# Stochastic Robust Team Tracking Control of Multi-UAV Networked System Under Wiener and Poisson Random Fluctuations

Bor-Sen Chen, *Life Fellow, IEEE*, Chun-Ping Wang, and Min-Yen Lee

***Abstract*—In this article, a robust leader–follower tracking control scheme is proposed to deal with the stochastic multi-unmanned aerial vehicle (UAV) networked team tracking control problem to achieve a prescribed $H_\infty$ robust tracking performance. By taking the leader's desired path and every UAV networked system into account, the leader–follower tracking error networked system is constructed by arranging the UAVs network systems into a leader–follower formation. To effectively reduce the effect of the external disturbance on the team tracking process, a robust $H_\infty$ controller is proposed. With the help of the Itô–Lévy formula, the robust team tracking control design of the multi-UAV system is transformed to a Hamilton–Jacobin inequality (HJI)-constrained optimization problem. Since the HJI constraint cannot be easily solved, the Takagi–Sugeno (T–S) fuzzy interpolation method is employed to approximate the nonlinear networked system with a set of local linearized stochastic networked systems to simplify the design procedure. By the proposed T–S fuzzy control method, the $H_\infty$ robust leader–follower tracking control design of the stochastic multi-UAV networked system can be transformed to equivalent linear matrix inequalities (LMIs)-constrained optimization problem which can be efficiently solved by using the convex optimization techniques. Finally, a design example is given with simulation to illustrate the design procedure of the robust team tracking control of desired attitude and path and validate the effectiveness of the proposed robust $H_\infty$ team tracking control method of the multi-UAV networked system.**

***Index Terms*—Leader–follower formation control, linear matrix inequality (LMI), networked control system, quadrotor unmanned aerial vehicles (UAVs) networked system, robust control, stochastic fuzzy system.**

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## I. Introduction

The UNMANNED aerial vehicle (UAV) technology has been attracting more researchers and institutes due to its fast-growing market and vast application area which is not limited to military [1], [2]. In general, a single UAV has many limitations on the applications, such as load capacity and battery power. Compared to a single UAV, a formation of multiple UAVs can accomplish more complex and challenging tasks at higher efficiency with multiple capabilities. Several authors have investigated the use of multiple aircrafts in cooperating tasks [3]. With the decreasing manufacturing cost, multi-UAV formations are becoming popular and practical in many applications, such as environmental monitoring, battlefield surveillance, target search and rescue, intruder detection and interception, exploration of unsecured regions, etc. [4], [5].

Quadrotor UAV, one of the popular rotor UAVs, is a typical underactuated system with six degrees of freedom (6DoF) and four independent control inputs. Although these underactuated characteristics can effectively reduce the manufacturing difficulty of the system, the nonlinearity of strong coupling system variables in the quadrotor UAV dynamic system makes the control system design become a great challenge [6]–[8]. Some linear and nonlinear control schemes have been proposed to deal with the attitude control and position control of the quadrotor UAV. Proportional–integral–derivative (PID) attitude control and linear quadratic regulator (LQR) attitude control were studied in [9]. The sliding-mode control [10] and backstepping control [11] were developed to be easy to implement practically. Some other control schemes were also proposed, such as adaptive control [12], predictor-based control [13], dynamic inversion control [14], etc.

Recently, because of the advances in data sensing and communication technology, the networked system of UAVs is constructed to achieve the desired goal of the team by exchanging the information among the UAVs through the network. In general, multiple UAVs operating in synergy need to form a formation [15]. The common formation control method is to establish the control strategies for each UAV to achieve their desired goal tracking. At present, there exist several formation control theories, such as the leader–follower law [16], virtual structure law [17], behavior-based control method [18], graph theory [19], etc. Among the methods mentioned above, the leader–follower approach has become one of the well-recognized and most popular strategies for the synchronization problem of the networked system. The idea of the leader–follower formation scheme is one of the UAV to be selected as the leader in response for tracking the reference path and guiding the team. The other UAVs, which are called the followers, are required to track the motion of the leader with corresponding formation distance. Since the leader–follower scheme only needs to specify the leader UAV information to represent the network team behavior, the team tracking design problem of the multi-UAV networked system can be simplified under the leader–follower scheme. To approach the leader–follower formation, the error dynamic system technique that has been used in the synchronization problem [20] or consensus problem [21], [22] is modified for the team reference tracking control of the stochastic multi-UAV networked systems in this article.

In the previous works of the UAV system, the external disturbances or internal fluctuations are usually ignored for the convenience of control design. However, in the realistic UAV system, the system will inevitably suffer from the unknown external disturbances and internal fluctuations. For example, the effect from the airflow to the UAV system can be regarded as an unknown external disturbance. Also the continuous fluctuation from the motors and the discontinuous jump from the sudden voltage jump of electric circuit or sensors [23], [24] can be described as a kind of stochastic internal fluctuation in the system. Hence, the flight control scheme should be further developed for the quadrotor UAV networked system with intrinsic continuous and discontinuous random fluctuations as well as external disturbances to improve flight control performance for the desired trajectory tracking in the practical application. Based on the Itô–Lévy formula [25], [26], the continuous Wiener process and discontinuous Poisson process are introduced to model the intrinsic continuous and discontinuous random fluctuations in the team tracking process of the multi-UAV networked system, respectively.

In this article, the $H_\infty$ robust multi-UAV team reference tracking design scheme is proposed to guarantee that the controlled multi-UAV networked system gradually converge to the desired attitude and reference path (target) with a specific formation under the effect of intrinsic random fluctuation and external disturbance [27]. With the help of the Itô–Lévy formula, the robust multi-UAV team tracking controller design is transformed to an equivalent Hamilton–Jacobin inequality (HJI)-constrained optimization problem. Since the HJI constraint is a nonlinear partial differential inequality constraint, there does not exist any numerical or analytic method to efficiently solve the HJI constraint. To simplify the HJI-constrained optimization problem for the $H_\infty$ robust leader–follower formation tracking control of the multi-UAV team system, the Takagi–Sugeno (T–S) fuzzy interpolation method is proposed to approximate the nonlinear stochastic system with several local stochastic linearized systems [28], [29]. Also, the HJI-constrained optimization problem for the $H_\infty$ robust leader–follower formation tracking design of the multi-UAV team system can be transformed to a linear matrix inequality (LMI)-constrained optimization problem which can be easily solved with the help of MATLAB LMI toolbox.

The main contributions of this article are described as follows.

1) A robust stochastic nonlinear multi-UAV team reference tracking problem is first formulated as a stochastic leader–follower $H_\infty$ reference tracking problem with the consideration of continuous Wiener fluctuations, discontinuous Poisson jump fluctuations, and external disturbances in the more realistic task of the multi-UAV team.
2) A stochastic nonlinear reference tracking error dynamic system is introduced for designing $H_\infty$ robust controller for the stochastic nonlinear multi-UAV team system to efficiently track the desired attitude and path in a leader–follower formation for more practical applications.
3) Robust $H_\infty$ team formation tracking optimization problem of the stochastic nonlinear multi-UAV networked system is transformed to an equivalent HJI-constrained optimization problem. Moreover, with the help of the T–S fuzzy interpolation method [31], the HJI-constrained optimization problem of leader–follower $H_\infty$ multi-UAV networked tracking system is transformed to an equivalent LMIs-constrained optimization problem so that the leader–follower $H_\infty$ multi-UAVs team tracking problem can be solved efficiently with the help of MATLAB LMI toolbox to simplify the design procedure.

The remainder of this article is organized as follows. The preliminaries and quadrotor UAV system are given in Section II. Section III presents the problem formulation of the leader–follower task in the team formation tracking control of the multi-UAV networked system. The asymptotical tracking analysis and the stochastic $H_\infty$ tracking analysis are also given in this section. In Section IV, the T–S fuzzy model is introduced to deal with the stochastic nonlinear leader–follower $H_\infty$ robust team tracking problem. The stochastic $H_\infty$ robust team tracking controller design and the design procedures are proposed in Section V. In Section VI, a design example is provided to illustrate the design procedures and demonstrate $H_\infty$ tracking performance of the stochastic leader–follower $H_\infty$ robust team tracking control of the multi-UAV networked system. Finally, some concluding remarks are made in Section VII.

*Notation:* $A^{\mathrm{T}}$: the transpose of matrix $A$; $A \geq 0 (A > 0)$: symmetric positive semidefinite (symmetric positive-definite) matrix $A$, respectively; $I_n$: $n$-dimensional identity matrix; $\|\boldsymbol{x}\|_2$: the Euclidean norm for the given vector $\boldsymbol{x} \in \mathbb{R}^n$; $C^2(\mathbb{R}^n)$: the class of functions $f(\boldsymbol{x})$ which is twice continuously differentiable with respect to $\boldsymbol{x}$ ; $f_{\boldsymbol{x}}$: the gradient column vector of continuously differentiable function $\boldsymbol{f}(\boldsymbol{x}) \in \mathbb{R}^1$ with respect to $\boldsymbol{x} \in \mathbb{R}^n$(i.e.,$([\partial f(\boldsymbol{x})]/\partial \boldsymbol{x})$.); $f_{\boldsymbol{xx}}$: the Hessian matrix with elements of second partial derivatives of twice continuously differentiable function $\boldsymbol{f}(\boldsymbol{x}) \in \mathbb{R}^1$ with respect to $\boldsymbol{x} \in \mathbb{R}^n$, (i.e.,$([\partial^2 f(\boldsymbol{x})]/\partial \boldsymbol{x}^2)$.); $\mathcal{L}^2_{\mathcal{F}}(\mathbb{R}^+, \mathbb{R}^n)$: the space of nonanticipative stochastic processes $y(t) \in \mathbb{R}^n$ with respect to an increasing $\sigma-$ algebras $\mathcal{F}_t (t \geq 0)$ satisfying $\|\boldsymbol{y}(t)\|_{\mathcal{L}^2(\mathbb{R}^+;\mathbb{R}^n)} \triangleq \boldsymbol{E}\{\int_0^\infty \boldsymbol{y}^{\mathrm{T}}(t)\boldsymbol{y}(t)dt\}^{(1/2)} < \infty$; and $\boldsymbol{E}$: the expectation operator.

## II. Preliminaries and Quadrotor UAV System

In [6] and [7], the position of the mass center of the quadrotor is denoted by the vector $\Xi = [x, y, z]^{\mathrm{T}}$. This position vector is expressed relatively with respect to an inertial frame ($I$) associated with the unit vector basis $(e_1, e_2, e_3)$. The attitude is denoted by $\Theta = [\phi, \theta, \psi]$. These three angles are the Euler angles, that is, roll $(-(\pi/2) < \phi < (\pi/2))$, pitch $(-(\pi/2) < \theta < (\pi/2))$, and yaw $(-\pi < \psi < \pi)$ that define the orientation vector of the quadrotor in space expressed in body frame ($B$) fixed to the body of the quadrotor UAV and associated with vector basis $(e_{b1}, e_{b2}, e_{b3})$. Fig. 1 illustrates the UAV dynamic model with the attitude and position in the inertial frame $(e_1, e_2, e_3)$ and the body frame $(e_{b1}, e_{b2}, e_{b3})$ respectively. The linear velocity and acceleration of the translational system are given, respectively, as $\dot{\Xi} = [\dot{x}, \dot{y}, \dot{z}]$ and $\ddot{\Xi} = [\ddot{x}, \ddot{y}, \ddot{z}]$. The angular velocity and acceleration of the attitude system are given by $\dot{\Theta} = [\dot{\phi}, \dot{\theta}, \dot{\psi}]$ and $\ddot{\Theta} = [\ddot{\phi}, \ddot{\theta}, \ddot{\psi}]$, respectively. The quadrotor dynamic equation will be written in the form of two subsystems corresponding to the translational motion (referring to the position of the mass center of the UAV) and angular motion (referring to the attitude of the UAV). With the Newton–Euler method, the $i$th quadrotor UAV dynamic of the multi-UAV networked system can be described as follows:

$$\begin{pmatrix} \dot{x}_1^i(t) \\ \dot{x}_2^i(t) \\ \dot{y}_1^i(t) \\ \dot{y}_2^i(t) \\ \dot{z}_1^i(t) \\ \dot{z}_2^i(t) \\ \dot{\phi}_1^i(t) \\ \dot{\phi}_2^i(t) \\ \dot{\theta}_1^i(t) \\ \dot{\theta}_2^i(t) \\ \dot{\psi}_1^i(t) \\ \dot{\psi}_2^i(t) \end{pmatrix} = \begin{pmatrix} x_2^i(t) \\ -\frac{K_x^i}{m^i}x_2^i(t) + \big(\cos\phi_1^i(t)\sin\theta_1^i(t)\cos\psi_1^i(t) \\ +\sin\phi_1^i(t)\sin\psi_1^i(t)\big)\frac{F^i(t)}{m^i} + v_x^i(t) \\ y_2^i(t) \\ -\frac{K_y^i}{m^i}y_2^i(t) + \big(\cos\phi_1^i(t)\sin\theta_1^i(t)\sin\psi_1^i(t) \\ -\sin\phi_1^i(t)\cos\psi_1^i(t)\big)\frac{F^i(t)}{m^i} + v_y^i(t) \\ z_2^i(t) \\ -\frac{K_z^i}{m^i}z_2^i(t) - g + \cos\phi_1^i(t)\cos\theta_1^i(t)\frac{F^i(t)}{m^i} + v_z^i(t) \\ \phi_2^i(t) \\ \frac{J_y^i - J_z^i}{J_x^i}\theta_1^i(t)\psi_1^i(t) - \frac{K_\phi^i}{J_x^i}\phi_2^i(t) + \frac{1}{J_x^i}\tau_\phi^i(t) + v_\phi^i(t) \\ \theta_2^i(t) \\ \frac{J_z^i - J_x^i}{J_y^i}\phi_1^i(t)\psi_1^i(t) - \frac{K_\theta^i}{J_y^i}\theta_2^i(t) + \frac{1}{J_y^i}\tau_\theta^i(t) + v_\theta^i(t) \\ \psi_2^i(t) \\ \frac{J_x^i - J_y^i}{J_z^i}\phi_1^i(t)\theta_1^i(t) - \frac{K_\psi^i}{J_z^i}\psi_2^i(t) + \frac{1}{J_z^i}\tau_\psi^i(t) + v_\psi^i \end{pmatrix} \tag{1}$$

where $x_1^i(t), y_1^i(t), z_1^i(t) \in \mathbb{R}^1$ are the locations of the $i$th UAV in the Cartesian coordinate space with respect to the inertial frame, and $\phi_1^i(t), \theta_1^i(t), \psi_1^i(t) \in \mathbb{R}^1$ are the attitudes of the $i$th UAV consisted of the Euler angles of the UAV with respect to the inertial frame. The total thrust $F^i(t) \in \mathbb{R}^1$ and the rotational force $\tau_\phi^i(t), \tau_\theta^i(t), \tau_\psi^i(t) \in \mathbb{R}^1$ of the $i$th UAV are produced by four rotors. $g$ is the gravity constant. $v_x^i(t)$, $v_y^i(t)$, and $v_z^i(t)$ are the external disturbances of the $i$th UAV in the three translation dynamics. $v_\phi^i(t)$, $v_\theta^i(t)$, and $v_\psi^i(t)$ are the external disturbances of the $i$th UAV caused by the unexpected rotation force in raw, pitch, and yaw dynamics. $m^i \in \mathbb{R}^+$ denotes the total mass of the $i$th UAV, $J_x^i$, $J_y^i$, $J_z^i \in \mathbb{R}^+$ are the moments of inertia of the $i$th UAV, $K_x^i$, $K_y^i$, $K_z^i$, $K_\phi^i$, $K_\theta^i$, $K_\psi^i \in \mathbb{R}^+$ represent the aerodynamic damping coefficient of the $i$th UAV, and $l \in \mathbb{R}^+$ denotes the distance between the epicenter of UAV and the rotor axis.

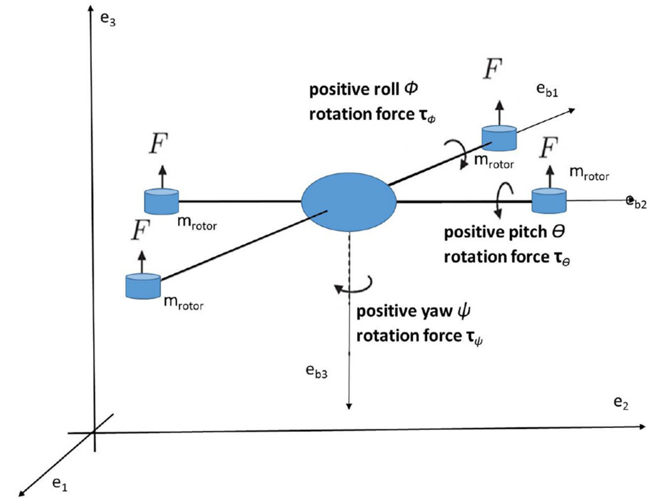



Fig. 1. UAV dynamic model with the attitude and position in the inertial frame$(e_1, e_2, e_3)$ and the body frame$(e_{b1}, e_{b2}, e_{b3})$.

Thus, the $i$th nonlinear quarotor UAV system of the $N$ UAVs team can be written as follows:

$$\dot{\boldsymbol{X}}_i(t) = f_i(X_i(t)) + g_i(X_i(t))U_i(t) + \boldsymbol{v}_i(t), \text{ for } i = 1, 2, \ldots, N \tag{2}$$

with

$$\begin{aligned} \boldsymbol{X}_i(t) &= \big[x_1^i(t), x_2^i(t), y_1^i(t), y_2^i(t), z_1^i(t), z_2^i(t) \\ &\quad \phi_1^i(t), \phi_2^i(t), \theta_1^i(t), \theta_2^i(t), \psi_1^i(t), \psi_2^i(t)\big]^{\mathrm{T}} \\ U_i(t) &= \big[F^i(t), \tau_\phi^i(t), \tau_\theta^i(t), \tau_\psi^i(t)\big]^{\mathrm{T}} \\ \boldsymbol{v}_i(t) &= \big[0, v_x^i(t), 0, v_y^i(t), 0, v_z^i(t), 0, v_\phi^i(t), 0, v_\theta^i(t), 0, v_\psi^i(t)\big]^{\mathrm{T}} \\ f_i(\boldsymbol{X}_i(t)) &= \Big[x_2^i(t), -\frac{K_x^i}{m^i}x_2^i(t), y_2^i(t), -\frac{K_y^i}{m^i}y_2^i(t), z_2^i(t), -g - \frac{K_z^i}{m^i}z_2^i(t) \\ &\quad \phi_2^i(t), \frac{J_y^i - J_z^i}{J_x^i}\theta_1^i(t)\psi_1^i(t) - \frac{K_\phi^i}{J_x^i}\phi_2^i(t), \theta_2^i(t), \frac{J_z^i - J_x^i}{J_y^i}\phi_1^i(t) \\ &\quad \times \psi_1^i(t) - \frac{K_\theta^i}{J_x^i}\theta_2^i(t), \psi_2^i(t), \frac{J_x^i - J_y^i}{J_z^i}\phi_1^i(t)\theta_1^i(t) - \frac{K_\psi^i}{J_z^i}\psi_2^i(t)\Big]^{\mathrm{T}} \end{aligned}$$

$$g_i(\boldsymbol{X}_i(t)) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \big(\cos\phi_1^i(t)\sin\theta_1^i(t)\cos\psi_1^i(t) + \sin\phi_1^i(t)\sin\psi_1^i(t)\big)\frac{1}{m^i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \big(\cos\phi_1^i(t)\sin\theta_1^i(t)\sin\psi_1^i(t) - \sin\phi_1^i(t)\cos\psi_1^i(t)\big)\frac{1}{m^i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \cos\phi_1^i(t)\cos\theta_1^i(t)\frac{1}{m^i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{J_x^i} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{J_y^i} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{J_z^i} \end{bmatrix}.$$

## III. Problem Formulation

In this article, the leader–follower architecture in [34] is employed to deal with the multi-UAV team tracking control problem and the corresponding schematic diagram is shown

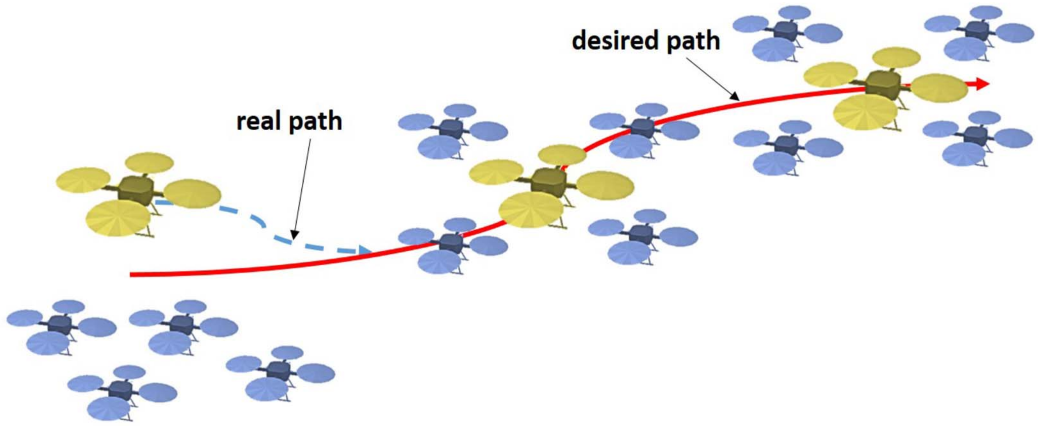



Fig. 2. Leader–follower team formation of the multi-UAV networked system to track a desired reference trajectory. The yellow UAV is the leader and the blue UAVs are the followers.

in Fig. 2. Under the concept of leader–follower architecture, the multi-UAV networked system can be separated into one leader system and the followers system. Since the leader has its own desired target and the each follower is asked to track the leader's trajectories with a specific distance, for the convenience of design, a specific error tracking augmented system is constructed via combining the leader's networked system and the followers networked system with their corresponding desired tracking path and attitude. To efficiently reduce the effect of external disturbance in the team tracking path on the desired target tracking, a robust controller is proposed to ensure the team tracking performance of the nonlinear multi-UAV system.

### A. Leader–Follower Networked System Model

Consider a connected team network consisting of $N$ UAVs and let the $H = \{h_1, h_2, \ldots, h_N\}$ denote the index set of UAVs. Without loss of generality, the $h_1$ UAV is assigned as a leader (it can receive and transmit information) and the other UAVs are assigned as the follower (they can only receive information). Hence, the set of multi-UAVs is partitioned into a set of leader $H_L$ and a set of followers $H_F$, that is, $H = H_L \cup H_F = \{h_1\} \cup \{h_2, \ldots, h_N\}$. The dynamic of each follower agent $h_i \in H_F$ based on the interactions with the leader is given as

$$\dot{X}_i(t) = f_i(X_i(t)) + g_i(X_i(t))U_i(t) + \boldsymbol{v}_i(t) \tag{3}$$

for $i = 2, 3, \ldots, N$, and the dynamic of the leader agent $h_1 \in H_L$ is represented as

$$\dot{X}_1(t) = f_1(X_1(t)) + g_1(X_1(t))U_1(t) + \boldsymbol{v}_1(t) \tag{4}$$

which will be controlled to track the desired trajectory $X_d$ generated by the following reference model:

$$\dot{X}_d(t) = A_r X_d(t) + r(t). \tag{5}$$

The team tracking control of the multi-UAV networked system is to specify the control input $U_1(t)$ of leader $h_1$ to track the desired trajectory $X_d(t)$ in (5) and to specify the control input $U_i(t)$ of each follower to track the trajectory of the leader in a specified leader–follower formation.

*Remark 1:* The reference model introduced in (5) is to generate the desired tracking trajectory of the leader UAV. The matrix $A_r$ denotes a specific asymptotically stable matrix. The vector $X_d(t)$ denotes the reference state trajectories which include the desired temporary position, velocity, angle, and angle velocity of leader. The vector $r(t)$ denotes the bounded reference input. At the steady state, $X_d(t) = -A_r^{-1} r(t)$. If we choose $A_r = -I$, the desired trajectory $X_d(t)$ is equivalent to the reference input $r(t)$ at the steady state. In this case, $X_d(t)$ will converge to $r(t)$ at the steady state.

Aggregating the states of all followers into a vector $X_F(t) \in \mathbb{R}^{12(N-1)}$, and the states of the leader into a vector $X_L(t) \in \mathbb{R}^{12}$, the followers networked system in (3) and the leader networked system in (4) can be augmented as the following dynamic system:

$$\dot{\mathbf{X}}(t) = f_0(\mathbf{X}(t)) + g_0(\mathbf{X}(t))\mathbf{u}(t) + \boldsymbol{v}_0(t) \tag{6}$$

where $X(t) = [X_1^{\mathrm{T}}(t), X_2^{\mathrm{T}}(t), \ldots, X_N^{\mathrm{T}}(t)]^{\mathrm{T}}, f_0(X(t)) = [f_1^{\mathrm{T}}(X_1(t)), f_2^{\mathrm{T}}(X_2(t)), \ldots, f_N^{\mathrm{T}}(X_N(t))]^{\mathrm{T}}, g_0(X(t)) = diag\{g_1(X_1(t)), g_2(X_2(t)), \ldots, g_N(X_N(t))\}$, and $u(t) = [U_1^{\mathrm{T}}(t), U_2^{\mathrm{T}}(t), \ldots, U_N^{\mathrm{T}}(t)]^{\mathrm{T}}$.

### B. Robust Leader–Follower Team Tracking Controller Design of Multi-UAV Networked System

In order to arrange the UAVs to a leader–follower formation $e_d$, we introduce a nonlinear tracking error dynamic of the leader–follower team tracking system. By shifting the origin of nonlinear tracking error dynamic system to the leader–follower formation $e_d$, the robust leader–follower team formation tracking problem can be transformed to an $H_\infty$ regulation control problem which can be easily solved.

Let $e_1(t) = X_1(t) - X_d(t)$, $e_i(t) = X_i(t) - X_1(t)$, for $i = 2, \ldots, N$, the tracking error dynamic of the multi-UAV networked leader–follower team tracking system is introduced as follows:

$$\dot{\mathbf{e}}(t) = \bar{f}(\mathbf{e}(t)) + \bar{g}(\mathbf{e}(t))\mathbf{U}(t) + \bar{\boldsymbol{v}}(t) \tag{7}$$

where

$$\mathbf{e}(t)=\begin{pmatrix}X_d(t)\\ e_1(t)\\ e_2(t)\\ \vdots\\ e_N(t)\end{pmatrix}=\begin{pmatrix}X_d(t)\\ X_1(t)-X_d(t)\\ X_2(t)-X_1(t)\\ \vdots\\ X_N(t)-X_1(t)\end{pmatrix}$$

$$\bar{f}(\mathbf{e}(t))=\begin{pmatrix}A_rX_d(t)\\ \bar{f}_1(e_1(t))\\ \bar{f}_2(e_2(t))\\ \vdots\\ \bar{f}_N(e_N(t))\end{pmatrix}=\begin{pmatrix}A_rX_d(t)\\ f_1(X_1(t))-A_rX_d(t)\\ f_2(X_2(t))-f_1(X_1(t))\\ \vdots\\ f_N(X_N(t))-f_1(X_1(t))\end{pmatrix}$$

$$\bar{g}(\mathbf{e}(t))=\begin{pmatrix}0 & 0 & 0 & \cdots & 0\\ 0 & \bar{g}_1(e_1(t)) & 0 & \cdots & 0\\ 0 & -\bar{g}_1(e_1(t)) & \bar{g}_2(e_2(t)) & & \vdots\\ \vdots & \vdots & & \ddots & \vdots\\ 0 & -\bar{g}_1(e_1(t)) & \cdots & \cdots & \bar{g}_N(e_N(t))\end{pmatrix}$$

$$\mathbf{U}(t)=\begin{pmatrix}0\\ U_1(t)\\ U_2(t)\\ \vdots\\ U_N(t)\end{pmatrix},\quad \bar{\boldsymbol{v}}(t)=\begin{pmatrix}r(t)\\ \boldsymbol{v}_1(t)-r(t)\\ \boldsymbol{v}_2(t)-\boldsymbol{v}_1(t)\\ \vdots\\ \boldsymbol{v}_N(t)-\boldsymbol{v}_1(t)\end{pmatrix}.$$

We define the target of the multi-UAVs leader–follower formation vector as $e_d=[0,0,e_{d2}^{\mathrm{T}},\ldots,e_{dN}^{\mathrm{T}}]^{\mathrm{T}}$. To arrange the UAVs in the leader–follower formation, the origin of nonlinear multi-UAV tracking error dynamic system in (7) should be shifted to the $e_d$. In this situation, if the shifted nonlinear multi-UAV tracking error dynamic system in (7) is robustly stabilized at the origin, then the robust asymptotic stability of $\tilde{\mathbf{e}}(t)=\mathbf{e}(t)-e_d$ is equivalent to asymptotically achieving the leader–follower formation (target) $e_d$.

Then, we obtain the following shifted nonlinear multi-UAV tracking error dynamic system as follows:

$$\dot{\tilde{\mathbf{e}}}(t)=\tilde{f}\big(\tilde{\mathbf{e}}(t)\big)+\tilde{g}\big(\tilde{\mathbf{e}}(t)\big)\mathbf{U}(t)+\bar{\boldsymbol{v}}(t) \tag{8}$$

where $\tilde{f}(\tilde{\mathbf{e}}(t))=\bar{f}(\tilde{\mathbf{e}}(t)+e_d)$, $\tilde{g}(\tilde{\mathbf{e}}(t))=\bar{g}(\tilde{\mathbf{e}}(t)+e_d)$ for the simplicity of notation.

Thus, the origin $\tilde{\mathbf{e}}(t)=0$ of the nonlinear multi-UAV team tracking error dynamic system in (8) is at the desired steady state (target of leader–follower formation) $e_d$ of the original nonlinear multi-UAV tracking error dynamic system in (7), that is, the multi-UAV team formation problem in (6) with the desired steady formation $e_d$ is transformed to the stabilization problem of the shifted nonlinear multi-UAV error dynamic system in (8).

In practice, the quadrotor UAV networked system always suffer from the parametric fluctuations due to the rotors, rigid bodies, electrical circuits, or sensors. To mimic the internal fluctuation of real quadrotor UAV networked system in the team tracking process, the dynamical model in (8) should be modified with continuous and discontinuous intrinsic random process. In the following text, the Wiener process $W_i(t)\in\mathbb{R}^1$ can be deemed as a continuous random fluctuation on the information transmitting via the uncertain network channel. On the other hand, the Poisson counting processes $N_i(t)\in\mathbb{R}^1$ are discontinuous abruptly random fluctuations and it is used to mimic the sudden voltage jump of electrical circuits and sensors during the flight, which will cause discontinuous changes of the system in (8).

Let $(\Omega,\mathcal{F},\mathcal{F}_t,\mathcal{P})$ be a filtration probability space, $\Omega$ denotes the sample space, $\mathcal{P}$ is the probability measure, $\mathcal{F}_t$ is $\sigma$-algebra generated by the following two mutually independent stochastic processes: 1) the Wiener process $\{W_i(s)\}_{i=1}^N$ and 2) the Poisson counting processes $\{N_i(s)\}\}_{i=1}^N$ for $s\le t$, $\mathcal{F}=\{\mathcal{F}_t : t\ge 0\}$.

The stochastic nonlinear leader–follower multi-UAV networked tracking error system is described as follows:

$$d\tilde{\mathbf{e}}(t)=\Big(\tilde{f}\big(\tilde{\mathbf{e}}(t)\big)+\tilde{g}\big(\tilde{\mathbf{e}}(t)\big)\mathbf{U}(t)+\bar{\boldsymbol{v}}(t)\Big)dt+\sum_{i=1}^N\tilde{\sigma}_i\big(\tilde{\mathbf{e}}(t)\big)dW_i(t)+\sum_{i=1}^N\tilde{\Gamma}_i\big(\tilde{\mathbf{e}}(t)\big)dN_i(t) \tag{9}$$

where $\tilde{f}:\mathbb{R}^{12(N+1)}\to\mathbb{R}^{12(N+1)}$, $\tilde{g}:\mathbb{R}^{12(N+1)}\to\mathbb{R}^{12(N+1)\times 4(N+1)}$, $\tilde{\sigma}_i:\mathbb{R}^{12(N+1)}\to\mathbb{R}^{12(N+1)}$, and $\tilde{\Gamma}_i:\mathbb{R}^{12(N+1)}\to\mathbb{R}^{12(N+1)}$ are the nonlinear Borel measurable continuous functions, which are satisfied with Lipschitz continuity. The input vector $\mathbf{U}(t)\in\mathcal{L}_{\mathcal{F}}^2(\mathbb{R}^+;\mathbb{R}^{4(N+1)})$ is the control force produced by the four rotors of the UAVs. The Wiener process $W_i(t)$ is the continuous but nondifferentiable stochastic process, and $\tilde{\sigma}_i(\tilde{\mathbf{e}}(t))dW_i(t)$ denotes the effect of continuous stochastic intrinsic noise caused by the $i$th UAV in the team tracking process. The Poisson counting process $N_i(t)$ is the discontinuous changes of the system at time instant $t$ caused by an incident, and $\tilde{\Gamma}_i(\tilde{\mathbf{e}}(t))dN_i(t)$ denotes the effect of discontinuous stochastic intrinsic noise caused by the $i$th UAV in the team tracking process. Assume $W_i(t)$ and $N_j(t)$ are independent for $i=1,2,\ldots,N$ and $j=1,2,\ldots,N$.

*Remark 2:* Some important properties of the Wiener process and Poisson jump process in (9) are given as follows [25], [26].

1) $\boldsymbol{E}\{W_i(t)\}=\boldsymbol{E}\{dW_i(t)\}=0$, for $i=1,\ldots,N$.
2) $\boldsymbol{E}\{dW_i(t)dW_j(t)\}=\begin{cases}dt, & \text{for } i=j\\ 0, & \text{for } i\ne j.\end{cases}$
3) $\boldsymbol{E}\{dN_i(t)\}=\lambda_i dt$, where the finite scalar number $\lambda_i>0$ is the Poisson jump intensity, for $i=1,\ldots,N$.

The analysis of the stochastic dynamical system in (9) is different from the deterministic case due to the indifferentiable property of these two stochastic processes. For the Wiener process, it is continuous but not differentiable at every time instant. For the Poisson jump process, there exists discontinuity when jump occurs. The indifferentiable property of these two stochastic processes in dynamical system is unable to be analyzed by the deterministic method. Fortunately, with the help of the Itô–Lévy formula, the stochastic differential property of the function of $\tilde{\mathbf{e}}(t)$ in (9) is able to be analyzed. Before Theorem 1, the Itô–Lévy formula is proposed in the following lemma.

*Lemma 1 [25]:* Let the Lyapunov function $V(\cdot)\in C^2(\mathbb{R}^{12(N+1)})$ with $V(0)=0$ and $V(\cdot)\ge 0$. For the stochastic nonlinear system in (9), the Itô–Lévy formula of $V(\tilde{e}(t))$ is given as follows:

$$dV\big(\tilde{\mathbf{e}}(t)\big) = \Bigg[V_{\tilde{\mathbf{e}}}^T\Big(\tilde{f}\big(\tilde{\mathbf{e}}(t)\big) + \tilde{g}\big(\tilde{\mathbf{e}}(t)\big)\mathbf{U}(t) + \bar{\boldsymbol{v}}(t)\Big) + \frac{1}{2}\sum_{i=1}^{N}\tilde{\sigma}_i^{\mathrm{T}}\big(\tilde{\mathbf{e}}(t)\big)V_{\tilde{\mathbf{e}}\tilde{\mathbf{e}}}\tilde{\sigma}_i\big(\tilde{\mathbf{e}}(t)\big)\Bigg]dt + \sum_{i=1}^{N}V_{\tilde{\mathbf{e}}}^T\tilde{\sigma}_i\big(\tilde{\mathbf{e}}(t)\big)dW_i(t) + \sum_{i=1}^{N}\Big\{V\Big(\tilde{\mathbf{e}}(t) + \tilde{\Gamma}_i\big(\tilde{\mathbf{e}}(t)\big)\Big) - V\big(\tilde{\mathbf{e}}(t)\big)\Big\}dN_i(t) \tag{10}$$

where $V_{\tilde{\mathbf{e}}} = ([\partial V(\tilde{\mathbf{e}}(t))]/[\partial\tilde{\mathbf{e}}(t)])$ and $V_{\tilde{\mathbf{e}}\tilde{\mathbf{e}}} = ([\partial^2 V(\tilde{\mathbf{e}}(t))]/[\partial\tilde{\mathbf{e}}^2(t)])$.

Also, another useful lemma could provide as an indirect method for the proof of the following theorems.

*Lemma 2 [30]:* For any matrices $M_1$ and $M_2$ with appropriate dimensions, we have the following inequality:

$$M_1^{\mathrm{T}}M_2 + M_2^{\mathrm{T}}M_1 \leq M_1^{\mathrm{T}}ZM_1 + M_2^{\mathrm{T}}Z^{-1}M_2 \tag{11}$$

where $Z$ is any positive-definite symmetric matrix.

Since the asymptotic tracking is one of the most important issues in the leader–follower team formation tracking control of the multi-UAV networked system. Therefore, in the following theorem, a controller is constructed to guarantee the asymptotic tracking performance of the multi-UAV networked tracking system in (9). Also, the design of the controller for the multi-UAV networked tracking system in (9) is transformed to an equivalent HJI problem.

*Theorem 1:* Consider the following stochastic nonlinear multi-UAV tracking error dynamic system in (9) without the external disturbances, that is, $\bar{\boldsymbol{v}}(t) = 0$

$$d\tilde{\mathbf{e}}(t) = \Big(\tilde{f}\big(\tilde{\mathbf{e}}(t)\big) + \tilde{g}\big(\tilde{\mathbf{e}}(t)\big)\mathbf{U}(t)\Big)dt + \sum_{i=1}^{N}\tilde{\sigma}_i\big(\tilde{\mathbf{e}}(t)\big)dW_i(t) + \sum_{i=1}^{N}\tilde{\Gamma}_i\big(\tilde{\mathbf{e}}(t)\big)dN_i(t). \tag{12}$$

If there exist a feedback controller $\mathbf{U}(t) = K(\tilde{\mathbf{e}}(t))$ and a Lyapunov function $V(\cdot) \in C^2(\mathbb{R}^{12(N+1)})$ with $V(0) = 0$ and $V(x) \geq 0$, $\forall x \in \mathbb{R}^{12(N+1)}$, satisfying the following inequalities:

$$m_1\big\|\tilde{\mathbf{e}}(t)\big\|_2^2 \leq V\big(\tilde{\mathbf{e}}(t)\big) \leq m_2\big\|\tilde{\mathbf{e}}(t)\big\|_2^2 \tag{13}$$

where $m_1,\ m_2 > 0$, and the following HJI:

$$V_{\tilde{\mathbf{e}}}^T\bigg(\tilde{f}\big(\tilde{\mathbf{e}}(t)\big) + \tilde{g}\big(\tilde{\mathbf{e}}(t)\big)\mathbf{K}\big(\tilde{\mathbf{e}}(t)\big) + \frac{1}{2}\sum_{i=1}^{N}\tilde{\sigma}_i^{\mathrm{T}}\big(\tilde{\mathbf{e}}(t)\big)V_{\tilde{\mathbf{e}}\tilde{\mathbf{e}}}\tilde{\sigma}_i\big(\tilde{\mathbf{e}}(t)\big) + \sum_{i=1}^{N}\lambda_i\Big\{V\Big(\tilde{\mathbf{e}}(t) + \tilde{\Gamma}_i\big(\tilde{\mathbf{e}}(t)\big)\Big) - V\big(\tilde{\mathbf{e}}(t)\big)\Big\} < 0 \tag{14}$$

then the stochastic nonlinear multi-UAV team networked system in (9) could achieve the asymptotic tracking in probability, that is, $\tilde{\mathbf{e}}(t) \to 0$ or $\bar{\mathbf{e}}(t) \to e_d$ in probability.

*Proof:* Please refer to Appendix A. ■

In general, the effect of external disturbance is unavoidable during the team tracking process for the multi-UAV networked system. Since the external disturbances are unpredictable, a robust $H_\infty$ tracking controller is constructed to efficiently attenuate the effect of external disturbances $\bar{\boldsymbol{v}}(t)$ on the team tracking performance of the stochastic nonlinear multi-UAV networked system. The definition of the robust $H_\infty$ team tracking performance is defined as follows:

$$J_\infty(\mathbf{U}(t)) = \sup_{\substack{\bar{\boldsymbol{v}}(t)\in \\ \mathcal{L}_{\mathcal{F}}^2\left(\mathbb{R}^+;\mathbb{R}^{\bar{N}}\right)}} \frac{E\Big\{\int_0^{t_f}\big[\tilde{\mathbf{e}}^{\mathrm{T}}(t)Q\tilde{\mathbf{e}}(t)\big]dt - V\big(\tilde{\mathbf{e}}_0\big)\Big\}}{E\Big\{\int_0^{t_f}\bar{\boldsymbol{v}}^{\mathrm{T}}(t)\bar{\boldsymbol{v}}(t)dt\Big\}} \tag{15}$$

where the term $-V(\tilde{\mathbf{e}}_0)$ is used to deduct the effect of initial condition $\tilde{\mathbf{e}}_0$ for some Lyapunov functions $V(\tilde{\mathbf{e}}(t))$, $V(\cdot) \in C^2(\mathbb{R}^{12(N+1)})$, and $V(\cdot) > 0$. $t_f$ is the terminal time of tracking control, $Q$ is a positive-definite weighting matrix, and $\bar{N} = 12(N+1)$. The design objective is to specify $\mathbf{U}(t)$ to attenuate the effect of any finite energy external disturbance on the leader–follower team formation tracking of the multi-UAV networked system under a prescribed level $\rho^2$. If one can specify a control $U^*(t)$ such that $J_\infty(U^*(t)) \leq \rho^2$ for some prescribed $\rho > 0$, then the effect of any external disturbance $\bar{\boldsymbol{v}}(t)$ on the tracking error $\tilde{\mathbf{e}}(t)$ must be attenuated below a desired level $\rho^2$ from the viewpoint of energy, that is, the $\mathcal{L}_2$ gain from $\bar{\boldsymbol{v}}(t)$ to $\tilde{\mathbf{e}}(t)$ must be equal or less than a prescribed value $\rho^2$.

*Theorem 2:* In the stochastic nonlinear multi-UAV networked tracking system (9), if one could specify a nonlinear tracking controller $\mathbf{U}(t) = \mathbf{K}(\tilde{\mathbf{e}}(t))$, such that the following HJI has a positive solution $V(\tilde{\mathbf{e}}(t)) \in C^2(\mathbb{R}^{12(N+1)})$ with $V(0) = 0$ and $V(\tilde{\mathbf{e}}(t)) \geq 0$

$$\tilde{\mathbf{e}}^{\mathrm{T}}(t)Q\tilde{\mathbf{e}}(t) + V_{\tilde{\mathbf{e}}}^T\tilde{f}\big(\tilde{\mathbf{e}}(t)\big) + V_{\tilde{\mathbf{e}}}^T\tilde{g}\big(\tilde{\mathbf{e}}(t)\big)\mathbf{K}\big(\tilde{\mathbf{e}}(t)\big) + \frac{1}{4\rho^2}V_{\tilde{\mathbf{e}}}^TV_{\tilde{\mathbf{e}}} + \frac{1}{2}\sum_{i=1}^{N}\tilde{\sigma}_i^{\mathrm{T}}\big(\tilde{\mathbf{e}}(t)\big)V_{\tilde{\mathbf{e}}\tilde{\mathbf{e}}}\tilde{\sigma}_i\big(\tilde{\mathbf{e}}(t)\big) + \sum_{i=1}^{N}\lambda_i\Big\{V\Big(\tilde{\mathbf{e}}(t) + \tilde{\Gamma}_i\big(\tilde{\mathbf{e}}(t)\big)\Big) - V\big(\tilde{\mathbf{e}}(t)\big)\Big\} \leq 0 \tag{16}$$

then the robust leader–follower $H_\infty$ team tracking control performance in (15) of the stochastic nonlinear multi-UAV networked system in (9) is guaranteed with a prescribed level $\rho^2$ for all possible $\bar{\boldsymbol{v}}(t) \in \mathcal{L}_{\mathcal{F}}^2(\mathbb{R}^+, \mathbb{R}^{12(N+1)})$. In the case $\bar{v}(t) = 0$, the asymptotic tracking of the multi-UAV networked system in (9) can be achieved in probability with the proposed controller $\mathbf{K}(\tilde{\mathbf{e}}(t))$, that is, $E[\tilde{\mathbf{e}}^{\mathrm{T}}(t)\tilde{\mathbf{e}}(t)] \to 0$, as $t \to \infty$.

*Proof:* Please refer to Appendix B. ■

## IV. T–S Fuzzy Model for Stochastic Multi-UAV Networked System

In general, the robust $H_\infty$ team tracking control problem for stochastic nonlinear multi-UAV networked system needs to solve an HJI in (16), which is difficult to be solved directly. Therefore, a T–S fuzzy model is introduced to approximate the nonlinear stochastic system by interpolating several local linear systems. Sequently, the controller design can be significantly simplified.

For the stochastic nonlinear multi-UAV shifted team tracking error dynamic system in (9), the nonlinear terms $\tilde{f}(\tilde{\mathbf{e}}(t))$, $\tilde{g}(\tilde{\mathbf{e}}(t))$, $\tilde{\sigma}(\tilde{\mathbf{e}}(t))$, and $\tilde{\Gamma}(\tilde{\mathbf{e}}(t))$ consist the functions of the leader's state $X_1(t)$ and the followers' state $X_i(t)$. Since all UAVs are identical, we can use a T–S fuzzy model to approximate $\tilde{f}_i(\tilde{e}_i(t))$, $\tilde{g}_i(\tilde{e}_i(t))$, $\tilde{\sigma}_i(\tilde{e}_i(t))$, and $\tilde{\Gamma}_i(\tilde{e}_i(t))$ by several local linear functions of $\tilde{e}_i(t)$. The T–S fuzzy model of $\tilde{f}_i(\tilde{e}_i(t))$, $\tilde{g}_i(\tilde{e}_i(t))$, $\tilde{\sigma}_i(\tilde{e}_i(t))$, and $\tilde{\Gamma}_i(\tilde{e}_i(t))$ can be described by several fuzzy If–Then rules.

The $k$th rule of the T–S fuzzy model for stochastic nonlinear multi-UAV shifted team tracking error dynamic system in (9) can be described as the follows [31]:

*Plant Rule k:*

$$\begin{aligned}
&\text{if } X_{1,1}(t) \text{ is } G_{k,1},\ldots,\ X_{1,g}(t) \text{ is } G_{k,g}\\
&\quad X_{i,1}(t) \text{ is } G_{k,g+1},\ldots,X_{i,g}(t) \text{ is } G_{k,2g}\\
&\text{then}\quad \tilde{f}_i(\tilde{e}_i(t)) = A_{ik}\tilde{e}_i(t), \tilde{g}_i(\tilde{e}_i(t)) = B_{ik}\\
&\quad\quad \tilde{\sigma}_{ij}(\tilde{e}_i(t)) = C_{ijk}\tilde{e}_i(t),\ \text{for } j = 1,2,\ldots,N\\
&\quad\quad \tilde{\Gamma}_{ij}(\tilde{e}_i(t)) = D_{ijk}\tilde{e}_i(t),\ \text{for } j = 1,2,\ldots,N
\end{aligned}$$

for $k = 1, 2, \ldots, L$. $g$ is the number of the premise variable of each UAV. $G_{k,l}$ are the fuzzy sets of plant rule $k$ for $l = 1, \ldots, 2g$. $A_{ik}, B_{ik}, C_{ijk}$, and $D_{ijk}$ are the local linearization system matrices of $\tilde{f}_i(\tilde{e}_i(t))$ , $\tilde{g}_i(\tilde{e}_i(t))$, $\tilde{\sigma}_{ij}(\tilde{e}_i(t))$, and $\tilde{\Gamma}_{ij}(\tilde{e}_i(t))$ for $i, j = 1, 2, \ldots, N$ at the corresponding fuzzy set. The premise variables $X_{1,1}(t)$, $X_{1,2}(t), \ldots, X_{1,N}(t)$ and $X_{i,1}(t)$, $X_{i,2}(t), \ldots, X_{i,N}(t)$ are the variables in the state vectors of the leader $X_1$ and the follower $X_i$, respectively, and the membership grades of the premise variables are defined as $\{G_{k,l}(X_{1,l}(t))\}_{l=1}^{2g}$.

We define the function $\mu_k(X_1, X_i)$ as [31]

$$\mu_k(X_1, X_i) = \prod_{l=1}^{g} G_{k,l}\big(X_{1,l}(t)\big)\prod_{s=1}^{g} G_{k,g+s}\big(X_{i,s}(t)\big)$$

where $G_{k,l}(X_{1,l}(t))$ denotes the grade of membership of leader $X_{1,l}(t)$ in $G_{k,l}$, and $G_{k,g+s}(X_{i,s}(t))$ denotes the grade of membership of the follower $X_{i,s}(t)$ in $G_{k,g+s}$. Assuming that

$$\mu_k(X_1, X_i)5 \geq 0,\ \sum_{k=1}^{L}\mu_k(X_1, X_i) > 0 \quad \text{for } k = 1,2,\ldots,L$$

then, the interpolation function $h_k(X_1, X_i)$ is defined as

$$h_k(X_1, X_i) = \frac{\mu_k(X_1, X_i)}{\sum_{k=1}^{L}\mu_k(X_1, X_i)},\ \text{with } \sum_{k=1}^{L} h_k(X_1, X_i) = 1 \quad \text{for } k = 1,2,\ldots,L.$$

With the If-then rules above, the overall stochastic nonlinear multi-UAV shifted team tracking error dynamic system can be denoted as

$$\begin{aligned}
d\tilde{\mathbf{e}}(t) = \sum_{k=1}^{L} H_k(\bar{X})\bigg[&\big(A_k\tilde{\mathbf{e}}(t) + B_k\mathbf{U}(t) + \bar{\mathbf{v}}(t)\big)dt\\
&+\sum_{i=1}^{N} C_{ik}\tilde{\mathbf{e}}(t)dW_i(t) + \sum_{i=1}^{N} D_{ik}\tilde{\mathbf{e}}(t)dN_i(t)\bigg]
\end{aligned} \tag{17}$$

where $H_k(\bar{X}) = \text{diag}\{I_{12}, h_k(X_1), h_k(X_1, X_2), \ldots, h_k(X_1, X_N)\}$, $A_k = \text{diag}\{A_r, A_{1k}, A_{2k}, \ldots, A_{Nk}\}$, $C_{jk} = \text{diag}\{0, C_{1jk}, C_{2jk}, \ldots, C_{Njk}\}$, $D_{jk} = \text{diag}\{0, D_{1jk}, D_{2jk}, \ldots, D_{Njk}\}$

$$B_k = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0\\ 0 & B_{1k} & 0 & \cdots & 0\\ 0 & -B_{1k} & B_{2k} & & \vdots\\ \vdots & \vdots & & \ddots & \vdots\\ 0 & -B_{1k} & \cdots & \cdots & B_{Nk}\end{pmatrix},\ \text{for } j = 1,2,\ldots,N.$$

Also, by the T–S fuzzy interpolation method, the $l$th control rule is given as follows:

*Control Rule l:*

$$\begin{aligned}
&\text{if } X_{1,1}(t) \text{ is } G_{l,1},\ldots,\ X_{1,g}(t) \text{ is } G_{l,g}\\
&\quad X_{i,1}(t) \text{ is } G_{l,g+1},\ldots,X_{i,g}(t) \text{ is } G_{l,2g}\\
&\text{then}\quad U_1(t) = K_{1l}\tilde{e}_1(t) \text{ for the leader}\\
&\quad\quad U_i(t) = K_{il}\tilde{e}_i(t)\ \text{ for the follower } i = 2,3,\ldots,N
\end{aligned}$$

for $l = 1, 2, \ldots, L$, and the fuzzy tracking controllers for the leader and followers are given as follows:

$$U_1(t) = \sum_{l=1}^{L} h_l(X_1, X_1)K_{1l}\tilde{e}_1(t) \tag{18}$$

$$U_i(t) = \sum_{l=1}^{L} h_l(X_1, X_i)K_{il}\tilde{e}_i(t),\ \text{for } i = 2,3,\ldots,N. \tag{19}$$

By the fuzzy controllers above, the overall stochastic nonlinear multi-UAV shifted team tracking error dynamic system in (17) can be denoted as

$$\begin{aligned}
d\tilde{\mathbf{e}}(t) = \sum_{k=1}^{L} H_k(\bar{X})\sum_{l=1}^{L} H_l(\bar{X})\bigg[&\big(A_k\tilde{\mathbf{e}}(t) + B_kK_l\tilde{\mathbf{e}}(t) + \bar{\mathbf{v}}(t)\big)dt\\
&+\sum_{i=1}^{N} C_{ik}\tilde{\mathbf{e}}(t)dW_i(t) + \sum_{i=1}^{N} D_{ik}\tilde{\mathbf{e}}(t)dN_i(t)\bigg]
\end{aligned} \tag{20}$$

where $K_l = \text{diag}\{0, K_{1l}, K_{2l}, \ldots, K_{Nl}\}$ .

*Remark 3:* In [31], Takagi and Sugeno have proposed the systematic method to build a T–S fuzzy model for nonlinear function by the system identification scheme, that is, the local system matrices can be identified by many kinds of identification methods such as least-square estimation method. Moreover, many studies have proven that the T–S fuzzy model can approximate a continuous function with any degree of accuracy [32]. However, there are still some fuzzy approximation errors in (17) and (20). In this design, the fuzzy approximation error is state dependent and very complex and can be merged into the intrinsic random fluctuation and external disturbance and its effect could be efficiently attenuated by the proposed robust $H_\infty$ team tracking control strategy.

## V. Robust $H_\infty$ Fuzzy Team Tracking Control Design of Multi-UAV Networked System

In the previous section, we have transformed the controller design for the multi-UAV networked system into an equivalent HJI-constraint optimization problem in (16). However, by the difficulties in solving the HJI constraint, there does not have any method to efficiently solve the HJI problem efficiently.

In this section, with the help of the T–S fuzzy interpolation method, the controller design for multi-UAVs networked system is transformed to an equivalent LMIs-constraint optimization problem which can be easily solved by using the MATLAB LMI toolbox. To prove the asymptotic tracking ability of the stochastic nonlinear multi-UAV networked system in (20) by the proposed fuzzy controller, a Lyapunov function for the team tracking system of (20) is selected as

$$V(t) = \tilde{\mathbf{e}}^{\mathrm{T}}(t)P\tilde{\mathbf{e}}(t) \tag{21}$$

where $P = P^{\mathrm{T}} > 0$ is the positive-definite matrix to be designed.

*Theorem 3:* In the stochastic nonlinear multi-UAV networked tracking system in (20), if we can find the matrices $W = W^{\mathrm{T}} > 0$ and $\{Y_l\}_{l=1}^{L}$ as the solution of the following LMIs:

$$\begin{bmatrix} \Phi_{lk} & WC_{1k}^{\mathrm{T}} & \cdots & WC_{Nk}^{\mathrm{T}} & WD_{1k}^{\mathrm{T}} & \cdots & WD_{Nk}^{\mathrm{T}} \\ * & -W & 0 & 0 & 0 & 0 & 0 \\ * & * & \ddots & 0 & 0 & 0 & 0 \\ * & * & * & -W & 0 & 0 & 0 \\ * & * & * & * & -\lambda_1^{-1}W & 0 & 0 \\ * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & -\lambda_N^{-1}W \end{bmatrix} < 0 \tag{22}$$

for $k, l = 1, 2, \ldots, L$, where $\Phi_{lk} = WA_k^{\mathrm{T}} + A_kW + B_kY_l + (B_kY_l)^{\mathrm{T}} + \sum_{i=1}^{N}\lambda_i(WD_{ik}^{\mathrm{T}} + D_{ik}W)$, then the stochastic nonlinear multi-UAV networked system (9) without external disturbance could achieve the asymptotic tracking in probability of the reference model (5) by the proposed fuzzy controller $K_l = Y_lW^{-1}$ in (18) and (19) in a leader–follower formation $e_d$.

*Proof:* Please refer to the online resource in [36]. ■

On the other hand, while the effect of external disturbance is taken into consideration, the design purpose of the fuzzy controller in this article is to specify the fuzzy controller in (18) and (19) for the multi-UAV networked system to achieve the robust $H_\infty$ tracking performance in (15) under the effect of the external disturbance. That is, the design objective is to specify $\mathbf{U}(t)$ to attenuate the effect of any external disturbance $\bar{\boldsymbol{v}}(t)$ on the tracking error $\tilde{\mathbf{e}}(t)$ below a desired level $\rho^2$ from the viewpoint of energy. Similar to Theorem 3, the robust fuzzy controller design problem is immediately transformed to an equivalent LMI-constrained optimization problem with the help of the T–S fuzzy approximation approach.

*Theorem 4:* In the stochastic nonlinear multi-UAV networked team formation tracking system (20), if we can find the matrices $P = P^{\mathrm{T}} > 0$ and $\{Y_l\}_{l=1}^{L}$ as solution of the following LMIs:

$$\begin{bmatrix} \Psi_{kl} & W & WC_{1k}^{\mathrm{T}} & \cdots & WC_{Nk}^{\mathrm{T}} & WD_{1k}^{\mathrm{T}} & \cdots & WD_{Nk}^{\mathrm{T}} \\ * & -Q^{-1} & 0 & 0 & 0 & 0 & & 0 \\ * & * & -W & & & & & 0 \\ * & * & * & \ddots & & & & 0 \\ * & * & * & * & -W & & & 0 \\ * & * & * & * & * & -\lambda_1^{-1}W & & 0 \\ * & * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & * & -\lambda_N^{-1}W \end{bmatrix} \leq 0 \tag{23}$$

for $k, l = 1, 2, \ldots, L$, where $W = P^{-1}$, $\Psi_{kl} = (1/\rho^2)I + A_kW + WA_k^{\mathrm{T}} + B_kY_l + Y_l^{\mathrm{T}}B_k^{\mathrm{T}} + \sum_{i=1}^{N}\lambda_i(WD_{ik}^{\mathrm{T}} + D_{ik}W)$, then the $H_\infty$ tracking control performance in (15) is guaranteed with a prescribed level $\rho^2$ by the proposed fuzzy controller $K_l = Y_lW^{-1}$ in (18) and (19) for all possible $\bar{\boldsymbol{v}}(t) \in \mathcal{L}_{\mathcal{F}}^2(\mathbb{R}^+, \mathbb{R}^{12(N+1)})$. In the case $\bar{\boldsymbol{v}}(t) = 0$, the asymptotic tracking is also achieved in probability by the proposed fuzzy controller (18) and (19), that is, $E[\tilde{\mathbf{e}}^{\mathrm{T}}(t)\tilde{\mathbf{e}}(t)] \to 0$, as $t \to \infty$.

*Proof:* Please refer to the online resource in [36]. ■

In order to obtain a better robust $H_\infty$ team tracking performance, the optimally robust $H_\infty$ team tracking control problem of the multi-UAV networked system is formulated as the following minimization problem, that is, we can achieve the optimal $H_\infty$ robust team tracking performance in (15) to efficiently attenuate the effect of random fluctuations and external disturbances as possible

$$\min_{W,\{Y_l\}_{l=1}^{L}} \rho^2$$
$$\text{subject to } W = W^{\mathrm{T}} > 0 \text{ and (23).} \tag{24}$$

The minimization problem (24) subject to the LMIs in (23) , is also called an eigenvalue problem (EVP) [35] which could be solved by decreasing $\rho$ until no solution $W > 0$ for the LMIs in (23). Based on the analyses above, the design procedure for the $H_\infty$ fuzzy team tracking control of stochastic multi-UAV networked system is summarized as follows.

*The Design Procedure of Optimal $H_\infty$ Robust Leader–Follower Team Tracking Control for Stochastic Multi-UAV Networked System:*

*Step 1:* Augment the leader UAV dynamic networked system in (4) and the follower UAV dynamic systems in (3) to an augmented system in (6).

*Step 2:* Transform the augmented system to a team tracking error dynamic system (8) between the leader and the desired trajectory with the relative position $e_d$ among the followers and the leader.

*Step 3:* Construct the T–S fuzzy system (20) with the T–S fuzzy identification method to approximate the nonlinear shifted team tracking error dynamic system by interpolating several local linearized systems. Then, we can obtain the LMIs in (23) for the LMI-constrained optimization problem in (24) for the optimal $H_\infty$ robust leader–follower team tracking design.

*Step 4:* Solve the EVP in (24) to obtain $W$ and $Y_l$, for all $l$ (thus $W = P^{-1}$, $K_l = Y_lW^{-1}$ can also be obtained) to obtain fuzzy controller gain $K_l$ in each local linearized system.

*Step 5:* Construct the fuzzy control law $\mathbf{U}(t) = \sum_{l=1}^{L}H_l(\bar{X})\bar{K}_l\tilde{\mathbf{e}}(t)$ by interpolating several local linearized fuzzy control laws. The fuzzy control law consists of the leader's control law (18) and the followers' control law (19).

## VI. Design Example

To illustrate the design procedure and confirm the effectiveness of the proposed robust controller, a robust $H_\infty$ leader–follower team tracking control scheme is proposed to deal with the formation of the practical quadrotor unmanned aerial vehicle (UAV) networked system consisted of one leader and four followers with continuous and discontinuous random

TABLE I
PARAMETERS OF QUADRATOR UAV

| Symbol | Value | Unit |
|---|---|---|
| $m$ | 2 | kg |
| $J_x$=$J_y$ | 1.25 | Ns$^2$/rad |
| $J_z$ | 2.2 | Ns$^2$/rad |
| $K_x$=$K_y$=$K_z$ | 0.01 | Ns/m |
| $K_\varphi$=$K_\theta$=$K_\psi$ | 0.012 | Ns/m |
| $l$ | 0.2 | m |
| $b$ | 2 | Ns$^2$ |
| $d$ | 5 | Nms$^2$ |
| $g$ | 9.81 | m/s$^2$ |

fluctuations in this design example. The stochastic shifted team tracking error dynamic model of the UAV team is shown in (9), and the parameters of each UAV of the networked system are listed in Table I [33].

To ensure the flight of UAVs is smooth and fast in practical applications, the design of attitude reference is a very important issue. In the designing of the attitude reference, we should consider not only the distance to the desired position but also the velocity of the UAV. For each UAV, the desired roll reference and the desired pitch reference are, respectively, constrained by the distance $e_x = x - x_d$ , $e_y = y - y_d$, and $e_z = z- z_d$ and the velocity $\dot{e}_x = \dot{x} - \dot{x}_d$, $\dot{e}_y = \dot{y} - \dot{y}_d$, and $\dot{e}_z = \dot{z} - \dot{z}_d$ while the desired yaw trajectory is set by the operator. The desired roll reference and desired pitch reference can be described as

$$\begin{aligned}\phi_d &= \sin^{-1}\left(\frac{m}{F(t)}\left(u_x \sin\psi_d - u_y \cos\psi_d\right)\right)\\ \theta_d &= \tan^{-1}\left(\frac{1}{u_z+g}\left(u_x \cos\psi_d + u_y \sin\psi_d\right)\right) \qquad (25)\end{aligned}$$

where $F(t)$ denotes total thrust, $u_x = a_1 e_x + a_2\dot{e}_x, u_y = b_1 e_y + b_2\dot{e}_y$, and $u_z = c_1 e_z + c_2\dot{e}_z$ are the force vectors of $x$-, $y$-, $z$-axis, respectively. By the roll and pitch angle planning scheme, the converging speed can be much faster than the scheme without roll and pitch angle planning. In this design example, we set $[a_1\ a_2\ b_1\ b_2\ c_1\ c_2] = [105105105]$. To make the control scheme much clear, the block diagram of control flowchart is shown in Fig. 3. In this simulation, we set the number of the followers to be four, and the team formation $e_d$, that is, the relative positions between the leader and four followers are set to be (0, 0, 0), (+0.1, +0.1, 0), (+0.1, −0.1, 0), (−0.1, +0.1, 0), and (− 0.1, −0.1, 0), respectively.

In this simulation example, the Wiener process is used to model the system parameter variations of the UAVs networked system and compensate the approximation error of fuzzy model. On the other hand, there exist some discontinuous fluctuation in the UAVs network system, such as drop out of data transmission, deformation and fault of mechanical elements in UAV, etc. Since the simulation has five independent UAVs, including a leader and four followers, we set five independent Poisson jump processes $\{P_i(t)\}_{i=1}^5$ and five independent Wiener processes $\{W_i(t)\}_{i=1}^5$ in this stochastic multi-UAV networked system. The following stochastic functions are used to mimic the aforementioned fluctuations on the three position velocities and accelerations as well as three attitude angular velocities and angular accelerations in the UAVs networked system

$$\begin{aligned}
\tilde{\sigma}_1(\tilde{\mathbf{e}}(t)) &= 0.01 * [0_{12\times1}, c_1(\tilde{\mathbf{e}}(t)), 0_{48\times1}]^{\mathrm{T}}\\
\tilde{\sigma}_2(\tilde{\mathbf{e}}(t)) &= 0.01 * [0_{24\times1}, c_2(\tilde{\mathbf{e}}(t)), 0_{36\times1}]^{\mathrm{T}}\\
\tilde{\sigma}_3(\tilde{\mathbf{e}}(t)) &= 0.01 * [0_{36\times1}, c_3(\tilde{\mathbf{e}}(t)), 0_{24\times1}]^{\mathrm{T}}\\
\tilde{\sigma}_4(\tilde{\mathbf{e}}(t)) &= 0.01 * [0_{48\times1}, c_4(\tilde{\mathbf{e}}(t)), 0_{12\times1}]^{\mathrm{T}}\\
\tilde{\sigma}_5(\tilde{\mathbf{e}}(t)) &= 0.01 * [0_{60\times1}, c_5(\tilde{\mathbf{e}}(t))]^{\mathrm{T}}\\
\tilde{\Gamma}_1(\tilde{\mathbf{e}}(t)) &= 0.1 * [0_{12\times1}, d_1(\tilde{\mathbf{e}}(t)), 0_{48\times1}]^{\mathrm{T}}\\
\tilde{\Gamma}_2(\tilde{\mathbf{e}}(t)) &= 0.1 * [0_{24\times1}, d_2(\tilde{\mathbf{e}}(t)), 0_{36\times1}]^{\mathrm{T}}\\
\tilde{\Gamma}_3(\tilde{\mathbf{e}}(t)) &= 0.1 * [0_{36\times1}, d_3(\tilde{\mathbf{e}}(t)), 0_{24\times1}]^{\mathrm{T}}\\
\tilde{\Gamma}_4(\tilde{\mathbf{e}}(t)) &= 0.1 * [0_{48\times1}, d_4(\tilde{\mathbf{e}}(t)), 0_{12\times1}]^{\mathrm{T}}\\
\tilde{\Gamma}_5(\tilde{\mathbf{e}}(t)) &= 0.1 * [0_{60\times1}, d_5(\tilde{\mathbf{e}}(t))]^{\mathrm{T}}
\end{aligned}$$

where

$$\begin{aligned}
c_1(\tilde{\mathbf{e}}(t)) = \Big[&0, -\frac{K_x}{m}x_2^1, 0, -\frac{K_y}{m}y_2^1, 0, -\frac{K_z}{m}z_2^1, 0, \frac{J_y - J_z}{J_x}\theta_1^1\psi_1^1 - \frac{K_\phi}{J_x}\phi_2^1, 0\\
&0, \frac{J_z - J_x}{J_y}\phi_1^1\psi_1^1 - \frac{K_\theta}{J_y}\theta_2^1, 0, \frac{J_x - J_y}{J_z}\phi_1^1\theta_1^1 - \frac{K_\psi}{J_z}\psi_2^1\Big]^{\mathrm{T}}\\
c_i(\tilde{\mathbf{e}}(t)) = \Big[&0, -\frac{K_x}{m}\left(x_2^i - x_2^1\right), 0, -\frac{K_y}{m}\left(y_2^i - y_2^1\right), 0, -\frac{K_z}{m}\left(z_2^i - z_2^1\right)\\
&0, \frac{J_y - J_z}{J_x}\left(\theta_1^i - \theta_1^1\right)\left(\psi_1^i - \psi_1^1\right) - \frac{K_\phi}{J_x}\left(\phi_2^i - \phi_2^1\right)\\
&0, \frac{J_z - J_x}{J_y}\left(\phi_1^i - \phi_1^1\right)\left(\psi_1^i - \psi_1^1\right) - \frac{K_\theta}{J_y}\left(\theta_2^i - \theta_2^1\right)\\
&0, \frac{J_x - J_y}{J_z}\left(\phi_1^i - \phi_1^1\right)\left(\theta_1^i - \theta_1^1\right) - \frac{K_\psi}{J_z}\left(\psi_2^i - \psi_2^1\right)\Big]^{\mathrm{T}}\\
d_1(\tilde{\mathbf{e}}(t)) = \Big[&0, -\frac{K_x}{m}x_2^1, 0, -\frac{K_y}{m}y_2^1, 0, -\frac{K_z}{m}z_2^1, 0, \frac{J_y - J_z}{J_x}\theta_1^1\psi_1^1 - \frac{K_\phi}{J_x}\phi_2^1\\
&0, 0, \frac{J_z - J_x}{J_y}\phi_1^1\psi_1^1 - \frac{K_\theta}{J_y}\theta_2^1, 0, \frac{J_x - J_y}{J_z}\phi_1^1\theta_1^1 - \frac{K_\psi}{J_z}\psi_2^1\Big]^{\mathrm{T}}\\
d_i(\tilde{\mathbf{e}}(t)) = \Big[&0, -\frac{K_x}{m}\left(x_2^i - x_2^1\right), 0, -\frac{K_y}{m}\left(y_2^i - y_2^1\right), 0, -\frac{K_z}{m}\left(z_2^i - z_2^1\right)\\
&0, \frac{J_y - J_z}{J_x}\left(\theta_1^i - \theta_1^1\right)\left(\psi_1^i - \psi_1^1\right) - \frac{K_\phi}{J_x}\left(\phi_2^i - \phi_2^1\right)\\
&0, \frac{J_z - J_x}{J_y}\left(\phi_1^i - \phi_1^1\right)\left(\psi_1^i - \psi_1^1\right) - \frac{K_\theta}{J_y}\left(\theta_2^i - \theta_2^1\right)\\
&0, \frac{J_x - J_y}{J_z}\left(\phi_1^i - \phi_1^1\right)\left(\theta_1^i - \theta_1^1\right) - \frac{K_\psi}{J_z}\left(\psi_2^i - \psi_2^1\right)\Big]^{\mathrm{T}}\\
&\text{for } i = 2, \ldots, 5.
\end{aligned}$$

In this design example, the state variables of $X_{i,1} = \phi_1^i, X_{i,2} = \theta_1^i$ are chosen as premise variables and it is assumed that these state variables are available, for $i = 1,, \ldots, 5$ . Since there are three fuzzy sets associated with each premise variable, totally there are 6561 fuzzy IF–THEN rules in the multiteam UAV system. The operation points of the T–S fuzzy model and their linearized local linear stochastic models are shown as follows.

The three operation points of $X_{i,1}$ are given as $op_1^1 = -\pi/6$, $op_1^2 = 0$, and $op_1^3 = \pi/6$, for $i = 1, 2, \ldots, 5$, and the other three operation points of $X_{i,2}$ are also given as $op_2^1 = -\pi/6$, $op_2^2 = 0$, and $op_2^3 = \pi/6$, for $i = 1, 2, \ldots, 5$.

The $q$th rule of this T–S fuzzy model for the shifted stochastic nonlinear multi-UAV team tracking error dynamic system in (9) is described as

$$\text{if } X_{i,1} \text{ is } op_1^{q_{i,1}} \text{ and } X_{i,2} \text{ is } op_2^{q_{i,2}}, \text{ for } i = 1, 2, \ldots, 5$$

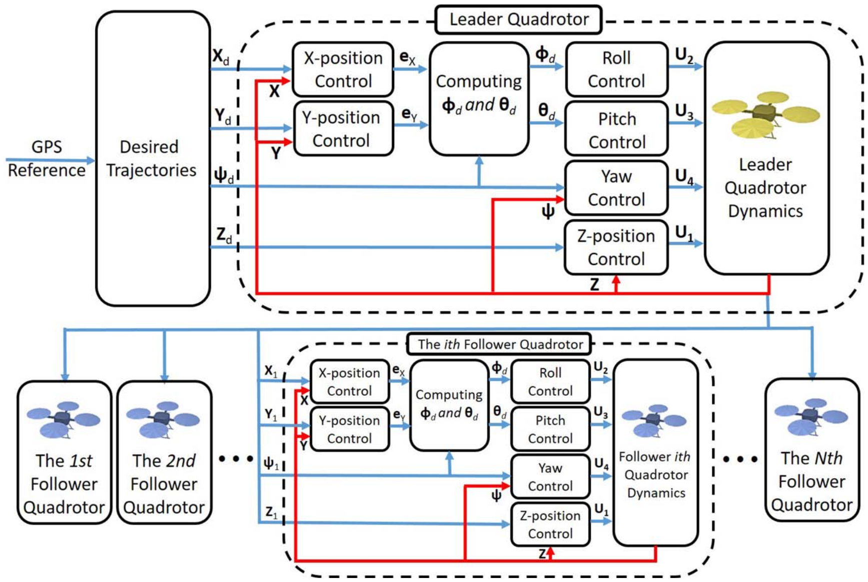



Fig. 3. Structure of leader–follower team tracking control scheme for the multiquadrotor UAVs networked system where the red line denotes the information flow inside the UAV and the blue line denotes the information flow between the UAVs.

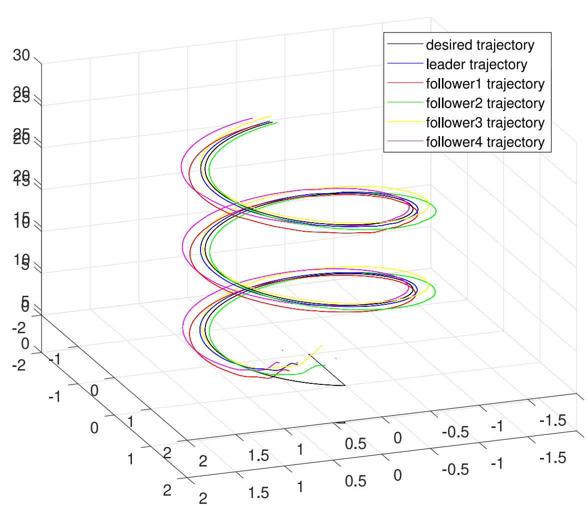



Fig. 4. 3-D graph shows that the actual flight path of the leader quadrotor UAV and the four follower quadrotor UAVs. In this figure, the leader's path trajectory converges to the desired trajectory quickly. Also, the leader–follower formation is achieved, that is, each follower successfully tracks the leader's path trajectory with a specific formation distance.

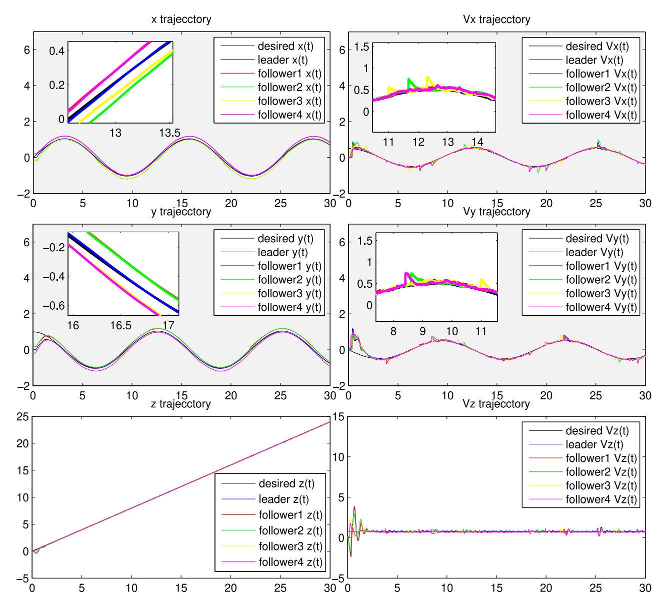



Fig. 5. Position trajectories of the leader quadrotor UAV and four follower quadrotor UAVs. In this figure, the proposed controller can effectively ensure the robust team formation tracking performance of each quadrotor UAV to achieved their desired tracking goal.

$$\text{then } d\tilde{e}(t) = \left(A_q\tilde{e}(t) + B_qU_q(t) + \bar{\boldsymbol{v}}(t)\right)dt + \sum_{i=1}^{5} C_{iq}\tilde{e}(t)dW_i(t) + \sum_{i=1}^{5} D_{iq}\tilde{e}(t)dN_i(t)$$

where $A_q, B_q, C_{iq}, D_{iq}$, for $i = 1, 2, \ldots, 5$, are obtained by fuzzy system identification with the help of MATLAB system identification toolbox.

Suppose the $H_\infty$ team tracking performance in (15) is used to design the multi-UAV networked system with the following weighting matrices. In this simulation example, to achieve the desired team formation tracking performance, the designer considers the position tracking performance more important than the velocity tracking control performance. Consequently, for the proposed weighting matrices, the weighting on the position trackings of each UAV is greater than the weighting on the velocity trackings of each UAV. The weighting matrices in this simulation are given as follows:

$$\begin{aligned} Q &= \mathrm{diag}\{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\} \\ Q_i &= 0.01\mathrm{diag}\{1, 0.001, 1, 0.001, 1, 0.001, \\ &\qquad 1, 0.001, 1, 0.001, 1, 0.001\} \\ &\text{for } i = 1, 2, \ldots, 6. \end{aligned}$$

In the reference model in (5), let $A_r = -I_{12\times12}$ and select the desired steady-state position trajectory and desired yaw

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.



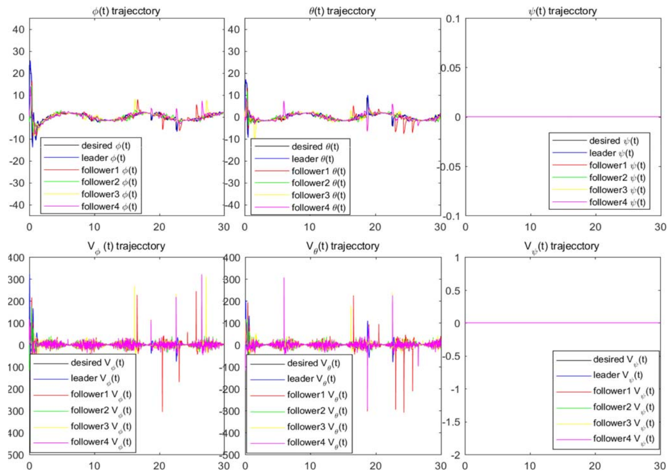
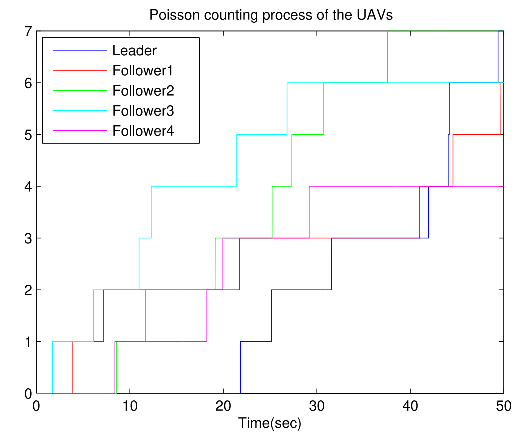
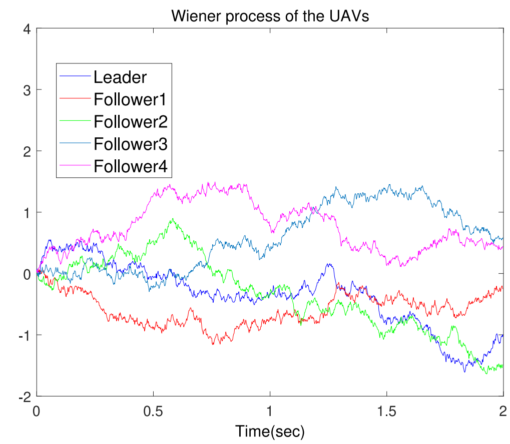



Fig. 6. Attitude trajectories of the leader quadrotor UAV and four follower quadrotor UAVs. The intense changes of the attitude are caused by the change of position and velocity due to intrinsic fluctuations of Wiener and Poisson processes.




(a)




(b)

Fig. 7. Stochastic random processes of the UAVs. (a) Poisson counting processes of the UAVs with intensity $\lambda_i = 0.1$. (b) Wiener processes of the UAVs.

angle trajectory as

$$x_d(t) = \sin(0.5t), y_d(t) = \cos(0.5t)$$
$$z_d(t) = 0.8t\ ,\ \ \psi_d(t) = 0.$$

The initial position of the leader and the followers is selected as

$$(0, 0, 0), (+0.2, +0.2, 0), (+0.2, -0.2, 0)$$
$$(-0.2, +0.2, 0),\ \ (-0.2, -0.2, 0)$$

and the external disturbances are set to be zero mean Gaussian noises with unit variance.

The simulation results are shown in Figs. 4–7. In Fig. 4, the 3-D trajectories graph shows that the actual flight path of the leader quadrotor UAV can track the trajectory of desired position quickly, and four follower quadrotor UAVs can also track the trajectory of leader quadrotor UAV position with the leader–follower formation quickly despite the external disturbances, intrinsic continuous Wiener fluctuation, and discontinuous Poisson jump fluctuation. The team of five quadrotor UAVs, including a leader and four followers, can complete the task of spiral upward maneuvering flight under the reference model tracking. It can be seen from Figs. 5 and 6 that the UAVs state trajectories of velocity and angular velocity could achieved the tracking performance under the influence of the continuous random Wiener fluctuations, discontinuous Poisson jump fluctuations, and external disturbances. In Fig. 7, we show the trajectories of discontinuous random fluctuations due to the Poisson jump counting process, and the continuous random fluctuations due to the Weiner process for each UAV system.

## VII. Conclusion

In this article, the stochastic robust $H_\infty$ leader–follower team tracking control design was proposed to deal with the stochastic nonlinear multi-UAV networked system. To make the UAVs be arranged in a fixed leader–follower formation simultaneously, a shifted team tracking error dynamic system was introduced to simplify the design procedure. By the proposed nonlinear reference tracking error dynamic system, the stochastic robust $H_\infty$ leader–follower team tracking control problem could be transformed to an equivalent HJI-constraint optimization problem. By the difficulties in solving HJI constraint for the controller design, the HJI-constrained optimization problem is replaced by an LMIs-constrained optimization problem by the stochastic T–S fuzzy identification method. Hence, the stochastic robust $H_\infty$ leader–follower team tracking control problem of the stochastic nonlinear multi-UAV networked system can be solved efficiently with the help of the MATLAB LMI-toolbox to simplify the design procedure. Finally, a design example of five UAVs team is given to validate the effectiveness of the proposed robust $H_\infty$ leader–follower team tracking control design of the stochastic nonlinear multi-UAV networked system. From the simulation results, the proposed stochastic robust $H_\infty$ leader–follower team tracking control can make both the leader and the followers robustly track the desired trajectories of attitude and path in a fixed leader–follower formation under the unknown external disturbances and intrinsic system fluctuations. Under the concept of Internet of Things (IoT) in future smart city, the multi-UAV networked control system can be connected with a ground computing unit to reduce the computational complexity of the multi-UAV networked control system. In this framework, the ground computing unit can receive the information of multiple UAVs and the suitable control strategy needs to be designed for the multiple UAVs through the wireless network to accomplish the team formation task. On the other hand, under the network-based structure, the ground computing unit may receive channel interferences or hostile attack signals from the wireless network. Since the interferences and attack signals will deteriorate the performance of the designed control strategy, the estimation and elimination of the aforementioned signals become a major issue of fault-tolerant design in the wireless network-based control problem. Hence, the future researches will focus on the network-based UAV team formation tracking problem and network-based UAV fault-tolerant control problem.

## Appendix A
## Proof of Theorem 1

By applying Lemma 1 in this article, we have

$$\begin{aligned} E\{dV(\tilde{\mathbf{e}}(t))\} &= E\{V_{\tilde{\mathbf{e}}}^T(\tilde{f}(\tilde{\mathbf{e}}(t)) + \tilde{g}(\tilde{\mathbf{e}}(t))\mathbf{K}(\tilde{\mathbf{e}}(t))) \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\tilde{\sigma}_i(\tilde{\mathbf{e}}(t))V_{\tilde{\mathbf{e}}\tilde{\mathbf{e}}}\tilde{\sigma}_i(\tilde{\mathbf{e}}(t)) + \sum_{i=1}^{N}\lambda_i\{V(\tilde{\mathbf{e}}(t) \\ &\quad + \tilde{\Gamma}_i(\tilde{\mathbf{e}}(t))) - V(\tilde{\mathbf{e}}(t))\}\}dt. \end{aligned} \tag{26}$$

If the inequality in (14) holds, then we can conclude

$$E\{dV(\tilde{\mathbf{e}}(t))\} \le E\left\{-m_3\|\tilde{\mathbf{e}}(t)\|_2^2\right\}dt \tag{27}$$

for some $m_3 > 0$. By using inequality (13), we obtain

$$\frac{dE\{V(\tilde{\mathbf{e}}(t))\}}{dt} \le \frac{-m_3}{m_2}E\{V(\tilde{\mathbf{e}}(t))\}. \tag{28}$$

Based on the inequality in (28), we have

$$E\{V(\tilde{\mathbf{e}}(t))\} \le E\{V(\tilde{\mathbf{e}}(0))\}\exp\left(\frac{-m_3 t}{m_2}\right) \tag{29}$$

and from (29), it implies

$$E\left\{\|\tilde{\mathbf{e}}(t)\|_2^2\right\} \le E\left\{\frac{V(\tilde{\mathbf{e}}(0))}{m_1}\right\}\exp\left(\frac{-m_3 t}{m_2}\right) \tag{30}$$

so that

$$\lim_{t\to\infty} E\left\{\|\tilde{\mathbf{e}}(t)\|_2^2\right\} = 0 \tag{31}$$

Also, the asymptotically stable of stochastic nonlinear multi-UAV shifted tracking error dynamic system implies that $\tilde{\mathbf{e}}(t) \to e_d$ in probability, that is, the leader–follower formation can be achieved. The proof is done.

## Appendix B
## Proof of Theorem 2

It is clear that

$$\begin{aligned} &E\left\{\int_0^{t_f}\tilde{\mathbf{e}}^{\mathrm{T}}(t)Q\tilde{\mathbf{e}}(t)dt\right\} \\ &= E\{V(\tilde{\mathbf{e}}_0)\} - E\{V(\tilde{\mathbf{e}}(t_f))\} \\ &\quad + E\left\{\int_0^{t_f}\tilde{\mathbf{e}}^{\mathrm{T}}(t)Q\tilde{\mathbf{e}}(t)dt + dV(\tilde{\mathbf{e}}(t))\right\} \le E\{V(\tilde{\mathbf{e}}_0)\} \\ &\quad + E\left\{\int_0^{t_f}\tilde{\mathbf{e}}^{\mathrm{T}}(t)Q\tilde{\mathbf{e}}(t)dt + dV(\tilde{\mathbf{e}}(t))\right\}. \end{aligned} \tag{32}$$

By applying Lemma 1 in this article, we have

$$\begin{aligned} &E\left\{\int_0^{t_f}\tilde{\mathbf{e}}^{\mathrm{T}}(t)Q\tilde{\mathbf{e}}(t)dt\right\} \\ &\le E\{V(\tilde{\mathbf{e}}_0)\} + E\left\{\int_0^{t_f}\tilde{\mathbf{e}}^{\mathrm{T}}(t)Q\tilde{\mathbf{e}}(t)\right. \\ &\quad + V_{\tilde{\mathbf{e}}}^T\tilde{f}(\tilde{\mathbf{e}}(t)) + V_{\tilde{\mathbf{e}}}^T\tilde{g}(\tilde{\mathbf{e}}(t))\mathbf{K}(\tilde{\mathbf{e}}(t)) + V_{\tilde{\mathbf{e}}}^T\bar{\boldsymbol{v}}(t) + \frac{1}{2}\sum_{i=1}^{N}\tilde{\sigma}_i^{\mathrm{T}}(\tilde{\mathbf{e}}(t)) \\ &\quad \left.\times V_{\tilde{\mathbf{e}}\boldsymbol{Q}}\tilde{\sigma}_i(\tilde{\mathbf{e}}(t)) + \sum_{i=1}^{N}\lambda_i\left\{V(\tilde{\mathbf{e}}(t) + \tilde{\Gamma}_i(\tilde{\mathbf{e}}(t))) - V(\tilde{\mathbf{e}}(t))\right\}dt\right\}. \end{aligned} \tag{33}$$

By using Lemma 2 in this article with $M_1^{\mathrm{T}} = (1/2)V_{\tilde{\mathbf{e}}}^T$, $M_2^{\mathrm{T}} = \bar{\boldsymbol{v}}(t)$ and $Z = \rho^2 I$, the term $V_{\tilde{\mathbf{e}}}^T\bar{\boldsymbol{v}}(t)$ from (33) can be bounded as follows:

$$\begin{aligned} V_{\tilde{\mathbf{e}}}^T\bar{\boldsymbol{v}}(t) &= \frac{1}{2}V_{\tilde{\mathbf{e}}}^T\bar{\boldsymbol{v}}(t) + \frac{1}{2}\bar{\boldsymbol{v}}^{\mathrm{T}}(t)V_{\tilde{\mathbf{e}}} \\ &\le \frac{1}{4\rho^2}V_{\tilde{\mathbf{e}}}^T V_{\tilde{\mathbf{e}}} + \rho^2\bar{\boldsymbol{v}}^{\mathrm{T}}(t)\bar{\boldsymbol{v}}(t). \end{aligned} \tag{34}$$

Replacing the term $V_{\tilde{\mathbf{e}}}^T\bar{\boldsymbol{v}}(t)$ in (33) by inequality (34), we obtain the following inequality:

$$\begin{aligned} &E\left\{\int_0^{t_f}\tilde{\mathbf{e}}^{\mathrm{T}}(t)Q\tilde{\mathbf{e}}(t)\right\} \\ &\le E\{V(\tilde{\mathbf{e}}_0)\} + E\left\{\int_0^{t_f}\tilde{\mathbf{e}}^{\mathrm{T}}(t)Q\tilde{\mathbf{e}}(t) + V_{\tilde{\mathbf{e}}}^T\tilde{f}(\tilde{\mathbf{e}}(t))\right. \\ &\qquad + V_{\tilde{\mathbf{e}}}^T\tilde{g}(\tilde{\mathbf{e}}(t))\mathbf{K}(\tilde{\mathbf{e}}(t)) + \frac{1}{4\rho^2}V_{\tilde{\mathbf{e}}}^T V_{\tilde{\mathbf{e}}} \\ &\qquad + \rho^2\bar{\boldsymbol{v}}^{\mathrm{T}}(t)\bar{\boldsymbol{v}}(t) + \frac{1}{2}\sum_{i=1}^{N}\tilde{\sigma}_i^{\mathrm{T}}(\tilde{\mathbf{e}}(t))V_{\tilde{\mathbf{e}}\tilde{\mathbf{e}}}\tilde{\sigma}_i(\tilde{\mathbf{e}}(t)) \\ &\qquad \left.+ \sum_{i=1}^{N}\lambda_i\left\{V\left(\tilde{\mathbf{e}}(t) + \tilde{\Gamma}_i(\tilde{\mathbf{e}}(t))\right) - V(\tilde{\mathbf{e}}(t))\right\}dt\right\}. \end{aligned} \tag{35}$$

If the following equation, HJI is satisfied:

$$\begin{aligned} &\tilde{\mathbf{e}}^{\mathrm{T}}(t)Q\tilde{\mathbf{e}}(t) + V_{\tilde{\mathbf{e}}}^T\tilde{f}(\tilde{\mathbf{e}}(t)) + V_{\tilde{\mathbf{e}}}^T\tilde{g}(\tilde{\mathbf{e}}(t))\mathbf{K}(\tilde{\mathbf{e}}(t)) \\ &\quad + \frac{1}{4\rho^2}V_{\tilde{\mathbf{e}}}^T V_{\tilde{\mathbf{e}}} + \frac{1}{2}\sum_{i=1}^{N}\tilde{\sigma}_i^{\mathrm{T}}(\tilde{\mathbf{e}}(t))V_{\tilde{\mathbf{e}}\tilde{\mathbf{e}}}\tilde{\sigma}_i(\tilde{\mathbf{e}}(t)) \\ &\quad + \sum_{i=1}^{N}\lambda_i\left\{V\left(\tilde{\mathbf{e}}(t) + \tilde{\Gamma}_i(\tilde{\mathbf{e}}(t))\right) - V(\tilde{\mathbf{e}}(t))\right\} \le 0 \end{aligned} \tag{36}$$

then we immediately have

$$E\left\{\int_0^{t_f}\tilde{\mathbf{e}}^{\mathrm{T}}(t)Q\tilde{\mathbf{e}}(t)\right\} \le E\{V(\tilde{\mathbf{e}}_0)\} + E\left\{\int_0^{t_f}\rho^2\bar{\boldsymbol{v}}^{\mathrm{T}}(t)\bar{\boldsymbol{v}}(t)\right\}$$

that is, $J_\infty(\mathbf{U}(t)) \le \rho^2$ is satisfied.

If the external disturbance is vanished in the system, that is, $\bar{\mathbf{v}}(t) = 0$, from (35) and (36), we immediately have

$$E\left\{\int_0^{t_f} \tilde{\mathbf{e}}^{\mathrm{T}}(t) Q \tilde{\mathbf{e}}(t) dt\right\} \leq E\left\{V\left(\tilde{\mathbf{e}}_0\right)\right\} \quad \forall t_f \in [0, \infty]. \tag{37}$$

Since (37) shows the total energy of $\tilde{\mathbf{e}}(t)$ on the interval $[0, \infty]$ is bounded, we have $E[\tilde{\mathbf{e}}^{\mathrm{T}}(t)\tilde{\mathbf{e}}(t)] \to 0$, as $t \to \infty$, that is, the asymptotic tracking for the multi-UAVs networked system is achieved. The proof is completed.

## References


[1] M. A. Ma'sum *et al.*, "Simulation of intelligent unmanned aerial vehicle (UAV) for military surveillance," in *Proc. Int. Conf. Adv. Comput. Sci. Inf. Syst. (ICACSIS)*, 2013, pp. 161–168.

[2] D. H. Lyon, "A military perspective on small unmanned aerial vehicles," *IEEE Instrum. Meas. Mag.*, vol. 7, no. 3, pp. 27–31, Sep. 2004.

[3] N. Nigam, "The multiple unmanned air vehicle persistent surveillance problem: A review," *Machines*, vol. 2, no. 1, pp. 13–72, Jan. 2014.

[4] J. J. Acevedo, B. C. Arrue, I. Maza, and A. Ollero, "Distributed approach for coverage and patrolling missions with a team of heterogeneous aerial robots under communication constraints," *Int. J. Adv. Robot. Syst.*, vol. 10, no. 28, pp. 13–72, Jan. 2014.

[5] J. J. Acevedo, B. C. Arrue, J. M. Diaz-Bañez, I. Ventura, I. Maza, and A. Ollero, "One-to-one coordination algorithm for decentralized area partition in surveillance missions with a team of aerial robots," *J. Intell. Robot. Syst.*, vol. 74, nos. 1–2, pp. 269–285, Apr. 2014.

[6] P. Pounds, R. Mahony, and P. Corke, "Modelling and control of a large quadrotor robot," *Control Eng. Pract.*, vol. 18, no. 7, pp. 691–699, Feb. 2010.

[7] T. Luukkonen, *Modelling and Control of Quadcopter*. School Sci., Aalto Univ., Espoo, Finland, Aug. 2011.

[8] T. Bresciani, *Modelling, Identification and Control of a Quadrotor Helicopter*, Dept. Autom. Control, Lund Univ., Lund, Sweden, Oct. 2008.

[9] L. M. Argentim, W. C. Rezende, P. E. Santos, and R. A. Aguiar, "PID LQR and LQR-PID on a quadcopter platform," in *Proc. IEEE Int. Conf. Inf. Electron. Vis.*, May 2013, pp. 1–6.

[10] M. Herrera, W. Chamorro, A. Gomez, and O. Camacho, "Sliding mode control: An approach to control a quadrotor," in *Proc. IEEE Asia–Pac. Conf. Comput.-Aided Syst. Eng.*, Jul. 2015, pp. 3–4.

[11] T. Madani and A. Benallegue, "Backstepping control for a quadrotor helicopter," in *Proc. IEEE Int. Conf. Intell. Robots Syst.*, Beijing, China, Jan. 2006, pp. 3255–3260.

[12] S. S. Ge, E. Hong, and T. H. Lee, "Robust adaptive control of nonlinear systems with unknown time delays," *Automatica*, vol. 41, no. 7, pp. 1181–1190, Jul. 2005.

[13] R. Lozano, P. Castillo, P. Garcia, and A. Dzul, "Robust prediction-based control for unstable delay systems: Application to the yaw control of a mini-helicopter," *Automatica*, vol. 40, no. 4, pp. 603–612, Apr. 2004.

[14] A. Das, K. Subbarao, and F. Lewis, "Dynamic inversion with zerodynamics stabilisation for quadrotor control," *IET Control Theory Appl.*, vol. 3, no. 3, pp. 303–314, Mar. 2009.

[15] D. J. Bennet, C. R. McInnes, M. Suzuki, and K. Uchiyama, "Autonomous three-dimensional formation flight for a swarm of unmanned aerial vehicles," *J. Guid. Control Dyn.*, vol. 34, no. 6, pp. 1899–1908, Dec. 2011.

[16] M. Defoort, T. Floquet, A. Kokosy, and W. Perruquetti, "Slidingmode formation control for cooperative autonomous mobile robots," *IEEE Trans. Ind. Electron.*, vol. 55, no. 11, pp. 3944–3953, Nov. 2008.

[17] J. R. T. Lawton, R. W. Beard, and B. J. Young, "A decentralized approach to formation maneuvers," *IEEE Trans. Robot. Autom.*, vol. 19, no. 6, pp. 933–941, Dec. 2003.

[18] P. Ogren, M. Egerstedt, and X. Hu, "A control Lyapunov function approach to multiagent coordination," *IEEE Trans. Robot. Autom.*, vol. 18, no. 5, pp. 847–851, Oct. 2002.

[19] W. Yu, G. Chen, W. Ren, J. Kurths, and W. X. Zheng, "Distributed higher order consensus protocols in multiagent dynamical systems," *IEEE Trans. Circuits Syst.*, vol. 58, no. 8, pp. 1924–1932, Aug. 2011.

[20] S.-J. Ho, C.-Y. Chen, and B.-S. Chen, "Analysis for the robust $H_\infty$ synchronization of nonlinear stochastic coupling networks through Poisson processes and core coupling design," *IEEE Trans. Control Netw. Syst.*, vol. 4, no. 2, pp. 223–235, Jun. 2017.

[21] Z. Li, Z. Duan, G. Chen, and L. Huang, "Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 57, no. 1, pp. 213–224, Jun. 2009.

[22] Z. Li, Z. Duan, and L. Huang, "Leader–follower consensus of multiagent systems," in *Proc. Amer. Control Conf.*, St. Louis, MO, USA, Jun. 2009, pp. 3256–3261.

[23] R. Çaglar, S. Ikizoglu, and S. Seker, "Statistical Wiener process model for vibration signals in accelerated aging processes of electric motors," *J. Vibroeng.*, vol. 16, no. 2, pp. 800–807, Mar. 2014.

[24] R. C. L. F. Oliveira, A. N. Vargas, J. B. R. do Val, and P. L. D. Peres, "Mode-independent $H_2$-control of a DC motor modeled as a Markov jump linear system," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 5, pp. 1915–1919, Sep. 2014.

[25] B. Øksendal and A. Sulem, *Applied Stochastic Control of Jump Diffusions*, 2nd ed. Heidelberg, Germany: Springer, 2007.

[26] F. Hanson, *Applied Stochastic Processes and Control for Jump-Diffusions: Modeling, Analysis and Computation*, 2nd ed. Philadelphia, PA, USA: SIAM, 2007.

[27] B. S. Chen, B. K. Lee, and L. B. Guo, "Optimal tracking design for stochastic fuzzy systems," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 6, pp. 796–813, Dec. 2003.

[28] K. Watanabe, "Stochastic fuzzy control. I. Theoretical derivation," in *Proc. IEEE Int. Conf. Fuzzy Syst.*, Yokohama, Japan, Mar. 1995, pp. 547–554.

[29] K. Watanabe, K. Izumi, and F. Han, "Stochastic fuzzy servo control using multiple linear dynamic models," in *Proc. IEEE Int. Conf. Knowl. Based Intell. Electron. Syst.*, Adelaide, SA, Australia, Apr. 1998, pp. 474–482.

[30] C.-S. Tseng, B.-S. Chen, and H.-J. Uang, "Fuzzy tracking control design for nonlinear dynamic systems via T–S fuzzy model," *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 3, pp. 381–392, Jun. 2001.

[31] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. SMC-15, no. 1, pp. 116–132, Feb. 1985.

[32] B. Kosko, "Fuzzy systems as universal approximators," *IEEE Trans. Comput.*, vol. 43, no. 11, pp. 1329–1333, Nov. 1994.

[33] J.-J. Xiong and E.-H. Zheng, "Position and attitude tracking control for a quadrotor UAV," *ISA Trans.*, vol. 53, no. 3, pp. 725–731, May 2014.

[34] Z. Lin, W. Ding, G. Yan, C. Yu, and A. Giua, "Leader–follower formation via complex Laplacian," *Automatica*, vol. 49, no. 6, pp. 1900–1906, Jun. 2013.

[35] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA, USA: SIAM, 1994.

[36] B. S. Chen, C. P. Wang, and M. Y. Lee. *Supplementary Document. Hsinchu, Taiwan*. Accessed: Dec. 31, 2019. [Online]. Available: https://www.dropbox.com/s/tzy7u0uzf7shnw9/ProofofUAVteamformation.pdf?dl=0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# Supplementary File

Bor-Sen Chen, *Life Fellow, IEEE,*, Chun-Ping Wang and Min-Yen Lee

***Abstract*—In this supplementary file, the proofs of Theorem 3 and 4 in the paper "Stochastic Robust Team Tracking Control of Multi-UAV Networked System under Wiener and Poisson Random Fluctuations" are given.**

### Appendix C- Proof of Theorem 3

From the result of Theorem 1, if the HJI in (14) is satisfied, the stochastic nonlinear multi-UAV networked system achieves asymptotic tracking in probability. However, the HJI is hard to be solved. We can transform the HJI to a set of LMIs by using the T-S fuzzy model proposed in the Section IV. With the T-S fuzzy model in (20) and Lemma 2, we can get the following equalities:

$$V_{\tilde{\mathbf{e}}}{}^T \tilde{f}(\tilde{\mathbf{e}}(t)) = \sum_{k=1}^{L} H_k(\bar{X})\tilde{\mathbf{e}}^T(t)[A_k^T P + PA_k]\tilde{\mathbf{e}}(t) \tag{A-13}$$

$$\begin{aligned} V_{\tilde{\mathbf{e}}}{}^T \tilde{g}(\tilde{\mathbf{e}}(t))\mathbf{U}(t) &= \sum_{k=1}^{L} H_k(\bar{X})\sum_{l=1}^{L} H_l(\bar{X})\tilde{\mathbf{e}}^T(t)2PB_kK_l\tilde{\mathbf{e}}(t) \\ &= \sum_{k=1}^{L} H_k(\bar{X})\sum_{l=1}^{L} H_l(\bar{X})\tilde{\mathbf{e}}^T(t)[PB_kK_l + (PB_kK_l)^T]\tilde{\mathbf{e}}(t) \end{aligned} \tag{A-14}$$

$$\begin{aligned} \tfrac{1}{2}\sum_{i=1}^{N}\tilde{\sigma}_i\left(\tilde{\mathbf{e}}(t)\right)V_{\tilde{\mathbf{e}}\tilde{\mathbf{e}}}\tilde{\sigma}_i\left(\tilde{\mathbf{e}}(t)\right) &= \sum_{i=1}^{N}(\sum_{k=1}^{L} H_k(\bar{X})C_{ik}\tilde{\mathbf{e}}(t))^T P \\ \times(\sum_{j=1}^{L} H_j(\bar{X})C_{ij}\tilde{\mathbf{e}}(t))^T &\leq \sum_{k=1}^{L} H_k(\bar{X})\tilde{\mathbf{e}}^T(t)[\sum_{i=1}^{N} C_{ik}{}^T PC_{ik}]\tilde{\mathbf{e}}(t) \end{aligned} \tag{A-15}$$

$$\begin{aligned} &\sum_{i=1}^{N}\lambda_i\{V(\tilde{\mathbf{e}}(t) + \tilde{\Gamma}_i\left(\tilde{\mathbf{e}}(t)\right)) - V(\tilde{\mathbf{e}}(t)\} = \sum_{i=1}^{N}\lambda_i\{(\tilde{\mathbf{e}}(t) + \tilde{\Gamma}_i\left(\tilde{\mathbf{e}}(t)\right))^T P \\ &\times(\tilde{\mathbf{e}}(t) + \tilde{\Gamma}_i\left(\tilde{\mathbf{e}}(t)\right)) - \tilde{\mathbf{e}}^T(t)P\tilde{\mathbf{e}}(t)\} = \sum_{i=1}^{N}\lambda_i\{(\tilde{\mathbf{e}}(t) + \sum_{k=1}^{L} H_k(\bar{X})D_{ik} \\ &\times\tilde{\mathbf{e}}(t))^T P(\tilde{\mathbf{e}}(t) + \sum_{k=1}^{L} H_k(\bar{X})D_{ik}\tilde{\mathbf{e}}(t)) - \tilde{\mathbf{e}}^T(t)P\tilde{\mathbf{e}}(t)\} \\ &\leq \sum_{k=1}^{L} H_k(\bar{X})\tilde{\mathbf{e}}^T(t)[\sum_{i=1}^{N}\lambda_i\{D_{ik}^T PD_{ik} + D_{ik}^T P + PD_{ik}\}]\tilde{\mathbf{e}}(t) \end{aligned} \tag{A-16}$$

By (A-13)-(A-16), the HJI in (14) can be replaced by:

$$\begin{aligned} &\sum_{k=1}^{L}\sum_{l=1}^{L} H_k(\bar{X})H_l(\bar{X})\tilde{\mathbf{e}}^T(t)[A_k^T P + PA_k + PB_kK_l + (PB_kK_l)^T \\ &+\sum_{i=1}^{N} C_{ik}{}^T PC_{ik} + \sum_{i=1}^{N}\lambda_i(D_{ik}^T PD_{ik} + D_{ik}^T P + PD_{ik})]\tilde{\mathbf{e}}(t) < 0 \end{aligned} \tag{A-17}$$

If the following algebraic Riccati like inequalities are satisfied:

$$\begin{aligned} &A_k^T P + PA_k + PB_kK_l + (PB_kK_l)^T + \sum_{i=1}^{N} C_{ik}{}^T PC_{ik} \\ &+\sum_{i=1}^{N}\lambda_i(D_{ik}^T PD_{ik} + D_{ik}^T P + PD_{ik}) < 0 \end{aligned} \tag{A-18}$$

for $k, l = 1, 2, ..., L$, then the HJI constraint in (14) the stochastic nonlinear multi-UAV networked system without external disturbance could achieve the asymptotic tracking in probability.

By pre-multiplying and post-multiplying $W = P^{-1}$ to (A-18), we have:

$$\begin{aligned} &WA_k^T + A_kW + B_kY_l + (B_kY_l)^T + \sum_{i=1}^{N} WC_{ik}{}^T W^{-1}C_{ik}W \\ &+\sum_{i=1}^{N}\lambda_i(WD_{ik}^T W^{-1}D_{ik}W + WD_{ik}^T + D_{ik}W) < 0 \end{aligned}$$

for $k, l = 1, 2, ..., L$ with $Y_l = K_lW$. Then, by Schur complement [1], the above Riccati-like inequalities are equivalent to the LMIs in (22). This completes the proof.

### Appendix D- Proof of Theorem 4

From the result of Theorem 2, the $H_\infty$ tracking performance (15) of stochastic nonlinear multi-UAV networked tracking system (9) is achieved when the inequality (16) is satisfied. By substituting the Lyapunov function in (21) and the T-S fuzzy model in (20), we can get the following equalities:

$$V_{\tilde{\mathbf{e}}}{}^T \tilde{f}(\tilde{\mathbf{e}}(t)) = \sum_{k=1}^{L} H_k(\bar{X})\tilde{\mathbf{e}}^T(t)[A_k^T P + PA_k]\tilde{\mathbf{e}}(t) \tag{A-19}$$

$$\begin{aligned} V_{\tilde{\mathbf{e}}}{}^T \tilde{g}(\tilde{\mathbf{e}}(t))\mathbf{U}(t) &= \sum_{k=1}^{L} H_k(\bar{X})\sum_{l=1}^{L} H_l(\bar{X})\tilde{\mathbf{e}}^T(t)2PB_kK_l\tilde{\mathbf{e}}(t) \\ &= \sum_{k=1}^{L} H_k(\bar{X})\sum_{l=1}^{L} H_l(\bar{X})\tilde{\mathbf{e}}^T(t)[PB_kK_l + (PB_kK_l)^T]\tilde{\mathbf{e}}(t) \end{aligned} \tag{A-20}$$

$$\begin{aligned} \tfrac{1}{2}\sum_{i=1}^{N}\tilde{\sigma}_i\left(\tilde{\mathbf{e}}(t)\right)V_{\tilde{\mathbf{e}}\tilde{\mathbf{e}}}\tilde{\sigma}_i\left(\tilde{\mathbf{e}}(t)\right) &= \sum_{i=1}^{N}(\sum_{k=1}^{L} H_k(\bar{X})C_{ik}\tilde{\mathbf{e}}(t))^T P \\ \times(\sum_{j=1}^{L} H_j(\bar{X})C_{ij}\tilde{\mathbf{e}}(t))^T &\leq \sum_{k=1}^{L} H_k(\bar{X})\tilde{\mathbf{e}}^T(t)[\sum_{i=1}^{N} C_{ik}{}^T PC_{ik}]\tilde{\mathbf{e}}(t) \end{aligned} \tag{A-21}$$

$$
\begin{aligned}
&\sum_{i=1}^{N}\lambda_i\{V(\tilde{\mathbf{e}}(t)+\tilde{\Gamma}_i\left(\tilde{\mathbf{e}}(t)\right))-V(\tilde{\mathbf{e}}(t))\}\\
&=\sum_{i=1}^{N}\lambda_i\{(\tilde{\mathbf{e}}(t)+\tilde{\Gamma}_i\left(\tilde{\mathbf{e}}(t)\right))^T P(\tilde{\mathbf{e}}(t)+\tilde{\Gamma}_i\left(\tilde{\mathbf{e}}(t)\right))-\tilde{\mathbf{e}}^T(t)P\tilde{\mathbf{e}}(t)\}\\
&=\sum_{i=1}^{N}\lambda_i\{(\tilde{\mathbf{e}}(t)+\sum_{k=1}^{L}H_k(\bar{X})D_{ik}\tilde{\mathbf{e}}(t))^T P(\tilde{\mathbf{e}}(t)\\
&\quad+\sum_{k=1}^{L}H_k(\bar{X})D_{ik}\tilde{\mathbf{e}}(t))-\tilde{\mathbf{e}}^T(t)P\tilde{\mathbf{e}}(t)\}\\
&\leq\sum_{k=1}^{L}H_k(\bar{X})\tilde{\mathbf{e}}^T(t)[\sum_{i=1}^{N}\lambda_i(D_{ik}^T P D_{ik}+D_{ik}^T P+PD_{ik})]\tilde{\mathbf{e}}(t)
\end{aligned}
\tag{A-22}
$$

By (A-19)-(A-22), (A-10) can be replaced by

$$
\begin{aligned}
&E\{\int_0^{t_f}\tilde{\mathbf{e}}^T(t)Q\tilde{\mathbf{e}}(t)\}\leq E\{V(\tilde{\mathbf{e}}_0)\}+E\{\int_0^{t_f}[\sum_{k=1}^{L}\sum_{l=1}^{L}H_k(\bar{X})\\
&\times H_l(\bar{X})\tilde{\mathbf{e}}^T(t)[Q+A_k^T P+PA_k+PB_kK_l+(PB_kK_l)^T+\tfrac{1}{\rho^2}PP\\
&+\sum_{i=1}^{N}{C_{ik}}^T PC_{ik}+\sum_{i=1}^{N}\lambda_i(D_{ik}^T PD_{ik}+D_{ik}^T P+PD_{ik})]\tilde{\mathbf{e}}(t)]dt\}\\
&+E\{\int_0^{t_f}\rho^2\bar{\boldsymbol{v}}^T(t)\bar{\boldsymbol{v}}(t)dt\}
\end{aligned}
\tag{A-23}
$$

If the following algebraic Riccati-like inequalities are satisfied:

$$
\begin{aligned}
&Q+A_k^T P+PA_k+PB_kK_l+(PB_kK_l)^T+\tfrac{1}{\rho^2}PP\\
&+\sum_{i=1}^{N}{C_{ik}}^T PC_{ik}+\sum_{i=1}^{N}\lambda_i(D_{ik}^T PD_{ik}+D_{ik}^T P+PD_{ik})\leq 0\\
&for\ k,l=1,2,\cdots,n
\end{aligned}
\tag{A-24}
$$

, then we immediately have

$$
E\{\int_0^{t_f}\tilde{\mathbf{e}}^T(t)Q\tilde{\mathbf{e}}(t)\}\leq E\{V(\tilde{\mathbf{e}}_0)\}+E\{\int_0^{t_f}\rho^2\bar{\boldsymbol{v}}^T(t)\bar{\boldsymbol{v}}(t)\}
$$

, i.e., $J_\infty(\mathbf{U}(t))\leq\rho^2$ is satisfied.

Let $W=P^{-1}$ and $Y_l=K_lW$; then multiplying $W$ to two sides of (A-24), we get

$$
\begin{aligned}
&WQW+WA_k^T+A_kW+B_kY_l+{Y_l}^T B_k^T+\tfrac{1}{\rho^2}I\\
&+\sum_{i=1}^{N}W{C_{ik}}^T W^{-1}C_{ik}W+\sum_{i=1}^{N}\lambda_i(WD_{ik}^T W^{-1}D_{ik}W\\
&\qquad+WD_{ik}^T+D_{ik}W)\leq 0
\end{aligned}
\tag{A-25}
$$

By Schur complement [1], the Riccati-like inequalities in (A-25) are equivalent to the LMIs in (23). The $H_\infty$ tracking control performance is achieved with a prescribed $\rho^2$.

If the external disturbance is vanished in the system, i.e., $\bar{\boldsymbol{v}}(t)=0$, from (A-24) , the (A-23) can be rewritten as follows:

$$
\begin{aligned}
&E\{\int_0^{t_f}\tilde{\mathbf{e}}^T(t)Q\tilde{\mathbf{e}}(t)dt\}\leq E\{V(\tilde{\mathbf{e}}_0)\}\\
&\forall t_f\in[0,\infty]
\end{aligned}
\tag{A-26}
$$

From (A-26), we have $E[\tilde{\mathbf{e}}^T(t)\tilde{\mathbf{e}}(t)]\to 0$ as t$\to\infty$, i.e., the asymptotic tracking for the multi-UAVs networked system is achieved. This completes the proof.

## References


[1] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan," *Linear Matrix Inequalities in System and Control Theory,*" Philadelphia, PA, USA: SIAM, 1994.