# Feature Analysis – Edges and Blobs
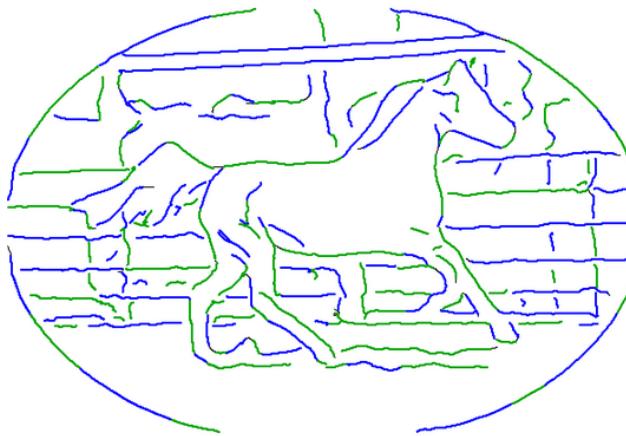
Gonzalez & Woods Digital Image Processing – Chapter 10

Feature Analysis
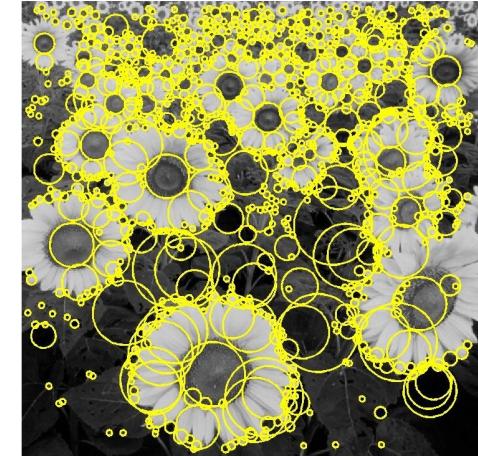
Regions/Segmentation
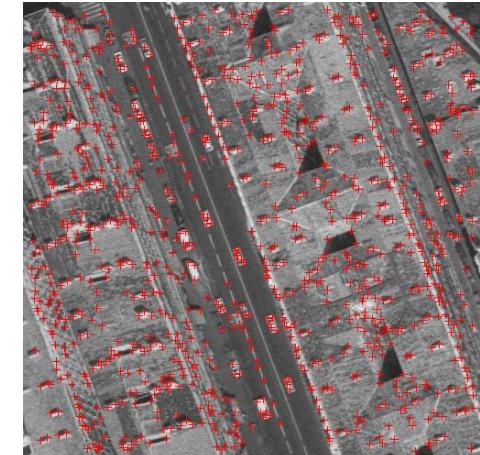

**Contours/Line segments**

Interest Points

Blobs

Corners

Slide adapted from by Cordelia Schmid
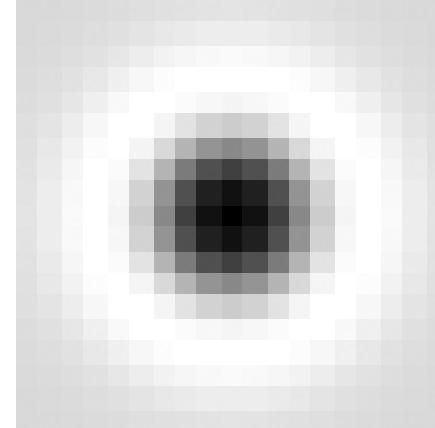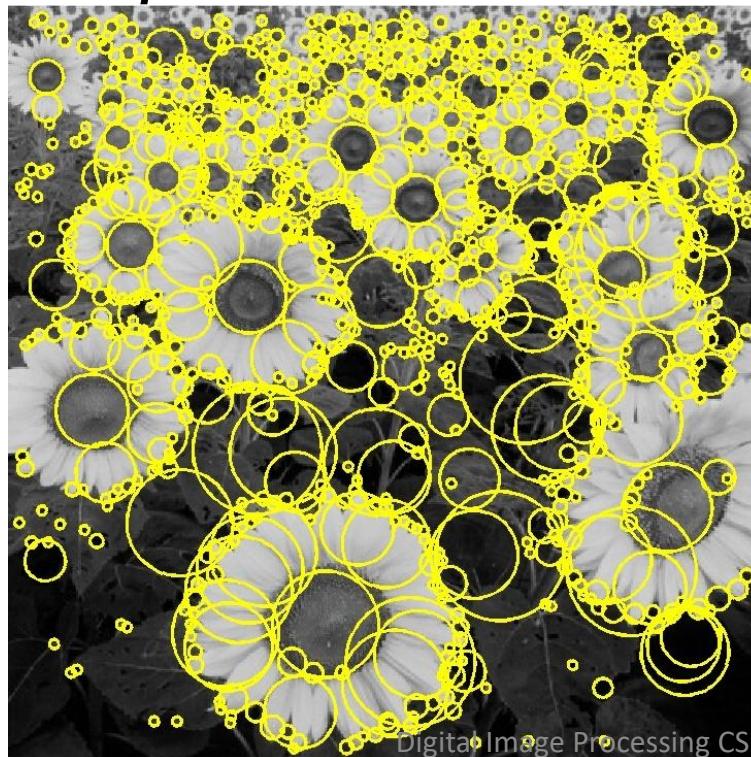
Blob detection: Basic idea
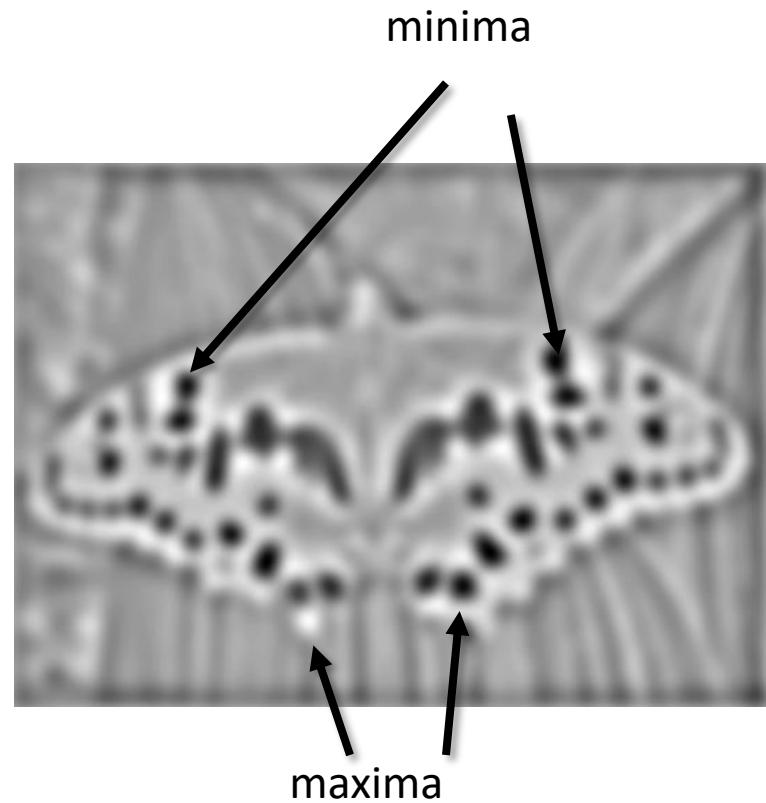
- To detect blobs, convolve the image with a "blob filter" at multiple scales and look for extrema of filter response in the resulting *scale space*

Slide Credit: Dr. Svetlana Lazebnik

Blob detection: Basic idea

minima

maxima

$*$ ⊙ $=$
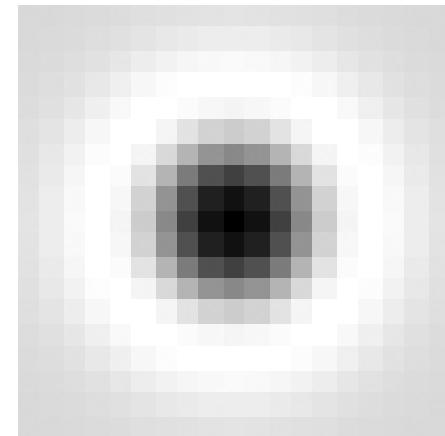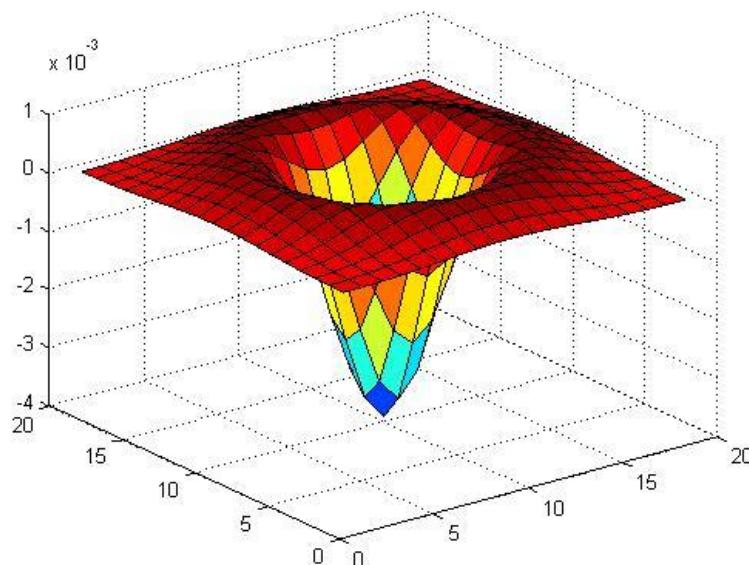
- Find maxima *and minima* of blob filter response in space *and scale*

Source: N. Snavely
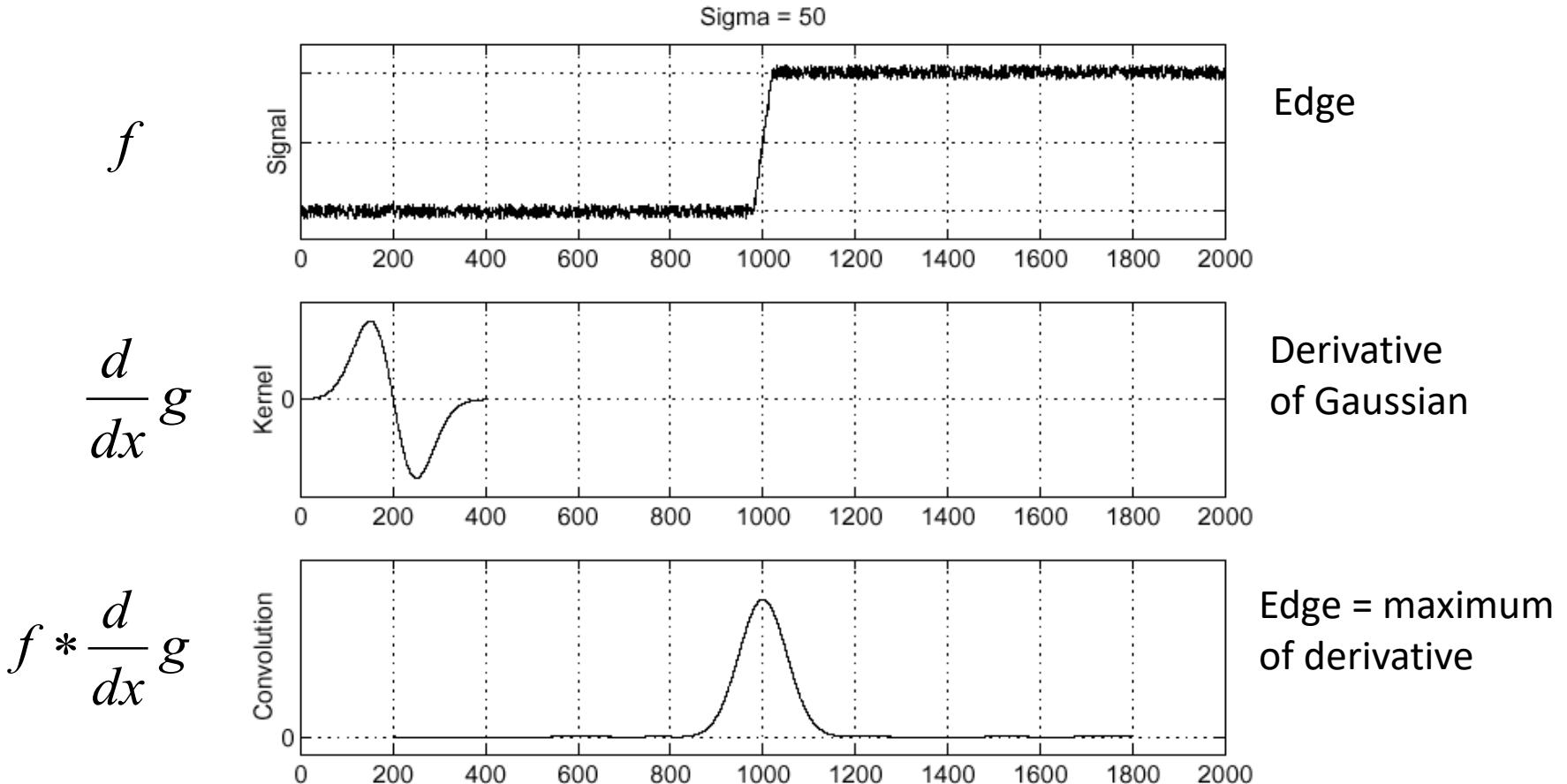
Blob filter

- Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D
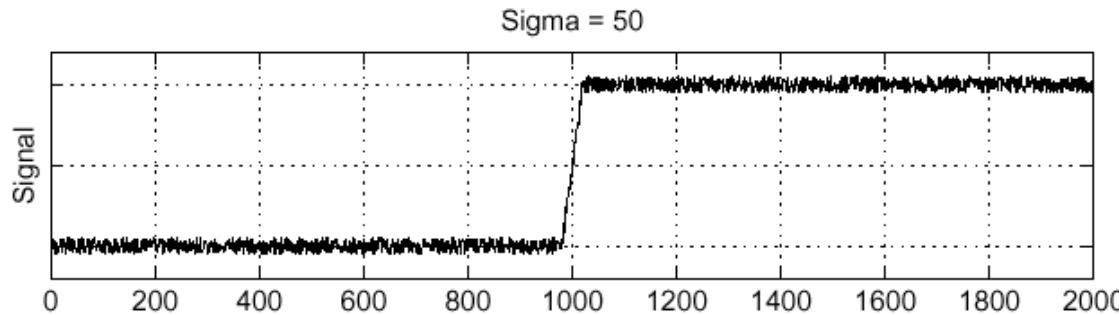
$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

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Slide Credit: Dr. Svetlana Lazebnik

Recall: Edge detection

f

Sigma = 50

Edge

$\dfrac{d}{dx} g$

Derivative
of Gaussian

$f * \dfrac{d}{dx} g$

Edge = maximum
of derivative

Source: S. Seitz

Edge detection, Take 2

Sigma = 50

f — Edge

$\dfrac{d^2}{dx^2}\,g$ — Second derivative of Gaussian (Laplacian)

$f * \dfrac{d^2}{dx^2}\,g$ — Edge = zero crossing of second derivative

Source: S. Seitz
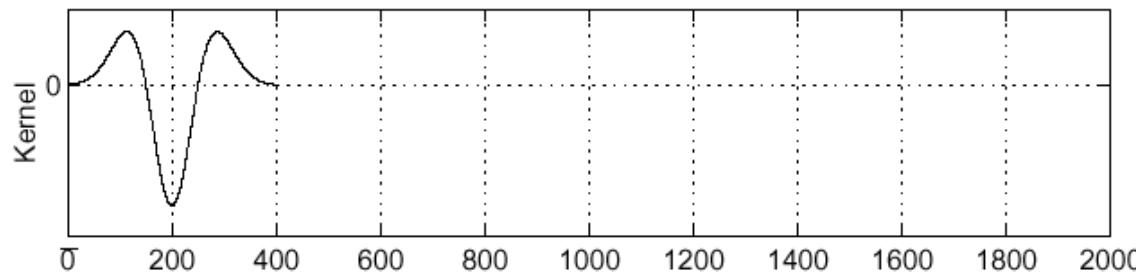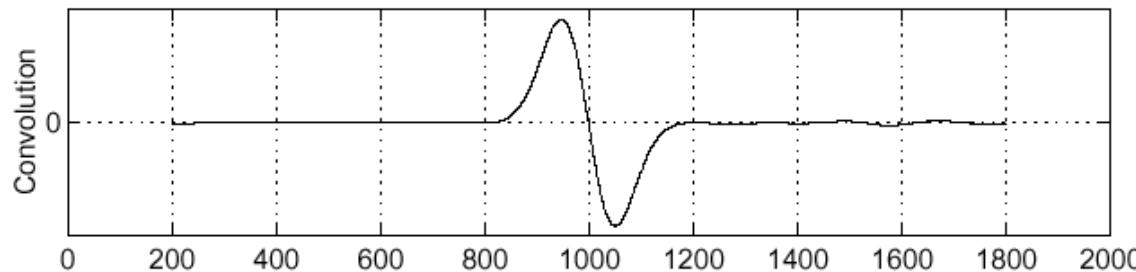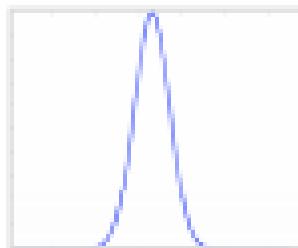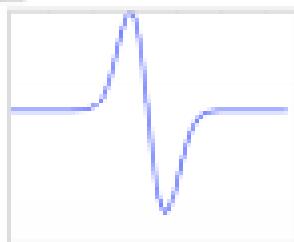
1D Gaussian and Derivatives
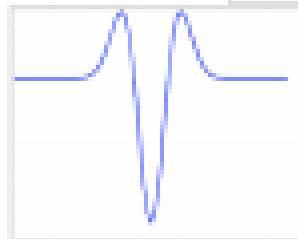
$$g(x) = e^{-\frac{x^2}{2\sigma^2}}$$
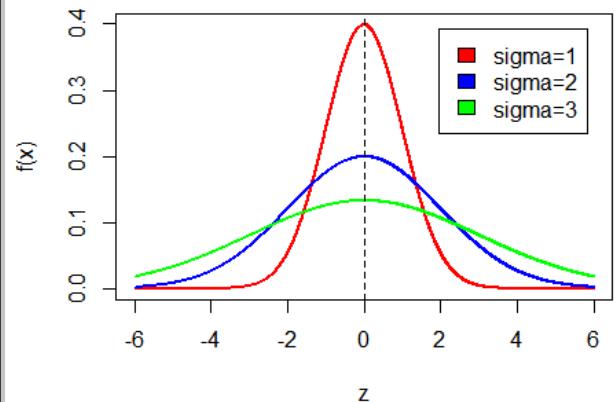
$$g'(x) = -\frac{1}{2\sigma^2} 2xe^{-\frac{x^2}{2\sigma^2}} = -\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$$

$$g''(x) = \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right)e^{-\frac{x^2}{2\sigma^2}}$$
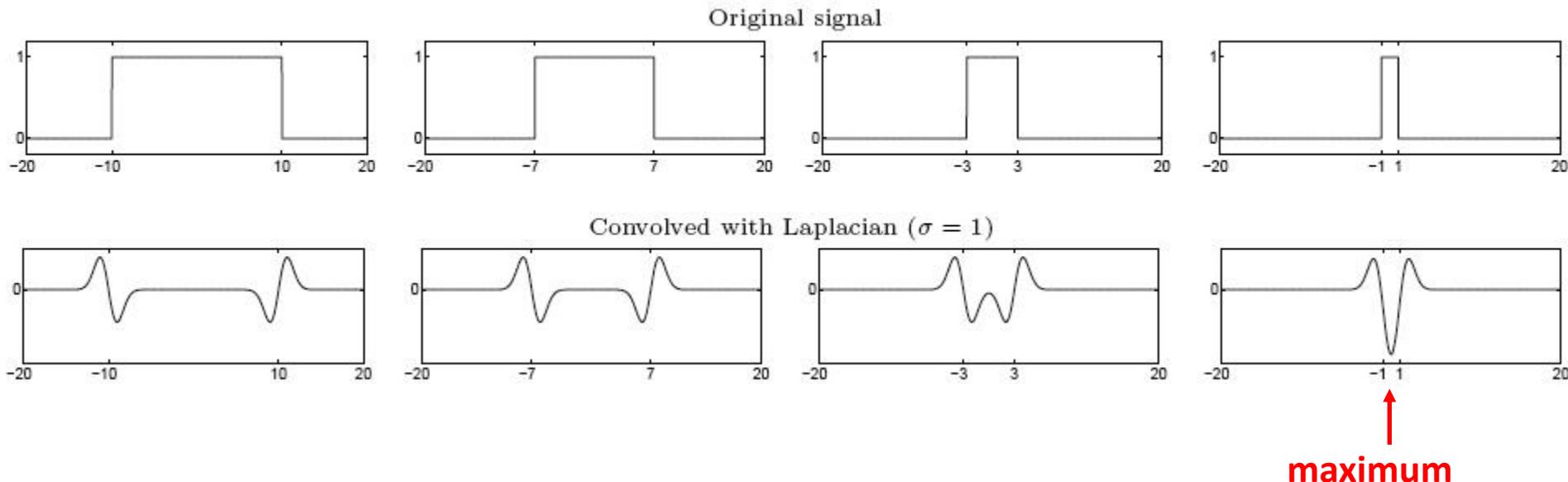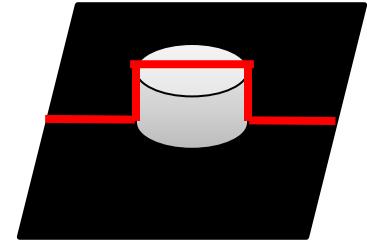
Normal density function with different variance
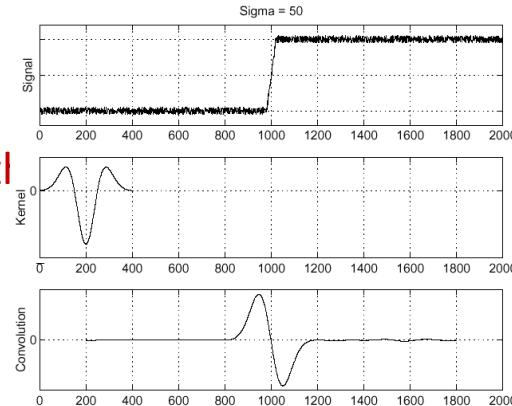
From edges to blobs

- Edge = ripple

- Blob = superposition of two ripples

Original signal

Convolved with Laplacian ($\sigma = 1$)
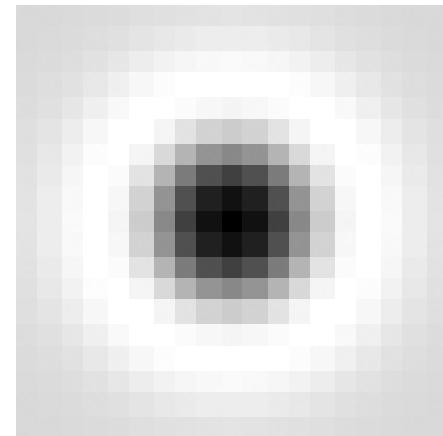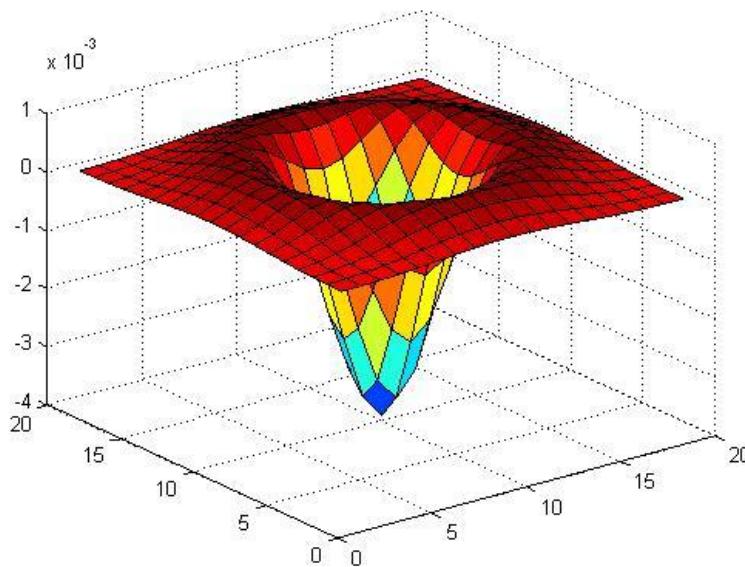
maximum

Spatial selection: the magnitude of the Laplacian response
will achieve a maximum at the center of the blob,
provided the scale of the Laplacian is "matched" to the scale of t

Sigma = 50

Slide Credit:

Blob detection in 2D

- Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D

Scale-normalized:

$$\nabla^2_{\text{norm}} g = \sigma^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$$

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Slide Credit: Dr. Svetlana Lazebnik
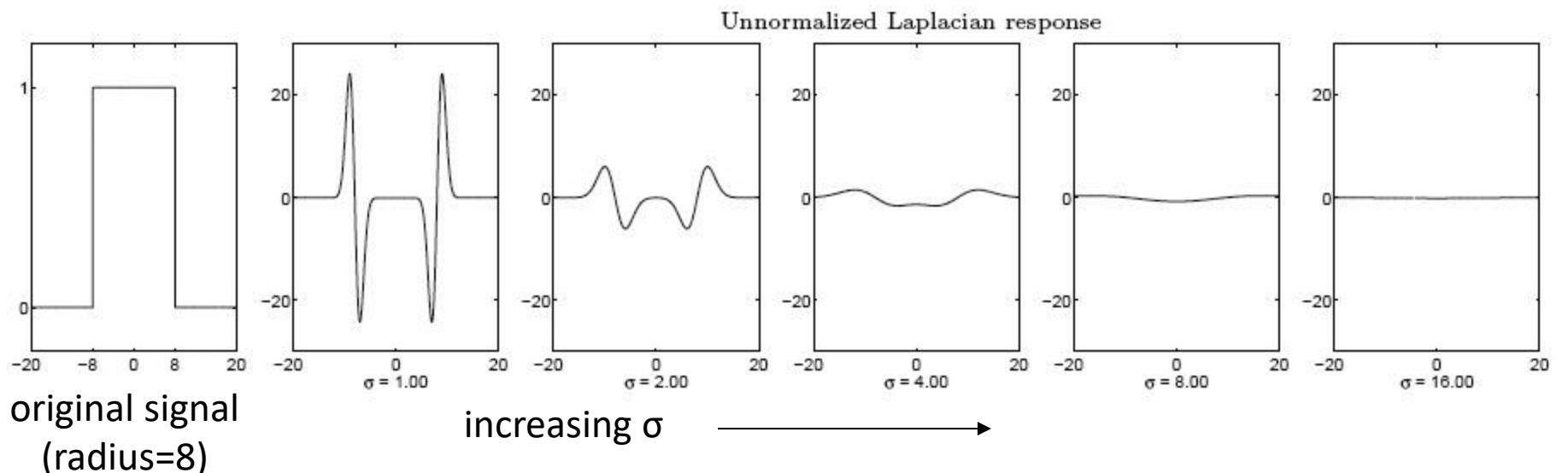
Scale selection

- We want to find the characteristic scale of the blob by convolving it with Laplacians at several scales and looking for the maximum response

- However, Laplacian response decays as scale increases:
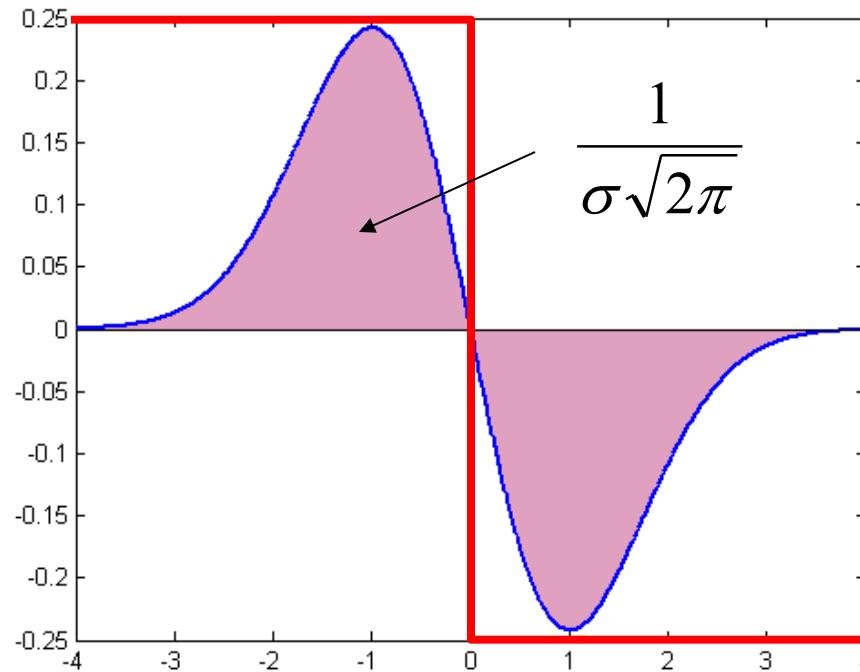
$$\nabla^2_{\text{norm}} g =$$

Unnormalized Laplacian response

original signal
(radius=8)

increasing σ →

Slide Credit: Dr. Svetlana Lazebnik

Scale normalization

- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases

- To keep response the same (scale-invariant), must <u>multiply Gaussian derivative by σ</u>

- Laplacian is the second Gaussian derivative, so it must be <u>multiplied by σ^2</u>

Slide Credit: Dr. Svetlana Lazebnik
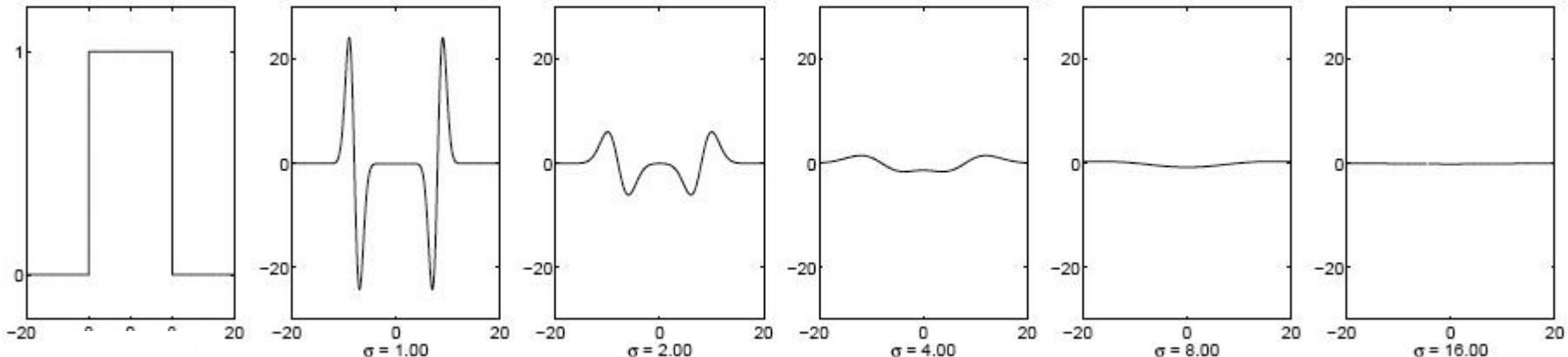
Scale normalization

- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases

$$\frac{1}{\sigma\sqrt{2\pi}}$$

Slide Credit: Dr. Svetlana Lazebnik

Effect of scale normalization
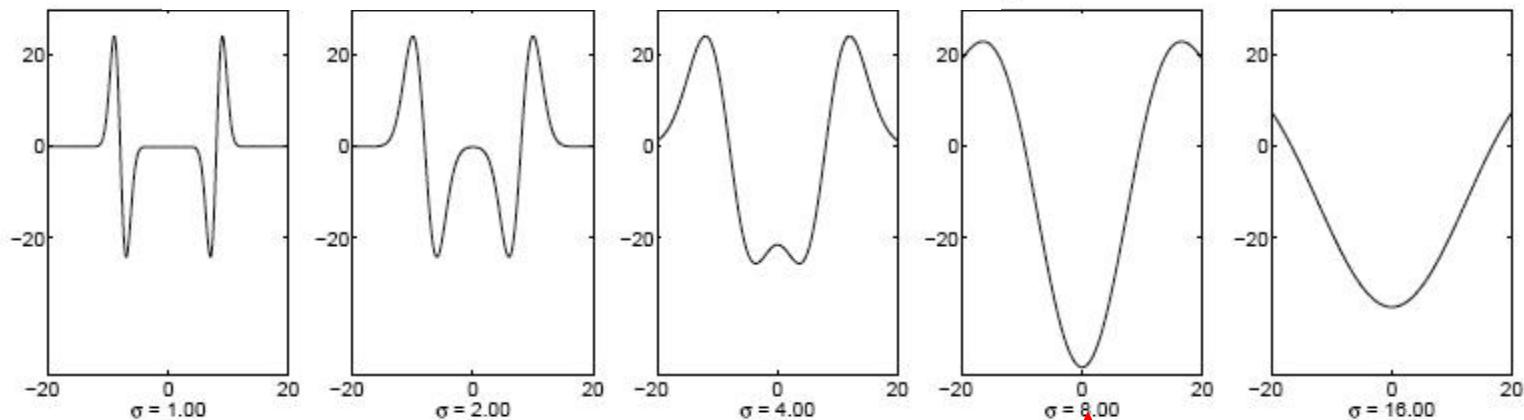
Unnormalized Laplacian response

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

Original signal

Scale-normalized Laplacian response

$$\nabla^2_{\text{norm}} g = \sigma^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$$
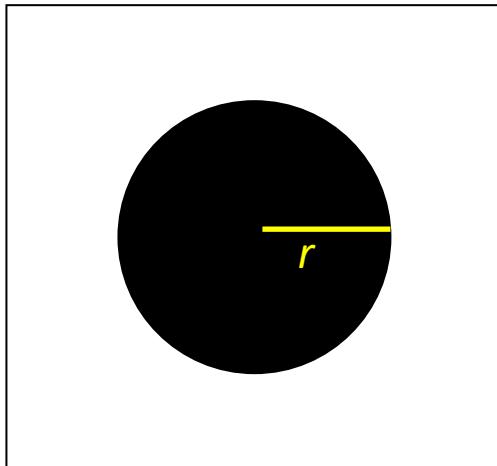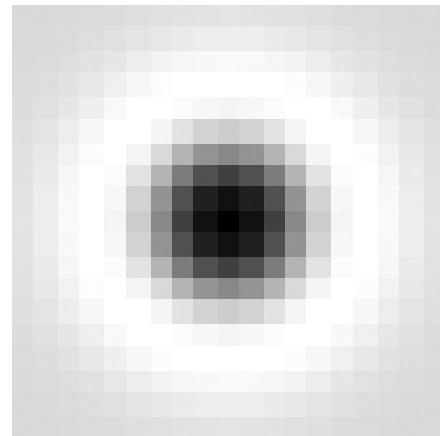
maximum

Scale selection
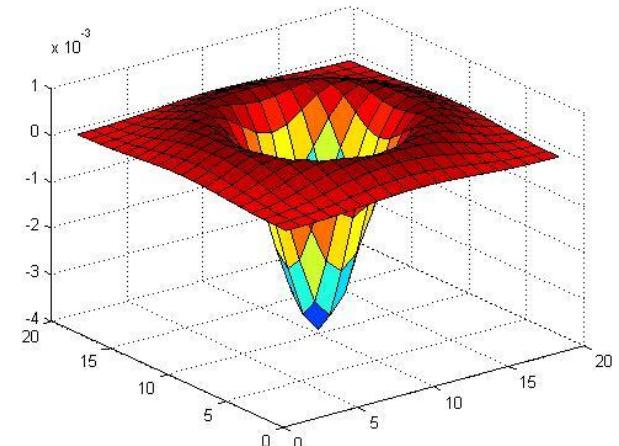
- At what scale does the Laplacian achieve a maximum response to a binary circle of radius r?


image

Laplacian
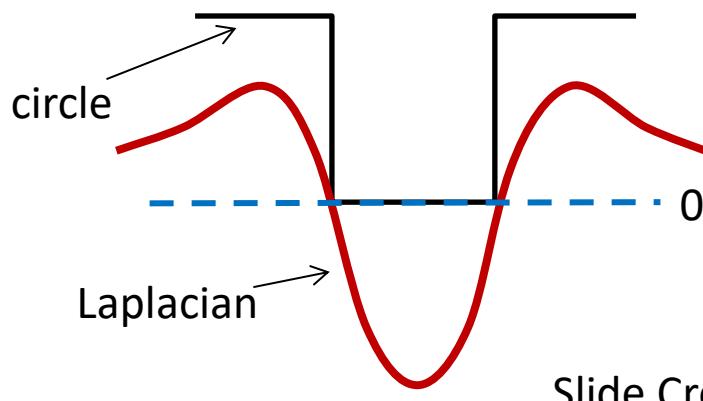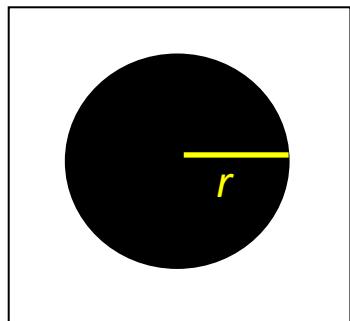
$$(x^2 + y^2 - 2\sigma^2)e^{-(x^2+y^2)/2\sigma^2}$$

Slide Credit: Dr. Svetlana Lazebnik

Scale selection

- At what scale does the Laplacian achieve a maximum response to a binary circle of radius r?

- To get maximum response, the zeros of the Laplacian have to be aligned with the circle

- The Laplacian is given by (up to scale): $(x^2 + y^2 - 2\sigma^2)e^{-(x^2+y^2)/2\sigma^2}$

- Therefore, the maximum response occurs at

$$0 = (r^2 - 2\sigma^2)e^{-\frac{r^2}{2\sigma^2}}$$
$$0 = (r^2 - 2\sigma^2)$$

$$\sigma = r / \sqrt{2}.$$

circle

Laplacian

0
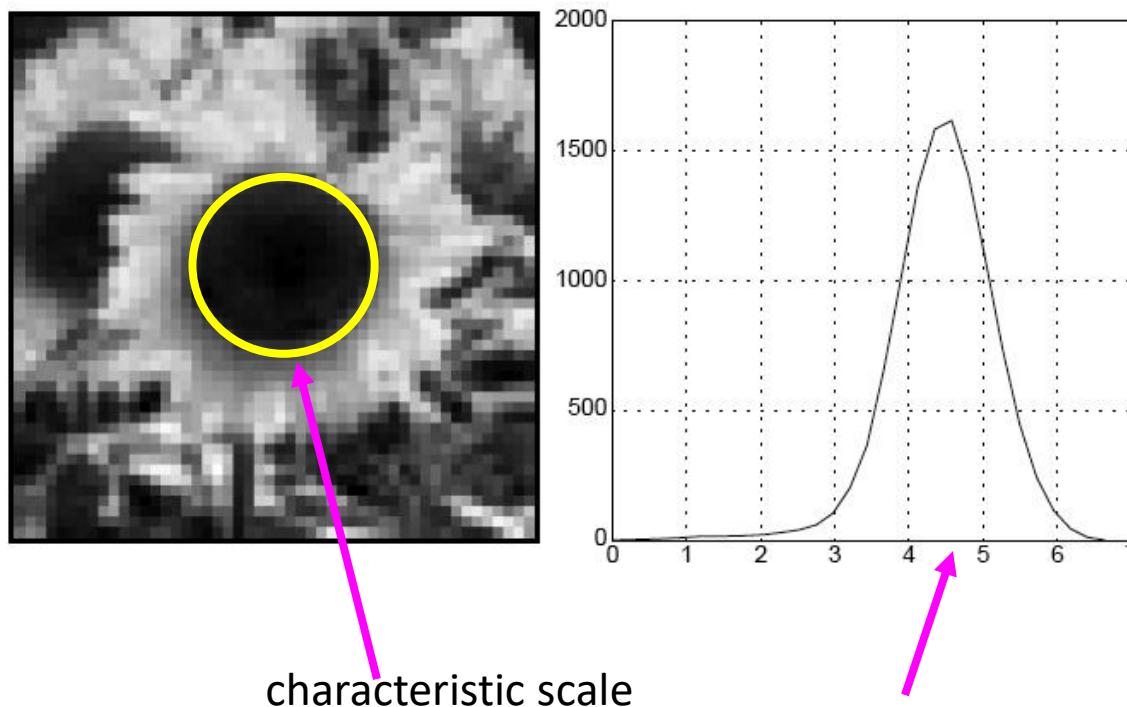
image

Characteristic scale

- We define the characteristic scale of a blob as the scale that produces peak of Laplacian response in the blob center

characteristic scale

T. Lindeberg (1998). "Feature detection with automatic scale selection." *International Journal of Computer Vision* **30** (2): pp 77--116.

Slide Credit: Dr. Svetlana Lazebnik

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales

Slide Credit: Dr. Svetlana Lazebnik

Scale-space blob detector: Example

Slide Credit: Dr. Svetlana Lazebnik

Scale-space blob detector: Example

sigma = 11.9912

Slide Credit: Dr. Svetlana Lazebnik
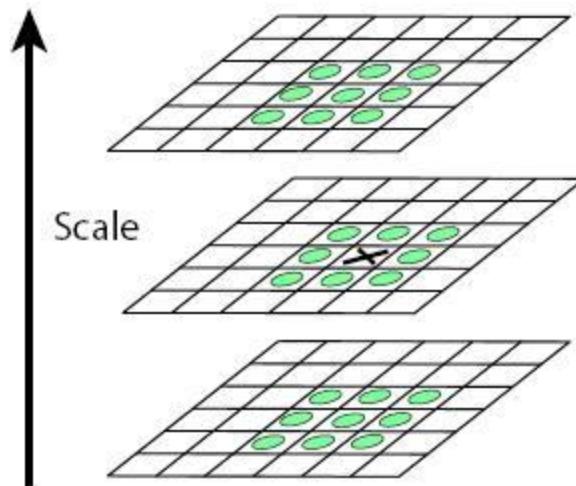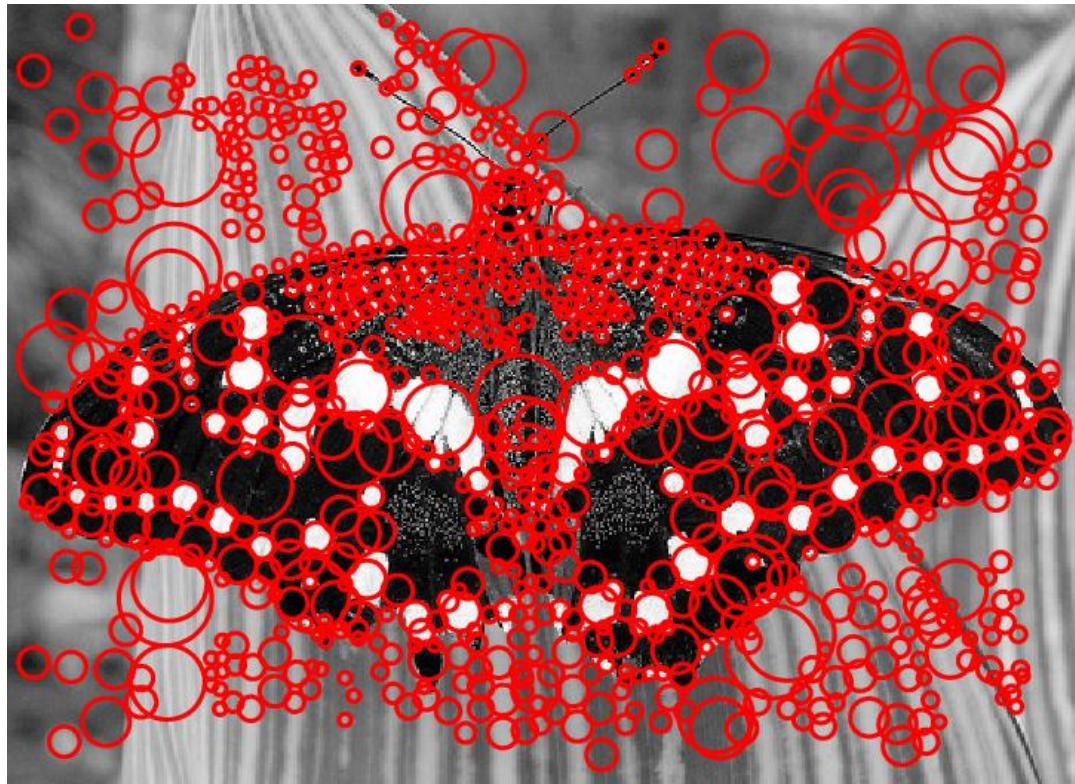
Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales

2. Find maxima of squared Laplacian response in scale-space

Slide Credit: Dr. Svetlana Lazebnik

Scale-space blob detector: Example

Slide Credit: Dr. Svetlana Lazebnik

Efficient implementation
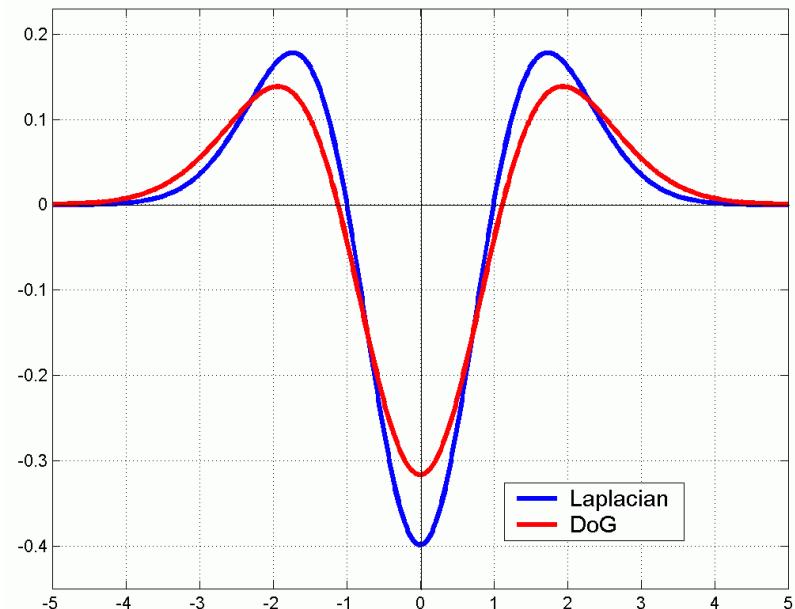
- Approximating the Laplacian with a difference of Gaussians:

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$
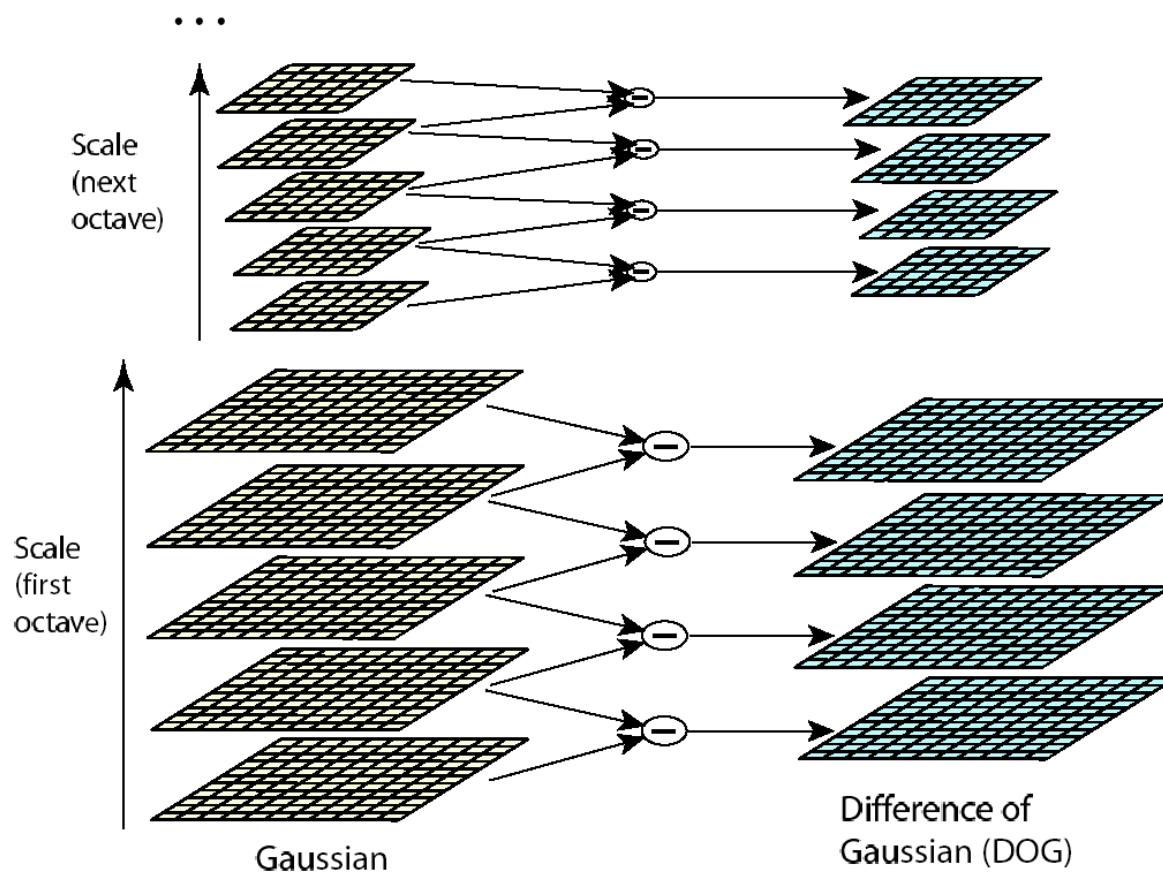
(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)

Slide Credit: Dr. Svetlana Lazebnik
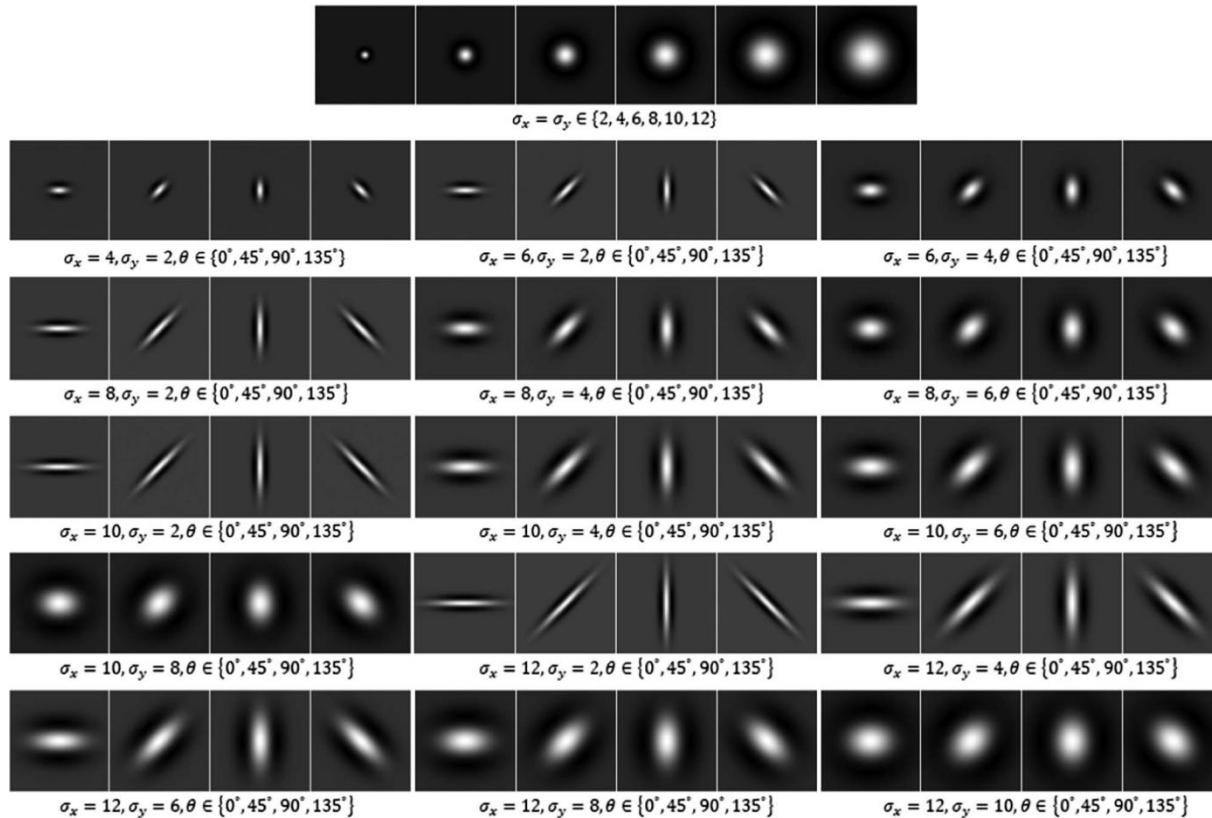
Efficient implementation
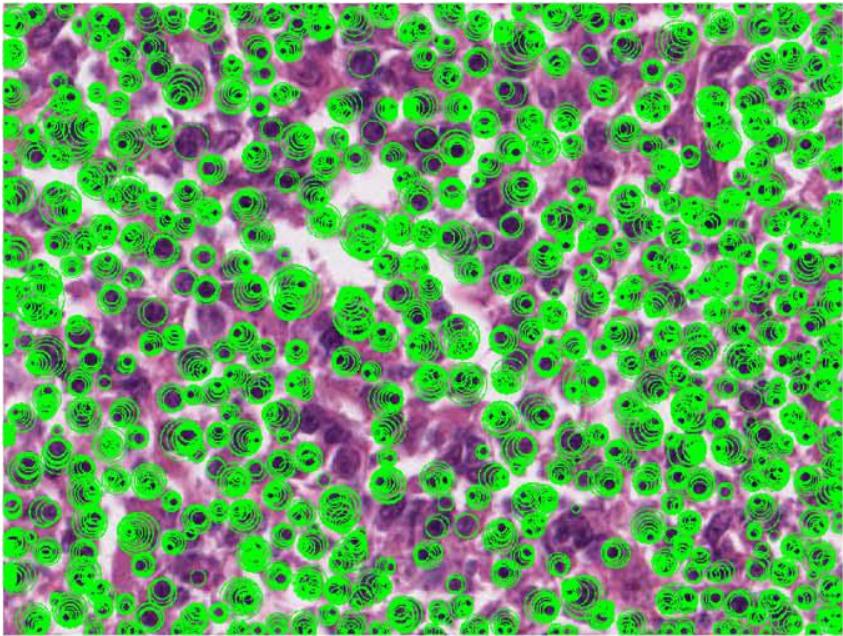
David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

Slide Credit: Dr. Svetlana Lazebnik
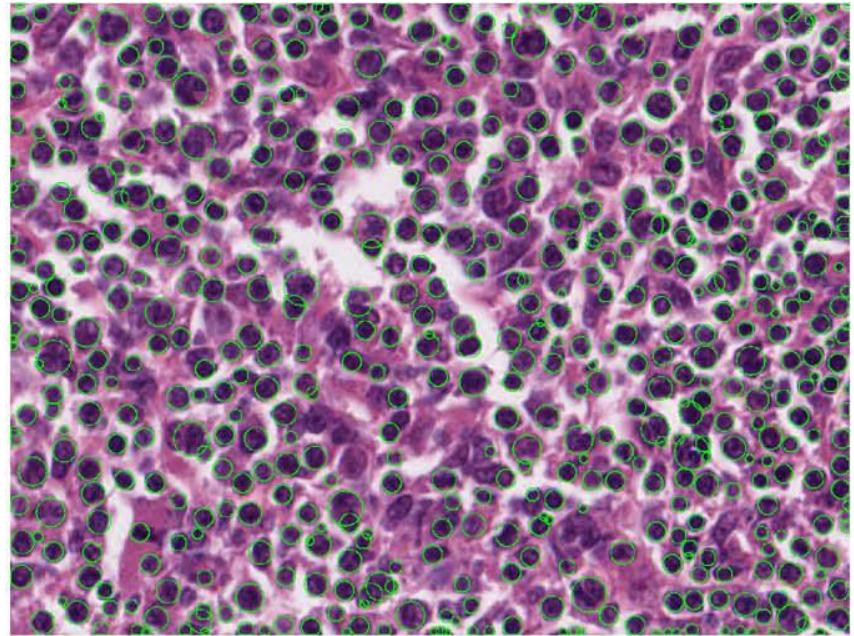
Generalized Laplacian of Gaussian filter

Kong, Hui, Hatice Cinar Akakin, and Sanjay E. Sarma. "A generalized Laplacian of Gaussian filter for blob detection and its applications." *IEEE transactions on cybernetics* 43.6 (2013): 1719-1733.
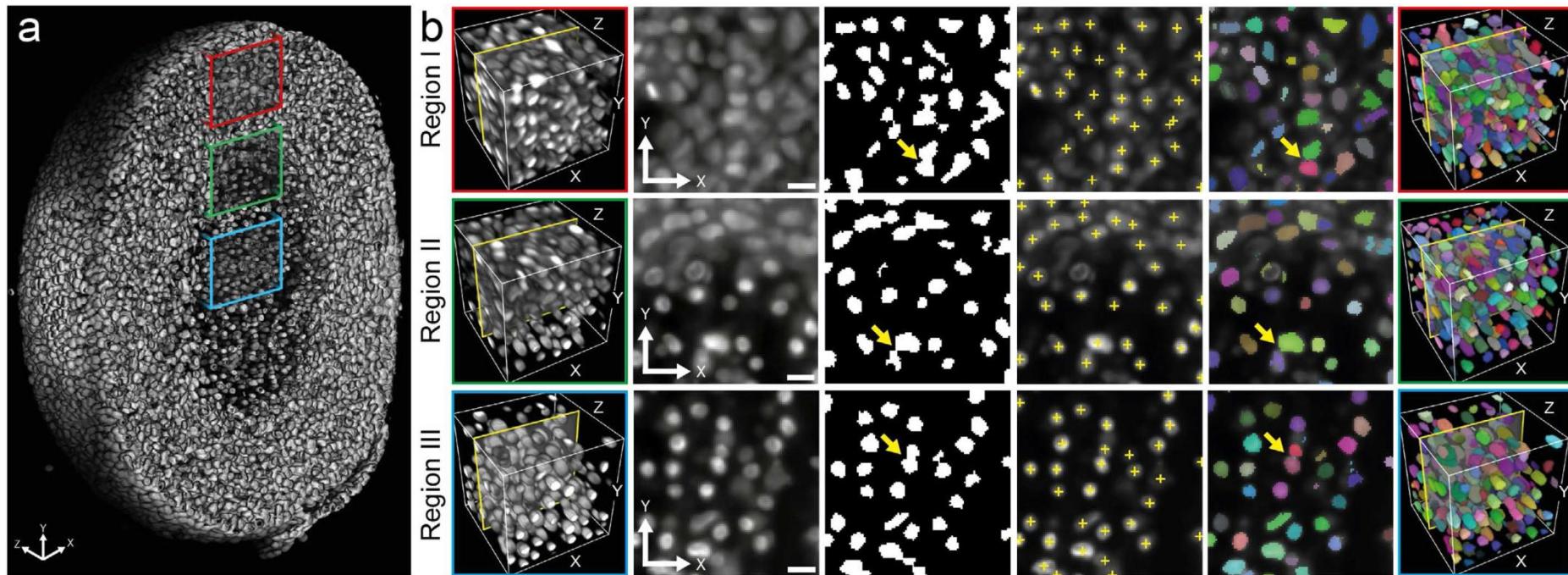
(a) and (b) Detected blobs by circular LoG blob detector before and after pruning, respectively.

Kong, Hui, Hatice Cinar Akakin, and Sanjay E. Sarma. "A generalized Laplacian of Gaussian filter for blob detection and its applications." *IEEE transactions on cybernetics* 43.6 (2013): 1719-1733.

Multiscale image analysis reveals structural heterogeneity of the cell microenvironment in homotypic spheroids
Alexander Schmitz
, Sabine C. Fischer
, Christian Mattheyer
, Francesco Pampaloni
 & Ernst H. K. Stelzer
Scientific Reports **7**, Article number: 43693

	Rendering	Single plane (X-Y)	Single plane (Z-Y)	Magnification (4×)
Dataset S9				
Dataset L3				