

Examples of Time Complexity

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Time Complexity Examples: $O(n)$

```
for (i=0; i<n; i++) {
    statements...
}
```

```
for (i=0; i<n; i=i+2) {
    statements...
}
```

```
for (i=0; i<n; i=i+50) {
    statements...
}
```

```
def factorial( n ):
    if n == 1:
        return 1
    return n * factorial( n - 1 )
```

$$\begin{cases} a_1 = 1 \\ a_n = 1 + a_{n-1} \end{cases}$$

Time Complexity Examples: $O(n^{1/2})$

```
p=0;
for (i=0; p<n; i++) {
    p=p+i;
}
```

i	p
0	0
1	0+1
2	0+1+2
3	0+1+2+3

$k = k(k+1)/2 = n$
 $k = O(n^{1/2})$

```
i=1;
k=1;
while (k<n) {
    statements...
    k=k+i;
    i++;
}
```

Time Complexity Examples: $O(n^2)$

```
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        statements...
    }
}
```

互相獨立

```
for (i=0; i<n; i++) {
    for (j=0; j<i; j++) {
        statements...
    }
}
```

depend on i
0+1+2+3...
 $n(n+1)/2 = O(n^2)$

Time Complexity Examples: $O(\log(\log(n)))$

```
p=0;
for (i=0; i<n; i=i*2) {
    p++
}
for (j=0; j<p; j=j*2) {
    statements...
}
```

	i	p
1	1	1
2	1*2	2
3	1*2*2	3

$k = 2^{(k-1)}$

$k-1 = \log n$
 $k = \log n$

$O(\log(p)) \Rightarrow$
 $O(\log(\log n))$

```
for (i=0; i<n; i=i*2) {
    statements...
}
```

1	n
2	n/2
3	n/2^2
k	n/2^k

```
for (i=n; i>=1; i=i/2) {
    statements...
}
```

Time Complexity Examples: $O(\log(\log(n)))$

```
for (i=2; i<n; i=i*i) {
    statements...
}
```

1	2
2	2^2
3	2^3

$2^{2^{k-1}} \Rightarrow k-1$
 $= \log(\log n)$

```
for (i=n; i>2; i=sqrt(i)) {
    statements...
}
```

1	n
2	$n^{1/2}$
3	$n^{1/4}$
4	$n^{1/8}$

$n^{1/2^{k-1}} = 2$

Time Complexity Examples: $O(n \log(n))$

```
for (i=0; i<n; i++) {
    for (j=1; j<n; j=j*2) {
        statements...
    }
}
```

i	j
1	1
2	1*2
3	1*2*2

$2^{(k-1)} = n$
 $k-1 = \log_2(n)$
 加上外迴圈 $O(n \log n)$

```
for (i=1; i<n; i++) {
    for (j=1; j<n; j=j+i) {
        statements...
    }
}
```

i	j
1	n
2	n/2
3	n/3

k	n/k
---	-----

$n + n/2 + n/3 + \dots + n/k \dots n/n$
 $= n(1 + 1/2 + 1/3 \dots)$

$$\int_1^n \frac{1}{x} dx = \ln(n)$$

$n \log n$

Time Complexity Examples: $O(2^n)$

```
int fibo(n){  
    if (n==1) return 1;  
    if (n==2) return 2;  
    return fibo(n-1)+fibo(n-2);  
}
```


References

○ References

- Time Complexity #1
- Time Complexity #2
- How to find time complexity of an algorithm