POLSCI 9592

Lecture 6: Unordered Categorical Data Models

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Goals for This Session

- 1. Develop Multinomial Logit Model
- 2. Effects and Effect Displays
- 3. Model Fit and Evaluation
- 4. IIA Assumption
- 5. Conditional Logit
- 6. Separation Problems



Multinomial Logit

- The MNL model is used when the dependent variable is a set of unordered categories.
- We can think of MNL as a generalization of the Ordered Logit model (i.e., the OL model is nested in the MNL model)
- Before, we suggested that there is very little difference between logits and probits that our inferences will likely be very similar. In the multinomial case, these are two quite different models that require different estimation techniques and make different sets of assumptions.
 - We will focus exclusively on the multinomial logit (MNL) model here.



The MNL Model

The MNL model basically works like a bunch of binary logit models. Assume our dependent variable y has $m=\{1,2,\ldots,M\}$ different values.

- Let's say we want to consider the probability of being a 2 versus the probability of being a 1. We could do the following:
 - \circ Subset the data to only those observations where $y=\{1,2\}$.
 - \circ Estimate the binary logit model: $\log\left(\frac{Pr(y=2|X)}{Pr(y=1|X)}\right) = \mathbf{Xb}$.
- The MNL model basically does this in turn for every value of y in $\{2,\ldots,M\}$.
 - \circ Each time, estimating $\log\Bigl(rac{Pr(y=m|X)}{Pr(y=1|X)}\Bigr) = \mathbf{X}\mathbf{b}_m$



The MNL Model

- The group against which we estimate all of the binary logits (i.e., 1 in the example above) is called the reference group.
- We can calculate probabilities of being in a single group as follows:

$$Pr(y=m|X) = rac{exp(\mathbf{X}eta_m)}{\sum_{j=1}^{J}exp(\mathbf{X}eta_j)}$$

• For identification purposes, we set $\beta_1 = \mathbf{0}$. That is, all of the coefficients for the first group are set to zero. Remember, exp(0) = 1.



Likelihood Function

Just like in our other models, the likelihood is simply the probability that the observation takes on its observed y value. We could write this as:

$$L(eta_1,eta_2,\ldots,eta_{J-1}|\mathbf{X}) = \prod_{m=1}^J \prod_{y_i=m} rac{exp(\mathbf{X}eta_m)}{\sum_{j=1}^J exp(\mathbf{X}eta_j)}$$

The Log-likelihood function, then, is:

$$\log L(eta_1,eta_2,\ldots,eta_{J-1}|\mathbf{X}) = \sum_{m=1}^J \sum_{y_i=m} \log\Biggl(rac{exp(\mathbf{X}eta_m)}{\sum_{j=1}^J exp(\mathbf{X}eta_j)}\Biggr)$$



France Data

```
dat <- import("data/france_mnl.dta")
dat <- factorize(dat)
labs <- sapply(1:ncol(dat), function(i)attr(dat[[i]], "label"))
data.frame(var = names(dat), description = labs)</pre>
```

```
description
##
           var
                                                           Party of vote
## 1
          vote
                                            Party number of chosen party
## 2
       votenum
## 3
          male
                                            R gender (0=female, 1=male)
                                                                Age of R
## 4
           age
## 5
         urban
                                          size of city in which R lives
                                                 Subjective social class
       soclass
## 6
      hhincome
                                     Household income (in francs/month)
## 7
                                                       Member of a union
## 8
         union
                            Retrospective national economic evaluations
## 9
        retnat
        demsat satisfaction with the functioning of democracy in france
## 10
## 11
                                 Good choice for France to belong to EU
         eusup
## 12
        lrself self-position on the left-right scale (0=left, 10=right)
## 13
          rp10
                      Respondent identified left-right placement of PCF
          rp20
                       Respondent identified left-right placement of PS
## 14
## 15
          rp50
                    Respondent identified left-right placement of Green
## 16
          rp70
                      Respondent identified left-right placement of UDF
                      Respondent identified left-right placement of UMP
## 17
          rp73
## 18
          rp80
                       Respondent identified left-right placement of FN
```



Estimation: multinom

age

The model can be estimated with the multinom function in the nnet package.

```
mod <- multinom(vote ~ lrself + male + retnat + age +</pre>
    union , data=dat, trace=F)
noquote(t(mnlSig(mod)))
                                        UDF
                                 Green
                                                UMP*
                                                        FΝ
## (Intercept)
                         3.809* 3.253* -0.996
                                               -1.504 -1.153
## lrself
                        0.227* 0.389* 0.925* 1.151* 0.832*
                                0.223
                                        0.727
                                                0.565
                                                        0.231
## male
                         0.574
                        -0.909 -1.306 -1.827 -1.514 -1.196
## retnatsame
                        -1.041 -1.712 -2.524* -2.635* -0.840
## retnatworse
```

You could also use broom::tidy(mod) to get a tidier model output.

 $-0.020 \times -0.047 \times -0.004 -0.010 -0.017$

unionyes, union member -1.325* -1.004* -1.272* -1.279* -1.485*



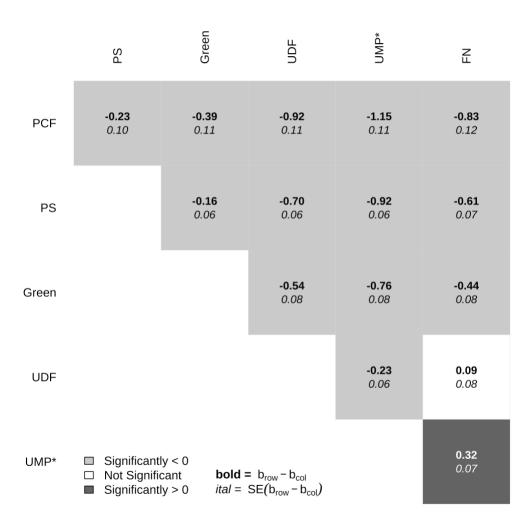
Interpretation: Comparing Coefficients

- Coefficients are in log-odds of voting for the party identified in the row relative to the base category (in R, the base category is always the first one).
- Note that the coefficients relating covariates to non-base-category choices must be considered separately.
- \bullet For example, the effect of left-right self-placement on the choice between Greens and Socialists is -0.174, meaning that as left-right increases, voters are more likely to vote for the Green party than the Socialists.



Pairwise Comparisons

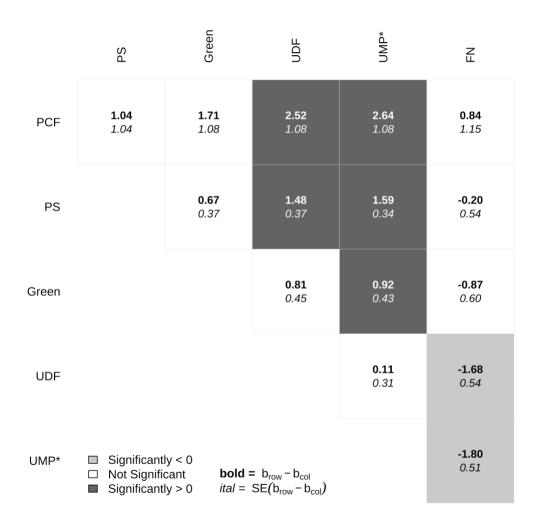
plot(factorplot(mod, variable="lrself"))





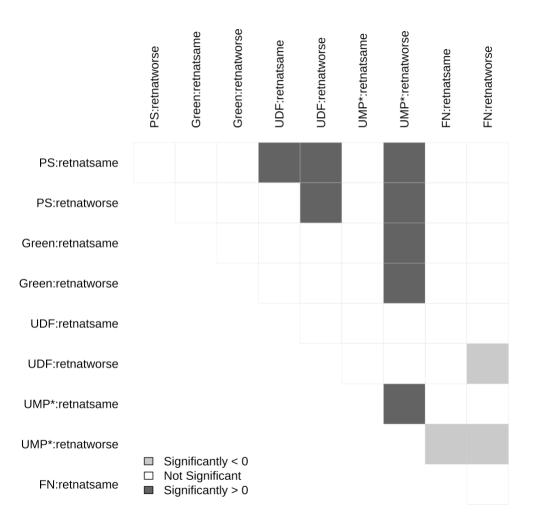
Pairwise Comparisons with Factors

plot(factorplot(mod,
 variable="retnatworse"))



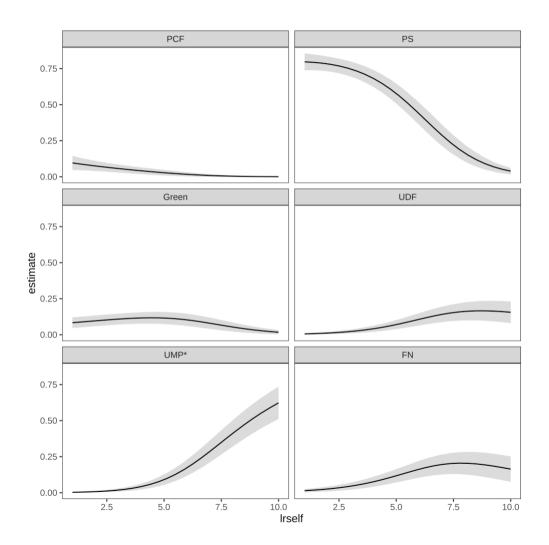


Pairwise Comparisons with Factors



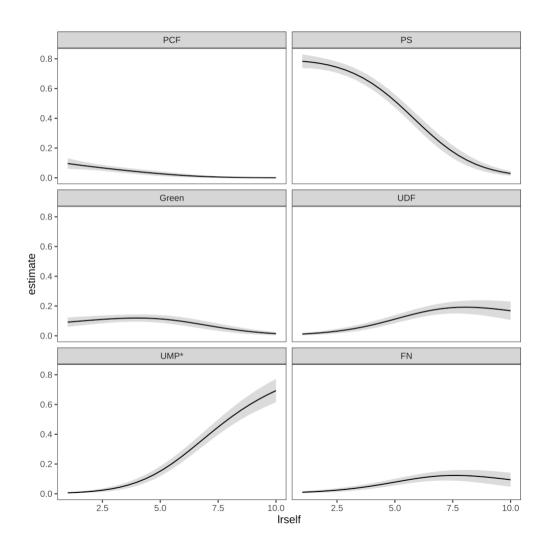


Effects Plots: MER



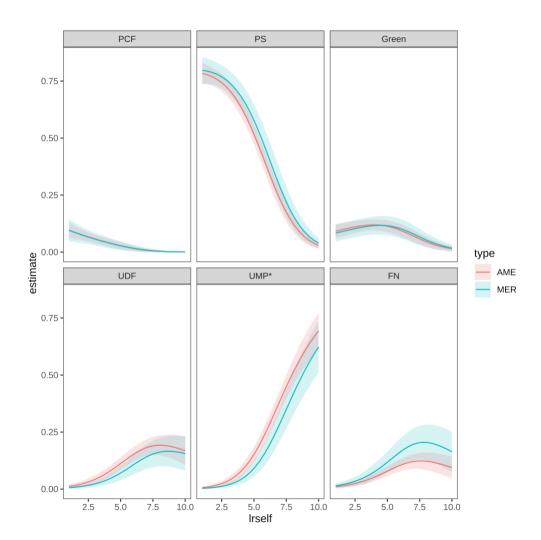


Effect Plot: AME





Comparing Effects





Discrete Changes: MER

```
comps <- comparisons(mod, newdata="median",</pre>
            variables = list(lrself = "2sd",
                             male = c(0,1),
                             retnat = "minmax",
                             age = "2sd",
                             union = "reference"))
comps %>%
  mutate(eff = sprintf("%.2f%s",
                        estimate,
                       ifelse(p.value < .05, "*", ""))) %>%
  select(2:4, eff) %>%
  pivot_wider(names_from="group", values_from="eff")
## # A tibble: 5 × 8
                                                                      `UMP*` FN
    term contrast
                                                           Green UDF
```



Discrete Changes: AME



Model Fit

mnlfit(mod)

```
## $result
##
                                Estimate
                                            p-value
## Fagerland, Hosmer and Bonfi 57.5035680 0.03593795
## Count R2
                               0.6354481
                                                 NA
## Count R2 (Adj)
                               0.2760000
                                                 NA
## ML R2
                               0.5289860
                                                 NA
## McFadden R2
                               0.2622133
                                                 NA
## McFadden R2 (Adj)
                               0.2334525
                                                 NA
## Cragg-Uhler(Nagelkerke) R2 0.5607412
                                                 NA
##
## attr(,"class")
## [1] "mnlfit"
```



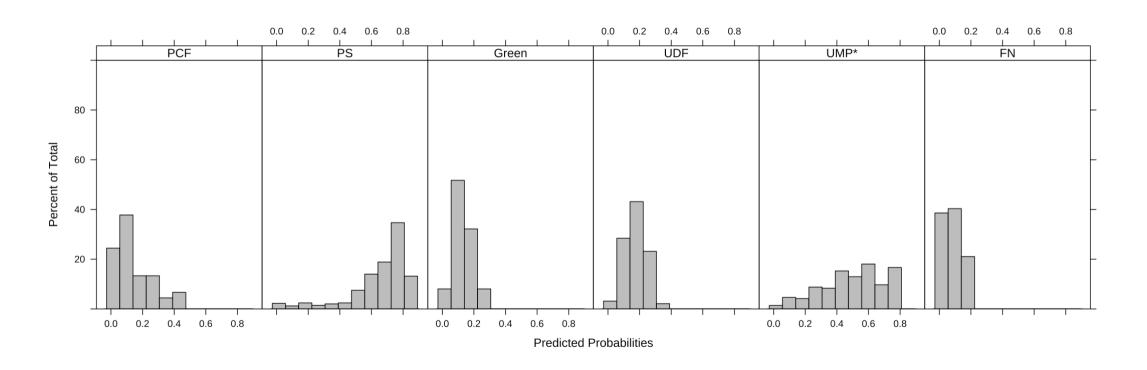
PRE

```
## mod1: vote ~ lrself + male + retnat + age + union
## mod2: vote ~ 1
##
## Analytical Results
## PMC = 0.496
## PCP = 0.635
## PRE = 0.276
## ePMC = 0.316
## ePCP = 0.478
## ePRE = 0.237
```



Probabilities by Group

probgroup(mod)





Classification Table

0 31

We could also look at the classification table where the rows are the actual votes and the columns are the predicted votes

```
y <- model.response(model.frame(mod))
yhat <- predict(mod)
table(y, yhat)

## yhat
## y PCF PS Green UDF UMP* FN
## PCF 0 44 0 0 1 0
## PS 0 457 0 0 36 0
## Green 0 82 0 0 5 0
## UDF 0 39 0 0 56 0
## UDF 0 39 0 0 56 0
## UMP* 0 42 0 0 174 0
```

Note that we predict most people will vote for the two biggest parties - socialists and UMP and we predict a few will vote for the national front (FN).



IIA Assumption

IIA = Independence of Irrelevant Alternatives

- The addition or removal of choices from the choice set should not change the odds of choosing one category over another.
- This is forced to be the case in MNL. Multinomial probit does not make this assumption.



Tests for IIA

There are tests for IIA, but Long and Freese argue they shouldn't be used.

- They are not particularly powerful and can often give conflicting results.
- Dow and Endersby suggest that the relaxing the assumption is rarely important and that doing so comes with its own potential set of problems.
- In my own work, I have found if trying to estimated predicted probabilities, both models work about the same.



Conditional Logit

In the MNL models above, all of the variables varied by individual (i.e., they are attributes of the individuals), but not by choice within individual.

- In this case, each there is a coefficient for each choice Occasionally, there are attributes of the choices we would like to incorporate.
- In this case, Conditional Logit can be used.

The Model

Remember, before we assumed that the relationship between y and \mathbf{X} was captured with eta_m

$$Pr(y=m|X) = rac{exp(\mathbf{X}eta_m)}{\sum_{j=1}^{J}exp(\mathbf{X}eta_j)}$$

where $\mathbf{X}\beta_m = \beta_{0,m} + \beta_{1,m}X_1 + \beta_{2,m}X_2 + \ldots + \beta_{k,m}X_k$. By introducing choice-specific information, we need to change the linear predictor to: $\mathbf{Z}_m\gamma + \mathbf{X}\beta_m$. Notice here, that there are some variables that have an m subscript and those variables only get one coefficient - that is, there is one coefficient relating variable Z_1 to \mathbf{y} because the information in Z_1 varies within observations.



Data Preparation

```
names(dat)[13:18]
## [1] "rp10" "rp20" "rp50" "rp70" "rp73" "rp80"
names(dat)[13:18] <- paste("rp", levels(dat$vote), sep=".")</pre>
dat2 <- dat %>%
  dplyr::select(c("vote", "lrself", "urban", "union", "retnat",
            "age", "male", starts_with("rp"))) %>%
  na.omit()
library(fastDummies)
library(dfidx)
dat2 <- dat2 %>%
  dummy_cols("vote", remove_selected_columns=TRUE)
names(dat2) <- gsub("vote_", "vote.", names(dat2))</pre>
dat2 <- dat2 %>%
  mutate(obs = row_number()) %>%
  pivot_longer(7:18,
                names_pattern = "(.*)\setminus (.*)",
                names_to = c(".value", "alt"))
dat2l <- dfidx(dat2, idx = c("obs", "alt"), choice = "vote")</pre>
```



Conditional Logit Model

```
mlogit2 <- mlogit(vote ~ 0 + I(abs(lrself - rp)) | urban + union + retnat +
   age + male, data=dat2l, reflevel="PS")
printCoefmat(summary(mlogit2)$CoefTable)</pre>
```

```
Estimate Std. Error z-value Pr(>|z|)
##
## (Intercept):FN
                                                          -3.0441 0.0023338 **
                                   -2.2506029 0.7393348
## (Intercept):Green
                                   -0.0489038 0.5260797
                                                          -0.0930 0.9259362
## (Intercept):PCF
                                   -4.2232729
                                              1.1722648
                                                          -3.6027 0.0003150 ***
## (Intercept):UDF
                                              0.5143635
                                                          -3.0103 0.0026102 **
                                   -1.5483687
## (Intercept):UMP**
                                   -0.5136178 0.4324413
                                                          -1.1877 0.2349450
## I(abs(lrself - rp))
                                   -0.5122424
                                              0.0284676 -17.9939 < 2.2e-16 ***
## urbansmall or medium city:FN
                                              0.3865481
                                                          -0.0412 0.9671092
                                   -0.0159390
## urbansmall or medium city:Green -0.1439434
                                              0.2796757
                                                          -0.5147 0.6067769
## urbansmall or medium city:PCF
                                    0.0577400
                                              0.3784491
                                                           0.1526 0.8787373
## urbansmall or medium city:UDF
                                   -0.0467703
                                               0.2977841
                                                          -0.1571 0.8751968
## urbansmall or medium city:UMP** -0.1612852
                                               0.2502350
                                                          -0.6445 0.5192288
## urbanbig city:FN
                                    0.3215769
                                               0.4453963
                                                           0.7220 0.4702934
## urbanbig city:Green
                                   -0.7541499
                                              0.3812699
                                                          -1.9780 0.0479293 *
## urbanbig city:PCF
                                   -0.2370372
                                              0.5087779
                                                          -0.4659 0.6412905
## urbanbig city:UDF
                                              0.3461118
                                                           0.1706 0.8645643
                                    0.0590352
## urbanbig city:UMP**
                                    0.2174351
                                              0.2829513
                                                           0.7685 0.4422174
## unionyes, union member:FN
                                    0.0342450
                                              0.5477225
                                                           0.0625 0.9501467
## unionyes, union member:Green
                                    0.2708221 0.3472815
                                                           0.7798 0.4354883
## unionyes, union member:PCF
                                    1.2122812 0.3615435
                                                           3.3531 0.0007992 ***
## unionyes, union member:UDF
                                   -0.0684448 0.4150341
                                                          -0.1649 0.8690119
## unionyes, union member:UMP**
                                   -0.0514542 0.3432224
                                                          -0.1499 0.8808316
## retnatsame:FN
                                    0.1536325
                                              0.6381573
                                                           0.2407 0.8097536
## retnatsame:Green
                                   -0.5581574
                                              0.4260458
                                                          -1.3101 0.1901661
## retnatsame:PCF
                                    0.9195162 1.1049479
                                                           0.8322 0.4053070
                                   -0.6974712 0.3764854
## retnatsame:UDF
                                                          -1.8526 0.0639419 .
```



Effect Plots

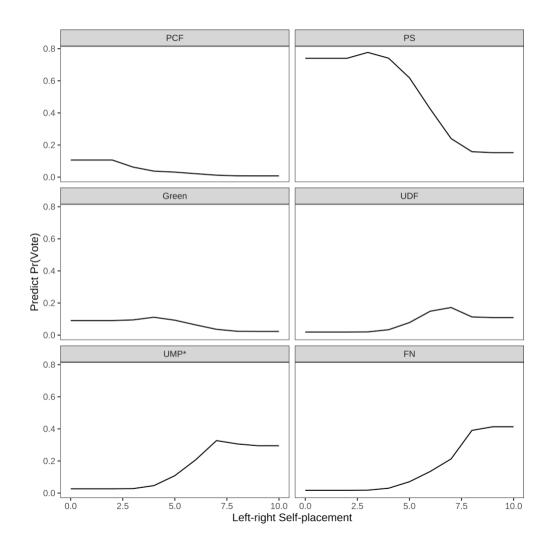
For conditional logit, there are no canned functions to make effects plots. Here are the steps you have to apply:

- 1. Make the data used to generate predictions (either one fake dataset holding all variables at some hypothetical value or a bunch of datasets each with the variable of interest held constant.)
- 2. Generate predictions using each dataset.
- 3. If confidence intervals are desired, use simulation to calculate sampling variability of the predicted probabilities.
- 4. Plot predictions



MER approach

```
lrs <- 0:10
fake <- makeFakeData(mlogit2,</pre>
                      dat2l,
                      change=list(lrself=lrs),
                      varying="rp")
probs <- predict(mlogit2, newdata=fake)</pre>
plot.dat <- data.frame(</pre>
  prob = c(t(probs)),
  party = factor(rep(colnames(probs),
                      length(lrs)),
                 levels=levels(dat$vote)),
  lrself = rep(lrs, each=6))
ggplot(plot.dat, aes(x=lrself, y=prob)) +
  geom_line() +
  facet_wrap(~party, ncol=2) +
  theme_bw() +
  theme(panel.grid=element_blank()) +
  labs(x="Left-right Self-placement",
       y="Predict Pr(Vote)")
```



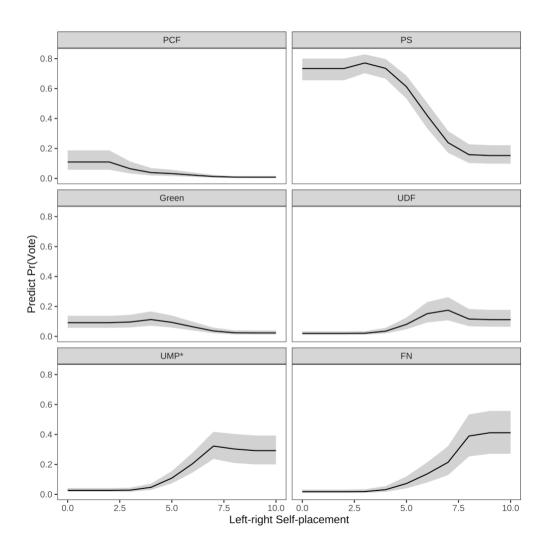


Confidence Intervals

```
B <- MASS::mvrnorm(2500, coef(mlogit2), vcov(mlogit2))</pre>
tmp <- mlogit2</pre>
probs <- NULL
for(i in 1:2500){
  tmp$coefficients <- B[i,]</pre>
  probs <- rbind(probs,</pre>
   cbind(data.frame(lrself = unique(fake$lrself),
                     sim=i),
         predict(tmp,
                  newdata=fake)))
plot.dat <- probs %>%
  pivot_longer(PS:`UMP*`,
               names_to="party",
               values_to="prob") %>%
  mutate(party = factor(party,
                         levels=levels(dat$vote))) %>%
  group_by(lrself, party) %>%
  summarise(p = mean(prob),
            lwr= quantile(prob, .025),
            upr = quantile(prob, .975))
```



Figure





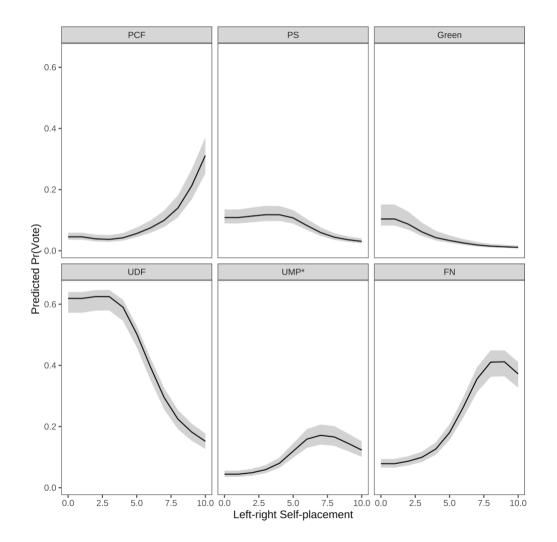
AME Approach

```
B <- MASS::mvrnorm(2500, coef(mlogit2), vcov(mlogit2))
lrs <- 0:10
out <- data.frame(
    lrself = rep(lrs, each=6),
    party = rep(levels(dat$vote), length(lrs)),
    p = NA,
    lwr = NA,
    upr = NA
)</pre>
```

```
for(j in lrs){
  tmp <- dat2l
  tmp$lrself <- j</pre>
  X <- model.matrix(</pre>
    update(mlogit2, data=tmp))
  b <- coef(mlogit2)</pre>
  XB < - X \% * \% t(B)
  Xb <- X %*% b
  p <- prop.table(matrix(exp(Xb), ncol=6, byrow=TRUE), 1)</pre>
  m <- colMeans(p)</pre>
  res <- NULL
  for(i in 1:ncol(XB)){
    exb <- exp(matrix(XB[,i], ncol=6, byrow=TRUE))</pre>
    p <- prop.table(exb, 1)</pre>
    res <- rbind(res, colMeans(p))</pre>
  l <- apply(res, 2, quantile, .025)</pre>
  u <- apply(res, 2, quantile, .975)</pre>
  out$p[which(out$lrself == j)] <- m</pre>
  out$lwr[which(out$lrself == j)] <- l</pre>
  out$upr[which(out$lrself == j)] <- u
out$party <- factor(out$party, levels=levels(dat$vote))</pre>
```



Figure





First Differences: MER

```
fake <- makeFakeData(mlogit2,</pre>
                      dat2l,
                      change=list(lrself=c(0,10)),
                      varying = "rp")
B <- MASS::mvrnorm(2500, coef(mlogit2), vcov(mlogit2))</pre>
tmp <- mlogit2</pre>
probs <- NULL
for(i in 1:2500){
  tmp$coefficients <- B[i,]</pre>
  probs <- rbind(probs,</pre>
   cbind(data.frame(lrself = unique(fake$lrself),
                     sim=i),
         predict(tmp,
                  newdata=fake)))
fd <- probs %>%
  group_by(sim) %>%
  summarise(across(PS:`UMP*`, ~diff(.x))) %>%
  ungroup %>%
  summarise(across(-sim, list(mean = ~mean(.x),
                                lwr = \sim quantile(.x, .025),
                                upr = \simquantile(.x, .975),
                                pval = \sim mean(.x < 0)))) %>%
  mutate(across(contains("pval"), ~ifelse(.x > .5, 1-.x, .x))) %;
  pivot_longer(everything(),
                names_pattern="(.*)_(.*)",
                names_to=c("party", ".value"))
```

```
## # A tibble: 6 × 5
    party mean lwr
                       upr pval
    <chr> <dbl> <dbl> <dbl> <dbl> <dbl>
          -0.58 -0.66 -0.5
           0.39 0.26 0.53
  2 FN
## 3 Green -0.07 -0.1 -0.04
                                0
        -0.1 -0.16 -0.05
## 4 PCF
                                0
## 5 UDF
           0.09 0.05 0.14
## 6 UMP* 0.27 0.18 0.37
                                0
```



First Differences: AME

```
fake <- makeFakeData(mlogit2,</pre>
                       dat2l,
                       change=list(lrself=c(0,10)),
                       varying="rp")
fake1 <- fake0 <- dat2l
fake0$lrself <- 0</pre>
fake1$lrself <- 10
B <- MASS::mvrnorm(100, coef(mlogit2), vcov(mlogit2))</pre>
tmp <- mlogit2</pre>
res <- NULL
for(i in 1:100){
  tmp$coefficients <- B[i,]</pre>
  p_hat0 <- colMeans(predict(tmp, newdata=fake0))</pre>
  p_hat1 <- colMeans(predict(tmp, newdata=fake1))</pre>
  res <- rbind(res, p_hat1-p_hat0)</pre>
fd_ame <- as.data.frame(res) %>%
  summarise(across(everything(),
                     list(p = \sim mean(.x),
                          lwr = \sim quantile(.x, .025),
                          upr = \simquantile(.x, .975),
                          pval = \sim mean(.x > 0)))) %>%
  mutate(across(contains("pval"),
                 \simifelse(.x > 0, 1-.x, .x))) %>%
  pivot_longer(everything(),
                names_pattern="(.*)_(.*)",
                names to=c("narty" " value"))
```

```
fd ame %>%
  mutate(across(-party,
                ~round(.x, 2)))
## # A tibble: 6 × 5
    party
              p lwr
                        upr pval
    <chr> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 PS
          -0.46 - 0.5 - 0.43
## 2 FN
           0.27 0.22 0.32
## 3 Green -0.08 -0.1 -0.06
                                0
## 4 PCF -0.1 -0.13 -0.07
           0.08 0.06 0.1
## 5 UDF
```

0

6 UMP* 0.29 0.26 0.32



Model Fit

PRE

```
yhat <- predict(mlogit2,newdata=dat2l)
yhat <- colnames(yhat)[apply(yhat, 1, which.max)]
yhat <- factor(yhat, levels=levels(dat$vote))
obs_vote <- dat2l %>% filter(vote == 1) %>% ungroup %>% unnest(idx) %>% select(alt) %>% pull()
tab <- table(obs_vote, yhat)
pcp <- sum(obs_vote == yhat)/sum(tab)
pmc <- max(table(dat$vote))/sum(tab)
(pcp-pmc)/(1-pmc)</pre>
```

[1] 0.251073

LR Test

```
ll1 <- logLik(mod)
ll2 <- logLik(mlogit2)
x2 <- -2*(ll1-ll2)
pchisq(x2, 9, lower.tail=FALSE)</pre>
```

```
## 'log Lik.' 2.629382e-28 (df=35)
```



Separation

Separation can be a problem in unordered categorical models, too. Let's add demsat to the model we ran in the beginning of the lecture:

```
mods <- multinom(vote ~ lrself + male + retnat + age +</pre>
    union + demsat, data=dat, trace=F)
mnlSig(mods)
         (Intercept) lrself
                              male retnatsame retnatworse
##
                                                                age
## PS
              3.572* 0.214* 0.601
                                       -0.977
                                                    -1.054 -0.021*
             1.840 0.388* 0.308
                                       -1.340
                                                   -1.783 -0.048*
## Green
                                       -1.847
                                                   -2.392 \times -0.005
## UDF
             -0.842 0.919 \times 0.705
## UMP*
             -1.251 1.151* 0.515
                                       -1.508
                                                   -2.438* -0.011
                                       -1.382
                                                   -1.373 -0.018
            -14.142* 0.802* 0.341
## FN
         unionyes, union member demsatsomewhat satisfied demsata little satisfied
##
## PS
                        -1.339*
                                                    0.490
                                                                              0.405
                                                                             1.552
## Green
                        -1.065*
                                                    1.616
## UDF
                        -1.234*
                                                   0.033
                                                                             -0.264
## UMP*
                        -1.231*
                                                                            -0.534
                                                  -0.118
                        -1.536*
## FN
                                                  13.139*
                                                                            13.949*
         demsatnot satisfied at all
##
## PS
                             -0.146
## Green
                             1.573
## UDF
                             -1.350
                            -1.464
## UMP*
## FN
                             14.195*
```



Bias Reduced Model

```
mods_br <- brmultinom(formula(mods), data=dat)</pre>
```

noquote(t(mnlSig(mods_br)))

```
##
                                      Green
                                             UDF
                                                             FΝ
## (Intercept)
                             3.045* 1.531
                                             -1.253
                                                     -1.664
                                                             -3.447
## lrself
                             0.201* 0.373*
                                             0.889*
                                                    1.115*
                                                             0.775*
## male
                             0.581
                                     0.295
                                             0.684
                                                     0.498
                                                             0.331
## retnatsame
                             -0.636 -1.011 -1.489 -1.157 -1.077
## retnatworse
                             -0.672 -1.414 -1.990* -2.034* -1.054
                             -0.021 \times -0.047 \times -0.005 -0.011 -0.017
## age
## unionyes, union member
                             -1.310* -1.020* -1.174* -1.194* -1.447*
## demsatsomewhat satisfied
                             0.585
                                     1.509
                                             0.126
                                                     -0.007 2.192
## demsata little satisfied
                                     1.441
                             0.488
                                             -0.167 -0.424 2.976
## demsatnot satisfied at all -0.065 1.462
                                             -1.161 -1.319 3.211*
```



Review

- 1. Develop Multinomial Logit Model
- 2. Effects and Effect Displays
- 3. Model Fit and Evaluation
- 4. IIA Assumption
- 5. Conditional Logit
- 6. Separation Problems



Replicating Anderson and Stephenson

Since Anderson and Stephenson used CES data, we can replicate their work. We won't do all the models, but we can look at one of them to compare the effects they got with effects in the different ways we have calculated them. The authors write:

"We expect that, if the issue is positional, we should see clear differentiation between the parties based on the left-right ideological split. If the environment is a valence issue, however, then we expect to find that environmental support has the greatest effect on the party perceived as best able to govern on "

- 1. How would you estimate these models?
- 2. How well do they fit?
- 3. What are the variable effects?
- 4. Which idea do you think is right?