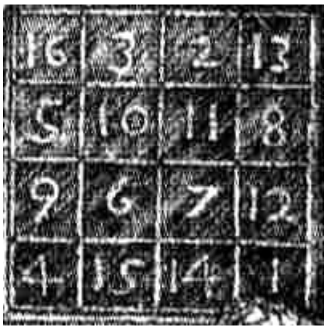
Final assignment: report to be sent by January 30th to:
codognet@is.s.u-tokyo.ac.jp
(please be sure that I acknowledge the receipt of your report)

The Magic Square Problem



16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

An instance of size *n* of the magic square problem consists in placing on a *n* × *n* square all the numbers in $\{1, \ldots, n^2\}$ such as the sum of the numbers in all rows, columns and the two main diagonals are the same. It is usually modeled by considering n^2 integer variables x_{ij} with domains $\{1, \ldots, n^2\}$ and the following constraints:

- a permutation constraint stating that all variables have a different value:

  $$\forall\,(\mathrm{i},\mathrm{j}) \in \{1,\ldots,\mathrm{n}\}^2,\ \forall\,(\mathrm{i}',\mathrm{j}') \in \{1,\ldots,\mathrm{n}\}^2 :\ (\mathrm{i},\mathrm{j}) \neq (\mathrm{i}',\mathrm{j}') \implies x_{ij} \neq x_{i'j'}$$

- 2n+2 linear equations for the sums on the rows, columns and the two main diagonals:
  - $\forall\,\mathrm{i} \in \{1,\ldots,\mathrm{n}\}\ :\ \sum_{j=1}^{n} x_{ij} = m$
  - $\forall\,\mathrm{j} \in \{1,\ldots,\mathrm{n}\}\ :\ \sum_{i=1}^{n} x_{ij} = m$
  - $\sum_{i=1}^{n} x_{ii} = m$
  - $\sum_{i=1}^{n} x_{n+1-i,i} = m$

  The value m can be easily computed to be: $m = \frac{n}{2}(n^2 + 1)$

Your task is to solve the Magic Square Problem with two different models and solving methods and to compare these approaches:

1. a constraint programming model using integer variables

   (program written in MiniZinc or Google OR-Tools with the CP-SAT solver)

2. a QUBO model (program to create the QUBO written in python with Fixstars Amplify SDK and using the Amplify Annealing Engine).

   Optionally, you can experiment with D-Wave Quantum Annealer and Hybrid solver. For using D-Wave, you need a D-Wave access token, to be obtained by following the instructions on the Fixstars Amplify website or directly at:
   https://cloud.dwavesys.com/leap/

Software resources:

- Fixstars Amplify:
  - Access: https://amplify.fixstars.com/
  - Documentation: https://amplify.fixstars.com/en/docs/quickstart.html
- MiniZinc :
  - Download : https://www.minizinc.org/
  - Documentation : https://www.minizinc.org/doc-2.6.4/en/index.html
- OR-tools :
  - Download : https://developers.google.com/optimization
  - Documentation : https://developers.google.com/optimization/introduction

Your report should be about 10-15 pages long and describe your programming project, that is, the models and their implementations (the programs). It should include experimental results and runtimes for solving the Magic Square problem for various problem instances (eg. N=4, 5, 6, 7, …). You should analyze the performance and scalability of the different approaches and comment on problems, if any.

In addition of the above report, files containing the codes of the two programs (QUBO, MiniZinc/OR-tools) are requested.