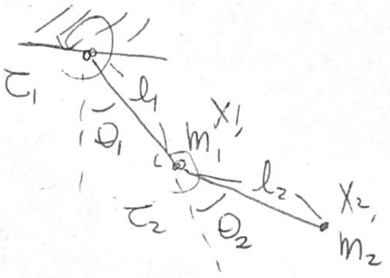
Underactuated Robotics

Today

- Motivations
- Definitions
- Review of dynamics
- Course outline



$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$ - joint coordinates

$\dot{q}$ - Joint velocities

$\ddot{q}$ - Joint accelerations

$u = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$ - control inputs

F = ma governing everything.

$\ddot{q} = f(q, \dot{q}, u, t)$ -- ①

$\ddot{q} = f_1(q, \dot{q}, t) + \underbrace{f_2(q, \dot{q}, t)\, u}$ - ②

linear by u.

Defin. Fully actuated.

If $\text{rank}[f_2(q,\dot{q},t)] = \dim[q]$.

"full rank"

$$u = \pi(q,\dot{q},t)$$

$$= f_2^{-1}(q,\dot{q},t)\,[-f_1(q,\dot{q},t) + u'] \quad -③$$

③ → ②: $\ddot{q} = u'$.

"Feedback Linearization"

Defin. Underactuated

If $\text{rank}[f_2(q,\dot{q},t)] < \dim[q]$

A control system is underactuated

if control input u cannot produce

$\ddot{q}$ in arbitrary direction

Kinematics

$$X_1 = \begin{bmatrix} l_1 \sin(\theta_1) \\ -l_1 \cos(\theta_1) \end{bmatrix} = \begin{bmatrix} l_1 s_1 \\ -l_1 c_1 \end{bmatrix}$$

$$X_2 = X_1 + \begin{bmatrix} l_2 \sin(\theta_1+\theta_2) \\ -l_2 \cos(\theta_1+\theta_2) \end{bmatrix} = X_1 + \begin{bmatrix} l_2 s_{1+2} \\ -l_2 c_{1+2} \end{bmatrix}$$

$$\dot{X}_1 = \begin{bmatrix} l_1 c_1 \dot{\theta}_1 \\ l_1 s_1 \dot{\theta}_1 \end{bmatrix}$$

$$\dot{X}_2 = \dot{X}_1 + \begin{bmatrix} l_2 c_{1+2} (\dot{\theta}_1 + \dot{\theta}_2) \\ l_2 s_{1+2} (\dot{\theta}_1 + \dot{\theta}_2) \end{bmatrix}$$

Dynamics (Lagrangian)

T (total kinetic energy) $= \frac{1}{2} \dot{X}_1^T m_1 \dot{X}_1 + \frac{1}{2} \dot{X}_2^T m_2 \dot{X}_2$

U (total potential energy) $= -m_1 g l_1 c_1 - m_2 g (l_1 c_1 + l_2 c_{1+2})$

$L = T - U$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = Q$$

↳ generalized force.

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derivation ?

$f_L(q, \dot{q}, \ddot{q}) = u.$

$\rightarrow \ddot{q} = f(q, \dot{q}, u).$

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau = \underset{?}{B(q)}u.$$

torques

Inertia matrix   Coriolis matrix.   Gravity ~~matrix~~ matrix

→ Manipulator equations

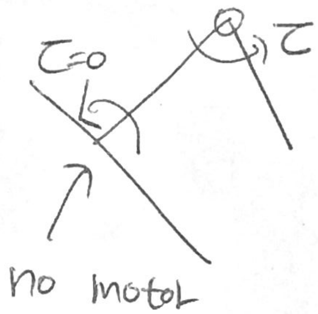
$$T = \frac{1}{2}\dot{q}^T H(q)\dot{q}. \quad \cancel{\text{tho}}$$

$H^T(q) = H(q) > 0.$ ($\because$ no negative kinetic energy)

$B(q) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$: fully actuated

$$\ddot{q} = \underbrace{H(q)^{-1}\left[B(q)u - C(q,\dot{q})\dot{q} - G(q)\right]}.$$

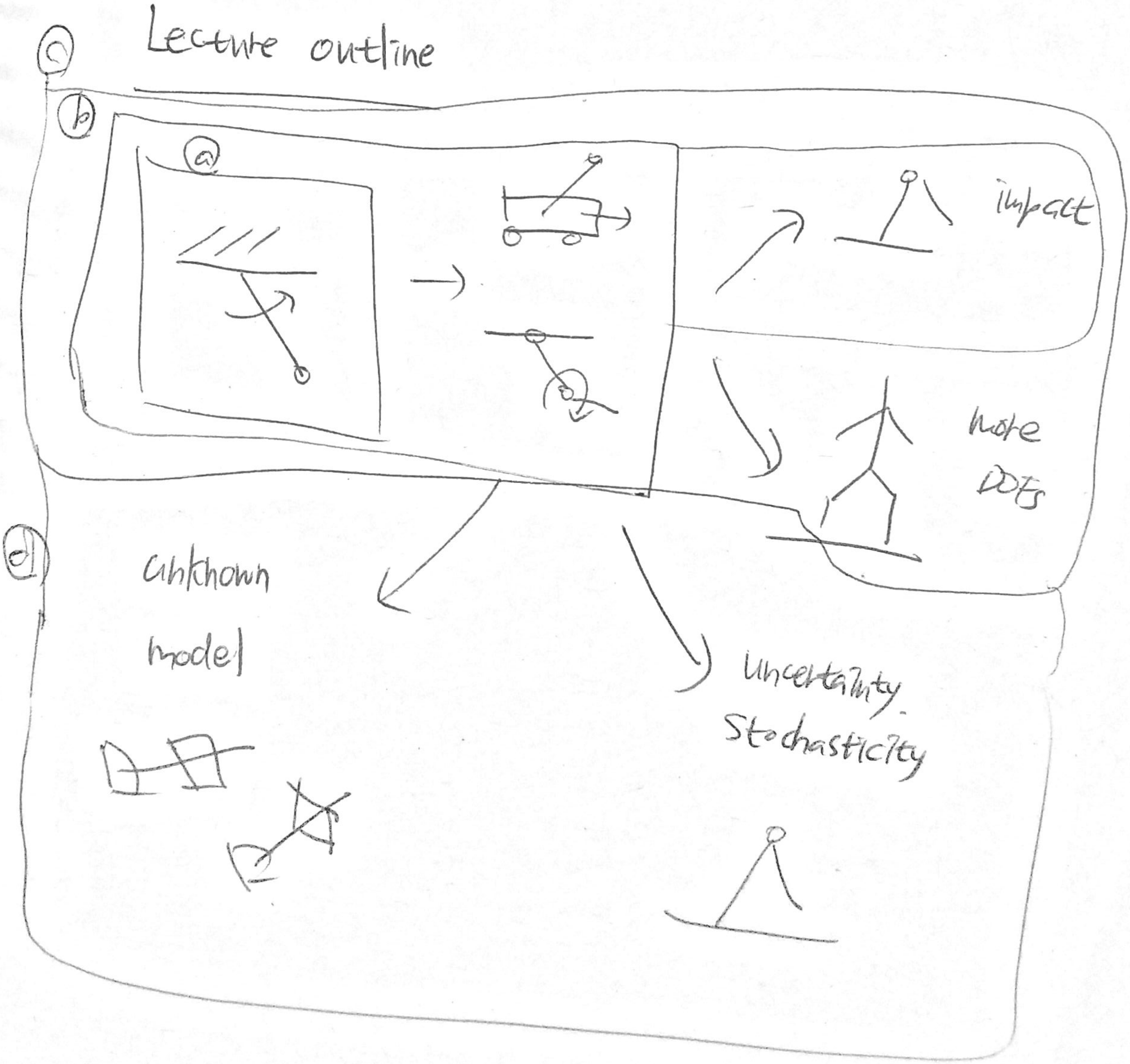
$B(q) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$: underactuated why underactuated?



walking
jumping
flying
throwing

robot

Lecture outline



ⓐ optimal control

- analytical ~~control~~
- numerical (based on dynamic programming)

ⓑ Numerical optimal control

- policy search

ⓒ Motion planning

ⓓ machine learning control (reinforcement learning)