POLSCI 9592

Lecture 3: Interactions

Dave Armstrong

Goals for This Session

1. Discuss Interaction Effects in the Linear Model
2. Discuss Interaction Effects in the GLM
3. Presentation of Interaction Results

Interaction Effects in LMs (1)

When the partial effect of one variable depends on the value of another variable, those two variables are said to "interact".

- For example, we may want to test whether age effects are different for men (coded 1) and women (coded 0).
- In such cases it is sensible to fit separate regressions for men and women, but this does not allow for a formal statistical test of the differences
- Specification of interaction effects facilitates statistical tests for a difference in slopes within a single regression

Interaction Effects in LMs (2)

Interaction terms are the *product of the regressors for the two variables*.

- The interaction regressor in the model below is $X_i D_i$:

$$Y_i = \alpha + \beta X_i + \gamma D_i + \delta(X_i D_i) + \varepsilon_i$$
$$\text{income}_i = \alpha + \beta\,\text{age}_i + \gamma\,\text{men}_i + \delta(\text{age}_i \times \text{men}_i) + \varepsilon_i$$

Ultimately we want to know two things:

- Is there a statistically significant interactive (i.e., multiplicative or conditional) effect?
- If the answer to #1 is "yes", what is the nature of that effect (i.e., what does it look like)?

Below, I will walk you through all of the possible two-way interaction scenarios and we will discuss how to answer these two questions.

ANOVA Type I Sums of Squares

Consider the model:

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_1 x_2 + e$$

In a type I test, the following tests are calculated.

1. The effect of x_1 not controlling for any other variables.
2. The effect of x_2 controlling for x_1.
3. The effect of x_3 controlling for x_1 and x_2.
4. The effect of the interaction, $x_1 x_2$ controlling for x_1, x_2 and x_3.

The results depend on the order in which the variables are included in the model.

The `anova()` function in the `stats` package does this kind of test.

ANOVA Type II Sums of Squares

Consider the model:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_1x_2 + e$$

In a type II test, the following tests are calculated.

1. The effect of x_1 controlling for x_2 and x_3.
2. The effect of x_2 controlling for x_1 and x_3.
3. The effect of x_3 controlling for x_1 and x_2 and x_1x_2.
4. The effect of the interaction, x_1x_2 controlling for x_1, x_2 and x_3.

When testing lower-order terms, they do not control for higher-order terms of the same variable(s).

The `ANOVA(..., type="II")` function in the `car` package does this test.

ANOVA Type III Sums of Squares

Consider the model:

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_1 x_2 + e$$

In a type III test, the following tests are calculated.

1. The effect of x_1 controlling for x_2, $x_1 x_2$ and x_3.
2. The effect of x_2 controlling for x_1, $x_1 x_2$ and x_3.
3. The effect of x_3 controlling for x_1, x_2 and $x_1 x_2$.
4. The effect of the interaction, $x_1 x_2$ controlling for x_1, x_2 and x_3.

When testing lower-order terms, they do control for higher-order terms of the same variable(s).

The `ANOVA(..., type="III")` function in the `car` package does this test.

Two Categorical Variables

With two categorical variables, essentially you are estimating a different conditional mean for every pair of values across the two categorical variables. You could do that as follows:

```
library(DAMisc)
library(car)
data(Duncan)
Duncan <- Duncan %>%
  mutate(inc.cat = cut(Duncan$income, 3),
         inc.cat = factor(as.numeric(inc.cat),
                          labels=c("Low", "Middl

mod <- lm(prestige~ inc.cat * type + education,
  data=Duncan)
```

```
S(mod, brief=TRUE)
```

```
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)               7.8827     3.4364   2.294 0.027915 *
## inc.catMiddle            22.4574     4.8792   4.603 5.30e-05 ***
## inc.catHigh              51.2807     9.4351   5.435 4.29e-06 ***
## typeprof                 55.6073    11.6800   4.761 3.30e-05 ***
## typewc                    2.5446     8.1162   0.314 0.755746
## education                 0.2799     0.1121   2.496 0.017411 *
## inc.catMiddle:typeprof -41.5789    11.2428  -3.698 0.000740 ***
## inc.catHigh:typeprof   -50.3567    13.3929  -3.760 0.000621 ***
## inc.catMiddle:typewc   -13.0171    10.3130  -1.262 0.215223
## inc.catHigh:typewc     -33.6407    13.1215  -2.564 0.014806 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard deviation: 9.115 on 35 degrees of freedom
## Multiple R-squared: 0.9334
## F-statistic: 54.54 on 9 and 35 DF,  p-value: < 2.2e-16
##    AIC    BIC
## 337.29 357.16
```

Anova

Q1: Is there an interaction Effect here?

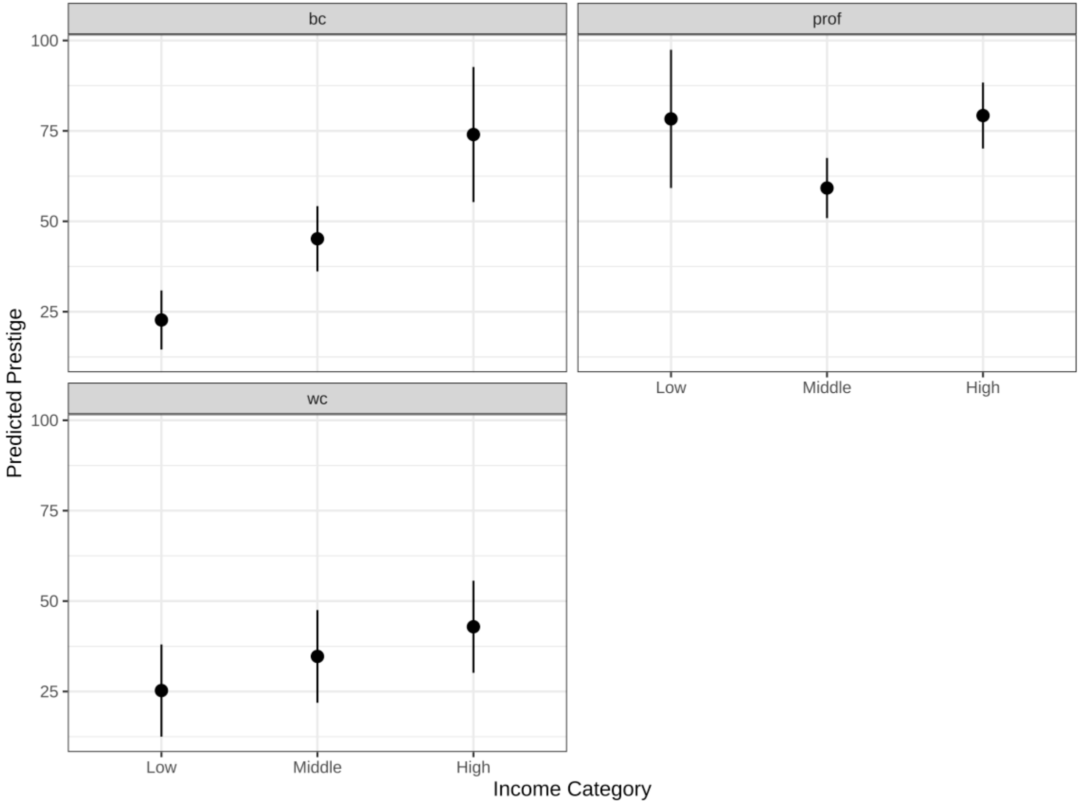
- An incremental (Type II) F-test will answer that question. We want to test the null hypothesis that all of the interaction dummy regressor coefficients are zero in the population.
- The `inc.cat:type` line of the output gives the results of this test.

```
Anova(mod)
```

```
## Anova Table (Type II tests)
##
## Response: prestige
##               Sum Sq Df F value    Pr(>F)
## inc.cat      3491.9  2 21.0159 1.010e-06 ***
## type         2856.0  2 17.1885 6.308e-06 ***
## education     517.7  1  6.2313  0.017411 *
## inc.cat:type 1644.4  4  4.9484  0.002871 **
## Residuals    2907.7 35
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Q2: What is the nature of the interaction?

```
library(marginaleffects)
p1 <- predictions(mod, newdata = "mean", variabl
ggplot(p1) +
  geom_pointrange(aes(x=inc.cat, y=estimate,
                  ymin=conf.low,
                  ymax=conf.high)) +
  facet_wrap(~type, ncol=2) +
  theme_bw() +
  labs(x="Income Category",
       y="Predicted Prestige")
```



Testing Differences

Imagine that you wanted to test whether the effect of moving from middle income to high income was the same for blue collar and white collar occupations.

```
p1 %>%
  mutate(param = paste0("b", row_number())) %>%
  select(param, inc.cat, type, estimate) %>%
  as.data.frame()
```

```
##   param inc.cat type estimate
## 1    b1    High prof 79.24763
## 2    b2    High   wc 42.90089
## 3    b3    High   bc 73.99703
## 4    b4  Middle prof 59.20218
## 5    b5  Middle   wc 34.70119
## 6    b6  Middle   bc 45.17372
## 7    b7     Low prof 78.32369
## 8    b8     Low   wc 25.26095
## 9    b9     Low   bc 22.71635
```

```
hypotheses(p1, "b3-b6 = b2 -b5",
           df = mod$df.residual)
```

```
##
##   Hypothesis Estimate Std. Error    t Pr(>|t|)   S 2.5 % 97.5 % Df
##  b3-b6=b2-b5     20.6       13.5 1.52    0.136 2.9 -6.83   48.1 35
```

One Categorical and One Continuous

With one categorical and one continuous variable, we want to show the conditional coefficients of the continuous variable (probably in a table) and we want to show the conditional coefficients of the dummy variables.

```
data(Prestige, package="carData")
Prestige$income <- Prestige$income/1000
mod <- lm(prestige ~ income*type + education,
        data=Prestige)
S(mod, brief=TRUE)
```

```
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      -6.7273     4.9515  -1.359   0.1776
## income            3.1344     0.5215   6.010 3.79e-08 ***
## typeprof         25.1724     5.4670   4.604 1.34e-05 ***
## typewc            7.1375     5.2898   1.349   0.1806
## education         3.0397     0.6004   5.063 2.14e-06 ***
## income:typeprof  -2.5102     0.5530  -4.539 1.72e-05 ***
## income:typewc    -1.4856     0.8720  -1.704   0.0919 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard deviation: 6.455 on 91 degrees of freedom
##   (4 observations deleted due to missingness)
## Multiple R-squared: 0.8663
## F-statistic: 98.23 on 6 and 91 DF,  p-value: < 2.2e-16
```

Anova

Q1: Is there a significant interaction?

```
Anova(mod)
```

```
## Anova Table (Type II tests)
##
## Response: prestige
##              Sum Sq Df F value    Pr(>F)
## income       1058.8  1 25.4132 2.342e-06 ***
## type          591.2  2  7.0947   0.00137 **
## education    1068.0  1 25.6344 2.142e-06 ***
## income:type   890.0  2 10.6814 6.809e-05 ***
## Residuals    3791.3 91
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Notice that the `income:type` line of the `Anova` output tells us that the interaction is significant. Thus, we should go on to calculate and explain the conditional coefficients.

Conditional Coefficients of Income

Q2: What is the nature of the interaction effect?

- The nature of the interaction has to be considered both for `income` and for `type`.
- We can calculate the conditional effects and variances of `income` as follows:

```
(s <- slopes(mod, variables="income", by = "type"))
```

```
##
##  type Estimate Std. Error    z Pr(>|z|)    S 2.5 % 97.5 %
##  bc      3.134      0.522 6.01  < 0.001 29.0 2.112   4.16
##  prof    0.624      0.222 2.82  0.00486  7.7 0.190   1.06
##  wc      1.649      0.709 2.33  0.02002  5.6 0.259   3.04
##
## Term: income
## Type: response
## Comparison: dY/dX
```

```
hypotheses(s, hypothesis = ~pairwise)
```

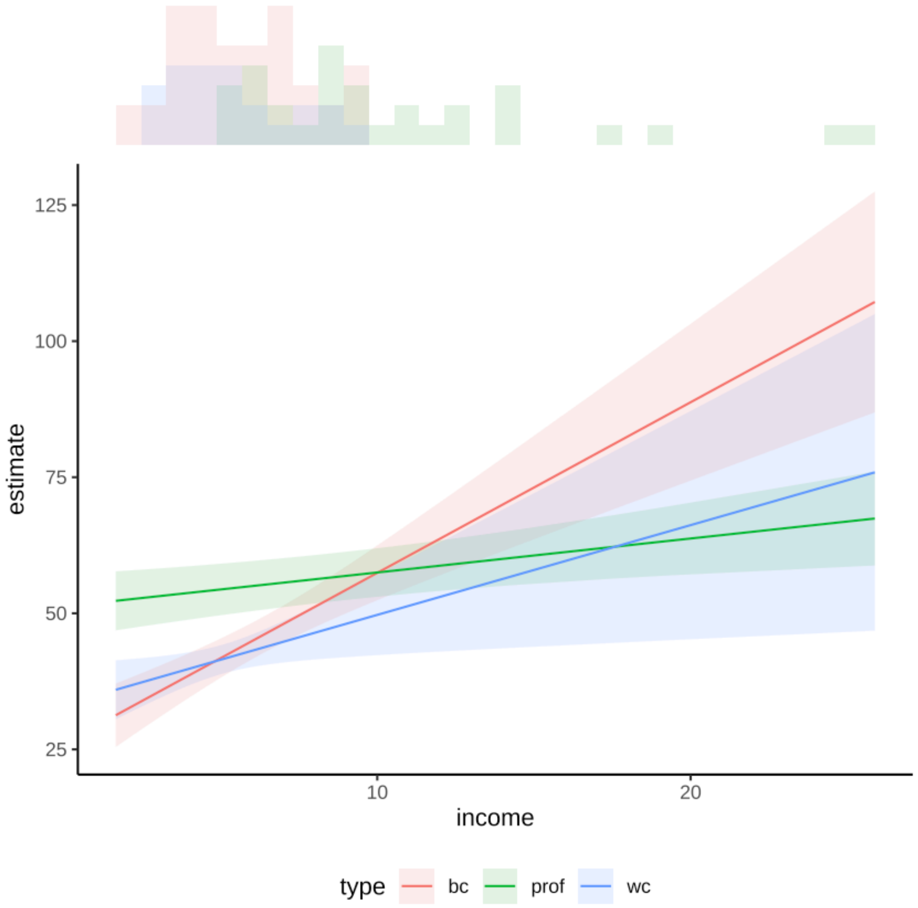
```
##
##   Hypothesis Estimate Std. Error     z Pr(>|z|)    S  2.5 % 97.5 %
##  (b2) - (b1)    -2.51      0.553 -4.54   <0.001 17.4 -3.594 -1.426
##  (b3) - (b1)    -1.49      0.872 -1.70   0.0885  3.5 -3.195  0.224
##  (b3) - (b2)     1.02      0.740  1.38   0.1664  2.6 -0.426  2.476
```

Conditional Effects of Income

```r
preds <- predictions(mod, newdata="mean",
            variables = list(income = unique,
                             type=unique))
library(patchwork)
g1 <- ggplot(preds,
             aes(x=income,
                 y=estimate,
                 ymin=conf.low,
                 ymax=conf.high,
                 fill=type,
                 color=type)) +
  geom_ribbon(alpha=.15,
              color="transparent") +
  geom_line()  +
  theme_classic() +
  theme(legend.position="bottom")

g2 <- ggplot(Prestige %>%
               filter(!is.na(type)),
             aes(x=income,
                 fill=type)) +
  geom_histogram(position="identity",
                 alpha=.15,
                 show.legend = FALSE) +
  theme_void()

g2 +
  g1 +
  plot_layout(nrow = 2,
              heights = c(1, 4))
```



Interpretation

- The slope is significant for all occupation types and is the biggest for blue collar.
- Confidence bounds for both blue collar and white collar occupation lines are very big at high levels of income (lack of data density).
- The only valid places where professional occupations can be compared to the others is between around 5,000 and 8,000 dollars.

Conditional Effect of Type

Q2: What is the nature of the interaction effect (this time for `type`)?

- The conditional effect of type (as we saw) is a bit more difficult. Here, We would presumably have to test each pairwise difference: BC vs Prof, BC vs WC and Prof vs WC for different values of education. First, let's think about what we need.

$$
\begin{aligned}
\text{BC vs Prof: } \frac{\partial \text{Prestige}}{\partial \text{Prof}} &= b_2 + b_5 \text{Income} \\
\text{BC vs WC: } \frac{\partial \text{Prestige}}{\partial \text{WC}} &= b_3 + b_6 \text{Income} \\
\text{Prof vs WC: } \frac{\partial \text{Prestige}}{\partial \text{Prof}} - \frac{\partial \text{Prestige}}{\partial \text{WC}} &= (b_2 - b_3) + (b_5 - b_6) \text{Income}
\end{aligned}
$$

Conditional Effect of Type: Numerical

```r
num3 <- function(x, mult=1){
  s <- sd(x, na.rm=TRUE)*mult
  m <- mean(x, na.rm=TRUE)
  c("Mean - SD" = m-s, "Mean" = m, "Mean + SD" = m+s)
}
p2 <- purrr::map(num3(Prestige$income), \(x)
             avg_predictions(mod, newdata= datagrid(income=x), variables=list(type=unique)))

bind_rows(lapply(p2, \(x)hypotheses(x, hypothesis=~pairwise)), .id="income") %>%
  as_tibble() %>%
  select(income, hypothesis, estimate, std.error, conf.low, conf.high)
```

```
## # A tibble: 9 × 6
##   income    hypothesis  estimate std.error conf.low conf.high
##   <chr>     <chr>          <dbl>     <dbl>    <dbl>     <dbl>
## 1 Mean - SD (b2) - (b1)    18.8       4.48     9.98     27.5
## 2 Mean - SD (b3) - (b1)     3.35      3.43    -3.38     10.1
## 3 Mean - SD (b3) - (b2)   -15.4       3.44   -22.2      -8.68
## 4 Mean      (b2) - (b1)     8.11      3.55     1.16     15.1
## 5 Mean      (b3) - (b1)    -2.96      2.60    -8.07      2.14
## 6 Mean      (b3) - (b2)   -11.1       2.81   -16.6      -5.55
## 7 Mean + SD (b2) - (b1)    -2.55      4.01   -10.4       5.32
## 8 Mean + SD (b3) - (b1)    -9.27      5.40   -19.9       1.32
## 9 Mean + SD (b3) - (b2)    -6.72      4.88   -16.3       2.84
```

Interpretation

In the previous table we see the following:

- The differences between Professional occupations and the other two groups is significant when income is at its mean and a standard deviation below its mean.
- There are no significant differences at the mean plus one SD.

Two continuous Variables

With two continuous variables the interpretation gets a bit trickier. For example, consider the following model:

$$\hat{Y}_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i1} X_{i2}$$

We want to know the partial conditional effect of both X_1 and X_2, but unlike above, neither can be boiled down to a small set of values. Just think about the equation:

$$\frac{\partial \hat{Y}}{\partial X_1} = \beta_1 + \beta_4 X_2$$
$$\frac{\partial \hat{Y}}{\partial X_2} = \beta_2 + \beta_4 X_1$$

Note, that β_4 is the amount by which the *effect* of X_1 goes up for every additional unit of X_2 and the amount by which the *effect* of X_2 goes up for every additional unit of X_1.

Testable Hypotheses

$$\hat{Y}_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i1} X_{i2}$$

Berry, Golder and Milton (2012) suggest that we should be able to test 5 hypotheses:

- $\mathbf{P}_{X_1|X_2=\text{min}}$ The marginal effect of X_1 is [positive, zero, negative] when X_2 takes its lowest value.
- $\mathbf{P}_{X_1|X_2=\text{max}}$ The marginal effect of X_1 is [positive, zero, negative] when X_2 takes its highest value.
- $\mathbf{P}_{X_2|X_1=\text{min}}$ The marginal effect of X_2 is [positive, zero, negative] when X_1 takes its lowest value.
- $\mathbf{P}_{X_2|X_1=\text{max}}$ The marginal effect of X_2 is [positive, zero, negative] when X_1 takes its highest value.
- $\mathbf{P}_{X_1 X_2}$ The marginal effect of each of X_1 and X_2 is [positively, negatively] related to the other variable.

Example

```r
mod <- lm(prestige ~ income*education + type, data=Prestige)
S(mod, brief=TRUE)
```

```
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)      -17.80359    7.59424  -2.344 0.021212 *
## income             3.78593    0.94453   4.008 0.000124 ***
## education          5.10432    0.77665   6.572 2.93e-09 ***
## typeprof           5.47866    3.71385   1.475 0.143574
## typewc            -3.58387    2.42775  -1.476 0.143303
## income:education  -0.21019    0.06977  -3.012 0.003347 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard deviation: 6.806 on 92 degrees of freedom
##   (4 observations deleted due to missingness)
## Multiple R-squared: 0.8497
## F-statistic:   104 on 5 and 92 DF,  p-value: < 2.2e-16
##    AIC    BIC
## 661.80 679.89
```

Example (2)

Q1: Is there a significant interaction?

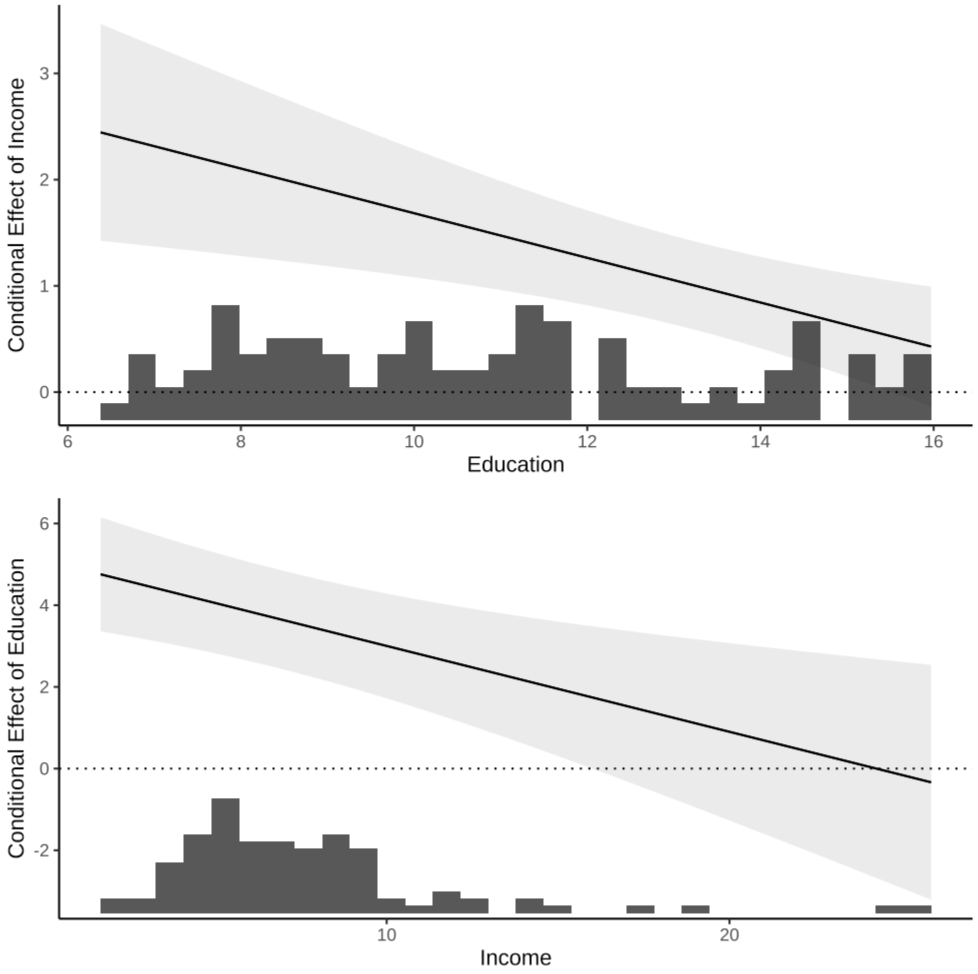
- The `income:education` line answers this question. If it is significant, then there is a significant interaction, otherwise there is not.
- This is counter to a minor, though still influential, point in Brambor, Clark and Golder (2006), but is consistent with Berry, Golder and Milton (2012).
- In this case, the interaction is significant, so we can move on to the next question

Q2: What is the nature of the interaction?

```
library(ggplotify)
h1 <- ggplot(Prestige, aes(x=education)) +
  geom_histogram() +
  theme_void()
m1 <- plot_slopes(mod, variable="income", condition="education")
  geom_hline(yintercept=0, linetype=3) +
  theme_classic() +
  labs(x="Education", y="Conditional Effect of Income")
m1 <- m1 - annotation_custom(as.grob(h1), ymax=calc_ymax(m1))

m2 <- plot_slopes(mod, variable="education", condition="income")
  geom_hline(yintercept=0, linetype=3) +
  theme_classic() +
  labs(x="Income", y="Conditional Effect of Education")
h2 <- ggplot(Prestige, aes(x=income)) +
  geom_histogram() +
  theme_void()
m2 <- m2 - annotation_custom(as.grob(h2), ymax=calc_ymax(m2))

m1 + m2 + plot_layout(nrow=2)
```



When Confidence Bounds Equal Zero

You may want to know when the confidence bounds are equal to zero. Consider the equation:

$$\hat{Y}_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i1} X_{i2}$$

- We know that the conditional effect of X_1 is $\beta_1 + \beta_4 X_2$ and that the lower bound is $(\beta_1 + \beta_4 X_2) - 1.96 \times SE(\beta_1 + \beta_4 X_2)$.
- Since those are all quantities that we know (or estimate), we could set it equal to zero and solve.
- This is what the `changeSig` function does.

Change in Significance

```
changeSig(mod, c("income", "education"))
```

```
## LB for B(income | education) = 0 when education=15.4979 (95th pctile)
## UB for B(income | education) = 0 when education=27.9396 (> Maximum Value in Data)
## LB for B(education | income) = 0 when income=15.9273 (96th pctile)
## UB for B(education | income) = 0 when income=59.5175 (> Maximum Value in Data)
```

Berry, Golder and Milton Hypotheses

```r
library(DAMisc)
BGMtest(mod, c("income", "education"))
```

```
##                est    se      t p-value
## P(X|Zmin)  2.445 0.520  4.698 0.000
## P(X|Zmax)  0.429 0.287  1.495 0.138
## P(Z|Xmin)  4.756 0.712  6.681 0.000
## P(Z|Xmax) -0.335 1.466 -0.229 0.820
## P(XZ)     -0.210 0.070 -3.012 0.003
```

Interpretation

- The effect of income is nearly always significant, though it gets smaller as education gets bigger. That is, as education increases, we expect smaller increases in prestige from increasing income
- The effect of education is significant and positive until around 16,000 dollars, which is around 2/3 the range of `income`, but is the 96^{th} percentile because of the skewness of income.
- This suggests that people tend to derive prestige from either higher incomes or higher education, but not really both.

Implicit Interaction

Consider the Following model:

$$\log \Omega = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3$$

In this case, the effect of x_1 is

$$\frac{\partial \Lambda(Xb)}{\partial x_1} = \lambda(Xb) b_1$$

where $\Lambda(\cdot)$ is the CDF, and $\lambda(\cdot)$ the PDF of the logistic distribution. - Note, that even when there is no product term, marginal effect is conditional on the values of the other variables through $\lambda(Xb)$.

Compression or Conditioning?

The effect noted above is often referred to as "compression".

- Compression happens necessarily as a function of the "S" shape of the logistic CDF.
- Changes in probabilities in the middle of the curve are bigger than changes out in the tails of the distribution.

The Debate

The debate, such as it is, in political science is whether or not compression constitutes a substantively interesting interaction.

- On the "compression is interesting" side is Berry, DeMeritt and Esarey (2010)
- On the "compression is not interesting" side is Nalger (1991)
- Rainey (2014)) has a nice discussion of the debate and the virtues of both approaches.

Rainey's Suggestion

Situtation	Description	Include a Product Term?	Quantity of Interest	Source
Interaction in Influencing the Latent Outcome	Guided by a strong theory, the analyst hypothesizes that X and Z interact in influencing the latent outcome variable Y^*. For example, it sometimes makes sense to conceptualize Y^* as utility and derive a probit model using a random utility framework (Train 2009). See Berry, DeMeritt, and Esarey (2010, esp.pp. 261-262) for more details and an example.	Yes	$\frac{\partial^2 Y^*}{\partial X \partial Z}$	Nagler (1991)
Interaction Due to Compression Alone	Guided by a strong theory, the analyst hypothesizes that X and Z interact in influencing $\Pr(Y)$ due to compression alone. That is, as the probability of an event approaches zero or one, the effect of any explanatory variable (including X and Z) have smaller effects. While researchers such as Frant (1991) and Nagler (1991) sometimes dismiss compression as an unimportant form of interaction, Berry, DeMeritt, and Esarey (2010) make a strong case that this type of interaction is often theoretically meaningful.	**No**	$\frac{\partial^2 \Pr(Y)}{\partial X \partial Z}$	Berry, DeMeritt, and Esarey (2010)
Specification Ambiguity	Guided only by weak theoretical intuition, the analyst hypothesizes that X and Z interact in influencing $\Pr(Y)$, but have no strong theoretical rational for the functional form. In this situation, the analyst lacks the theoretical guidance necessary to theorize about interaction in terms of the latent variable or on the basis of compression alone.	Yes	$\frac{\partial^2 \Pr(Y)}{\partial X \partial Z}$	Berry, DeMeritt, and Esarey (2014)

Rainey's Suggestions

1. Clearly state the interactive theory and provide a model that can represent both the theoretically expected and null relationships.
2. Always include the product term (if you propose an interaction could be present).

Second difference

To figure out if there is an interaction, you need to calculate the second difference (or cross-derivative) in the outcome for the two variables in the interaction. If the two variables are X and Z, you would calculate: s

$$\begin{aligned}\Delta\Delta Pr(Y=1|X,Z) = &[Pr(Y=1|X=\text{high}, Z=\text{low}) \\ &- Pr(Y=1|X=\text{low}, Z=\text{low})] \\ &- [Pr(Y=1|X=\text{high}, Z=\text{high}) \\ &- Pr(Y=1|X=\text{low}, Z=\text{high})]\end{aligned}$$

We'll use R to do this for us below.

Example

Consider vote for Obama in 2012. I hypothesize:

- For people on the right, income decreases the likelihood of voting for Obama. For those on the left, income increases the probability of voting for Obama.

```
dat <- import("data/anes2012.dta")
mod <- glm(votedem ~ black + evprot + incgroup_num *lrself +
             econ_retnat, data=dat, family=binomial)
summary(mod)
```

```
##
## Call:
## glm(formula = votedem ~ black + evprot + incgroup_num * lrself +
##     econ_retnat, family = binomial, data = dat)
##
## Coefficients:
##                      Estimate Std. Error z value Pr(>|z|)
## (Intercept)          4.629952   0.398818  11.609  < 2e-16 ***
## black                4.995479   0.284023  17.588  < 2e-16 ***
## evprot              -0.468927   0.156150  -3.003 0.002673 **
## incgroup_num         0.056964   0.021800   2.613 0.008975 **
## lrself              -0.216037   0.059100  -3.655 0.000257 ***
## econ_retnat         -1.480114   0.077297 -19.148  < 2e-16 ***
## incgroup_num:lrself -0.019020   0.003707  -5.131 2.89e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 4017.9  on 2918  degrees of freedom
## Residual deviance: 2006.6  on 2912  degrees of freedom
## AIC: 2020.6
##
## Number of Fisher Scoring iterations: 6
```

Second Difference in Probabilities

Below, we answer the question: is there an interaction that is interesting?

```
avg_predictions(mod, variables = list(incgroup_num = c(9,22), lrself=c(3,8)),
                hypothesis="(b3-b1)-(b4-b2)=0")
```

```
##
##         Hypothesis Estimate Std. Error    z Pr(>|z|)    S  2.5 % 97.5 %
##   (b3-b1)-(b4-b2)=0    0.137     0.0289 4.74   <0.001 18.8 0.0802  0.193
##
## Type: response
```

Since this is significant (the 2.5% value is greater than zero), it suggests there is a significant interaction between income and left-right self-placement.

Alternative Specification

```
avg_comparisons(mod,
                newdata = datagrid(lrself=c(3,8), grid_type = "counterfactual"),
                variables = list(incgroup_num = c(9,22)), by="lrself",
                hypothesis = ~ pairwise)
```

```
##
##  Hypothesis Estimate Std. Error     z Pr(>|z|)    S  2.5 %  97.5 %
##   (8) - (3)   -0.137     0.0289 -4.74   <0.001 18.8 -0.193 -0.0802
##
## Type: response
```

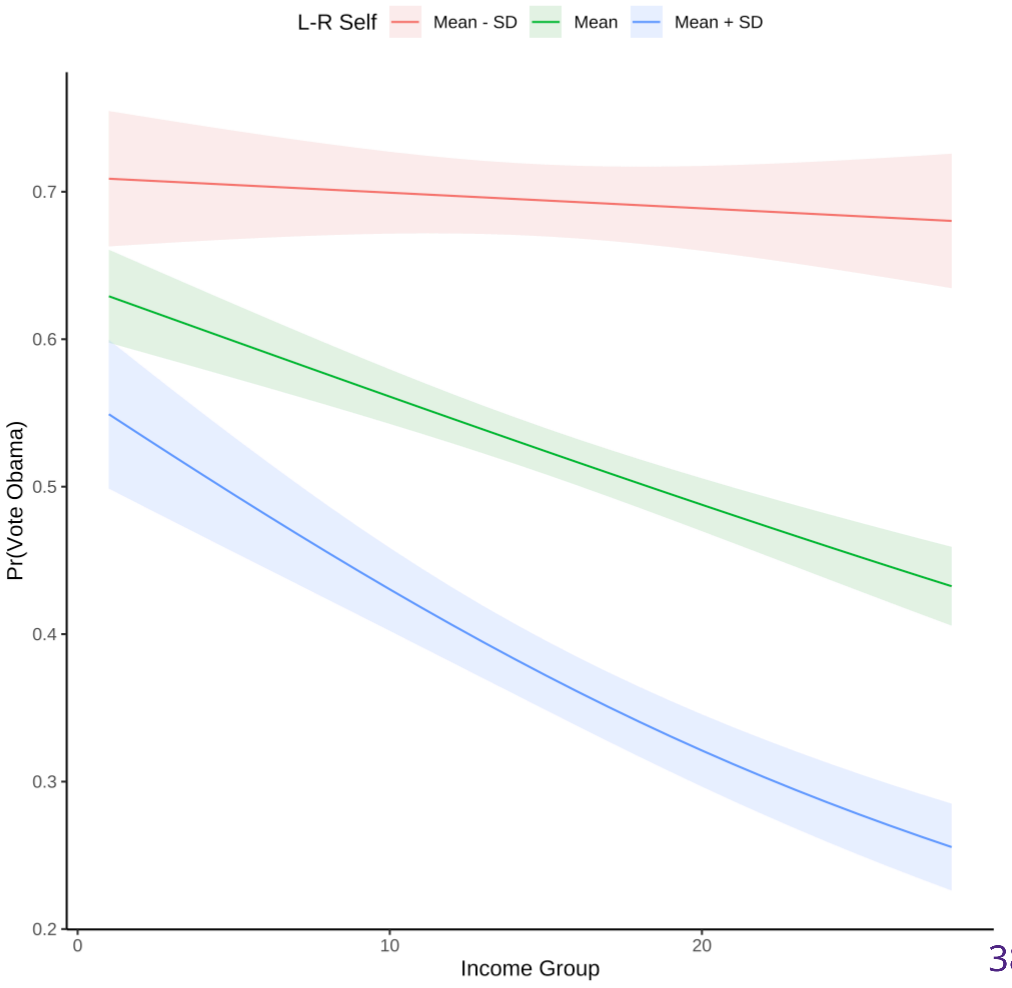
```
avg_comparisons(mod,
                newdata = datagrid(lrself=c(3,8), grid_type = "counterfactual"),
                variables = list(incgroup_num = c(9,22)), by="lrself",
                hypothesis = ~ revpairwise)
```

```
##
##  Hypothesis Estimate Std. Error    z Pr(>|z|)    S  2.5 % 97.5 %
##   (3) - (8)    0.137     0.0289 4.74   <0.001 18.8 0.0802  0.193
##
## Type: response
```

Nature of Interaction

```
pv <- avg_predictions(mod,
                      variables=list(incgroup_num=unique,
                                     lrself=num3)) %>%
  mutate(cond = as.factor(lrself))
levels(pv$cond) <- c("Mean - SD", "Mean", "Mean + SD")

ggplot(pv, aes(x=incgroup_num,
               y=estimate,
               ymin=conf.low,
               ymax=conf.high,
               fill=cond)) +
  geom_ribbon(alpha=.15, color="transparent") +
  geom_line(aes(color=cond)) +
  theme_classic() +
  theme(legend.position="top") +
  labs(x="Income Group",
       y="Pr(Vote Obama)",
       colour="L-R Self", fill="L-R Self")
```



Review

We covered the following topics:

1. Discuss Interaction Effects in the Linear Model
2. Discuss Interaction Effects in the GLM
3. Presentation of Interaction Results

Exercises

In 2012, Jaroslav Tir and Douglas Stinnett published an article arguing that international water treaties can reduce the impact of water scarcity on conflict. You can find the replication data at the *JPR* replication archive here's a link directly to the data's zip archive.

1. In the model they label "River and Water Variables", they predict `cwmid` with `lnwaterpcmin`, `instcoop`, `numbtreaties`, `anyupdown`, `power1`, `alliance`, `gdpmax`, `interdep`, `dyaddem`, `contig`, `peaceyrs1`, `_spline1`, `_spline2` and `_spline3` included additively.
    - What are the effects of the water variables?
2. In their third model, they add an interaction between `lnwaterpcmin` and `instcoop`. I want you to do the same.
    - Evaluate the need for the interaction.
    - Plot the nature of the interaction.
    - How does this model fit relative to the previous model?