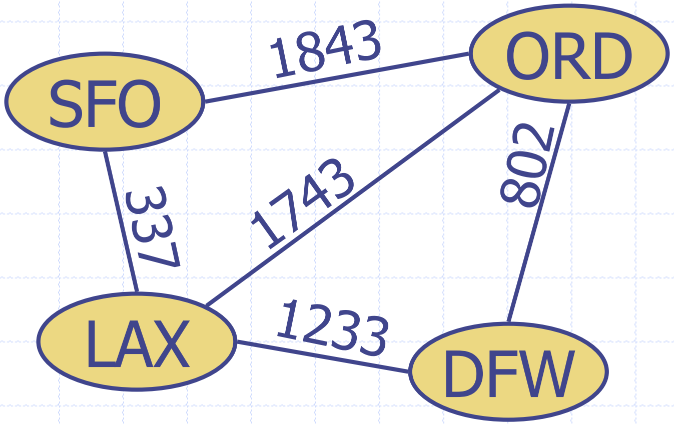
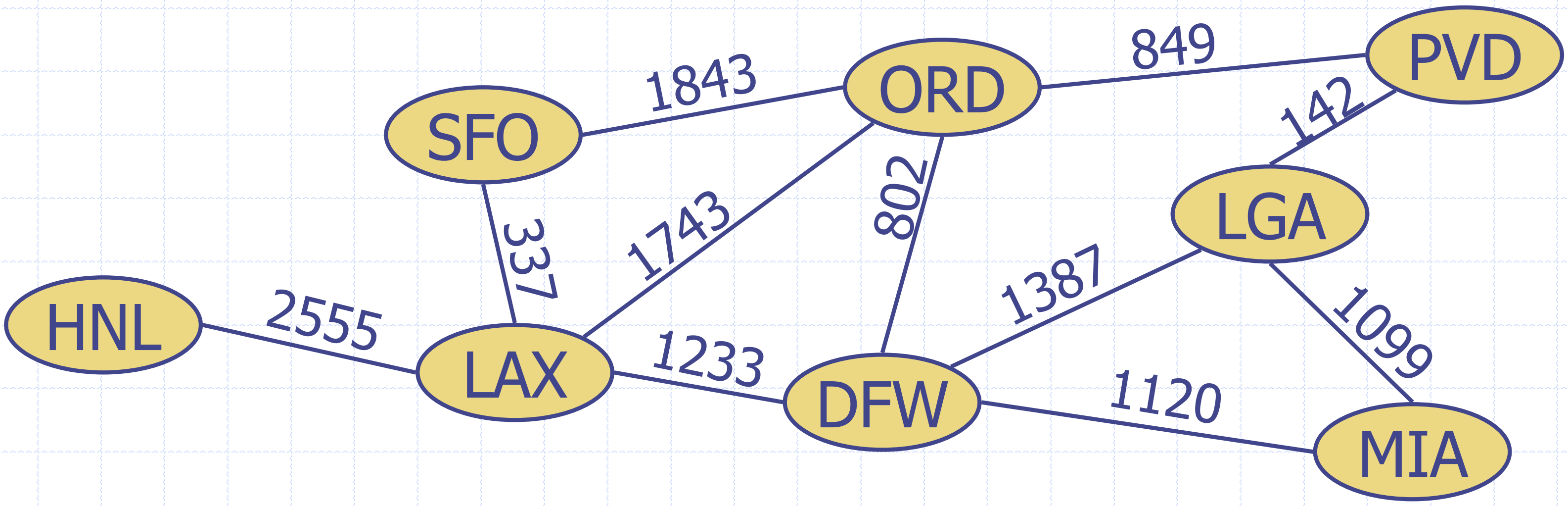
Graphs



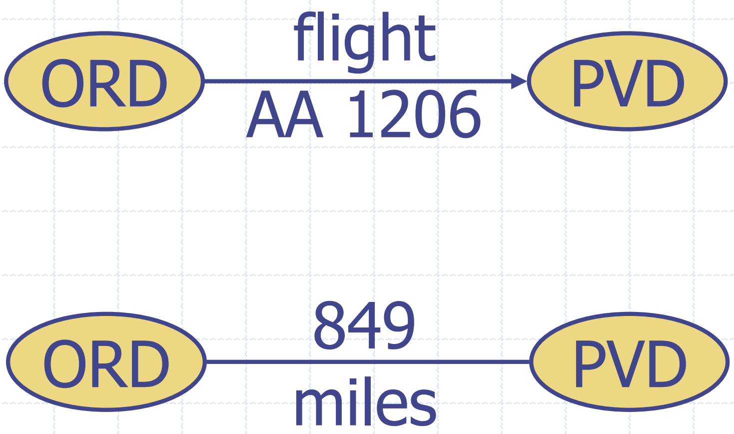
Graphs

- A graph is a pair (V, E), where
  - V is a set of nodes, called vertices
  - E is a collection of pairs of vertices, called edges
- Example:
  - A vertex represents an airport and stores the three-letter airport code
  - An edge represents a flight route between two airports and stores the mileage of the route



Edge Types

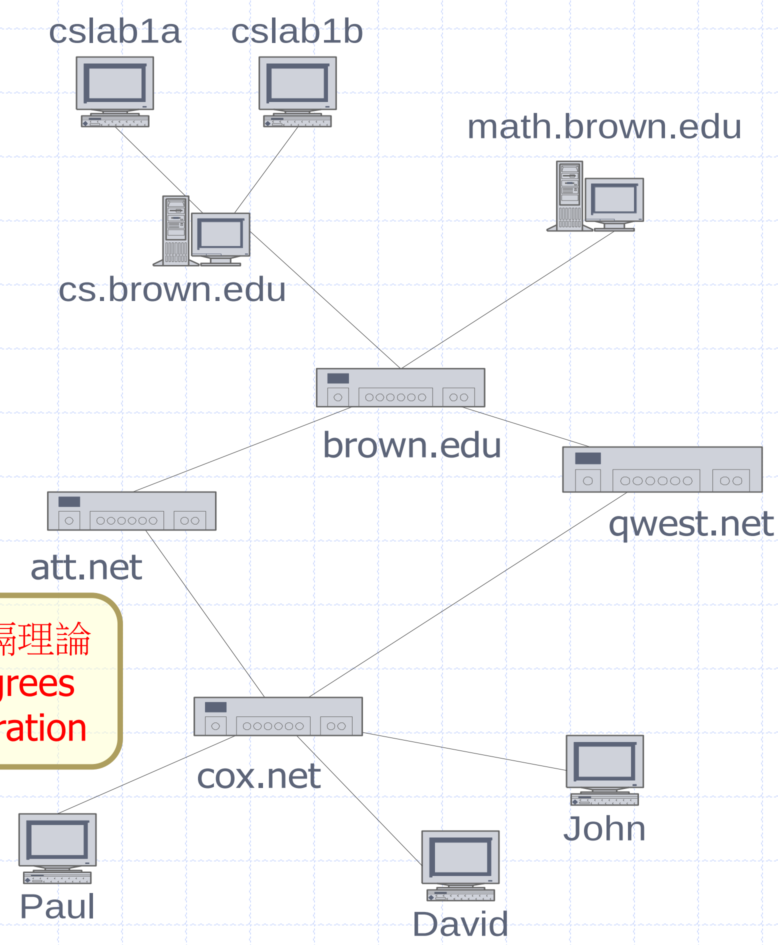
- Directed edge
  - ordered pair of vertices (u,v)
  - first vertex u is the origin or source
  - second vertex v is the destination
  - e.g., a flight number
- Undirected edge
  - unordered pair of vertices (u,v)
  - e.g., a flight mileage
- Directed graph (digraph)
  - all the edges are directed
- Undirected graph
  - all the edges are undirected



Applications

- Electronic circuits
  - Printed circuit board
  - Integrated circuit
- Transportation networks
  - Highway network
  - Flight network
- Computer networks
  - Internet
- Social networks
  - Facebook, line, etc.
- Databases
  - Entity-relationship diagram

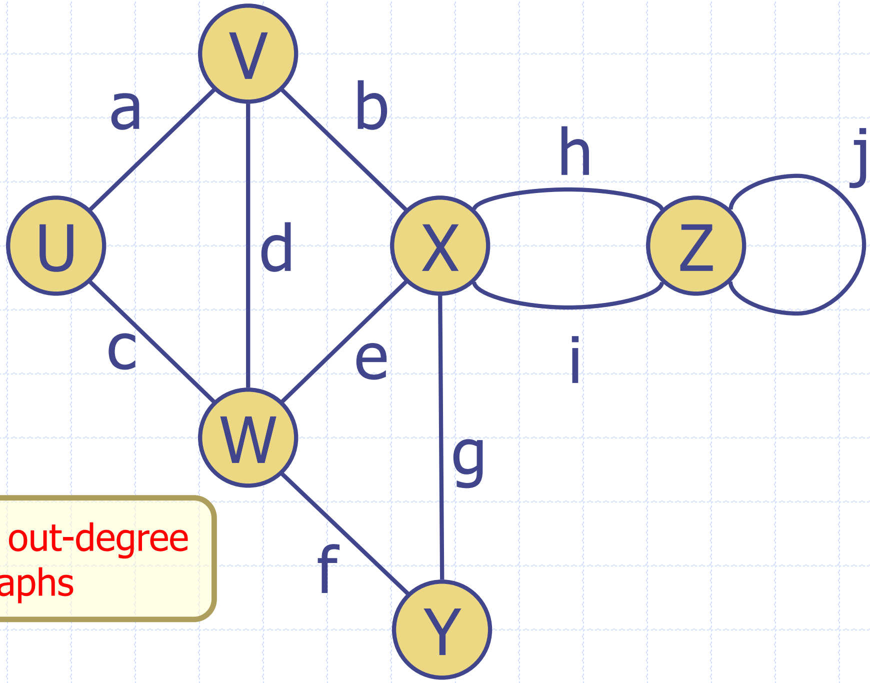
六度分隔理論
Six degrees
of separation



Terminology

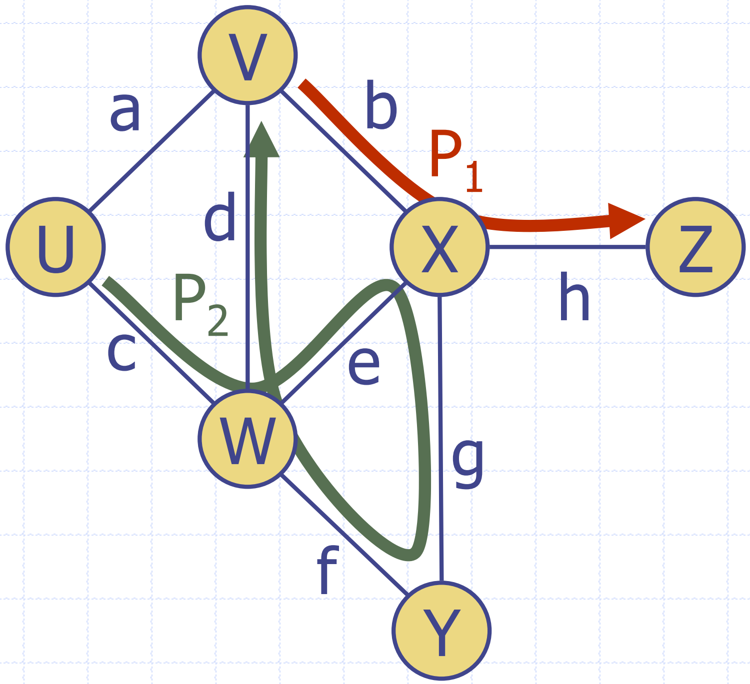
- End vertices (or endpoints) of an edge
  - U and V are the endpoints of a
- Edges incident on a vertex
  - a, d, and b are incident on V
- Adjacent vertices
  - U and V are adjacent
- Degree of a vertex
  - X has degree 5
- Parallel edges
  - h and i are parallel edges
- Self-loop
  - j is a self-loop

In-degree and out-degree for digraphs



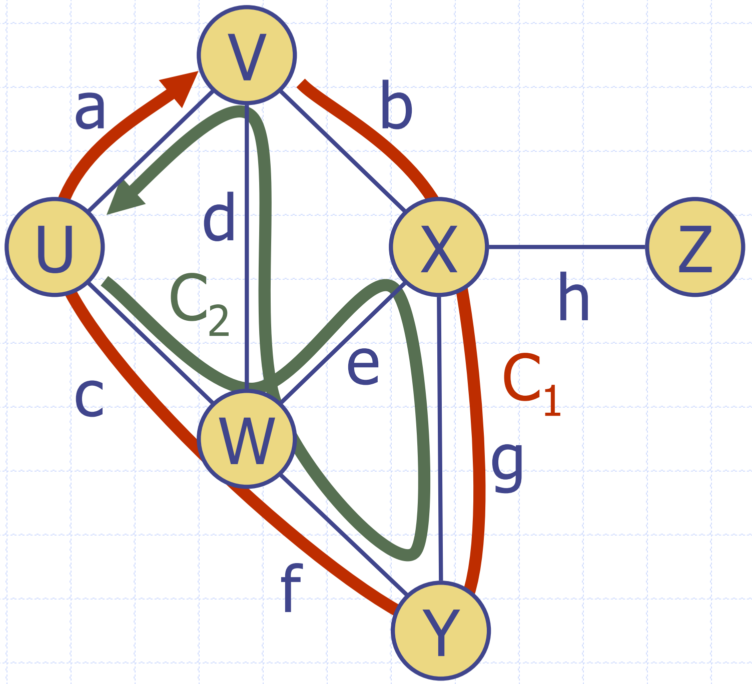
Terminology (cont.)

- Path
  - sequence of alternating vertices and edges
  - begins with a vertex
  - ends with a vertex
  - each edge is preceded and followed by its endpoints
- Simple path
  - path such that all its vertices and edges are distinct
- Examples
  - P_1=(V,b,X,h,Z) is a simple path
  - P_2=(U,c,W,e,X,g,Y,f,W,d,V) is a path that is not simple



Terminology (cont.)

- Cycle
  - circular sequence of alternating vertices and edges
  - each edge is preceded and followed by its endpoints
- Simple cycle
  - cycle such that all its vertices and edges are distinct
- Examples
  - C_1=(V,b,X,g,Y,f,W,c,U,a,↵) is a simple cycle
  - C_2=(U,c,W,e,X,g,Y,f,W,d,V,a,↵) is a cycle that is not simple



Properties

Property 1

$\sum_v \deg(v) = 2m$

Proof: each edge is counted twice

Property 2

In an undirected graph with no self-loops and no multiple edges

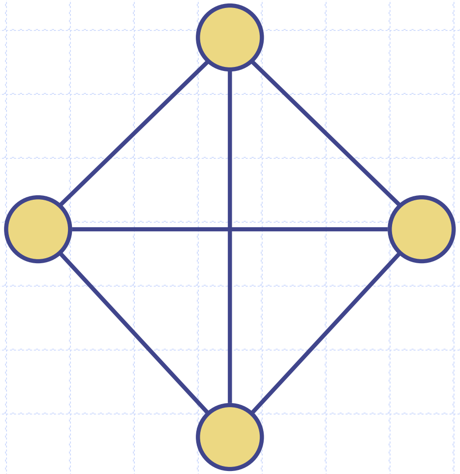
$m \leq n\,(n-1)/2$

Proof: each vertex has degree at most $(n-1)$

What is the bound for a directed graph?

Notation

n	number of vertices
m	number of edges
$\deg(v)$	degree of vertex v



Example

- $n = 4$
- $m = 6$
- $\deg(v) = 3$

Main Methods of the Graph ADT

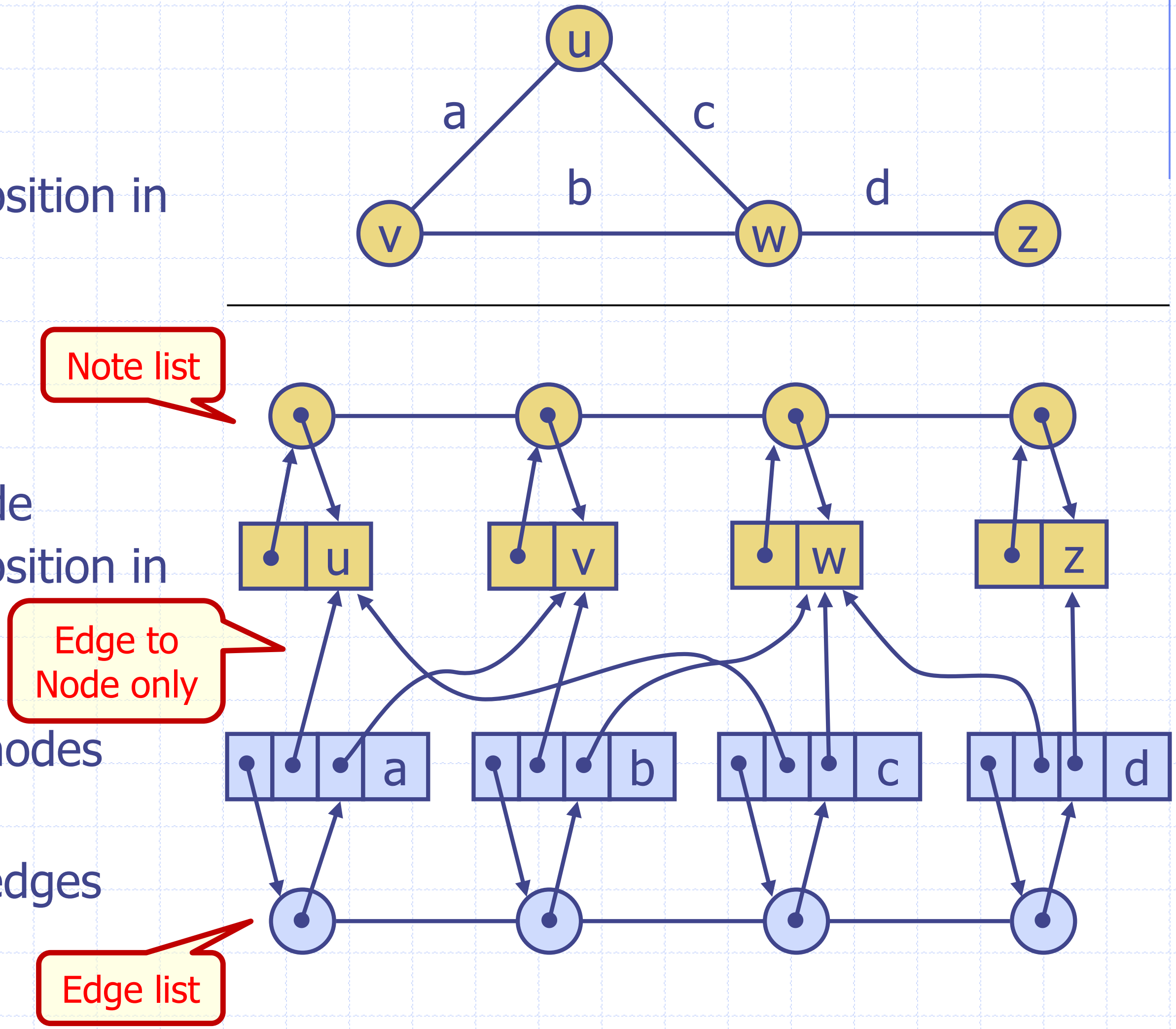
- Vertices and edges
  - are positions
  - store elements
- Accessor methods
  - e.endVertices(): a list of the two endvertices of e
  - e.opposite(v): the vertex opposite of v on e
  - u.isAdjacentTo(v): true iff u and v are adjacent
  - *v: reference to element associated with vertex v
  - *e: reference to element associated with edge e
- Update methods
  - insertVertex(o): insert a vertex storing element o
  - insertEdge(v, w, o): insert an edge (v,w) storing element o
  - eraseVertex(v): remove vertex v (and its incident edges)
  - eraseEdge(e): remove edge e
- Iterable collection methods
  - incidentEdges(v): list of edges incident to v
  - vertices(): list of all vertices in the graph
  - edges(): list of all edges in the graph

Data Structures for Graphs

- Edge-list structure
- Adjacency-list structure
- Adjacency-matrix structure

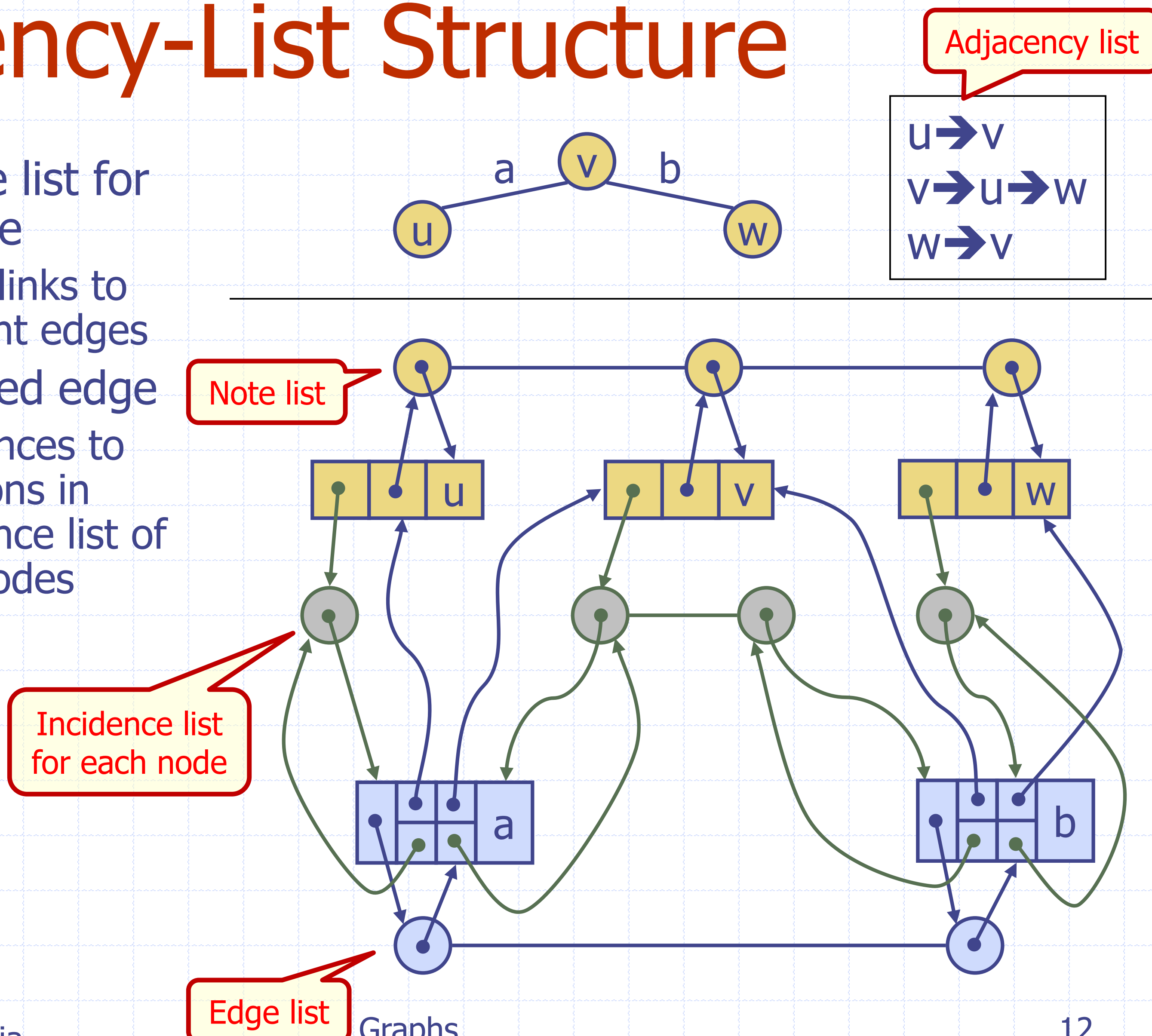
Edge-List Structure

- Node
  - element
  - reference to position in node list
- Edge
  - element
  - origin node
  - destination node
  - reference to position in edge list
- Node list
  - list of links to nodes
- Edge list
  - list of links to edges



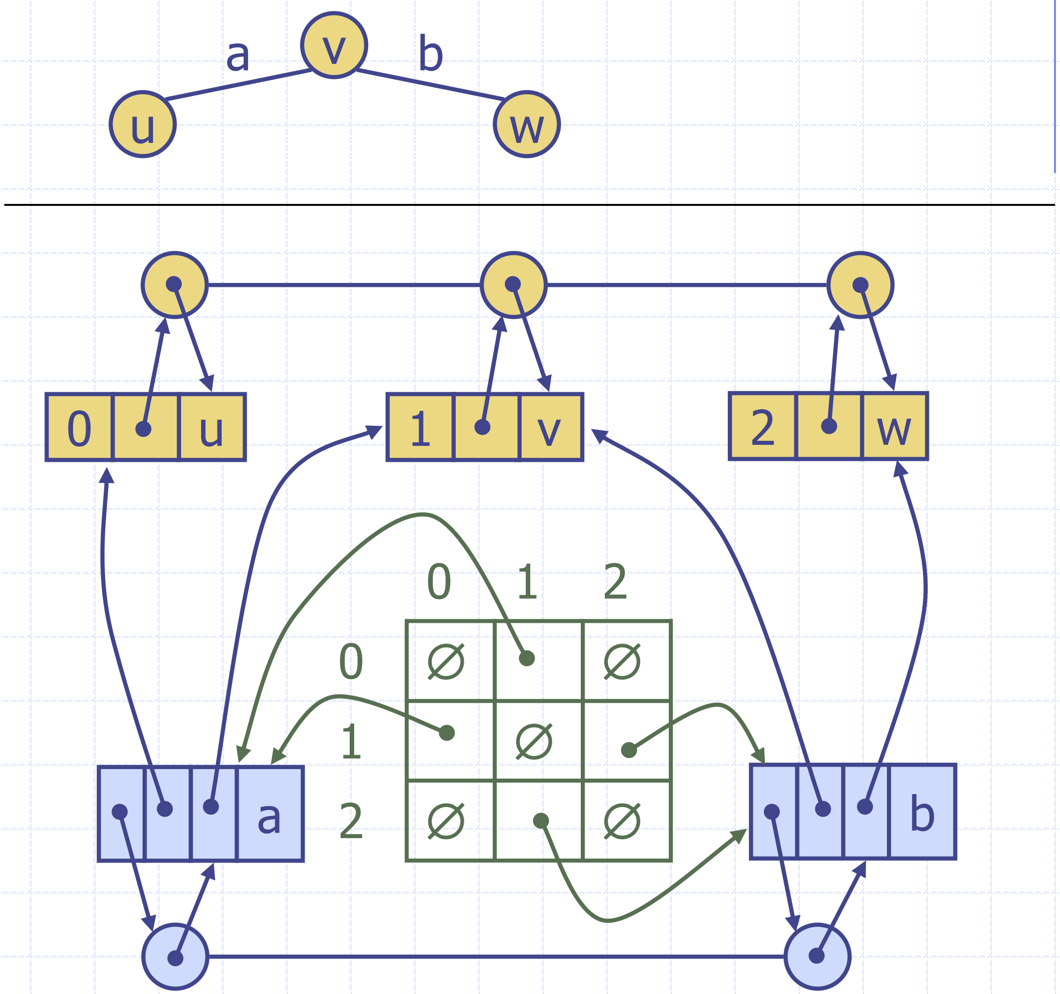
Adjacency-List Structure

- Incidence list for each node
  - list of links to incident edges
- Augmented edge
  - references to positions in incidence list of end nodes



Adjacency-Matrix Structure

- Augmented nodes
  - Integer index associated with node
- Adjacency matrix
  - Reference to edge for adjacent nodes
- The "old fashioned" matrix just has 0 for no edge and 1 for edge



Performance

We shall stick to this structure!

▪ n vertices, m edges ▪ no parallel edges ▪ no self-loops	Edge List	Adjacency List	Adjacency Matrix
Space	$n + m$	$n + m$	n^2
v.incidentEdges()	m	$\deg(v)$	n
u.isAdjacentTo(v)	m	$\min(\deg(u), \deg(v))$	1
insertVertex(o)	1	1	n^2
insertEdge(v, w, o)	1	1	1
eraseVertex(v)	m	$\deg(v)$	n^2
eraseEdge(e)	1	1	1

Assume V & E are stored in doubly linked lists

Create a new matrix