Assignment 4

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Outline

1. Problem

2. Solution

Problem Statement

(Papoulis chapter 9 ,ex 9.29) show that $R_{XX}(t_1, t_2) = q(t_1)\delta(t_1, t_2)$
$E\{y^2(t)\} = I(t)$ a)
$y(t) = \int_0^I h(t, \alpha)X(\alpha)d\alpha then I(t) = \int_0^I h^2(t, \alpha)q(\alpha)d\alpha$
b)$y'(t) + c(t)y(t) = x(t) then I'(t) + 2c(t)I(t) = q(t)$

Solution

- a) $I(t) = E\int_0^t \int_0^t h(t,\alpha)x(\alpha)h(t,\beta)x(\beta)d\alpha d\beta$
- $I(t) = \int_0^t \int_0^t h(t,\alpha)h(t,\alpha)q(\alpha)\delta(\alpha-\beta)d\alpha d\beta = \int_0^t h^2(t,\alpha)q(\alpha)d\alpha$
- b) if $y'(t) + c(t)y(t) = x(t)$ then y(t) is the output of a linear time varying system as in a) with impulse response h(t,α) such that
- $\dfrac{dh(t,\alpha)}{dt} + c(t)h(t,\alpha) = \delta(t-\alpha)$, $h(\alpha^-,\alpha) = 0$
- or equivalently $\dfrac{dh(t,\alpha)}{dt} + c(t)h(t,\alpha) = 0$,$t{>}0$,$h(\alpha^+,\alpha) = 1$
- this yields $h(t,\alpha) = e^- \int_a^t c(\tau)d\tau$
- hence,if $I(t) = \int_0^t h^2(t,\alpha)q(\alpha)d\alpha$ then $I'(t) + 2c(t)I(t) = q(t)$
- because the impulse response of the of this equation equals $e^- 2\int_a^t c(\tau)d\tau = h^2(t,\alpha)$
- hence proved