$$\ln p(\mathbf{t}|\mathbf{x}, \mathbf{w}, \beta) = -\frac{\beta}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{N}{2}\ln\beta - \frac{N}{2}\ln(2\pi). \qquad (1.62)$$

$$p(\mathbf{w}|\alpha) = \mathcal{N}(\mathbf{w}|\mathbf{0}, \alpha^{-1}\mathbf{I}) = \left(\frac{\alpha}{2\pi}\right)^{(M+1)/2}\exp\left\{-\frac{\alpha}{2}\mathbf{w}^{\mathrm{T}}\mathbf{w}\right\} \qquad (1.65)$$

$$p(\mathbf{w}|\mathbf{X}, \mathbf{T}) \propto p(\mathbf{T}|\mathbf{X}, \mathbf{w})\, p(\mathbf{w})$$

$$\propto \exp\left\{-\frac{\beta}{2}\sum_{n=1}^{N}\left\{\mathbf{w}^{\mathrm{T}}\boldsymbol{\phi}(\mathbf{x}_n) - t_n\right\}^2 - \frac{\alpha}{2}\mathbf{w}^{\mathrm{T}}\mathbf{w}\right\}$$

$$= \exp\left\{-\frac{\beta}{2}\sum_{n=1}^{N}\left(\mathbf{w}^{\mathrm{T}}\boldsymbol{\phi}(\mathbf{x}_n)\boldsymbol{\phi}(\mathbf{x}_n)^{\mathrm{T}}\mathbf{w} - 2t_n\mathbf{w}^{\mathrm{T}}\boldsymbol{\phi}(\mathbf{x}_n) + t_n^2\right) - \frac{\alpha}{2}\mathbf{w}^{\mathrm{T}}\mathbf{w}\right\}$$

$$= \exp\left\{-\frac{1}{2}\mathbf{w}^{\mathrm{T}}\left(\beta\sum_{n=1}^{N}\boldsymbol{\phi}(\mathbf{x}_n)\boldsymbol{\phi}(\mathbf{x}_n)^{\mathrm{T}} + \alpha\mathbf{I}\right)\mathbf{w} + \beta\sum_{n=1}^{N}t_n\mathbf{w}^{\mathrm{T}}\boldsymbol{\phi}(\mathbf{x}_n) - \frac{\beta}{2}\sum_{n=1}^{N}t_n^2\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\mathbf{w}^{\mathrm{T}}\mathbf{S}^{-1}\mathbf{w} + \beta\mathbf{w}^{\mathrm{T}}\sum_{n=1}^{N}t_n\boldsymbol{\phi}(\mathbf{x}_n)\right\}$$

$$\Rightarrow \mathcal{N}\left(\mathbf{w}\,|\,\mathbf{S}\beta\sum_{n=1}^{N}\boldsymbol{\phi}(\mathbf{x}_n)t_n,\ \mathbf{S}\right)$$

$$p(\mathbf{x}) = \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda}^{-1}) \qquad (2.113)$$
$$p(\mathbf{y}|\mathbf{x}) = \mathcal{N}(\mathbf{y}|\mathbf{A}\mathbf{x} + \mathbf{b}, \mathbf{L}^{-1}) \qquad (2.114)$$
$$p(\mathbf{y}) = \mathcal{N}(\mathbf{y}|\mathbf{A}\boldsymbol{\mu} + \mathbf{b}, \mathbf{L}^{-1} + \mathbf{A}\boldsymbol{\Lambda}^{-1}\mathbf{A}^{\mathrm{T}}) \qquad (2.115)$$

$$p(t|\mathbf{x}, \mathbf{X}, \mathbf{T}) = \int p(t|\mathbf{x}, \mathbf{w})\, p(\mathbf{w}|\mathbf{X}, \mathbf{T})\, d\mathbf{w}$$

$$p(\mathbf{y}) = \int p(\mathbf{y}|\mathbf{x})\, p(\mathbf{x})\, d\mathbf{x}$$

$$p(t|x, \mathbf{w}, \beta) = \mathcal{N}\left(t|y(x, \mathbf{w}), \beta^{-1}\right) \qquad (1.60)$$

$$\Rightarrow \boldsymbol{\mu} = \mathbf{S}\beta\sum_{n=1}^{N}\boldsymbol{\phi}(\mathbf{x}_n)t_n \qquad \Rightarrow p(t|\mathbf{x}, \mathbf{X}, \mathbf{T})$$

$$\boldsymbol{\Lambda} = \mathbf{S}^{-1} \qquad = \mathcal{N}\left(t\,|\,m(\mathbf{x}),\ s^2(\mathbf{x})\right)$$

$$\mathbf{A} = \boldsymbol{\phi}(\mathbf{x})^{\mathrm{T}} \qquad \text{where } m(\mathbf{x}) = \mathbf{S}\boldsymbol{\phi}(\mathbf{x})^{\mathrm{T}}\beta\sum_{n=1}^{N}\boldsymbol{\phi}(\mathbf{x}_n)t_n$$

$$\mathbf{b} = 0 \qquad s^2(\mathbf{x}) = \beta^{-1} + \boldsymbol{\phi}(\mathbf{x})^{\mathrm{T}}\mathbf{S}\,\boldsymbol{\phi}(\mathbf{x})$$

$$\mathbf{L} = \beta$$