it's impossible (like a cat turning into a dinosaur —no way that's happening!). - A probability of **1** means it's certain (like the sun rising tomorrow morning —it always does!). - Anything in between, like 0.5, means it's a 50-50 chance, like flipping a coin and getting heads.

Let's break it down with an example: If you flip a fair coin (one that's not rigged), there are two possible outcomes—heads or tails. Since each has an equal shot, the chance of getting heads is 1 out of 2, or $\frac{1}{2}$, which is 50

The Magic Formula

Your Probability Calculator!

Now, let's learn a super-easy formula to calculate probability. It's like a recipe for guessing chances! Here it is:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

- **What does this mean?** - $P(A)$ is the probability of some event A happening (like getting heads). - "Favorable outcomes" are the results you want (e.g., heads). - "Total number of outcomes" is all the possible things that could happen (e.g., heads or tails).

Let's try it with a fun example: Imagine you have a bag with 52 playing cards (a standard deck), and 26 of them are red (hearts and diamonds). What's the probability of picking a red card? - Favorable outcomes = 26 (the red cards). - Total outcomes = 52 (all the cards). - So, $P(\text{red card}) = \frac{26}{52} = \frac{1}{2}$ or 50

This formula works for anything with clear options, like rolling a die or picking marbles. Practice it, and you'll feel like a math wizard!

Meet the Sample Space!

The World of Possibilities!

The **sample space** is like a big menu listing every single thing that could happen in an experiment. An experiment is just something you do to see a result, like flipping a coin or rolling a die. The sample space includes all the possible outcomes.
- **Example 1:** If you roll a six-sided die, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$ because those are all the numbers you could get. - **Example 2:** If you flip a coin, the sample space is $S = \{\text{Heads, Tails}\}$.
Now, let's talk about different types of events within this space: - **Simple Event:** Just one outcome, like rolling a 5. - **Compound Event:** More than one outcome, like rolling an even number (2, 4, or 6). - **Mutually Exclusive:** Two outcomes that can't happen together, like rolling a 2 and a 5 at the same time (you can only get one number!). - **Exhaustive:** Covers all possible outcomes, like rolling any number from 1 to 6.
Think of the sample space as your playground—everything you can do is listed there!

Types of Probability

Three Flavors of Probability!

Probability comes in different "flavors," like ice cream! Here are the three main types, explained super simply: 1. **Classical Probability:** This is when everything has an equal chance, and you can figure it out with logic. For example, rolling a die has 6 equal outcomes, so the chance of rolling a 3 is $\frac{1}{6}$ 2. **Empirical Probability:** This comes from doing experiments and counting what happens. If you flip a coin 100 times and get heads 45 times, the empirical probability of heads is $\frac{45}{100} = 0.45$ or 453. **Subjective Probability:** This is your best guess based on feelings or experience. For example, you might say there's a "70
Each type is useful depending on what you're trying to figure out!

The Rules of the Game

Probability Axioms!

These are the golden rules that probability always follows, like the laws of a superhero world: 1. **Non-Negativity:** The probability of anything can't be negative. So, $0 \leq P(A) \leq 1$—it's always between 0 and 1. 2. **Total Probability = 1:** If you add up the probabilities of all possible outcomes in the sample space, it must equal 1. For a coin flip, $P(\text{Heads}) + P(\text{Tails}) = \frac{1}{2} + \frac{1}{2} = 1$. 3. **For Mutually Exclusive Events:** If two events can't happen together (like getting heads and tails on one coin flip), the probability of one or the other is just their probabilities added together: $P(A \cup B) = P(A) + P(B)$.
These rules keep probability fair and consistent!

Set Notation Cheat Sheet

Your Set Superpowers!

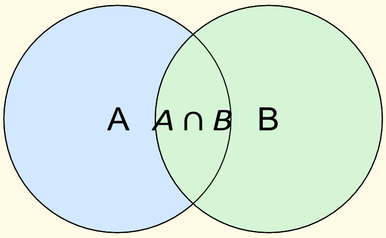
Sets are like teams that help organize events. Here's what the symbols mean: - $A \cup B$: The "union" means either A or B or both happen (like picking a red or black card). - $A \cap B$: The "intersection" means both A and B happen (like picking a red ace). - A^c: The "complement" is everything that's not A (like not getting heads). - $\emptyset$: The "null set" is impossible (like rolling a 7 on a six-sided die). - S: The "universal set" is the whole sample space (all possible outcomes).
These symbols are like shortcuts to talk about probabilities!

Venn Diagrams = Visual Magic!

Picture It!

Venn diagrams are like colorful maps that show how events overlap. Imagine two circles: - One circle is event A (e.g., rolling an even number). - The other is event B (e.g., rolling a number less than 4). - Where they overlap is $A \cap B$ (e.g., 2), and the rest shows $A \cup B$ (e.g., 2, 4, 6, or 1, 2, 3).

Draw it like this:



It's a fun way to see how events connect!

Handy Theorems

Your Probability Toolkit!

These are cool tricks to solve problems easily: - **Complement Rule:** If you know the probability of something happening ($P(A)$), the chance of it not happening is $P(A^c) = 1 - P(A)$. Example: If $P(\text{Heads}) = 0.5$, then $P(\text{not Heads}) = 1 - 0.5 = 0.5$. - **Odds in Favor:** This is the ratio of happening to not happening, $\frac{P(A)}{P(A^c)}$. If $P(\text{Heads}) = 0.5$, odds are $\frac{0.5}{0.5} = 1$ (even odds!). - **Odds Against:** The opposite, $\frac{P(A^c)}{P(A)}$, so $\frac{0.5}{0.5} = 1$.

These help when you need to compare chances!

Conditional Probability

When Events Depend on Each Other!

This is about what happens when one event affects another. The formula is:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- **What's this?** It's the probability of B happening given that A already happened. Think of it like: "What's the chance of rain if it's cloudy?" - **Example:** You have a deck of 52 cards. You pick one, and it's black (26 black cards left). What's the chance the next card is red? Initially, $P(\text{red}) = \frac{26}{52}$, but after taking a black card, only 51 cards remain with 26 red, so $P(\text{2nd red }|\text{1st black}) = \frac{26}{51} \approx 0.51$.

It's like updating your guess based on new info!

Addition Rules

Adding Probabilities!

These rules help combine events: - **General Addition Rule:** For any two events, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. This avoids counting the overlap twice. Example: Probability of rolling a 2 or an even number on a die ($P(2) = \frac{1}{6}$, $P(\text{even}) = \frac{3}{6}$, overlap = $\frac{1}{6}$, so $\frac{1}{6} + \frac{3}{6} - \frac{1}{6} = \frac{3}{6}$). - **Mutually Exclusive:** If events can't happen together, just add them: $P(A \cup B) = P(A) + P(B)$. Example: Rolling a 1 or a 2 ($\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$).

Easy peasy!

Multiplication Rules

Multiplying Probabilities!

This is for events happening together: - **Independent Events:** If one doesn't affect the other, $P(A \cap B) = P(A) \cdot P(B)$. Example: Flipping heads and rolling a 3 ($\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$). - **Dependent Events:** If one affects the other, $P(A \cap B) = P(A) \cdot P(B|A)$. Example: Drawing two red cards ($P(\text{1st red}) = \frac{26}{52}$, $P(\text{2nd red}|\text{1st red}) = \frac{25}{51}$, so $\frac{26}{52} \cdot \frac{25}{51}$).

It's like chaining probabilities!

Independent vs. Dependent Events

Free or Tied?

- **Independent:** One event doesn't change the other. Example: Flipping a coin and rolling a die—each happens on its own. - **Dependent:** One changes the next. Example: Drawing cards without replacing them—the first pick affects the second.

Know the difference to use the right rule!

Tree Diagrams

Map Your Probabilities!

Tree diagrams are like adventure maps with branches. For flipping a coin twice: - First flip: Heads or Tails (2 branches). - Second flip: Heads or Tails from each (4 total: HH, HT, TH, TT). - Multiply probabilities along paths (e.g., $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ for HH).

It's a fun way to see all outcomes!

Quick Recap Table (Chapter 1)

Your Probability Cheat Sheet!

Concept	Key Formula	Note
Classical Probability	$P(A) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$	Equal chances
Conditional Probability	$P(B\|A) = \frac{P(A \cap B)}{P(A)}$	Depends on A
General Addition Rule	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	Includes overlap
Multiplication Rule	$P(A \cap B) = P(A) \cdot P(B)$	Independent only

Welcome to Chapter 2!

Dive into Distributions!

Chapter 2 is your next quest into probability distributions! We'll learn about different types of variables and fun distributions like binomial, Poisson, and normal. Get ready to level up!

2.1 Types of Variables

Meet the Variables!

A **variable** is like a placeholder for something that can change, like your score in a game. A **random variable (RV)** depends on random outcomes, like rolling a die. - **Discrete Random Variable:** Counts things you can list, like 0, 1, 2... Example: Number of heads in 3 coin flips. - **Continuous Random Variable:** Measures things over a range, like height or temperature.
Think of discrete as steps and continuous as a smooth slide!

2.2 Probability Distribution Function (PDF)

The Probability Map!

This shows how probabilities spread across a random variable's values. - **Discrete RV (PMF - Probability Mass Function):** $P(X = x)$ tells the chance of each exact value. It must add up to 1 (e.g., for a die, $\frac{1}{6} + \frac{1}{6} + \ldots = 1$). - **Continuous RV (PDF - Probability Density Function):** $f(x)$ describes a curve where the area under it is 1 (e.g., heights).
It's like a map showing where the "probability treasure" is!

2.3 Mean and Variance (Discrete RV)

Measure the Spread!

- **Mean (Expected Value):** The average you'd expect, $E(X) = \sum x \cdot P(X = x)$. For a die, $1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \ldots = 3.5$. - **Variance:** How spread out it is, $Var(X) = E(X^2) - [E(X)]^2$ or $\sum (x - E(X))^2 \cdot P(X = x)$. For a die, it's about 2.92. - **Standard Deviation:** Square root of variance, $\sqrt{Var(X)}$, about 1.71 for a die.

It's like finding the center and width of your data cloud!

2.4 Binomial Distribution

Success or Fail!

This is for experiments with: - Fixed trials (n, like 10 flips). - Success/failure outcomes (e.g., heads or tails). - Independent trials (each flip doesn't affect the next). - Constant success probability (p, like 0.5 for a fair coin). - **Formula:** $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$, where $\binom{n}{k}$ is the number of ways to choose k successes. - **Mean:** $E(X) = np$, **Variance:** $Var(X) = np(1 - p)$.

Example: 3 flips, $p = 0.5$, chance of 2 heads: $\binom{3}{2} \cdot 0.5^2 \cdot 0.5^1 = 3 \cdot 0.25 \cdot 0.5 = 0.375$.

2.5 Multinomial Distribution

More Than Two Outcomes!

This is like binomial but for more categories (e.g., red, blue, green marbles). It counts ways to get specific numbers of each outcome.

2.6 Poisson Distribution

Count the Events!

Models rare events in a fixed time (e.g., calls in an hour). - Used when trials are many and success rare. - **Formula:** $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$, where λ is the average rate. - **Mean and Variance:** Both λ.

Example: 2 calls per hour, chance of 3 calls: $\frac{2^3 e^{-2}}{3!} = \frac{8 \cdot 0.135}{6} \approx 0.18$.

2.7 Normal Distribution

The Bell Curve!

A bell-shaped curve for continuous data (e.g., heights). - **Formula:** $f(x) = \sqrt{\frac{1}{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, where μ is mean, σ is standard deviation. - **Properties:** Area = 1, mean = median = mode. - **Empirical Rule:** 68

It's like a perfect hill of data!

2.8 Sampling Distributions

Stats in Action!

This is the distribution of a stat (like mean) from many samples. It helps with error and testing.

Quick Recap Table (Chapter 2)

Your Distribution Cheat Sheet!

Concept	Key Formula	Note
Binomial Distribution	$P(X = k) = {n \atop k} p^k(1-p)^{n-k}$	Fixed trials
Poisson Distribution	$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$	Rare events
Normal Distribution	$f(x) = \sqrt{\frac{1}{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	Bell-shaped

Welcome to Chapter 3!

Master Joint Probabilities!

Chapter 3 explores how two variables, X and Y, work together! It's like teaming up superheroes. Let's level up!

3.1 Joint Probability Mass Function (JPMF)

X and Y Team Up!

The JPMF tells the chance that X = x AND Y = y at the same time for discrete variables. - **Definition:** $f(x, y) = P(X = x \text{ and } Y = y)$. - Example: If X is dice 1 (1-6) and Y is dice 2, $f(2, 3)$ is the chance both show 2 and 3.
It's like finding the odds of two things happening together!

3.2 Properties of JPMF

The Rules!

Every JPMF must follow: - **Non-negativity:** $P(X = x, Y = y) \geq 0$ (no negative chances!). - **Total Probability = 1:** $\sum_x \sum_y P(X = x, Y = y) = 1$ (all pairs add to 100- **Defined over Support:** Only works for allowed (x, y) values.
These keep it real!

3.3 The Support of a Joint Distribution

Where It Lives!

The support is where $P(X = x, Y = y)$ is non-zero. - **Rectangular Support:** X and Y vary freely (e.g., X 1, 2, Y 1, 2, 3 6 pairs). - **Triangular Support:** Limited (e.g., $x + y \leq 4$ (0,0), (0,1), etc.).
It's the playing field for X and Y!

3.4 Marginal Distributions

Focus on One!

Get X or Y alone by summing over the other: - **Marginal of X:** $f_X(x) = \sum_y f(x, y)$. - **Marginal of Y:** $f_Y(y) = \sum_x f(x, y)$.
Example: If $f(1, 1) = 0.1, f(1, 2) = 0.2, f_X(1) = 0.1 + 0.2 = 0.3$.
It's like ignoring one teammate's score!

3.5 Conditional Distributions

Given One, What's the Other?

Chance of Y = y given X = x: - **Formula:** $P(Y = y|X = x) = \frac{f(x,y)}{f_X(x)}$ (if $f_X(x) = 0$).
Example: If $f(1, 1) = 0.1, f_X(1) = 0.3,\ P(Y = 1|X = 1) = \frac{0.1}{0.3} \approx 0.33$.
It's updating based on what you know!

3.6 Independence of Random Variables

Do They Work Alone?

X and Y are independent if $f(x, y) = f_X(x) \cdot f_Y(y)$ for all pairs. If not, they're linked.

3.7 Mean and Variance of X and Y

Measure the Means!

Use marginals: - $E(X) = \sum_x x \cdot f_X(x)$. - $Var(X) = E(X^2) - [E(X)]^2$.
Example: If $f_X(0) = 0.3, f_X(1) = 0.7, E(X) = 0 \cdot 0.3 + 1 \cdot 0.7 = 0.7$.

3.8 Covariance

How Do They Move?

Shows if X and Y move together: - **Formula:** $Cov(X, Y) = E(XY) - E(X)E(Y)$. - $E(XY) = \sum_x \sum_y x \cdot y \cdot f(x, y)$.
Example: From a table, $E(XY) = 0.4$, $E(X) = 0.7$, $E(Y) = 0.6$, $Cov = 0.4-0.42 = -0.02$ (weak negative).

3.9 Transformation of Variables

Create New Variables!

Make $Z = X + Y$ by summing pairs and their probabilities.

Quick Recap Table (Chapter 3)

Your Joint Cheat Sheet!

Concept	Key Formula	Note
JPMF	$f(x, y) = P(X = x, Y = y)$	Joint chance
Marginals	$f_X(x) = \sum_y f(x, y)$	X alone
Conditional	$P(Y = y\|X = x) = \frac{f(x,y)}{f_X(x)}$	Given X
Independence	$f(x, y) = f_X(x) \cdot f_Y(y)$	Must hold
E(X)	$\sum_x x \cdot f_X(x)$	Mean of X
Cov(X, Y)	$E(XY) - E(X)E(Y)$	Movement link
Transformation	$Z = X + Y$	New variable

Welcome to Chapter 4!

Explore Your Data!

Chapter 4 is about Exploratory Data Analysis (EDA)—like meeting your data for the first time! We'll spot patterns and get ready for stats. Let's go!

4.1 What is EDA?

Meet Your Data!

EDA is the first step in analyzing data. It's like saying hi to your data—looking at it, summarizing it, and drawing pictures before doing hard math like probabilities. The goal? Understand patterns and trends without guessing too much yet.

4.2 Types of Data

Know Your Data Types!

Data comes in two big types: - **Qualitative (Categorical):** Words or groups, not numbers (e.g., gender, color). - **Quantitative (Numerical):** Numbers you can measure (e.g., age, height). - **Discrete:** Countable (e.g., number of kids). - **Continuous:** Measurable ranges (e.g., temperature).
Know this to pick the right tools!

4.3 Techniques Used in EDA

Tools of the Trade!

Use pictures (graphical) and numbers (numerical) to explore.

4.4 Graphical Tools

See the Story!

Pictures tell the tale: - **Bar Chart:** Compare categories (e.g., males vs. females). - **Histogram:** Shape of data (e.g., test scores). - **Pie Chart:** Proportions (e.g., population). - **Box Plot:** Outliers (e.g., salaries). - **Scatter Plot:** Two-variable links (e.g., age vs. income).
Example: A histogram shows score clumps!

4.5 Numerical Tools

Crunch the Numbers!

Numbers summarize: - **Central Tendency:** Center of data. - Mean: Average ($\bar{x} = \frac{\sum x}{n}$). - Median: Middle value. - Mode: Most common. - **Dispersion:** Spread. - Range: Max - Min. - Variance: $s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$. - Standard Deviation: $s = \sqrt{s^2}$. - IQR: Q3 - Q1.

4.6 The Five Number Summary

Quick Data Snapshot!

Min, Q1, Median, Q3, Max. Used for boxplots and outliers.

4.7 Skewness

Shape It Up!

Symmetrical, positive (right tail), or negative (left tail). Use histograms to see.

4.8 Relationship Between Variables

Two-Variable Fun!

Use scatter plots and correlation ($-1 \leq r \leq 1$) to see links.

4.9 Outliers

Spot the Odd Ones!

Detect with boxplots or IQR rule ($x < Q1 - 1.5 \cdot IQR$ or $x > Q3 + 1.5 \cdot IQR$).

Quick Recap Table (Chapter 4)

Your EDA Cheat Sheet!

Concept	**Tool**	**Purpose**
Center	Mean, Median, Mode	Data's middle
Spread	Range, SD, IQR	Data's width
Shape	Histogram, Boxplot	Symmetry or skew
Outliers	Boxplot, IQR	Weird values
Relationship	Scatter plot, Correlation	Two-variable tie

Welcome to Chapter 5!

Master Counting!

Chapter 5 teaches permutations and combinations—counting ways to arrange or pick things for probability! Let's have fun!

5.1 What is Counting?

Counting Adventures!

Counting helps with arrangements (e.g., seating friends) or selections (e.g., picking winners).

5.2 Fundamental Counting Principle

Multiply the Fun!

If one choice has m options and another has n, total ways are $m \times n$. Example: 3 shirts, 4 pants = $3 \times 4 = 12$ outfits.

5.3 Permutations

Order Matters!

Arrangements where order counts: $nPr = \frac{n!}{(n-r)!}$ - Example: 3 books from 5 = 60 ways. - With repetition: $\frac{n!}{p_1! \cdot p_2! \cdot \ldots}$, e.g., BALLOON = 1260 ways.

5.4 Combinations

Order Doesn't Matter!

Selections where order doesn't count: $nCr = \frac{n!}{r! \cdot (n-r)!}$. - Example: 3 students from 5 = 10 ways.

5.5 Difference Between Permutations and Combinations

Know the Difference!

Permutations care about order; combinations don't.

5.6 Applications in Probability

Put It to Work!

Example: 6 red, 4 green balls, 2 red, 1 green probability = 0.5.

5.7 Useful Identities

Handy Tricks!

$nCr = nC(n-r)$, $nPr = nCr \cdot r!$.

5.8 Tips for Exams

Ace It!

Use permutations for order, combinations for selection.

Quick Recap Table (Chapter 5)

Your Counting Cheat Sheet!

Topic	Key Formula	Use When
Fundamental Rule	Multiply options	Independent choices
Permutation	$nPr = \frac{n!}{(n-r)!}$	Order matters
Combination	$nCr = \frac{n!}{r!\cdot(n-r)!}$	Order doesn't matter
Perm. w/ repetition	$\frac{n!}{p_1!\cdot p_2!\cdot\ldots}$	Repeated items

You're a Stats Superhero Now!