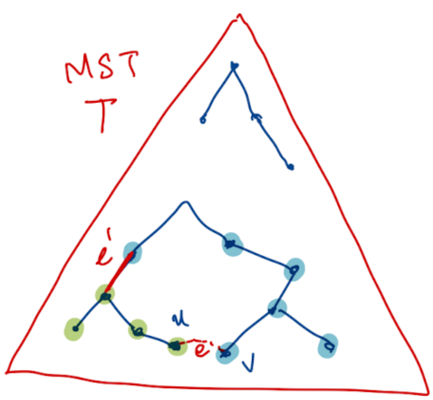
CSCI 323 Lecture 12 Note Wednesday, 03/16/2022

Announcements:
- The midterm will be on Wednesday, March 23, 2022.
- The Lecture Recordings will be available on the following YouTube Playlists Link: https://youtube.com/playlist?list=PLZaTmV9UMKlgYpo2cAiMaEWxqyvbiXDFd

Greedy Algorithm
References:
Algorithm Design - Chapter 4.5

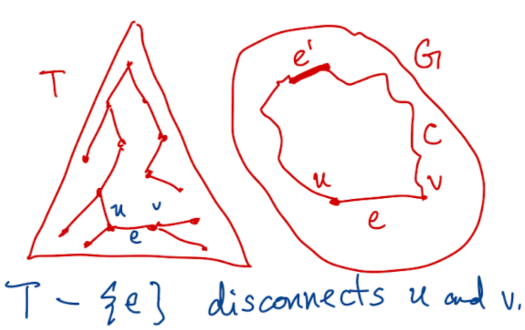
Minimum Spanning Tree [4.5]
- Property 1 – Cut Property
  - A *cut* is a partition of the vertices into two disjoint subsets.
  - Assume that all edge costs are distinct. Let S be any subset of nodes that is neither empty or equal to V and let e be the cheapest edge with one edge in S and the other in $V - S$, the complement of S. (e is the cheapest edge that cross the cut.)
  - The cut property states that, then e must be in every minimum spanning tree.
  - Proof by Contradiction:



Assume there is a cut $(S, V-S)$ such that the cheapest edge e across the cut is not the MST T.
If we add e to the MST T, it creates a cycle.
We can construct a new tree by removing a edge e' in the cycle such that $(T \cup \{e\}) - \{e'\}$
1) is a spanning tree.
2) has lower cost than T.
(since e is the cheapest, e' must be more expensive than e', swapping e and e' will lower the cost.)

Contradiction! New tree is cheaper.

- Property 2 – Cycle Property (will prove optimality of reverse-delete)
  - Consider any cycle C in the graph. Let e be the most expensive edge in C. Then e cannot be in any minimum spanning tree.
  - Proof by Contradiction:



Assume the most expensive edge e in a cycle is in the MST T.
If we remove the edge $e = (u, v)$ from T, it will disconnect u and v.
Since e is part of a cycle, there must be an edge e' in the cycle that connect u and v after removing e.
Then $(T - \{e\}) \cup \{e'\}$ will be cheaper than T, and it's also a spanning tree.
Contradiction!

What to expect or prepare for the next class:
- Review for Midterm