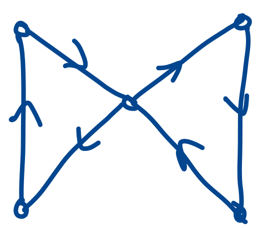
Reductions & NP-completeness [chapter 8]

Hamiltonian Cycle

Given G, a cycle in G is Hamiltonian if it goes through every vertex exactly once.



← Does G have H.C.?

No.

Show 3-SAT $\leq_p$ Hamiltonian Cycle.

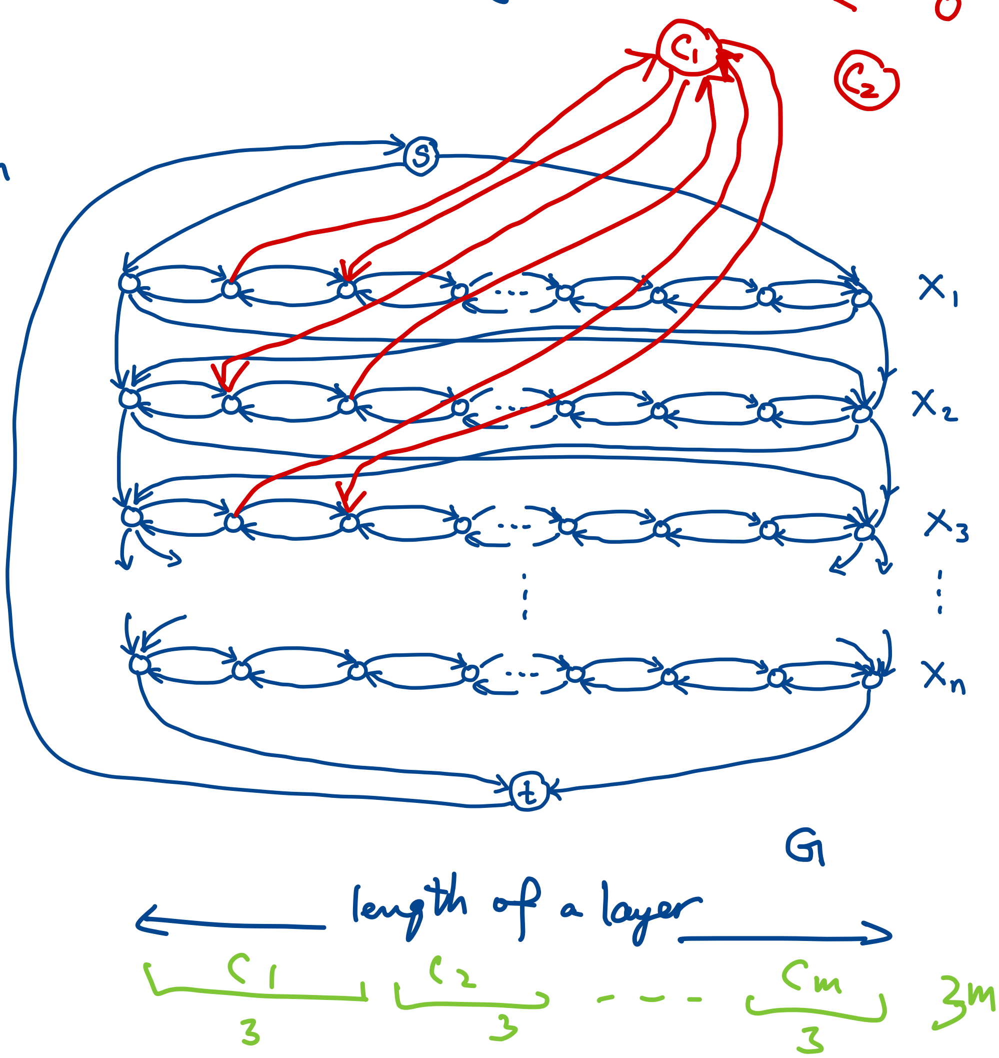
→ 1
← 0

$x_1, x_2, \ldots x_n$

$F = C_1 \wedge C_2 \wedge \ldots \wedge C_m$

$C_i = (x_i \vee x_j \vee \overline{x_k})$

$C_1 = (x_1 \vee \overline{x_2} \vee x_3)$

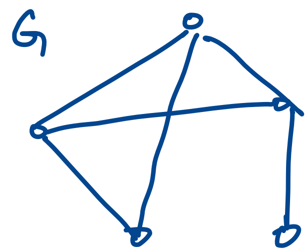


$\Rightarrow$ G has a HC if and only if F is satisfiable.

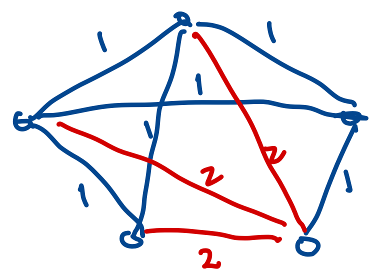
HC $\leq_p$ Traveling Salesman Problem.

TC : Given a weighted complete graph, find the shortest path that go though every node and back to the starting node.

HC: G



TSP: weighted. Complete Graph



To the TSP algorithm, we ask, is there a tour (cycle) of length at most n. (# vertices).

We say "Yes" to HC. if TSP algorithm says "Yes". and "No" otherwise.

Summary

$3SAT \leq_p MIS \leq VC \leq SC$ and $3SAT \leq_p HC \leq_p TSP$

Defining NP and NP-complete.

NP - Set of problems such that given a solution to the problem, it can be verified in polynomial time.

P - Set of problems such that there exists a polynomial time algorithm for it.

$P \subseteq NP$.