Assignment 4

AI1110: Probability and Random Variables

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paupolis ex 9.28

(paupolis Exercise 9.28) (Papoulis chapter 9 ,ex 9.28) show that $R_{XX}(t_1, t_2)$ =$q(t_1)\delta(t_1, t_2)$ $E\{y^2(t)\} = I(t)$ a) $y(t) = \int_0^I h(t, \alpha)X(\alpha)d\alpha$ then $I(t) = \int_0^I h^2(t, \alpha)q(\alpha)d\alpha$ b)$y'(t) + c(t)y(t) = x(t)thenI'(t) + 2c(t)I(t) = q(t)$

I. Solution

Solution

- a) $I(t) = E\int_0^t \int_0^t h(t, \alpha)x(\alpha)h(t, \beta)x(\beta)d\alpha d\beta$
- $I(t) = \int_0^t \int_0^t h(t, \alpha)h(t, \alpha)q(\alpha)\delta(\alpha - \beta)d\alpha d\beta = \int_0^t h^2(t, \alpha)q(\alpha)d\alpha$
- b) if $y'(t) + c(t)y(t) = x(t)$ then y(t) is the output of a linear time varying system as in a) with impulse response h(t,α) such that
- $\dfrac{dh(t, \alpha)}{dt} + c(t)h(t, \alpha) = \delta(t-\alpha)$, $h(\alpha^-, \alpha) = 0$
- or equivalently $\dfrac{dh(t, \alpha)}{dt} + c(t)h(t, \alpha) = 0$,t>0 ,$h(\alpha^+, \alpha) = 1$
- this yields $h(t, \alpha) = e^- \int_a^t c(\tau)d\tau$
- hence,if $I(t) = \int_0^t h^2(t, \alpha)q(\alpha)d\alpha$ then $I'(t) + 2c(t)I(t) = q(t)$
- because the impulse response of the of this equation equals $e^-2\int_a^t c(\tau)d\tau = h^2(t, \alpha)$
- hence proved