Circuit and Electronics II

Design Lab I

Fourth-order Butterworth Filter Design

工海系 二年級

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Section I

Design structure

Two serial sallen-key second-order active filter

1.8kHz fourth-order active low pass filter

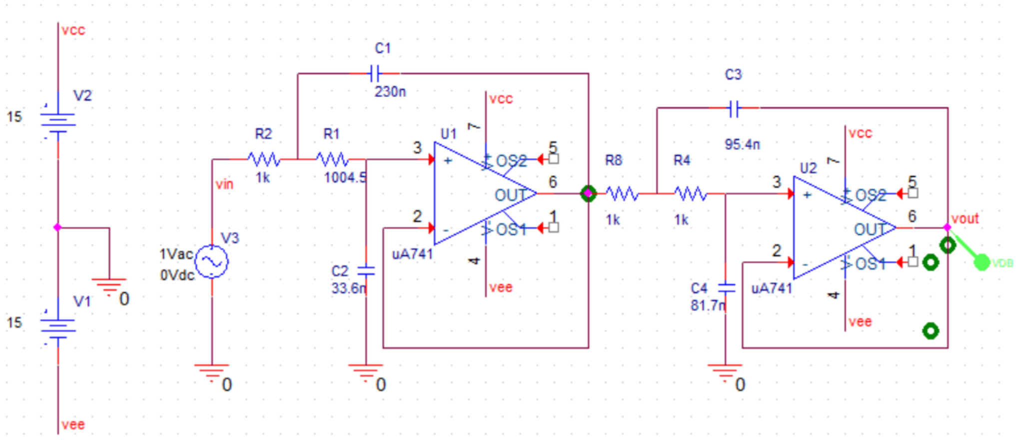

Fig.1 Design Structure of filter (select resistors)

Bode Plot

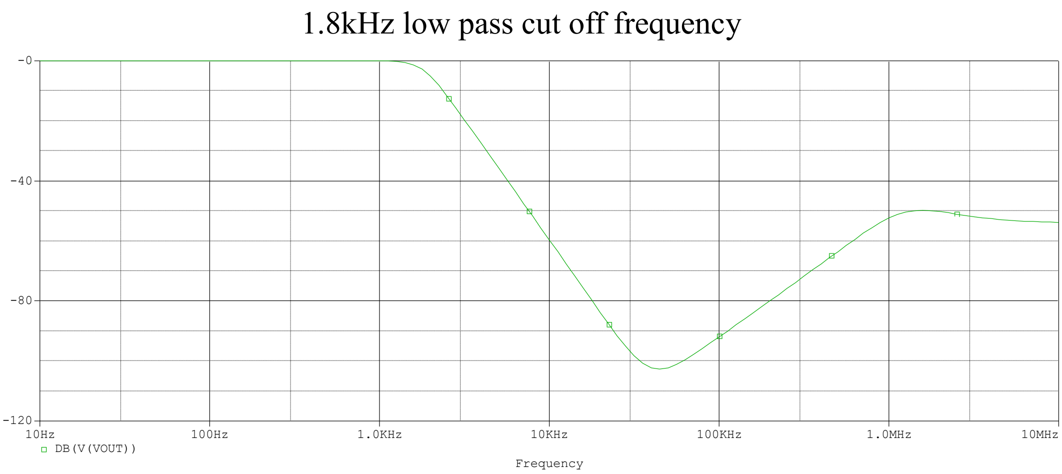

Fig.2 Bode Plot of 1.8kHz low pass filter from Pspice

Evaluate	Measurement	Value
☑	Cutoff_Lowpass_3dB(V(vout)/V(vin))	1.80002k

Fig.3 -3dB cut off frequency data on Pspice

Phase Angle

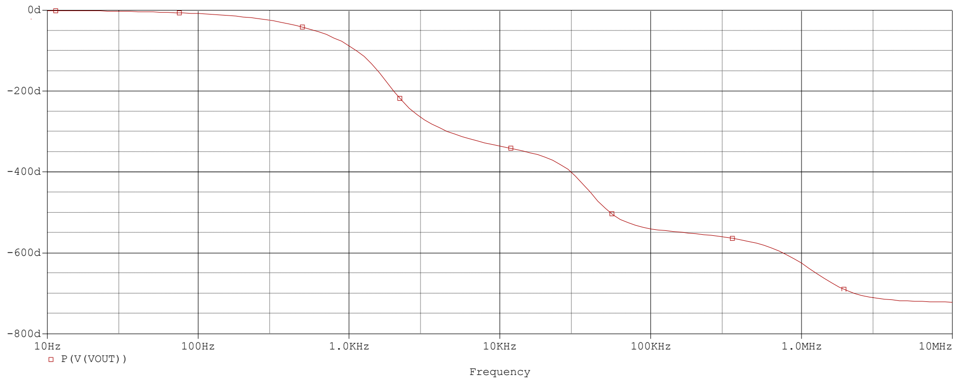

Fig.4 Phase Angle of 1.8kHz low pass filter export from Pspice

Section II

The amplitude response

(1) Normalize filter

$$H(jw) = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}}$$

$$|H(jw)|^2 = \frac{1}{1+\omega^{2n}} = H(j\omega)H(-j\omega)$$

(2) Transfer function

$$\mathrm{H(j\omega)} \rightarrow H(s), \qquad s = \sigma + j\omega \ \ (\sigma = 0)$$

$$\Rightarrow \omega = \frac{s}{j}$$

$$|H(s)|^2 = \frac{1}{1+(\frac{s}{j})^{2n}}$$

(3) Pole

$$H(s)H(-s) = \frac{1}{1+\frac{s^{2n}}{j}}$$

$$\text{The pole of } H^2(S) \rightarrow \left(\frac{S}{J}\right)^{2n} = -1$$

$$s^{2n} = -j^{2n}$$

By Eular formula,

$$-1 = e^{j\pi(2k-1)}$$

$$j = e^{\frac{2\pi}{2}}$$

$$s^{2n} = e^{j\pi(2k-1+n)}$$

That is,

$$S_{kn} = e^{\frac{j\pi(2k+n-1)}{2n}} = \cos\left(\frac{\pi}{2n}(2k+n-1)\right) + j\ \sin\left(\frac{\pi}{2n}(2k+n-1)\right)$$

(4) Transfer function for Normalized Butterworth filter

$$\mathrm{H(s)} = \frac{1}{(s-s_1)(s-s_2)\ldots(s-s_n)}$$

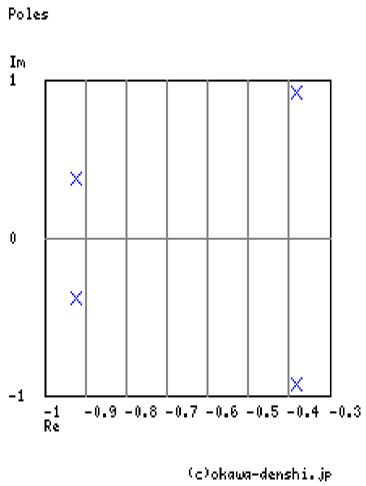

Fig.5 Pole Zero plot of H(s) in fourth order Butterworth filter

For fourth order butterworth filter, the transfer function for stage one and two should be:

$$\frac{1}{({s_n}^2 - 0.765\ s_n + 1)} \times \frac{1}{({s_n}^2 - 1.484\ s_n + 1)}$$

Section III

Fourth-order sallen-key Butterworth filter design follow the principles below:

(1) $$\mathrm{H(s)} = \frac{\frac{1}{R_{1n}C_{1n}R_{2n}C_{2n}}}{s^2+\left(\frac{1}{R_{1n}C_{1n}}+\frac{1}{R_{2n}C_{1n}}\right)s+1} \times \frac{\frac{1}{R_{3n}C_{3n}R_{4n}C_{4n}}}{s^2+\left(\frac{1}{R_{3n}C_{3n}}+\frac{1}{R_{4n}C_{3n}}\right)s+1} =$$

$$\frac{1}{({s_n}^2 - 0.765\ s_n + 1)} \times \frac{1}{({s_n}^2 - 1.484\ s_n + 1)}$$

(2) $$\frac{1}{R_{1n}C_{1n}R_{2n}C_{2n}} = \frac{1}{R_{3n}C_{3n}R_{4n}C_{4n}} = 1 \ \ (because\ \ of\ \ normalization)$$

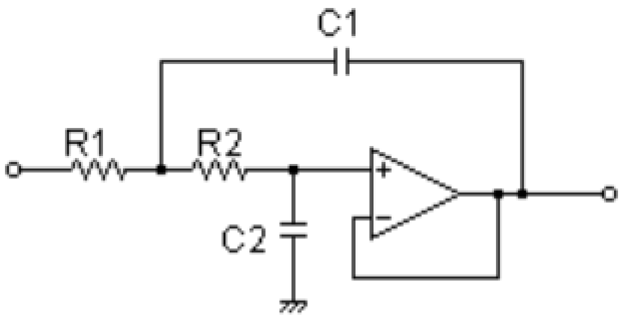

Fig.6 A sallen-key active filter model

Design pattern:

There are totally 8 variables, but only have 4 equations, so we have to set value into some variables. We can choose to select capacitor value, or select resistor value we want.

If we choose to select resistor, set $R_{1n}, R_{2n}, R_{3n}, R_{4n}$ all to 1k, then we can get the following equations:

$$\frac{1}{R_{1n}C_{1n}}+\frac{1}{R_{2n}C_{1n}} \equiv \frac{C_{2n}(1000+1000)}{R_{1n}C_{1n}R_{2n}C_{2n}} = 0.765$$

$$\frac{1}{R_{3n}C_{3n}}+\frac{1}{R_{4n}C_{3n}} \equiv \frac{C_{4n}(1000+1000)}{R_{4n}C_{3n}R_{4n}C_{4n}} = 1.848$$

$$\frac{1}{1000 \times C_{1n} \times 1000 \times C_{2n}} = \frac{1}{1000 \times C_{3n} \times 1000 \times C_{4n}} = 1$$

Then we get

$$C_{2n} = \frac{0.765}{2000}, C_{2d} = \frac{C_{2n}}{2\pi f} \ (while \ \ f \ \ is \ \ 1.8kHz) = 33.8nF$$

$$C_{1n} = \frac{1}{1000^2 \times \frac{0.765}{2000}} = \ 2.61 \times 10^{-3}, C_{1d} = \frac{C_{1n}}{2\pi f} = 230nF$$

$$C_{4n} = \frac{1.848}{2000}, \ \ C_{4d} = \ 81.7nF$$

$$C_{3n} = \frac{1}{1000^2 \times \frac{1.848}{2000}}, \ \ C_{3d} = \ 95.4nF$$

$$C_{1d} = 230nF, C_{2d} = 33.8nF, \ C_{3d} = \ 95.4nF, \ C_{4d} = \ 81.7nF$$

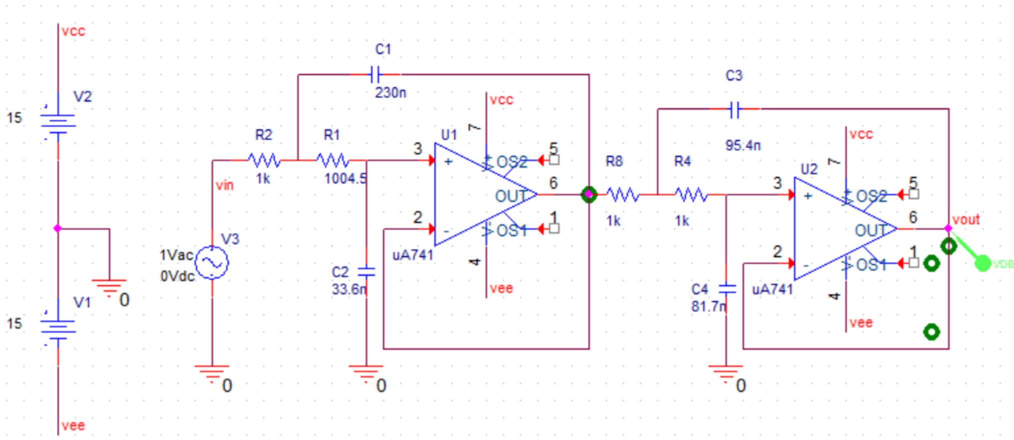

Fig.7 the same picture as Fig.1, the resistor R1 had been tuned into 1004.5Ω

Since resistors have variety of types, values and orders, it is easier to dig up random value of resistors than capacitors in the laboratory. In this case, we select capacitors value instead of resistors, here comes the following equations:

$$\frac{C_{2n}(R_{1n}+R_{2n})}{R_{1n}C_{1n}R_{2n}C_{2n}} = 0.765$$

$$\frac{1}{R_{1n}C_{1n}R_{2n}C_{2n}} = 1$$

$$\Rightarrow \frac{(R_{1n}+R_{2n})}{R_{1n}R_{2n}} = 0.765 \times C_{1n}, \quad R_{1n}R_{2n} = \frac{1}{C_{1n}C_{2n}}$$

Notice

$$C_n = C_d \times 2\pi f$$

If we try to calculate,

$$R_{2n} = \frac{1}{C_{1n}C_{2n}R_{1n}}$$

$$\frac{\left(R_{1n} + \frac{1}{C_{1n}C_{2n}R_{1n}}\right)}{R_{1n}\frac{1}{C_{1n}C_{2n}R_{1n}}} = 0.765 \times C_{1n}$$

$$\left(R_{1n} + \frac{1}{C_{1n}C_{2n}R_{1n}}\right) = \frac{0.765}{C_{2n}}$$

$$R_{1n}{}^2 - \left(\frac{0.765}{C_{2n}}\right)R_{1n} + \frac{1}{C_{1n}C_{2n}} = 0$$

the solution of R_{1n} must be real number

$$b^2 - 4ac \geq 0$$

$$\left(\frac{0.765}{C_{2n}}\right)^2 \geq 4 \times \frac{1}{C_{1n}C_{2n}}$$

$$\frac{C_{1n}}{C_{2n}} = \frac{C_{1d}}{C_{2d}} \geq 6.84$$

$$\frac{C_{3n}}{C_{4n}} = \frac{C_{3d}}{C_{4d}} \geq 4.86$$

If the two capacitors we picked do not match with the ratio above, it is impossible for us to get the right value of resistors.

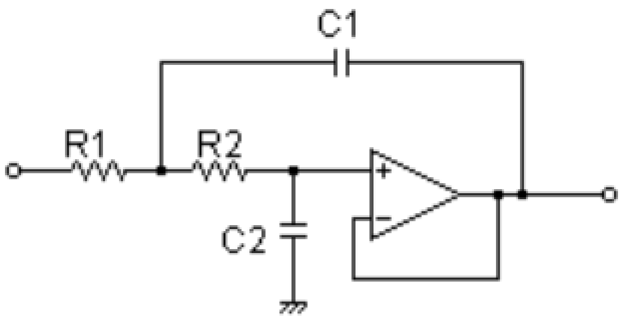

Fig.8 A sallen-key active filter model

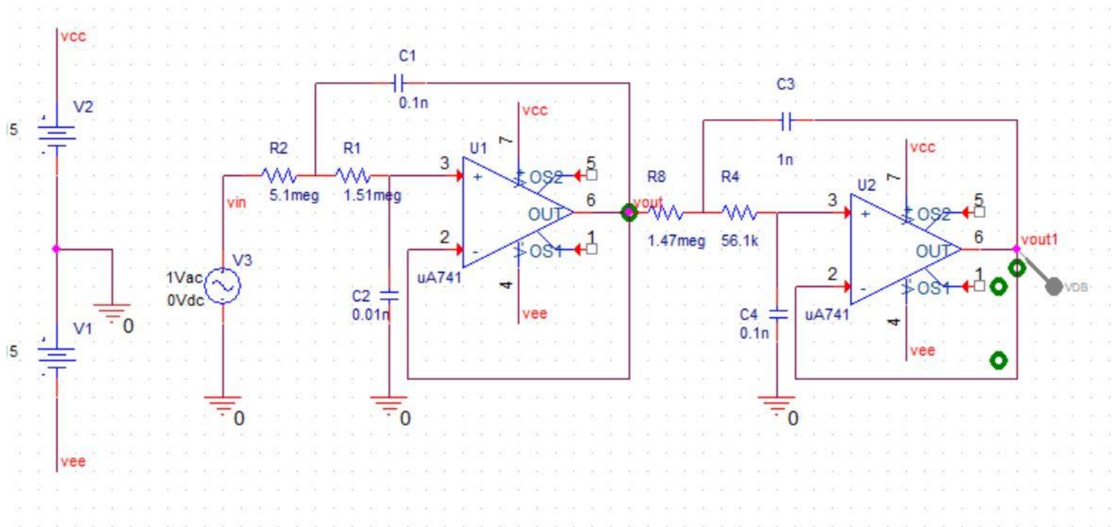

Fig.9 A sallen-key fourth order low pass active filter (select capacitor instead of resistor) design by myself, with cut off frequency 1.805kHz. Notice that this model only need 6 resistors and 4 capacitors in practice.

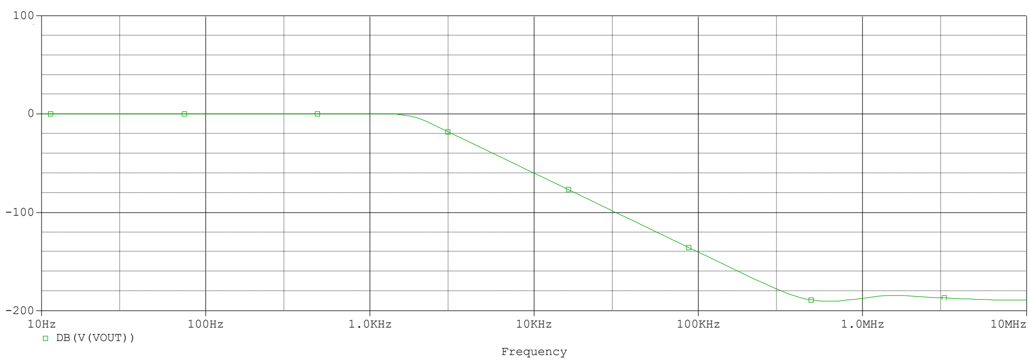

Fig.10 Bode Plot of sallen-key low pass active filter (select capacitor) with cut off frequency 1.805kHz

Evaluate	Measurement	Value
☑	Cutoff_Lowpass_3dB(V(vout)/V(vin))	1.80575k

Fig.11 -3dB cut off frequency data on Pspice