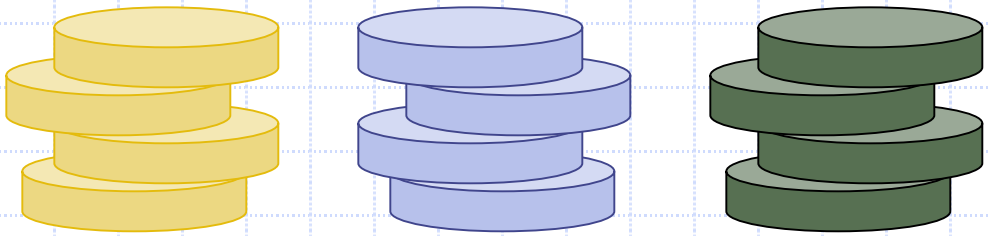
Stacks

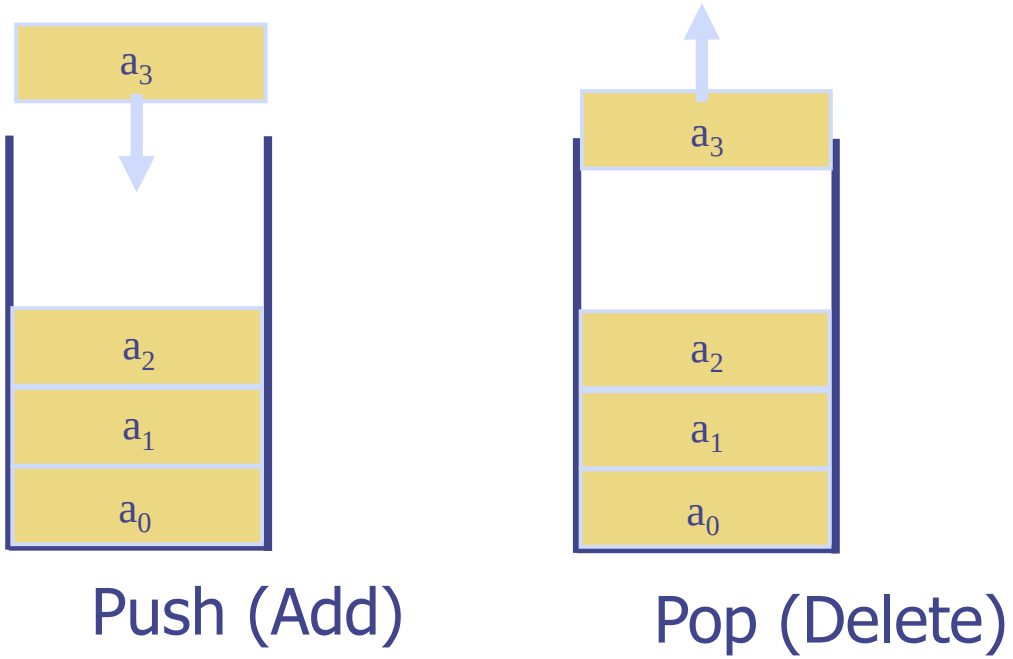


Stack

- What is a stack?
  - An ordered list where insertions and deletions occur at one end called the top.
  - Also known as last-in-first-out (LIFO) list.
- Examples

Stack Representation

- Given a stack $S = (a_0, \ldots, a_{n-1})$, a_0 is the bottom element, a_{n-1} is the top element, and a_i is on top of element a_{i-1}, $0 < i < n$.



Abstract Data Type (ADT)

- ADT (abstract data type) is an abstraction of a data structure which specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations
- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

ADT of Stacks

- Data
  - Arbitrary objects
- Operations
  - push(object): inserts an element
  - pop(): removes the last inserted element
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean empty(): indicates if no elements are stored

- Exceptions (Error conditions)
  - pop() and top() cannot be performed if the stack is empty.
  - push(object) cannot be performed if the stack is full.

Stack Interface in C++

- C++ interface corresponding to our Stack ADT
- Uses an exception class StackEmpty
- Different from the built-in C++ STL class stack

```
template <typename E>
class Stack  {
public:
   int size() const;
   bool empty() const;
   const E& top() const
        throw(StackEmpty);
   void push(const E& e);
   void pop() throw(StackEmpty);
}
```

No change in member variables (for compiler only)

給compiler看不能改到member variable

More info about const:
http://tw.tonytuan.org/2010/03/c-constconst-pointer-pointer-to-const.html

Applications of Stacks

- Page-visited history in a web browser
- Undo sequence in a text editor
- Chain of function calls in the C++ run-time system

- Infix to postfix conversion
- Postfix expression evaluation
- Parenthesis matching
- HTML tag matching
- Maze solution finding

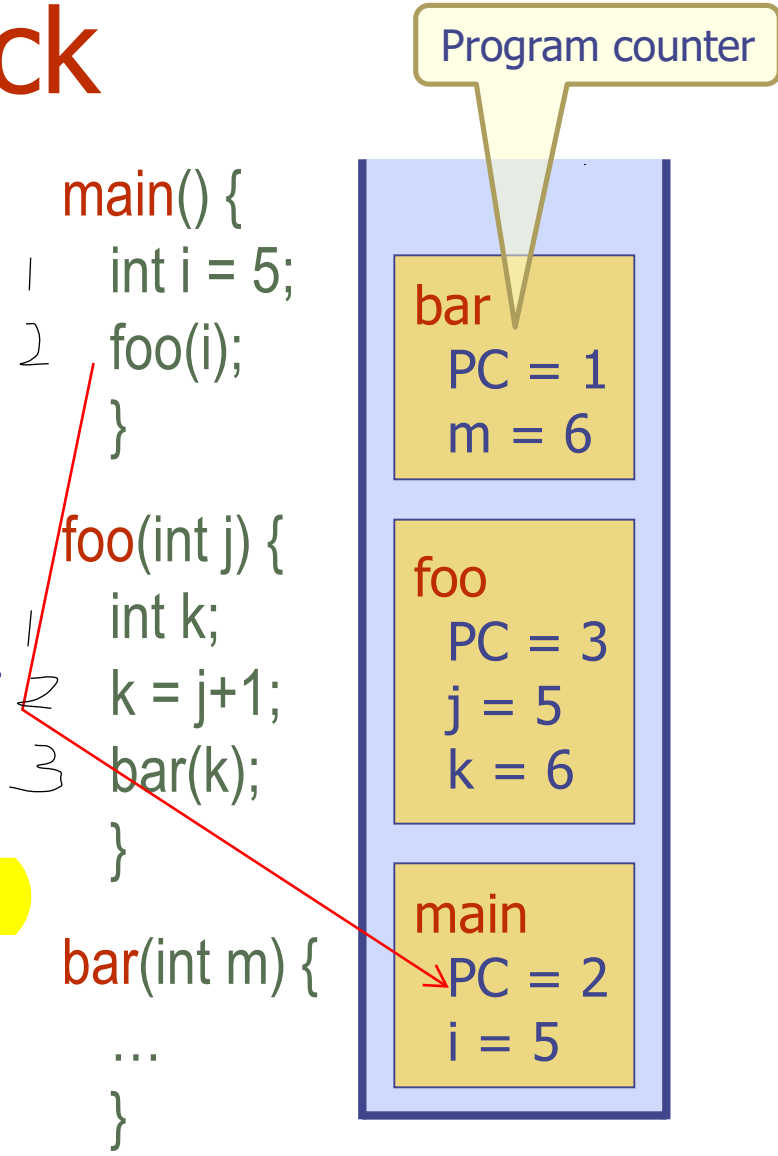
Not in textbook

Not in textbook

Not in textbook

C++ Run-Time Stack

- The C++ run-time system keeps track of the chain of active functions with a stack
- When a function is called, the system pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When the function ends, its frame is popped from the stack and control is passed to the function on top of the stack
- Allows for recursion

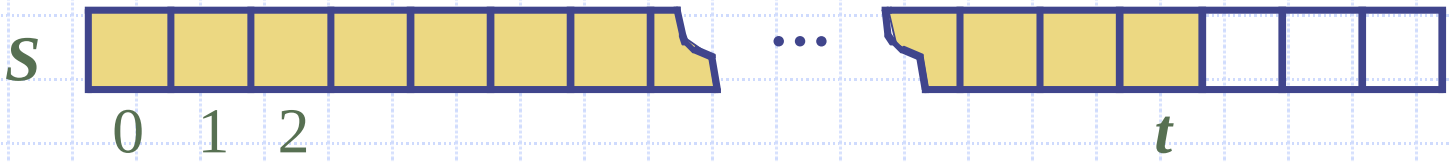


Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

Algorithm *size*()
  return $t + 1$

Algorithm *pop*()
  if *empty*() **then**
    throw *StackEmpty*
  else
    $t \leftarrow t - 1$
    return $S[t + 1]$



Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then throw a StackFull exception

```
Algorithm push(obj)
  if t = S.size() – 1 then
    throw StackFull
  else
    t ← t + 1
    S[t] ← obj
```

Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

Array-based Stack in C++

```
template <typename E>
class ArrayStack {
private:
   E* S; // array holding the stack
   int cap; // capacity
   int t; // index of top element
public:
   // constructor given capacity
   ArrayStack( int c) :
       S(new E[c]), cap(c), t(-1) { }
```

```
   void pop() {
     if (empty()) throw StackEmpty
            ("Pop from empty stack");
      t--;
    }
   void push(const E& e) {
      if (size() == cap) throw
          StackFull("Push to full stack");
      S[++ t] = e;
}
…  (other methods of Stack interface)
```

Example use in C++


```
ArrayStack<int> A;                      // A = [ ], size = 0
A.push(7);                              // A = [7*], size = 1
A.push(13);                             // A = [7, 13*], size = 2
cout << A.top() << endl; A.pop();       // A = [7*], outputs: 13
A.push(9);                              // A = [7, 9*], size = 2
cout << A.top() << endl;                // A = [7, 9*], outputs: 9
cout << A.top() << endl; A.pop();       // A = [7*], outputs: 9
ArrayStack<string> B(10);               // B = [ ], size = 0
B.push("Bob");                          // B = [Bob*], size = 1
B.push("Alice");                        // B = [Bob, Alice*], size = 2
cout << B.top() << endl; B.pop();       // B = [Bob*], outputs: Alice
B.push("Eve");                          // B = [Bob, Eve*], size = 2
```

Parentheses Matching

- Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “[”
  - correct: ( )(( )){([( )])}
  - correct: (( )(( )){([( )])})
  - incorrect: (( ))){([( )])}
  - incorrect: ({[ ])}
  - incorrect: ((])

Parentheses Matching Algorithm

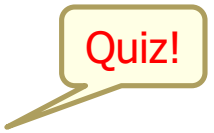
Algorithm ParenMatch(*X*,*n*):
Input: An array *X* of *n* tokens, each of which is either a grouping symbol, a variable, an arithmetic operator, or a number
Output: **true** if and only if all the grouping symbols in *X* match
Let *S* be an empty stack
for *i*=0 to *n*-1 **do**
    if *X*[*i*] is an opening grouping symbol **then**
        S.push(*X*[*i*])
    else if *X*[*i*] is a closing grouping symbol **then**
        if *S*.empty() **then**
            return false {nothing to match with}
        if *S*.pop() does not match the type of *X*[*i*] **then**
            return false {wrong type}
if *S*.empty() **then**
    return true {every symbol matched}
else return false {some symbols were never matched}

Expression Evaluation

- Expressions are converted into postfix notation for evaluation
  - Infix ➔ A/B–C+D*E–A*C
  - Postfix ➔ AB/C-DE*+AC*-

Operation	Postfix
$T_1 = A / B$	T_1C-DE*+AC*-
$T_2 = T_1 - C$	T_2 DE*+AC*-
$T_3 = D * E$	T_2T_3+AC*-
$T_4 = T_2 + T_3$	T_4AC*-
$T_5 = A * C$	T_4 T_5 -
$T_6 = T_4 - T_5$	T_6

Postfix Notation



- Advantages of postfix notation
  - No need to use parentheses
  - No need to consider precedence of operators
- Two questions
  - How to change infix notation into postfix notation?
  - How to evaluate an expression in postfix notation?

Infix to Postfix Conversion by Hand

- Three-pass algorithm:

(1) Fully parenthesize expression

a / b - c + d * e - a * c ➔

((((a / b) - c) + (d * e)) - a * c))

(2) All operators replace their corresponding right parentheses.

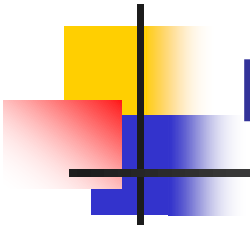
((((a / b) - c) + (d * e)) – (a * c)))

(3) Delete all parentheses.

ab/c-de*+ac*-

Rules of Conversion

- Two rules
  - Operators are taken out of the stack as long as their precedence is higher than or equal to the precedence of the incoming operator.
  - The left parenthesis is placed in the stack whenever it is found in the expression, but it is unstacked only when its matching right parenthesis is found.



Example: Infix to Postfix

The order of operands in infix and postfix are the same.

a+b*c ➔ abc*+

Token	Stack [0] [1] [2]	Output
a		a
+	+	
b	+	b
*	+ *	
c	+ *	c
		*+

Example: Infix to Postfix

Quiz!

a*(b+c)/d ➔ abc+*d/

Token	Stack [0]	[1]	[2]	Output
a				a
*	*			
(	*	(		
b	*	(		b
+	*	(	+	
c	*	(	+	c
)	*			+
/	/			*
d	/			d
				/

More Examples

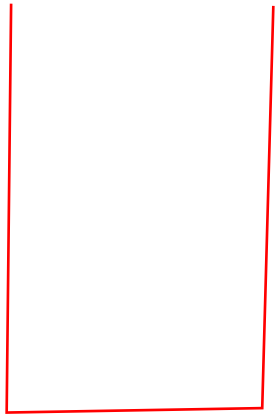
Quiz!

Infix	Postfix
2+3*4	234*+
a*b+5	ab*5+
(1+2)*7	12+7*
a*b/c	ab*c/
(a/(b-c+d))*(e-a)*c	abc-d+/ea-*c*
a/b-c+d*e-a*c	ab/c-de*+ac*-

Walk-through Examples

Quiz!

- a/b-c+d*e-a*c ➔ ab/c-de*+ac*-
- (a/(b-c+d))*(e-a)*c ➔ abc-d+/ea-*c*

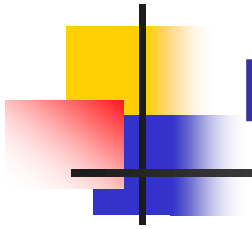


More Examples

Quiz!

- (a+b)*(c-d)/(e+f) ➔ ab+cd-*ef+/
- (a+b)*(c-d)/((e-f)*(g+h)) ➔ ab+cd-*ef-gh+*/

Evaluating postfix expressions



- Evaluation process

O(n)

  - Make a single left-to-right scan of the expression.
  - Place the operands on a stack until an operator is found.
  - Remove, from the stack, the correct numbers of operands for the operator, perform the operation, and place the result back on the stack.

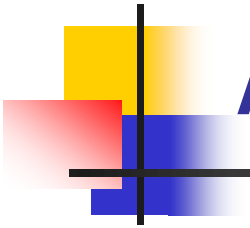
Example of Evaluating postfix expressions

Example: 6 2 / 3 – 4 2 * +

Token	Stack [0]	[1]	[2]
6	6		
2	6	2	
/	6/2		
3	6/2	3	
-	6/2-3		
4	6/2-3	4	
2	6/2-3	4	2
*	6/2-3	4*2	
+	6/2-3+4*2		

Walk-through Examples

- a/b-c+d*e-a*c ➔ ab/c-de*+ac*-
- (a/(b-c+d))*(e-a)*c ➔ abc-d+/ea-*c*



Animation

- Animation
  - Infix to postfix conversion
  - Postfix evaluation