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AGING AND MEMORY EFFECTS IN SOCIAL AND ECONOMIC DYNAMICS

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Doctor by the Universitat de les Illes Balears

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A en Manuel Miranda
pel seu suport i ajuda
durant tots aquests anys.
Sempre estaràs amb mi.
i recordare sempre
el que em vas ensenyar.

Dr José Javier Ramasco of the Consejo Superior de Investigaciones Científicas (CSIC) and Dr Maxi San Miguel of the Universitat de les Illes Balears (UIB)

WE DECLARE:

That the thesis titles *Dynamics of social interactions*, presented by David Abella Bujalance to obtain a doctoral degree, has been completed under my supervision and meets the requirements to opt for an International Doctorate.

For all intents and purposes, I hereby sign this document.

Signature

Dr. José Javier Ramasco Sukia	Dr. Maxi San Miguel
Thesis Supervisor	Thesis Supervisor

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Tambe afegir que aquest projecte no hagués estat possible sense l'ajuda de tots els companys que han fet possible que aquest projecte sigui una realitat. Jo que soc un dels que ha fet possible que aquest projecte sigui una realitat, vull agrair a tots els companys que han fet possible que aquest projecte sigui una realitat, per tot el suport que m'han donat durant tot el projecte.

Resum

En els sistemes complexos distribuïts, els sistemes de memòria transaccional distribuïda (DTM) són una eina molt útil per a la programació concurrent. Aquests sistemes permeten als desenvolupadors de software escriure codi concurrent sense haver de preocupar-se per la gestió de la memòria compartida. A més, els DTM ofereixen una interfície molt senzilla per a la programació concurrent, ja que permeten als desenvolupadors de software escriure codi concurrent de forma semblant a com ho farien si el codi fos seqüencial. Tot i això, els DTM no són una eina perfecta, ja que tenen un rendiment molt inferior al de les estructures de dades distribuïdes. A més, els DTM no són capaços de gestionar estructures de dades distribuïdes de forma eficient. Per aquest motiu, els DTM no són una eina adequada per a la programació de sistemes distribuïts.

Resumen

En los sistemas complejos distribuidos, los sistemas de memoria transaccional distribuida (DTM) son una herramienta muy útil para la programación concurrente. Estos sistemas permiten a los desarrolladores de software escribir código concurrente sin tener que preocuparse por la gestión de la memoria compartida. Además, los DTM ofrecen una interfaz muy sencilla para la programación concurrente, ya que permiten a los desarrolladores de software escribir código concurrente de forma similar a como lo harían si el código fuera secuencial. Sin embargo, los DTM no son una herramienta perfecta, ya que tienen un rendimiento muy inferior al de las estructuras de datos distribuidas. Además, los DTM no son capaces de gestionar estructuras de datos distribuidas de forma eficiente. Por este motivo, los DTM no son una herramienta adecuada para la programación de sistemas distribuidos.

Abstract

In complex systems distributed transactional memory (DTM) systems are a very useful tool for concurrent programming. These systems allow software developers to write concurrent code without having to worry about managing shared memory. In addition, DTM systems offer a very simple interface for concurrent programming, as they allow software developers to write concurrent code in a similar way to how they would if the code were sequential. However, DTM systems are not a perfect tool, as they have a much lower performance than distributed data structures. In addition, DTM systems are not able to manage distributed data structures efficiently. For this reason, DTM systems are not a suitable tool for programming distributed systems.

The list of articles detailed below, in chronological order by date of publication, form the basis of the present thesis.

1. David Abella, Maxi San Miguel, and José J. Ramasco. "Aging effects in Schelling segregation model". In: *Scientific Reports* 12.1 (Nov. 2022). ISSN: 2045-2322. DOI: 10.1038/s41598-022-23224-7. URL: http://dx.doi.org/10.1038/s41598-022-23224-7

2. David Abella, Maxi San Miguel, and José J. Ramasco. "Aging in binary-state models: The Threshold model for complex contagion". In: *Phys. Rev. E* 107 (2 Feb. 2023), page 024101. DOI: 10.1103/PhysRevE.107.024101. URL: https://link.aps.org/doi/10.1103/PhysRevE.107.024101

3. David Abella et al. "Ordering dynamics and aging in the symmetrical threshold model". In: *New Journal of Physics* 26.1 (Jan. 2024), page 013033. DOI: 10.1088/1367-2630/ad1ad4. URL: https://dx.doi.org/10.1088/1367-2630/ad1ad4

4. Idealista model for complex systems housing

5. Idealista spatial segmentation of the real state market

Other publications published during the PhD period are also included in the following list.

- David Abella, Giancarlo Franzese, and Javier Hernández-Rojas. "Many-Body Contributions in Water Nanoclusters". In: *ACS Nano* 17.3 (Jan. 2023), pages 1959–1964. ISSN: 1936-086X. DOI: 10.1021/acsnano.2c06077. URL: http://dx.doi.org/10.1021/acsnano.2c06077

- David Abella et al. "Unraveling higher-order dynamics in collaboration networks". In: *arXiv preprint arXiv:2306.17521* (2023)

This thesis provides a general overview of the research that I have been developing since the beginning of my PhD studies in September, 2021. I could define myself as a curious, creative and open-minded person, following the so called *IFISC attitude*, which means that I am always willing to learn new methods and address new problems, even though they are not directly related to my field of expertise. That is why, through this thesis many topics will be covered, from the study of human behavior and social systems, to the study of complex systems and network theory........

1.1 Scientific Landscape

This thesis adress the study of human behavior and social systems from a *complex systems* perspective, which studies the emergence of collective phenomena that arise from the interactions of many individuals, and that cannot be understood by studying the behavior of individual agents in isolation (the so-called *reductionist* approach) (0). The study of collective phenomena has a long history in the natural sciences, specially in the branch of statistical physics (0). This branch traditionally studies the emergence of collective phenomena in physical systems, such as the phase transitions in magnetic materials via spin models (0), the turbulence in fluids (0), the synchronization in oscillatory systems (0), or percolation (0). However, in recent years, the study of complex systems has evolved into the study of emergent phenomena beyond physical systems, such as biological (0), ecological (0), economic (0), and social systems (0). From the migration of birds (0) to the spreading of a fake news through social media (0), there are many examples of collective phenomena at which the study of complex systems can be applied.

The cascade of failures in power grids (0), the spread of a disease in a population (0), the consensus in political elections (0), the emergence of social norms (0) are some examples of social collective behavior in which the global phenomena cannot be understood by looking at a single individual. Social and economic collective phenomena has been studied from a variety of perspectives (sociology, psychology, economics, political sciences...), which often relies on qualitative methods, such as interviews, surveys, or ethnographic studies (0). However, the study of social systems from a complex systems perspective aims to provide a quantitative framework to understand the collective behavior, based on methodologies from statistical mechanics and network theory (0, 0). Nevertheless, this approach needs for a large amount of detailed data to validate theories and develop models, which historically has been a limitation for the study of social systems. It is in fact surprising how other branches of physics, where the typical scale of the phenomena is very large, as astrophysics, or very small, as particle physics, do not suffer from a lack of data, while the study of social systems, where the typical scale is human, has been historically limited by the lack of data.

Thankfully, the digital revolution has changed this picture, allowing the storage of large amounts of data generated by human activities, such as social media, mobile phones, or online platforms. Nowadays, every two year, more human socio-economic data is produced than during all the preceding years of human history together (0). This data, often referred to as *Big data*, has opened a new era for the study of social systems at a large scale, together with a paradigm shift in the way we understand human behavior (0). Nevertheless, this new era comes with an awareness, as the use of big data for the study of human behavior raises important ethical and privacy concerns, which need to be addressed in order to ensure the responsible use of data for the study of social systems (0). Moreover, from the technical point of view, this huge amount of data needs for a set of computational and mathematical resources to be analyzed and modeled.From this demand, the field of *Computational Social Science* has emerged, with the aim to develop new methods to study human behavior (0). This branch of the complex systems science was born as a combination of methodologies borrowed from social sciences, such as sociology, psychology, or economics, with computational methods from computer science, such as machine learning, data mining, or network theory (0). This interdisciplinary approach has allowed to develop new methods for forecasting social phenomena and understanding the basic mechanism behind human interactions.

One can differentiate two main approaches to build a representation of the reality from the data source. The first one is to focus on the prediction and forecasting of a certain social phenomena, such as the spread of a disease or the price of a stock. In this approach, the data is seen as a necessary input to our methodology to make quantitative predictions (0). However, in this approach, the mechanisms behind the phenomena are often hidden in the data, and the model is seen as a black box that provides accurate predictions (0). In this context, the use of machine learning (0) and deep learning (0) models are very popular, as they are able to capture complex patterns in the data. The second approach is to focus on the understanding of the mechanisms behind the phenomena. In this approach, the data is seen as a problem to be understood, an observation from which we can extract qualitative behaviors and patterns (0). In this context, the aim is to develop very simple models that are able to reproduce the main features of the data, and to extract the basic mechanisms behind the phenomena.

Following the later approach, network science has a critical role in the study of socio-economic systems, as it provides a natural framework to study the interactions between individuals. A network, or graph, is a mathematical representation of a set of nodes (individuals) connected by links (interactions), which allows to study the structure of the interactions and the dynamics of the system. The study of networks has a long history in the natural sciences, from the neurons network in the brain () to food webs in an ecosystem (). However, in recent years, new data sources lead to the discovery that complex properties and heterogeneities, present in most social systems, need for a topological descrition in terms of a complex network (). A complex network can be defined is a network that exhibits non-trivial topological properties, which we will explain later on this thesis. These properties are often found in social networks, such as a social media (), the collaboration network of scientists (), or the trade network of countries (). In particular, the study of information spreading as a dynamical system on networks has allowed to understand how information spreads through a social system and how consensus emerges ().

Contagion of information has been a topic of interest for social scientist. Early theoretical frameworks,influenced by psychological and sociological theories, show how individuals in a crowd lose their sense of self and are more susceptible to the ideas and emotions of the crowd (). Social imitation of behaviors and ideas was proposed as a mechanism for social change, facilitated by close contact and communication among individuals ().......................

- In particular, human interactions exhibit complex activity patterns that are difficult to understand and to model, and that are not present in the study of physical systems.

1.2 Challenges of Computational Social Science

- The study of human behavior and social systems is a complex problem that requires the use of computational methods to study human behavior and social systems.

- There are some challenges that are unique to the study of human behavior and social systems, and that are not present in the study of physical systems.

1.2.1 Data availability

- The main problem is the data availability, and the fact that the data that is generated by human activities is not always available for study.

- Notice that the data sources typically used for the study of human behavior does not come from controlled experiments, but from the digital traces that are generated by human activities.

1.2.2 Data analysis

- The second problem is the data analysis, and the fact that the data that is generated by human activities is not always easy to analyze.

- The data source to analyze usually is a piece of a larger dataset, so we need to be careful to avoid biases in the analysis driven by the data size.

- Temporal windows are also a problem, because when we analyze the dynamics of a system, we need to be careful to avoid biases in the analysis driven by the temporal window.

1.2.3 Modeling

- The third problem is the modeling, and the fact that the data that is generated by human activities is not always easy to model.

- Deterministic models are not always useful to model human behavior, and we need to use stochastic models to model human behavior.

- Also, mechanistic models and data driven models is something that we need to consider when we model human behavior.

- Another possibility is to use agent-based models to model human behavior. With the advent of computational methods in the latter half of the 20th century, researchers gained powerful tools to simulate and analyze complex social systems. Agent-based modeling (ABM) emerged as a particularly influential approach, enabling scientists to create and study systems of interacting agents (individuals or collective entities) and observe emergent behaviors from simple rules of interaction.

1.2.4 Applications

- Computational social science has many applications, and it is being used to study human behavior and social systems.

- Sociotechnical systems, social networks, and human dynamics are some of the applications of computational social science.

- fake news detection, information spreading, and social influence are some of the applications of computational social science.

1.3 Terminology and general concepts

- In this section, we introduce some terminology and general concepts that are used in the study of human behavior and social systems.

Complex networks, interface density, and community structure are some of the concepts that are used in the study of human behavior and social systems.

binary state models, random networks, configuration models, and preferential attachment are some of the models that are used in the study of human behavior and social systems.

1.4 Datasets

- We used the idealista dataset

- The strong point of the idealista dataset is that it contains information about the real estate market in Spain, and that it is a large dataset that contains information about the real estate market in Spain.

- The missing point of the idealista dataset is that it contains information about the real estate market in Spain, and that it is a large dataset that contains information about the real estate market in Spain.

The methodology to study social and economic systems has been significantly influenced by the development of mathematical models that describe the essential features of these systems. In this chapter, we differentiate between simple and complex contagion models, two different information transmission mechanisms that have been widely studied in the literature. We also introduce the Granovetter-Watts threshold model, a fundamental model for understanding the dynamics of complex contagion in social networks, and the Sakoda-Schelling model, a segregation model that was a precursor of the nowadays agent-based simulations. We also introduce topics such as the bursty dynamics in human interactions and the concept of aging, highlighting how these factors influence the dynamics in social systems. These insights will be helpful to understand the results in the following chapters of the thesis.

2.1 Introduction

The contagion of ideas is a process that has been studied for many years and is present in many social systems, ranging from small groups and communities to large networks and societies at a global scale. This process, often referred to as social contagion (0), involves the spread of ideas, behaviors, innovations, and emotions (spread of "information") among individuals and groups through various forms of social interaction. The metaphor of contagion highlights the similarities between the spread of infectious diseases and the transmission of information, where a single "infected" individual can influence multiple others, leading to widespread information.

In this context, binary-state models have emerged as a versatile tool to describe a variety of natural and social phenomena in systems formed by many interacting agents. Each agent is considered to be in one of two possible states: susceptible/infected, adopters/non-adopters, democrat/republican, etc., depending on the context of the model. In all cases, one of the states represent the presence/spreading of information and the other the absence of it. The interaction among agents is determined by the underlying network and the update rules of the model. Examples of binary-state models include processes of opinion formation and consensus (0, 0, 0, 0), disease or social contagion (0, 0), among others.

With the advent of network theory and the increasing availability of large-scale data from online platforms, researchers have been able to study the contagion of ideas with unprecedented precision and detail. Duncan Watts and Steven Strogatz's small-world model (0) and Albert-László Barabási and Réka Albert's work on scale-free networks (0) provided foundational insights into the structure of social networks and their role in facilitating or hindering the spread of information and ideas.

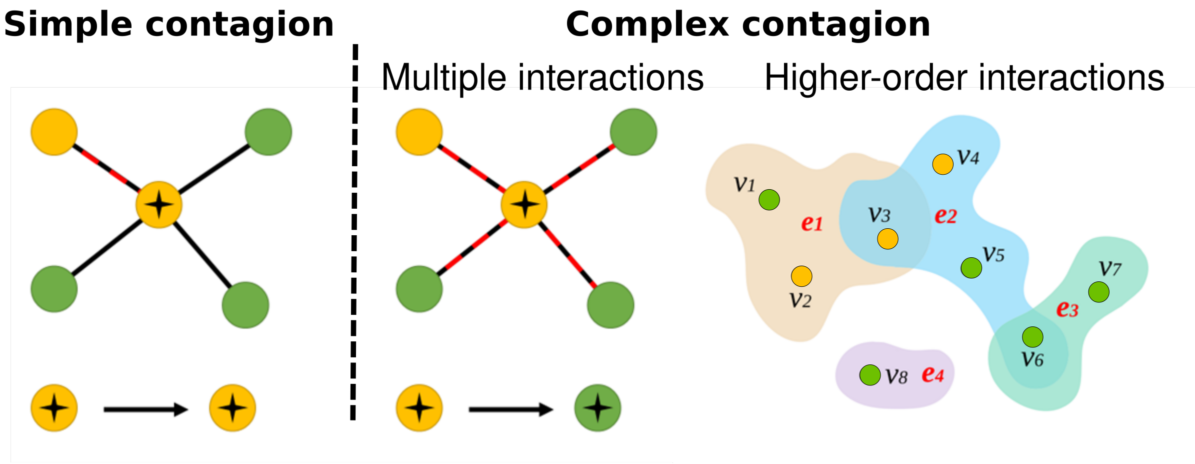

Figure 2.1: Comparison between the different types of social interaction. **Simple contagion**, where the agent considers just the pairwise interaction with one social contact (interaction highlighted with a dashed red line) and **Complex contagion**, where the agent considers the interaction with multiple social contacts. There are two distinguishable types of Complex contagion: **Multiple pairwise interactions**, where the agent considers the interaction with all social contacts (interactions highlighted with dashed red lines) and **Higher-order interactions**, where the agent considers the interaction with a group of social contacts, all at once, in a single interaction (not pairwise). The green, yellow colors represent the state (idea, position, political party...). (The hypergraph representation is from Ref. (0)).

On the other hand, the desicion-making process in social systems is influenced by a variety of factors, including social media influence (0, 0), peer pressure (0), emotional engagement (0, 0) and individual preferences. Peer effects and social influence have been shown to play a significant role in the adoption of new technologies, with individuals more likely to adopt new products or services if they see others in their social network doing the same (0, 0, 0).

In this chapter, we introduce the terms simple and complex contagions, two different mechanisms that describe how information spreads through social networks. Once we have defined these concepts, we move forward to introduce the Granovetter-Watts and the Sakoda-Schelling models, two fundamental models which update rules are based on a threshold mechanism, a particular case of complex interactions. We will introduce a theoretical framework useful to treat threshold models in complex networks and finally, we will introduce the concepts of bursty human dynamics and aging mechanism, which show that the Markovian assumption is not always valid in the study of social dynamics.

2.2 Simple and Complex Contagion

In the study of social contagion, researchers distinguish between two main types of contagion processes: simple contagion and complex contagion. Simple contagion refers to the spread of ideas, behaviors, or innovations primarily through single exposures or interactions, much like the transmission of infectious diseases. This process is characterized by the principle that an individual's likelihood of adopting a new idea or behavior increases with each additional exposure to that idea or behavior within their social network (0, 0). In contrast, complex contagion involves multiple exposures or reinforcements from different sources within the network, often requiring a critical mass of adopters before an individual is influenced to adopt the idea or behavior (0, 0, 0).

Simple contagion is often described as a process that involves only dyadic interactions, where the adoption of an idea or behavior is facilitated by direct contact between two individuals (see Fig. ??). This type of contagion is fundamental to understanding how information, beliefs, or diseases spread through populations via direct, pairwise connections (0, 0). Features of simple contagion include the rapid dissemination of information and the efficient spread of both beneficial and detrimental behaviors across social ties (0, 0).

In contrast, Complex contagion takes place in scenarios where adoption is not merely a result of dyadic interactions but also involves group dynamics and/or the reinforcement from multiple sources within the network. This type of contagion often requires a critical mass or threshold of adopters at the individual's surroundings to trigger the adoption of information (0, 0). This condition that characterizes complex contagion can be understood in two ways: (i) as a reinforcement of the idea or behavior from multiple pairwise (dyadic) interactions (0, 0), or (ii) as a reinforcement from multiple sources in a single group interaction (higher-order interaction) (0, 0, 0). In the first case, the peer pressure, characteristic of complex contagion processes, is included into the model, which is designed to be used a simple network of dyadic social contacts. In the second case, the group interaction is included in a higher-order network or hypergraph (0), which is a more general representation of the social contacts where the interactions are not restricted to dyads. In this case, the complex contagion process takes place via a single group interaction. See Fig. **??** for a graphical representation of the different examples of complex contagion.

Moreover, real-world processes are influenced not solely by either simple or complex contagion mechanisms but by a complex interaction between the two (Hybrid contagion). Such multifaceted interactions give rise to varied outcomes, including phenomena like discontinuous transitions, tricriticality, and echo chambers emergence (0, 0, 0), all of which profoundly affect how information is spread, how behaviors are adopted, and how collective actions are formed.

There have been attempts to extract the simple/complex nature of a process from real data. For example, by analyzing the correlation between the infection order of network nodes and their local topology, it is possible to infer the type of contagion process that is taking place (0). Nevertheless, the classification of contagion processes remains a challenging task, as the dynamics of social contagion are influenced by a multitude of factors and high-quality data related to the infection process is often scarce.

2.3 Granovetter-Watts threshold model

In this thesis, we are interested in the dynamics of complex contagion driven by multiple interactions in a network of dyadic social contacts. In particular, we focus on a particular category of models called **threshold models**.

Threshold models represent a critical conceptual framework in understanding how individual behaviors aggregate to produce collective outcomes, especially in contexts where decisions are influenced by the actions of others (0, 0). By defining a "threshold" — the point at which an individual's perception of the collective behavior of others prompts them to act — these models offer insights into the pivotal role of social influence and network structure in driving large-scale changes from small initial actions (0). Rooted in the interdisciplinary nexus of sociology, economics, and network theory, these models illuminate the mechanics behind phenomena as diverse as social movements, technological adoption, market dynamics, and even cascading failures within infrastructures. All these phenomena share a common thread: the need for a critical mass of adopters to trigger a response, a threshold that must be crossed to initiate a cascade of adoption (0, 0).

When we talk about threshold models, the model that comes to our minds is the threshold model introduced by Mark Granovetter in 1978 (0), exploring how individual thresholds for adoption depend on the proportion of others adopting the behavior, highlighting the nonlinear nature of social influence and the importance of group interaction in complex contagion processes. In this model, each individual has a threshold that determines the number of neighbors they need to observe adopting a behavior before they themselves adopt it. This threshold can be interpreted as a measure of an individual's susceptibility to social influence,

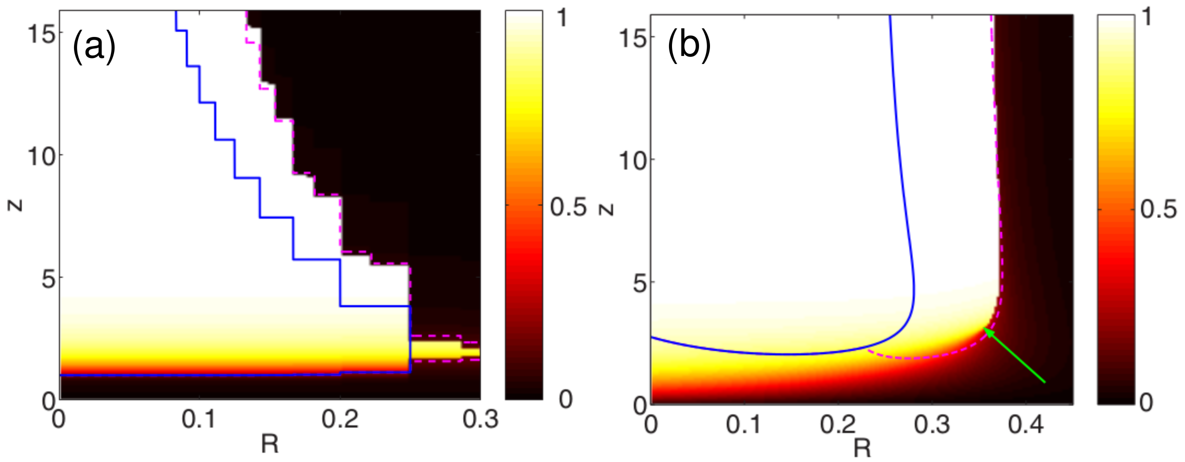

Figure 2.2: Average density n of active nodes as a heatmap for the Granovetter-Watts model. The simulations run in a Poisson random graph of mean degree z and uniform threshold value R **(a)** and threshold distributed is Gaussian with mean R and standard deviation 0.2 **(b)**. Seed fraction is set $n_0 = 0.01$. Lines show approximations to the global cascade boundaries. The phase transition is discontinuous. Image from Ref. (0).

capturing the idea that some people are more likely to adopt a behavior if they see many others doing the same, while others may require more convincing or reinforcement before they act. Duncan J. Watts, in 2002, built upon Granovetter's concept, applying mathematical analysis to explore the model within complex networks (0) . His work, particularly on how minor initial actions can lead to large cascades, further elucidated the relationship between individual thresholds and network structures. This model, named as the Granovetter-Watts threshold model, has since become a cornerstone of research on complex contagion and collective behavior, offering a powerful lens through which to study the cascade dynamics in complex networks.

Update rules — Granovetter-Watts model. An individual time step of the model is defined as follows:

1. Each node i has a threshold R_i.
2. At each time step, a node i is selected at random.
3. If the fraction of active neighbors of i is greater than R_i, then i becomes active.

The Granovetter-Watts model exhibits a phase transition from a regime where the adoption is rare, where there are only small cascades of adoption and none of them is global, to a regime where the adoption is widespread, where there are large cascades that reach all the system. This phase transition is discontinuous (0, 0), and it is characterized by a critical threshold value R_c that separates the two regimes (refer to Fig. **??**). The regime where the global cascades are rare,small and localized is a supercritical regime $R > R_c$ while the regime where cascades are fast and global is subcritical $R \leq R_c$. The discontinuous transition between the two regimes is driven by the interplay between the individual thresholds and the network structure, and it is a result of the collective dynamics of the system (see dependence of R_c with the average degree in Fig. **??**).

The exploration of this model has been widespread, encompassing studies on various types of networks including regular lattices and small-world networks (0), as well as on random graphs (0). It has also been examined within the contexts of networks with modular and community structures (0), networks that exhibit clustering (0, 0), hypergraphs (0), and networks characterized by homophily (0), among others. In addition, the literature has expanded to cover the effects of varying the rules for adoption, such as incorporating social reinforcement across multiple layers (0), examining the influence of opinion leaders and initial seed size on the process (0, 0), the introduction of on-off thresholds (0), and analyzing the dynamics when simple contagions compete with complex ones (0, 0, 0). Further, empirical data have been used to test the predictions of the Granovetter-Watts model, demonstrating its applicability across a wide range of real-world situations (0, 0, 0, 0, 0, 0, 0, 0).

2.4 The Sakoda-Schelling model

Thomas C. Schelling's segregation model (0), illustrates how individual preferences regarding neighbors can lead to significant segregation in urban areas, even when these preferences are relatively mild. The model utilizes a checkerboard setup where each agent (representing a household) prefers to live in a neighborhood where at least a certain percentage of neighbors are of the same type (see Fig. ??). Agents move to a new location if their tolerance threshold is not met [1]. This simple rule leads to complex patterns, showing that even a slight preference for similar neighbors can result in highly segregated communities, an insight that has profound implications for understanding social dynamics and urban planning.

Update rules — Schelling's model. An individual time step of the model is defined as follows:

1. Each node i has a tolerance threshold T_i.
2. At each time step, a node i is selected at random.
3. If the fraction of different kind neighbors of i is greater than T_i, then i moves to a neighboring location where the fraction of different kind neighbors is less than T_i.
   - If there is no available location, then i remains in the same location.

On the other hand, James M. Sakoda's model, initially conceptualized in his 1949 dissertation and fully introduced in Ref. (0), offers a more nuanced approach to modeling social interactions using a similar checkerboard framework. Unlike Schelling's, Sakoda's model incorporates a broader range of social interactions by allowing agents to exhibit positive, neutral, or negative attitudes towards their neighbors. These attitudes influence the agents' movements across the board, aiming to optimize their local environment according to specific utility functions that aggregate the effects of surrounding agents. Sakoda's model is capable of simulating a variety of social phenomena beyond segregation, such as the formation of stable social clusters and the dynamics of group interactions (0).

An important contribution associated to both models is the use of a checkerboard setup as the computational space where agents (or tokens) reside and interact according to predefined rules. The "hand-made" simulations performed by Sakoda and Schelling using this discrete spatial representation has since become a standard framework for studying agent-based models in social systems (0). This structure allows for the exploration of local interactions, emergence of global patterns and interface analysis, providing a powerful tool for understanding the dynamics of social systems.

Update rules — Sakoda's model. An individual time step of the model is defined as follows:

1. Each node i has an attitude matrix A_i, that defines the attitudes of i towards all other agents on the board.
2. At each time step, a node i is selected at random.
3. i evaluates the total utility for each neighboring location based on the sum of influences (according to the attitudes) from all other agents on the board, weighted by distance.
4. i moves to the available location with the highest utility.

The results of the Schelling's model demonstrated how even mild personal preferences can unexpectedly lead to significant spatial segregation. Its insights have been applied across economics, sociology, urban planning, and complexity science, profoundly influencing both academic research and practical policy discussions. Schelling's model became a foundational

[1] Note that this tolerance threshold acts as a measure of a critical mass of different neighbors that the individual tolerates (large tolerance allows for more diverse neighborhood while small tolerance requires several similar neighbors to be satisfied). This threshold definition is complementary to the one in the Granovetter-Watts model, where it is a measure of the number of similar neighbors required to adopt (small threshold allows to adopt with a more diverse neighborhood while large threshold requires several similar neighbors to adopt).

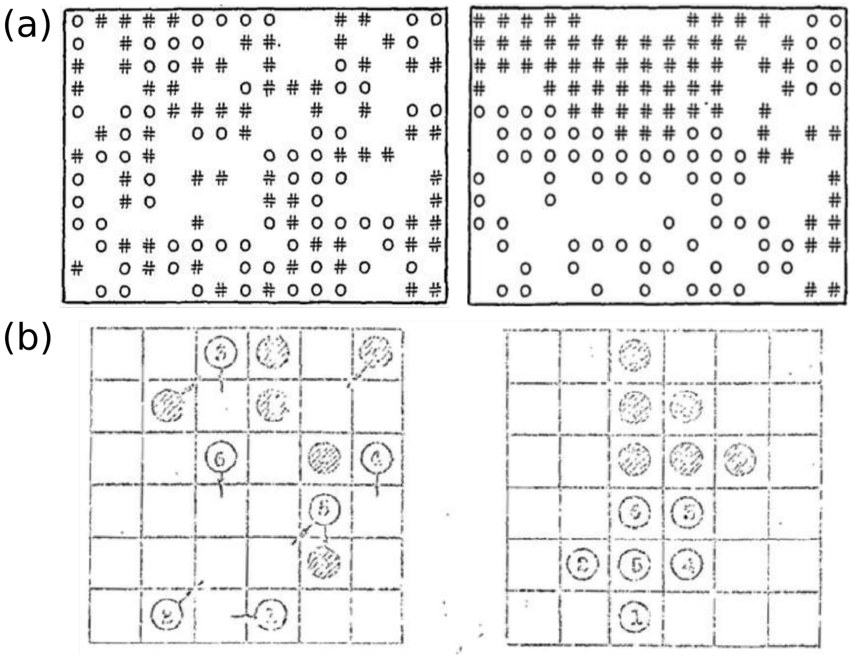

Figure 2.3: (a) Examples of the dynamics in Schelling's Segregation Model (from Schelling's original work (0)). **(b)** Examples of the dynamics in Sakoda's Checkerboard Conceptual Model (from Sakoda's original work (0)). In both cases, the author show at the left the initial configuration of the board and at the right the final segregated configuration after several iterations.

example in agent-based modeling, helping to educate countless researchers and practitioners about the impact of individual actions on broader social patterns. This contribution was one of the key reasons Schelling was awarded the Nobel Prize in Economics in 2005, underscoring the model's enduring influence and importance. Nevertheless, when we check the update rules, we observe that the Schelling's model is a particular case of the previous Sakoda's model, where agents have a negative attitude towards different-kind agents and a fixed tolerance threshold T_i. To honor the original contributions of both authors, we refer to this model through the thesis as the Sakoda-Schelling model.

In particular, the Sakoda-Schelling model has been studied from a Statistical Physics point of view due to its close relation to different forms of Kinetic Ising-like models (0, 0), and also addressing general questions of clustering and domain growth phenomena, as well as for the existence of phase transitions from segregated to non-segregated phases. For example, the relation with phase separation in binary mixtures has been considered (0, 0), as well as the connection with the phase diagram of spin-1 Hamiltonians (0, 0, 0, 0). In this context, a useful classification of models is to distinguish between two possible types of dynamics (0): **"constrained"**, where agents just move to satisfying vacancies (if possible), and **"unconstrained"**, where agents' motion does not prevent them to remain unsatisfied. In addition, the motion can be short-range (only to neighboring sites, as in the original model) or long-range. Constrained motion has been named "solid-like" because it generally leads to frozen small clusters, while unconstrained motion has been considered "liquid-like" because it allows for large growing clusters (0). Including the motion of satisfied agents leads to a noisy effect playing the role of temperature in a statistical physics approach (0, 0).

Despite there has been many attempt in the literature, the description of the phase diagram and the transitions in the Sakoda-Schelling model is a difficult task. In Ref. (0), the authors need to reduce the model to a simpler binary-state version that allows to obtain the phase transition from a segregated to a mixed phase.

2.5 Theoretical Framework

To explain the emergent properties exhibited by the agent based models and its computer simulations, we need to develop a theoretical framework that captures the essential features of the system. This framework should provide a mathematical description of the dynamics, allowing us to analyze the system's behavior and predict its evolution over time. In the context of social contagion and collective behavior, the theoretical framework typically involves a set of differential equations or master equations that describe the evolution of the system's state variables.

The theoretical framework for agent-based models running in complex networks can be broadly classified into two main categories: mean-field approaches and network-based approaches (0). Mean-field approaches treat the system as a homogeneous entity, where each agent interacts with the average behavior of the entire population. These approaches are well-suited for capturing the macroscopic dynamics of the system and are particularly useful for understanding the collective behavior that emerges from individual interactions. Network-based approaches, on the other hand, explicitly model the interactions between agents as a network structure, where nodes represent agents and edges represent interactions between them. These approaches are valuable for capturing the influence of the underlying network structure on the system's dynamics and for studying the impact of network properties on the spread of information and ideas.

2.5.1 Approximate Master Equation

A general framework for binary-state models in complex networks was developed by J. P. Gleeson (0, 0), which provides a general set of differential equations, the Approximate Master Equation (AME), to describe the dynamics of any Markovian binary-state model on a generic network. This framework has been widely used to study the dynamics of social contagion, opinion formation, consensus problems, and other collective behaviors in complex networks. The framework is particularly very useful in the context of thresholds models, which are not well suited for a mean-field approach (0), allowing us to identify of phase transitions, compute critical thresholds, and the predict the final state of the system. The full AME description and derivation can be found in Ref. (0), but we will provide a brief summary of the main concepts here.

Consider a system of N nodes in a network, where each node can be in one of two states: $+1$ or -1. The state of each node evolves over time according to a set of rules that depend on the states of its neighbors. Let us consider a node i, with a degree k (i.e., k connections to other nodes) and m neighbors of i in state -1 (e.g., "adopter"). If node i is in state $+1$ (e.g., "non-adopter"), the rate $T^{+}_{k,m}$ defines the probability per unit time that i will switch to state -1. Similarly, $T^{-}_{k,m}$ defines the probability per unit time for i, in state -1, to switch to state $+1$. These rates are, in general, functions of both the degree k and the number m of neighbors in state -1, reflecting how the local network configuration influences state transitions.

Taking into account this framework, the AME can be written as:

$$\frac{d}{dt}x^{\pm}_{k,m} = -T^{\pm}_{k,m}x^{\pm}_{k,m} + T^{\mp}_{k,m}x^{mp}_{k,m} - (k-m)\beta^{\pm}x^{\pm}_{k,m} + (k-m+1)\beta^{\pm}x^{\pm}_{k,m-1} - m\gamma^{\pm}x^{\pm}_{k,m} + (m+1)\gamma^{\pm}x^{\pm}_{k,m+1} \tag{2.1}$$

Here, $x^{+}_{k,m}$ and $x^{-}_{k,m}$ represent the fractions of nodes with degree k and m infected neighbors that are in state $+1$ and -1, respectively. $\beta^{\pm}$ and $\gamma^{\pm}$ are rates that describe how the adoption process spreads and recedes across the edges of the network, encapsulating the network's dynamic connectivity and its influence on the spread of states.

A key advantage of the AME is its ability to capture the complex dynamics of networks by considering the interactions between neighboring nodes, making it more accurate than simpler models like the mean-field theory, which assumes independence between nodes. Moreover, from the AME, one can make approximate the shape of the solutions $x^{\pm}_{k,m}(t)$ to reduce the number of differential equations, recovering the pair approximation and the heterogeneous mean field (0, 0).

On the other hand, the AME assumes a tree-like structure with negligible levels of clustering. This assumption implies that there are very few short loops in the network. This tree-like assumption simplifies the calculation and application of the AME by reducing the network's complexity, and becomes very useful for networks generated with the configuration model (0), with any given degree distribution, at the limit $N \to \infty$. Another limitation of the AME is based on the formulation itself, since framework is built assuming binary-state Markovian dynamics, which may not always accurately capture the real-world dynamics of social contagion. In fact, in next section, we will introduce the concept of bursty human dynamics, which is is an empirical evidence of the presence of non-Markovian effects. This effects can significantly impact the dynamics of social contagion processes.

2.6 Bursty Human Dynamics

Bursty behavior refers to the irregular and sporadically temporal patterns of interactions that include natural phenomena, like earthquakes and neuron firing, as well as human activities, such as email communication, mobility, and social dynamics. This section delves into the characteristics of bursty behavior, highlighting by empirical evidence, and discusses its significant implications for modeling human behavior.

Human activities often exhibit complex temporal patterns characterized by bursts—short periods of high activity interspersed with longer periods of inactivity (see examples Fig. ??(a-c)). This non-Poissonian behavior, referred to as burstiness, manifests across diverse human-driven processes and is extensively documented in communication dynamics, web browsing habits, and social interactions (0, 0). The seminal work, by A. L. Barabási (0), shown from email communication's data that activity periods do not follow a regular pattern but are clustered in bursts. This phenomenon has since been observed universally across various platforms such as mobile phone calls, text messaging and social media (0, 0, 0, 0, 0, 0, 0, 0, 0). Further research has analyzed burstiness, focusing on the persistence and periodicity of human interactions (0) or the effects of circadian rhythms (0), and has extended these analyses to web activity to predict behaviors across different online platforms (0).

An important evidence of bursty dynamics is the heavy-tailed distribution of inter-event times, indicating that the probability of short inter-event times is higher than expected from a Poisson process (see Fig. ??(c)). As a result of this bursty human behavior, there is an emergence of heterogenous degree distributions (0), which have been observed in many social systems (0). Further insights into the impact of burstiness on system dynamics come from studies linking it to memory and the structured nature of human dialogues, enhancing our understanding of how past interactions influence future activities (0, 0, 0).

Traditional models based on Poisson processes are often inadequate for capturing the real dynamics of human interaction. To address this, non-Poissonian models are necessary to provide a better fit for empirical observations (0). We differentiate two main approaches to include bursty human dynamics in our models:

- **Activity-driven models (nodes get activated):** These models incorporate the temporal aspects of human activity by assigning activity potentials to nodes within a network, dictating the likelihood of interactions based on observed human activity patterns (0).
- **Temporal networks (links get activated):** These models incorporate time-stamped interactions, such that at each time step our interaction network changes (0).

While both approaches have been successful to include bursty human dynamics, they offer different perspectives on the underlying mechanisms driving these behaviors: activity-driven models emphasize the burstiness of individual attempts to interact with others, while temporal networks focus on the burstiness of the interactions themselves. The choice of model depends on the specific research question and the level of detail required to capture the dynamics of interest.

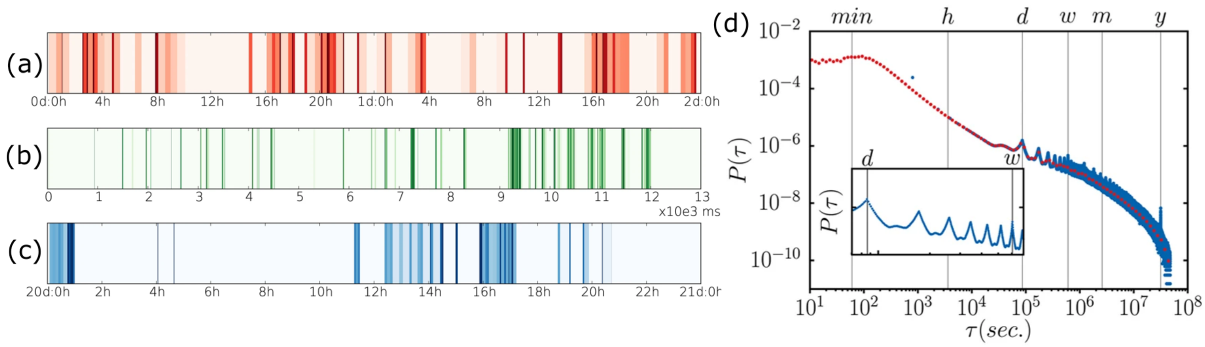

Figure 2.4: (a) Sequence of earthquakes with magnitude larger than two at a single location (South of Chishima Island, 8th–9th October 1994). **(b)** Firing sequence of a single neuron (from rat's hippocampal). **(c)** Outgoing mobile phone call sequence of an individual. Shorter the time between the consecutive events darker the color (a,b,c from Ref. (0)). **(d)** Distribution of inter-event times between tweets for several users (from Twitter). Blue and red dots represent the lin- and log-binned scales in the τ axis. The localized maxima in the tail of the distribution correspond to circadian rhythms, as shown in the bottom inset (from Ref. (0)). The distribution is heavy-tailed, indicating bursty behavior.

It have been shown that implications of bursty behavior are dramatic, influencing the dynamics of network processes such as the spread of epidemics and information diffusion (0, 0). Understanding the mechanism behind this burstiness allows us to improve our predictions, aligning them more closely with natural human activity patterns.

2.7 Aging mechanism

"Aging" is one form of memory effect on which the rate of interactions depends on the persistence time of an agent in a state, modifying the transition to a different state (0, 0, 0).

This concept of aging was introduced in (0) taking as inspiration the non-equilibrium dynamics of spin glasses, where the effective temperature of the system changes with the time since a given perturbation was applied (0). In a context of social systems, this resistance to change can be interpreted as conviction. In models of species competition (0), this would imply that neighboring species are less likely to be displaced at a later stage of growth (0). In our words, the motivation behind the aging mechanism is to capture the tendency of individuals to stick to their previous beliefs or habits, a common feature in human behavior (0). In other words, the longer an individual holds a particular habit, the more he/she will accumulate experience, leading to a higher self-involvement and resistance to change (0). Moreover, this emotional attachment balances the memory-less and purely rational considerations of traditional models (0).

The aging mechanism is a non-Markovian effect that can be included in a model via an activation function that modifies the transition rates between states. This function depends on the time since the last transition, allowing us to include bursty dynamics in the individuals' attempts to interact with others (given a proper choice of the activation function (0)). This activation function is build such that probability of an individual to interact with another individual decreases with the time since the last interaction, even though there are studies that also account for anti-aging mechanisms (probability to interact increases) (0, 0).

2.7.1 Aging in pairwise interactions

Aging effects have been already shown to modify drastically the dynamics in the Voter model (0), a popular framework for exploring consensus formation in statistical physics and social dynamic. Fig. **??**(a) shows an schematic example of a Voter model with aging, in which a

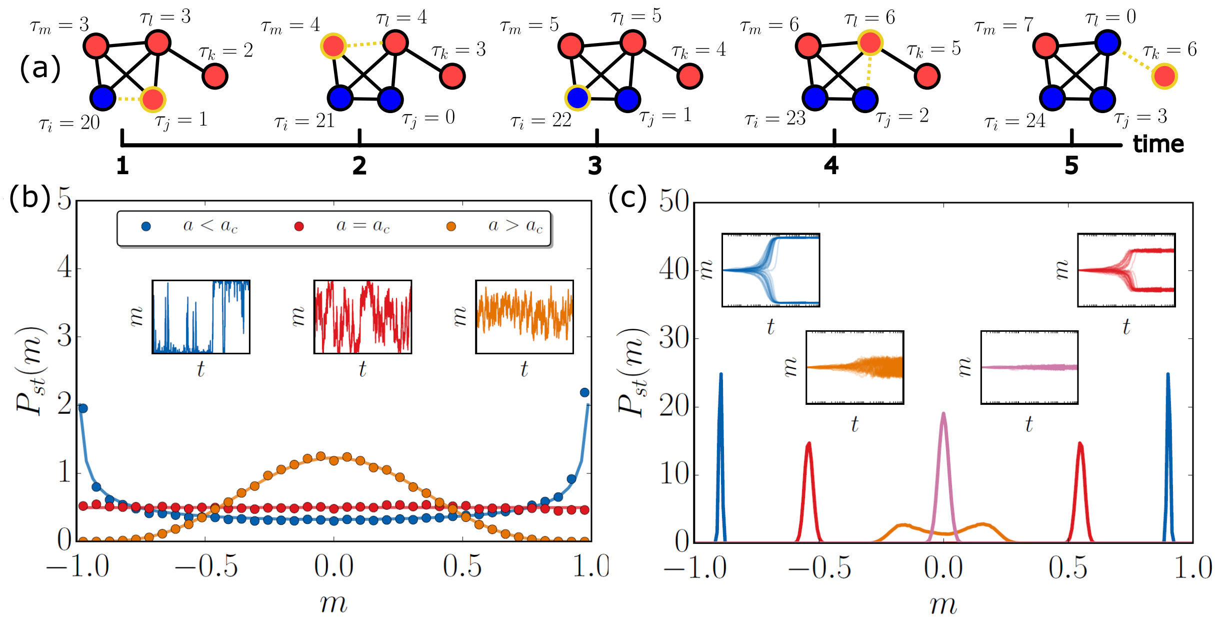

Figure 2.5: (a) Schematic representation of the evolution of the Voter model with aging: At 1, node j activates, copies i and resets age. At 2, node m activates and copies m (same state). At 3, i does not activate due to the age. At 4, node l copies j and resets age. At 5, k activates, copies l and resets age. **(b)** Stationary probability density function (pdf) of the magnetization in the three different regimes. Points come from simulations, solid lines are the theoretical curves. The insets show one typical trajectory of the dynamics, in each of the regimes. **(c)** Stationary pdf for the noisy voter model with aging, in the different regimes. The insets show 50 trajectories of the magnetization. (b-c from Ref. (0)).

minority is able to spread through the system due to aging. It is shown that, while aging can decelerate microdynamics by making state changes less frequent as agents' states age, it can accelerate macrodynamics, thus shortening the time required for the system to reach consensus. This counterintuitive phenomenon, observed across different network topologies, highlights the complex role of temporal elements in dynamic systems (0, 0, 0, 0).

In terms of stability, in Voter-like models, incorporating aging exhibit a higher tendency toward reaching stable configurations than their non-aging counterparts. The persistence of the majority state, reinforced by aging, contributes to this stabilization, making aging a significant factor in determining the system's equilibrium state (0). As an example, aging modifies the nature of the phase transition in the noisy Voter model. Specifically, it transforms a finite-size discontinuous transition between ordered and disordered phases into a continuous transition that falls into Ising universality class (0) (see discontinuous transition for noisy Voter model in Fig. **??**(b) and the continuous transition when aging is included in Fig. **??**(c)).

Moreover, this mechanism promotes longer persistence of the current majority state, thereby limiting the influence of fluctuating minority opinions over time and demonstrating a robust method for maintaining stability within a system (0). These insights highlight the complex interactions between temporal dynamics and the update rules, offering a richer understanding of how consensus and order emerge in social and physical systems.

Regarding to models of multiple pairwise interactions or higher-order interactions, the aging implications are still an open question and it is a topic of current research. Just for the specific case of the noisy majority vote model (0), it is known aging mechanism is able to modify the critical point of the disordered-ordered phase transition. Further research is needed to understand the joint effect of aging and multiple interactions.

The results in this chapter are published as:

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We incorporate aging into the Sakoda-Schelling model by making the probability of agents to move inversely proportional to the time they have been satisfied in their present location. This mechanism simulates the development of an emotional attachment to a location where an agent has been satisfied for a while. The introduction of aging has several major impacts on the model statics and dynamics: the phase transition between a segregated and a mixed phase of the original model disappears, and we observe segregated states with a high level of agent satisfaction even for high values of tolerance. In addition, the new segregated phase is dynamically characterized by a slow power-law coarsening process similar to a glassy-like dynamics.

3.1 Introduction

As it was introduced in section **??**, a robust result of the Sakoda-Schelling model is that segregation occurs even when individuals have a very mild preference for neighbors of their own type, so collective behavior is not to be understood in terms of individual intentions. In addition, the model introduced the concept of behavioral threshold that inspired a number of other models of collective social behavior (0). But still currently, Schelling's model is at the basis of fundamental studies of the micro-macro paradigm in Social Sciences (0), while it continues to have important implications for social and economic policies addressing the urban segregation problem (0, 0, 0, 0). A main limitation of the Sakoda-Schelling model is that it has no history or memory by which, for example, residents might prefer to maintain their present location (0).

As a result of the notable implications of this model and the robustness of the emerging segregation, there exists a vast literature around Schelling's results. Many variants of the original Sakoda-Schelling model have been reported modifying the rules that govern the dynamics, the satisfaction condition, or including other mechanisms, network effects, or specific applications (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0).

With the motivation of established relevant effects of aging in the previous chapter, our goal is to characterize how "aging" modifies the segregation dynamics of the Sakoda-Schelling model. In this context, aging must be understood as an emotional/economic attachment to a certain location linked to the persistence time in this location. This attachment balances the memory-less and purely rational considerations of the original model (0). The aging-induced inertia, which results in resistance to movement, is minimalist modeling of behavior with many different possible causes. Besides the moving out cost due to the housing market fluctuations, aging accounts for the links established with the neighborhood's public goods, venues, schools, etc, which are known to be highly relevant in this context (0, 0, 0). These urban elements are also a major consideration when households locate (0, 0, 0, 0) and aging also accounts for the memory of this decision.

In this chapter, aging is introduced in the Sakoda-Schelling model by considering that agents are less prone to change their location as they get older in a satisfying place. In other words, aging is introduced giving a smaller probability for satisfied agents to "move-out" the longer they have remained in a satisfying neighborhood. We implement this aging mechanism in the long-range noisy constrained version of the Schelling Model (0), for which a detailed phase diagram was reported. We study how this phase diagram is modified by the aging mechanism, finding that aging inhibits a segregated-mixed phase transition. This implies that aging favors segregation, a counter-intuitive result. We also describe the coarsening dynamics in the segregated phase showing that aging gives rise to a slower coarsening that breaks the time-translational invariance.

3.2 Aging in the Sakoda-Schelling model

The model considered here is a variant of the noisy constrained Sakoda-Schelling model (0) in which we explicitly include aging effects. For simplicity, we refer to this variant as the Sakoda-Schelling model during the rest of the paper to compare with the model presented here: the Sakoda-Schelling model with aging. For both, the system is established on a $L \times L$ Moore lattice with 8 neighbors per site and periodic boundary conditions, where agents of two kinds (representing, for instance, wealth levels, race, language, etc) occupy the sites. There are also empty sites (vacancies), where agents can move to, depending on their state and on the vacancy neighborhood. The condition of each site i of the lattice will be described with a variable σ_i that takes three possible values: $\sigma_i = \pm 1$ for the two kinds of agents and $\sigma_i = 0$ for vacancies. In addition, depending on the local environment, agents can be in two states: satisfied or unsatisfied. In our case, agents are satisfied if their neighborhood is constituted by a fraction of unlike agents lower than a fixed homogeneous threshold T. Otherwise, they are unsatisfied. Therefore, this control parameter T is a measure of how tolerant the population of the system is. We also need a non-zero vacancy density, $n_0 > 0$, for agents to change their location. This n_0 is understood as an extra parameter of the model. The initial configuration is built by randomly distributing the agents ($N_{\text{agents}} = L^2(1-n_0)$). We always consider initially one half of agents of each kind.

In the Sakoda-Schelling model considered in this study, an agent chosen by chance moves to a random satisfying vacancy (if any exists) independently of his/her initial state and of the distance. This process is repeated until the system reaches a stationary state. The movement of unsatisfied agents behaves as a driver for the system dynamics, while the motion of satisfied agents plays the role of noise. When tolerance T becomes larger, more satisfying vacancies are present in the system and the noise consequently increases.

The aging mechanism in our model is introduced by considering an activation probability of the agents inversely proportional to the time spent at a satisfied location, motivated by the definition for opinion dynamics (0). This methodology was proposed to mimic the power-law like inter-event time distributions observed in real-world social systems (0, 0). If an agent j is initially satisfied in her neighborhood, the internal time is set $\tau_j = 0$. Then, in every time step, a randomly chosen agent j follows different rules depending on whether she is originally satisfied or not. If unsatisfied, j moves to any random satisfying vacancy of the system. If satisfied, she moves to another satisfying vacancy with an activation probability $p_j = 1/(\tau_j + 2)$. In both cases, if no vacancy has a satisfying neighborhood, the agent j remains in the initial site. As before, these rules are iterated until the system reaches a stationary state (if possible). The time is counted in Monte-Carlo steps; after each Monte-Carlo step, that is after N_{agents} iterations, the internal time increases for all satisfied agents in one unit, $\tau_j \to \tau_j + 1$. Notice that, when an unsatisfied agent becomes satisfied due to the neighbor's motion, an internal time $\tau_j = 0$ is set for that agent. As for the Sakoda-Schelling model, there is a noise effect associated with the motion of satisfied agents. In this case, the intensity of this noise is related not only to the tolerance parameter T, but to the presence of aging as well. In fact, aging introduces more constraints to the movements and contributes to decreasing the noise.

Given the number of neighbors available in the Moore lattice, numerical simulations are only performed for a finite set of meaningful tolerance values: $\{1/8, 1/7, 1/6, \cdots, 6/7, 7/8\}$. During all our analysis, we focus on the low vacancy density region of the phase diagram.

3.3 Segregation coefficient

Many metrics have been introduced in the literature to discern if the final state is segregated or not (0, 0, 0, 0). The number of clusters is known to be directly related to the segregation because a high presence of small clusters indicates a mixing between agents. As for the Sakoda-Schelling model(0), we compute the following metric related to the second moment of the cluster size distribution:

$$s = \frac{2}{\left(L^2\left(1-n_0\right)\right)^2} \sum_{\{c\}} m_c^2, \tag{3.1}$$

where the index of the sum c runs over all the clusters $\{c\}$ and m_c is the number of agents in the cluster c. The average of s over realizations after reaching a stationary state is defined as the segregation coefficient $\langle s \rangle$. This metric is bounded between 0 and 1: $\langle s \rangle \to 1$ if there are only 2 equally-sized clusters, and $\langle s \rangle \to 0$ if the number of clusters tends to the number of agents. The cluster detection is performed using the Hoshen-Kopelman algorithm (0).

Another metric of segregation is the interface density (0), defined as the fraction of links connecting agents of different kinds. The calculation is done in two steps: estimating the interface density for each agent j, ρ_j, and then the average over all the agents ρ:

$$\rho_j = \frac{1}{2}\left(1 - \frac{\sigma_j \sum_{k\in\Omega_j} \sigma_k}{\sum_{k\in\Omega_j} \sigma_k^2}\right) \quad \text{and} \quad \rho = \frac{1}{\mathrm{N_{agents}}} \sum_{\mathrm{j=1}}^{\mathrm{N_{agents}}} \rho_{\mathrm{j}}, \tag{3.2}$$

where the indices k run over the neighborhood of agent j, Ω_j. If an agent j is surrounded only by vacant sites, we define by convention $\rho_j = 0$. Performing a realization average of ρ, we obtain the average interface density $\langle \rho \rangle$ in the stationary state is denoted as $\langle \rho_{\mathrm{st}} \rangle$. The time evolution of this metric, not present in literature, allows us to study the coarsening process.

3.4 Results

3.4.1 Phase diagram

To discuss the phase diagram of our model, we focus on the region of parameters with a vacancy density $n_0 < 50\%$ to avoid diluted states with a majority of vacancies. For this region, the Sakoda-Schelling model presents 3 different phases (0): frozen, segregated and mixed. For low tolerance values, the system freezes in a disordered state, given that there are no satisfying vacancies for any kind of agent. With increasing tolerance, the system undergoes a transition toward a segregated state, which is characterized by a 2-clusters dynamical final state. Finally, for high values of T, after another transition, we find a dynamical disordered (mixed) state, in which a vast majority of vacancies are satisfying for both kinds of agents, and small clusters are continuously created and annihilated.

These three phases are characterized by measuring the segregation coefficient $\langle s \rangle$ and the average interface density $\langle \rho_{\mathrm{st}} \rangle$ at the final state. The results for the original model are depicted as a function of the tolerance T in Fig. **??**a for the interface density and in Fig. **??**b for the segregation coefficient. At low values of T, both indicators show a disordered state that falls in the frozen phase. We also observe a dependence of the transition point with the vacancy

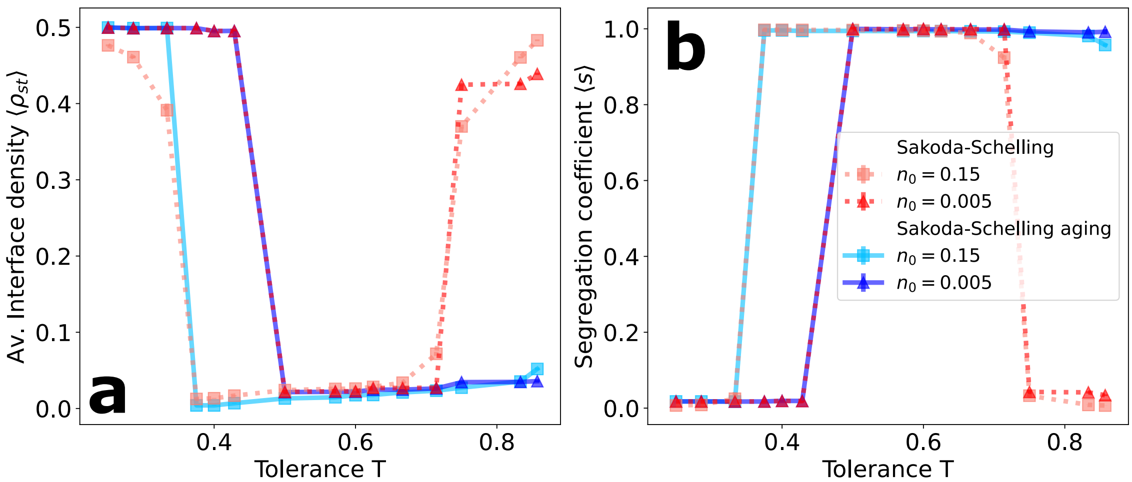

Figure 3.1: Average interface density $\langle \rho_{st} \rangle$ (**a**) and segregation coefficient $\langle s \rangle$ (**b**) at the stationary regime as a function of the tolerance parameter T for two values of the vacancy density $n_0 = 0.5\%$ and 15%. Results are shown for both the Sakoda-Schelling model and the variant with aging introduced in this paper. Simulations are performed on an 80×80 lattice and averaged over $5 \cdot 10^4$ realizations.

density. On the other hand, for high T values, the transition point between segregated and mixed states has no dependence on the parameter n_0. Notice that mixed and frozen states present a very similar value of $\langle s \rangle$ but can be differentiated by the stationary value of the average interface density $\langle \rho_{st} \rangle$. These results are in agreement with the results reported for the Sakoda-Schelling model(0), with the extra information provided by the average interface density.

The first quite dramatic effect of including aging in the system is the disappearance of the mixed state from the phase diagram. In both metrics, the difference between the models with and without aging is clearly manifested. For low T values, the frozen-segregated transition behaves similarly to the original model since aging has no implications as the system gets quickly frozen. Nevertheless, for high values of the tolerance $T > 0.5$, the segregated-mixed transition disappears, and the segregated phase is always present. This is not an intuitive effect and one would think that aging, contributing to difficult agent's mobility, should prevent the system from forming fully developed segregated clusters. However, it is just the opposite, and it favors cluster prevalence.

3.4.2 Segregated phase: final state

To gain further insights into the differences in the system dynamics that lead to the extended segregated phase, we compute the fraction of unsatisfied agents at the stationary regime n_u (see Fig. **??**a). This metric plays a role as a marker for the frozen-segregated transition, as shown for the 1D Sakoda-Schelling model (0). The frozen phase presents a big majority of unsatisfied agents for both models. After the transition, this parameter decays to very low values in the segregated phase, where a majority of agents are satisfied. In this phase, we observe a step-like increasing behavior of the unsatisfied agents with T. As the tolerance grows, the number of satisfying vacancies increases and the noisy movement of satisfied agents drives the system evolution, creating eventual unsatisfied agents in the sites that they abandon or target. However, in the Sakoda-Schelling model, the transition to a mixed state at $T = 0.75$ inhibits the creation of clear fronts between agents of different kinds, and it is also associated to a sharp increase of $n_u \simeq 0.05$ (red squares in Fig. **??**a). The Sakoda-Schelling model with aging, on the other hand, shows a lower fraction of unsatisfied agents during all values of the tolerance above the frozen-segregated transition (blue triangles in Fig. **??**a). So much so, that many realizations reach

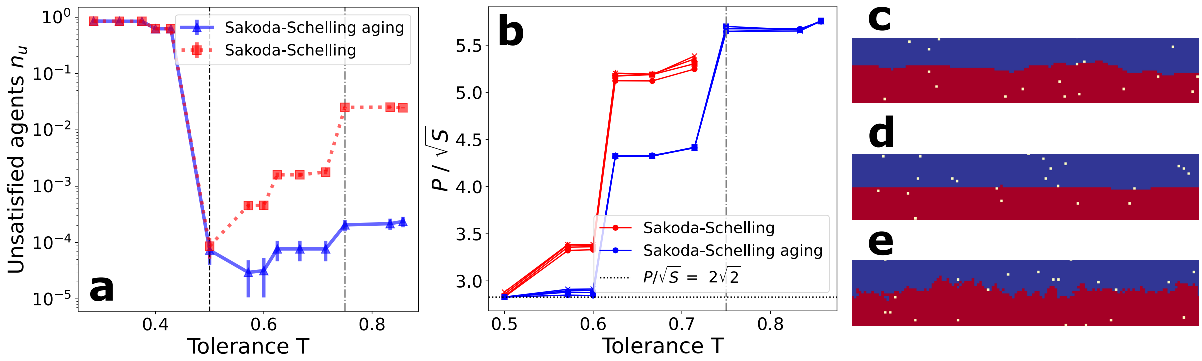

Figure 3.2: (**a**) Fraction of unsatisfied agents n_u at the stationary regime as a function of the tolerance parameter T. (**b**) Measure of the interface roughness between clusters of different kind of agents at the final stationary state $P/\sqrt{S}$ as a function of the tolerance parameter T. Different markers indicate different system sizes: $L = 40$ (circles), 60 (squares), 80 (triangles) and 100 (crosses). Results are shown for both the Sakoda-Schelling model with and without aging. Numerical simulations are performed for $n_0 = 0.5\%$ and averaged over $5 \cdot 10^4$ realizations. The frozen-segregated transition (dashed black line) and the segregated-mixed transition (gray dot-dashed line) are highlighted to differentiate the phases that the Sakoda-Schelling model exhibits. There are no values of $P/\sqrt{S}$ for the Sakoda-Schelling model above $T = 3/4$ because the segregated-mixed transition occurs. (**c**) Final state interface zoom snapshot for $T = 0.57$ using the original model. (**d**) Final state interface zoom snapshot for $T = 0.57$ using the model with aging. (**e**) Same as c for $T = 0.86$.

$n_u = 0$ and this causes the large error bars in Fig. **??**a after the transition. In a counterintuitive way, the introduction of aging causes a higher global satisfaction when compared with the original model in both the segregated and the mixed phases.

The creation of new unsatisfied agents at the final stationary state occurs at the interface, where different kind agents meet. This is why we study the interface roughness (perimeter) P as a function of the tolerance parameter. To compute this measure, we compute the number of agents of one kind in contact with different kind agents. To perform this calculation, we smooth the interface by considering vacancies surrounded by a majority of agents of a certain kind as members of that kind. In our system of $L \times L$ with periodic boundary, the minimum interface size (perimeter) P between clusters of agents of different kind is $P = 2L$. To avoid the L dependency, we calculate an adimensional magnitude $P/\sqrt{S}$, where S is the number of agents of each kind $S = N_{\text{agents}}/2 = L^2(1-n_0)/2$ (surface). This metric $P/\sqrt{S}$ is computed starting from a flat interface as an initial condition and evolving it for $t_{\max} = 10^4$ MC steps to reach well within the stationary state. With the metric $P/\sqrt{S}$, we are able to estimate how close is the final state interface of our system to the flat interface ($P/\sqrt{S} = 2\sqrt{2}$). The results show an increasing dependence of roughness with the tolerance parameter T (see Fig. **??**b). This growth can be explained as an increase in tolerance means that agents are satisfied with fewer "same-kind" neighbors. Therefore, the interface is able to be rougher, keeping the agents in a satisfied state. In addition, notice that all values with different L collapse, so the dependence on the system size has been eliminated.

Comparing both models, one observes a lower interface roughness for the Sakoda-Schelling model with aging, regardless of the value of T. The closest value to the flat interface occurs for the first values of T after the frozen-segregated phase transition (shown in Fig. **??**d). In the original model, we observe higher values of $P/\sqrt{S}$ due to the noise produced by the satisfied agents' behavior (see Fig. **??**c). Moreover, aging allows us to obtain a segregated phase with even larger interface roughness than the maximum observed in the original model for large values of T (see Fig. **??**e). We remark that, when aging is introduced, agents try to join those of their own kind but are less and less prone to change location as time passes. Thus, in the Sakoda-Schelling model with aging, agents in the bulk of the clusters mainly do not move and

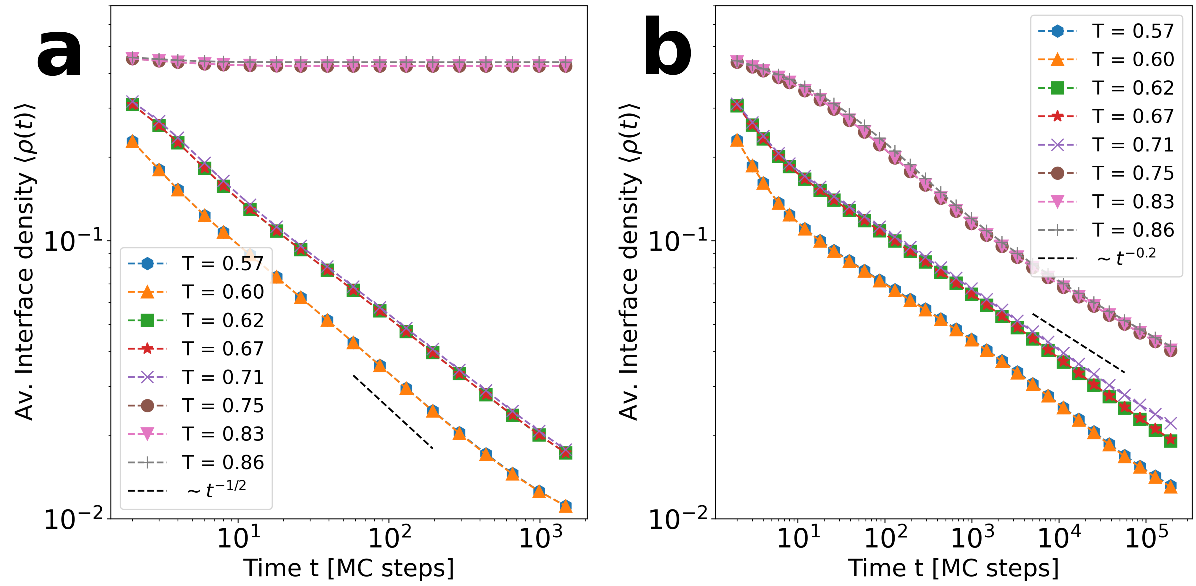

Figure 3.3: Average interface density $\langle\rho(t)\rangle$ as a function of time steps for different values of the tolerance parameter T using the Sakoda-Schelling model (**a**) and the version with aging (**b**). Average performed over $5 \cdot 10^3$ realizations. Fitted power-law in a black dashed line highlighting the estimated exponent value. We set system size $L = 200$ and $n_0 = 0.005$.

those moving more often are located at the interface between agent kinds. At medium and large scales, this phenomenon leads to ergodicity breaking in the final state dynamics.

3.4.3 Segregated phase: coarsening dynamics

Diverse versions of the original Schelling Model exhibit different behaviors in terms of coarsening dynamics. Recent publications report a power-law like domain growth (0, 0). We monitor here the evolution of the interface density $\langle\rho(t)\rangle$, which, in the segregated phase, decreases as $\langle\rho(t)\rangle \sim t^{-\alpha}$ so the domains should grow in our model following a power-law with time.

The coarsening process of the Sakoda-Schelling model at the segregated phase ($0.5 \leq T < 0.75$) is displayed in Fig. **??**a and Fig. **??**. We find that the average interface density follows a power-law decay with an exponent $\alpha \simeq 0.5$ for the limit of small vacancy density $n_0 \to 0$, in agreement with the value reported for close variants of the Sakoda-Schelling model (0). This exponent value is curious since the coarsening in the presence of a conserved quantity (but with local interactions) exhibits an exponent $\alpha = 1/3$ (0). Nevertheless, the interactions in this model are not local, and the coarsening exponent is more similar to the one in systems with a non-conserved order-parameter ($\alpha = 1/2$). Fig. **??**a shows as well how coarsening changes with the tolerance parameter. Even though the exponent α does not depend on T, we observe a certain delay when increasing T from 0.6 to 0.62. In the system evolution of Fig. **??**, one can see how the behavior of the satisfied agents for higher tolerance values is translated into rougher interfaces, causing such delay. For $T > 0.75$, the system exhibits a transition towards a mixed state where the interface density fluctuates around $\rho = 0.5$, indicating that the state is constantly disordered.

The Sakoda-Schelling model with aging shows very different behavior (Fig. **??**b). As expected, the average interface density exhibits a power-law decay with time for all values of the tolerance T after the frozen-segregated transition. Still, the decay is slower than for the Sakoda-Schelling model, with $\langle\rho(t)\rangle \sim t^{-0.2}$. A mechanism that could be behind this behavior is that the model with aging counts more satisfied agents than the original model, and their probability to move becomes lower as time goes by. Moreover, satisfied agents inside a cluster will not move and the

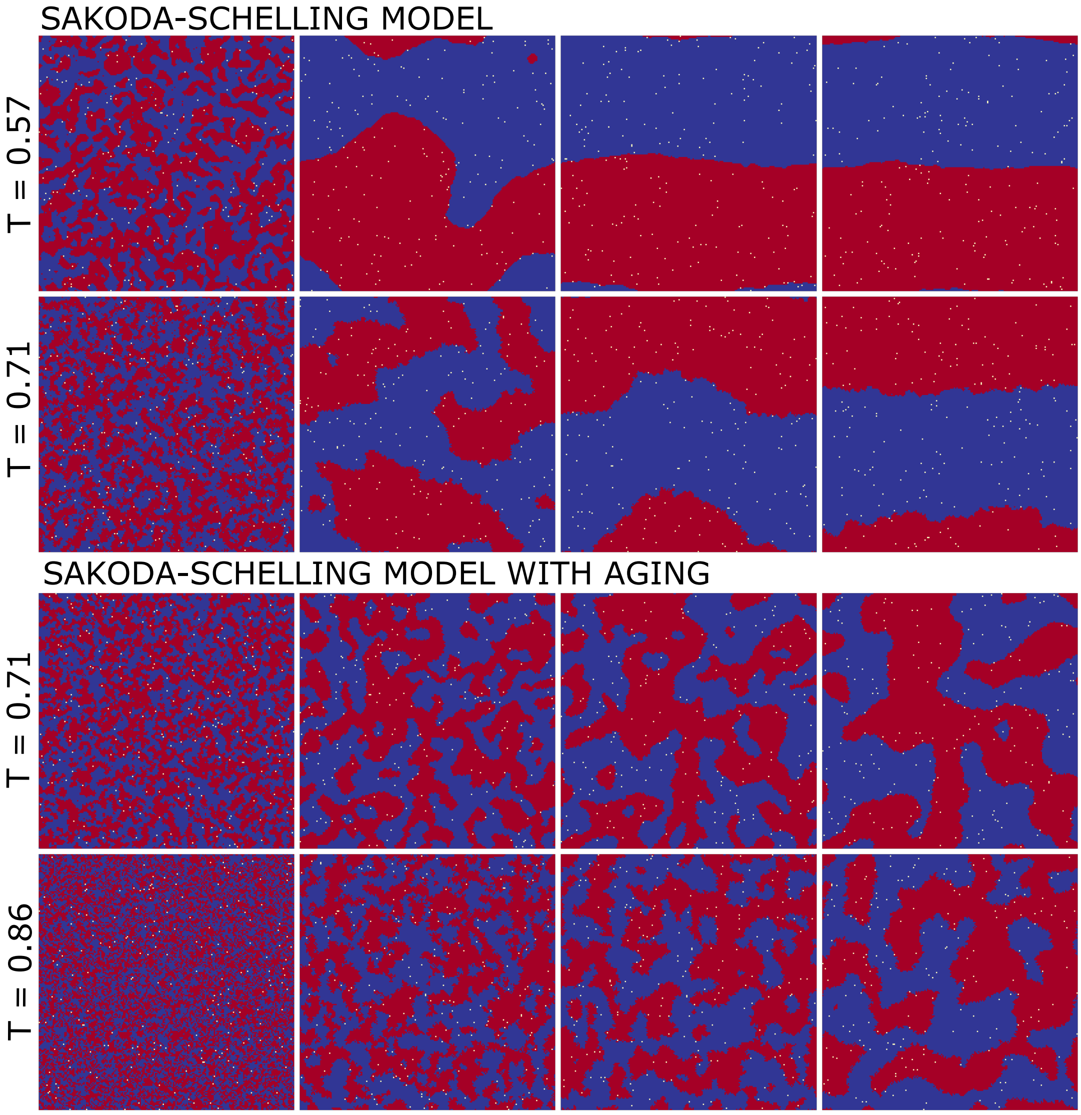

Figure 3.4: Coarsening towards the segregated state at two different values of T for both models. Snapshots are taken for 5, 500, 5000 and 50000 time steps ordered from left to right. We set system size $L = 200$ and $n_0 = 0.005$.

dynamics of the model take place at the interface. It is, therefore, more difficult for separated clusters to collide and merge, an effect that slows down the decay of the interface density. The persistence of small clusters becomes clear when the snapshots' evolution is compared for both models at the same tolerance value $T = 0.71$ (see Fig. **??**). Moreover, while for the original model the initial clustering for $t = 500$ steps does not determine the final state, in the case of aging the bigger clusters present at the beginning of the evolution are the ones that keep growing, determining the shape of the system configuration after 50000 time steps. This is a dynamical effect, because the system in both cases tends to a final configuration with 2-clusters.

In the case of the Sakoda-Schelling model with aging, we observe an early cross-over in the dynamics (Fig. **??**b). For $T < 0.75$, the coarsening starts with an initial decay of $\langle \rho(t) \rangle$ faster than $t^{-0.2}$. This occurs because in this regime it is necessary sometimes for the aging effects to become relevant, and initially the system behaves as in the original model. Similarly, for $T \geq 0.75$, $\langle \rho(t) \rangle$ decays slowly for a moment before reaching the power-law behavior for large t values. Confirming this scenario, Fig. **??** shows that for $T = 0.86$, the system starts evolving similarly to a mixed state until some clusters are created. At this moment, aging prevents the clusters'

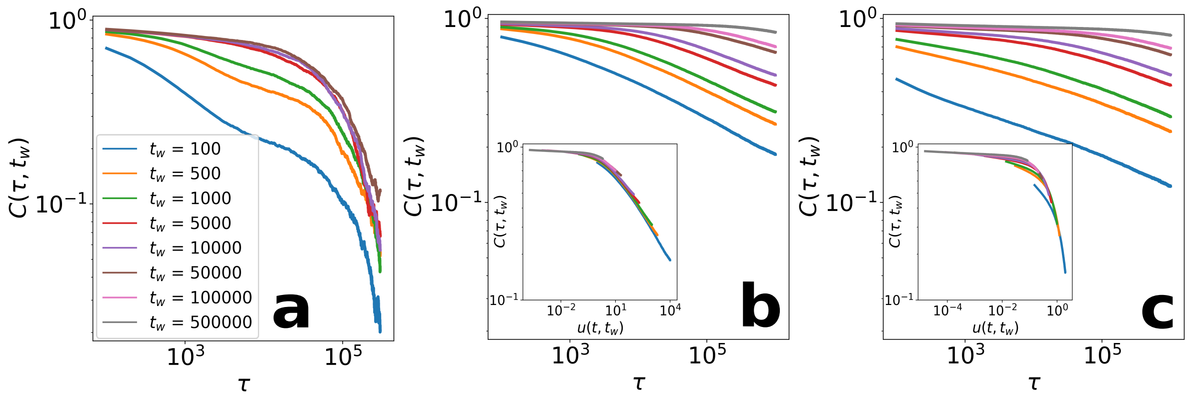

Figure 3.5: Two-times autocorrelation $C(\tau,t_\mathrm{w})$ as a function of the time period passed since the waiting time t_w. First, the autocorrelation is shown for the Sakoda-Schelling model at $T = 0.71$ in **a**, and for the version with aging at $T = 0.71$ in **b** and $T = 0.86$ in **c**. The insets are the result of the collapse using $u(\tau,t_w) = \tau/t_w$ (**b**) and $u(\tau,t_w) = \log(\tau+t_w)/\log(t_w) - 1$ (**c**). The curves correspond to different values of the waiting time t_w. Calculations performed on a 100×100 lattice averaged over $5\cdot10^4$ realizations.

desegregation, leading the system very slowly to coarsening dynamics and, eventually, to a fully segregated state.

Regarding the relaxation time to the final state, we see in Fig. **??** how for $T = 0.71$, the stationary state of the Sakoda-Schelling model is reached after approximately $t = 5000$ time steps. In contrast, the version with aging needs much more than 50000 steps to attain it. This highlights the important temporal difference between both models in terms of domain growth dynamics, which strongly increases the computational cost of the study of the stationary state of the model with aging. We have been thus able to study only medium and small system sizes in this final regime (see videos in Ref. (0)).

The dynamics studied thus far are performed considering the limit $n_0 \to 0$, but the analysis can be extended to higher vacancy densities. For the particular case of high n_0 and low T, aging leads to the formation of a vacancy cluster at the interface between domains (see details in Appendix **??**).

3.4.4 Aging breaks the asymptotic time-translational invariance

Here, we explore further time translational invariance (TTI) in the model dynamics. For this, we start by defining the two-time autocorrelation function $C(\tau,t_\mathrm{w})$ (0) as

$$C(\tau,t_\mathrm{w}) = \left\langle \frac{1}{M}\sum_{i=1}^{N}\sigma_i(t_\mathrm{w}+\tau)\,\sigma_i(t_\mathrm{w}) \right\rangle, \tag{3.3}$$

where N is the system size, $\langle\cdot\rangle$ refers to averages over realizations, t_w is the waiting time to start the autocorrelation measurements, τ a time interval after t_w and M is a normalization factor defined as

$$M = \sum_{i=1}^{N}(\sigma_i(t_\mathrm{w}+\tau)\,\sigma_i(t_\mathrm{w}))^2. \tag{3.4}$$

which is computed at each realization.

The autocorrelation function is displayed for the Sakoda-Schelling model with $T = 0.75$ in Fig. **??**a. We observe the curves decreasing with τ as expected, and that after a characteristic time period ($t^*_\mathrm{w} \approx 5000$ for a system size of 80×80) they collapse into a single curve. This is the regime in which the dynamics becomes TTI, implying that the autocorrelation function does not depend any more on the waiting time, $C(\tau,t_\mathrm{w}) = C(\tau)$ for $t_\mathrm{w} > t^*_\mathrm{w}$.

For the Sakoda-Schelling model with aging, the dynamics show some different features (Figs. **??**b and **??**c). First, the autocorrelation functions decay slower with τ in all the cases, which is connected to the long-lived small clusters mentioned previously. We do not find in the simulations any value of t_{w}^{*} for the systems to fall into a TTI regime. Not only that, but a scaling relation including both τ and t_{w} can be applied to collapse the autocorrelation curves (see insets Figs. **??**b and **??**c). This behavior is similar to glassy systems (0), therefore it is useful to use the mathematical description for those systems in our case. In this type of dynamics, a final stationary state is not attainable in the thermodynamic limit, and it is possible to decompose the autocorrelation function into an equilibrium part and an "aging" part (aging in the sense of non-equilibrium dynamics in glassy systems) (0, 0):

$$C(\tau, t_{\mathrm{w}}) \simeq C_{\mathrm{eq}}(\tau)\, C_{\mathrm{aging}} u(\tau, t_w) = C_{\mathrm{eq}}(\tau)\, C_{\mathrm{aging}} \left(\frac{h(\tau)}{h(t_{\mathrm{w}})} \right), \tag{3.5}$$

where C_{eq} describes the fast relaxation of the system components within each domain (TTI term), C_{aging} is a scaling function and $u(\tau, t_w)$ is a normalization factor which, in some cases, can be written as the quotient of an unknown function $h(t)$ at the two times τ and t_{w}. This function $h(t)$ is known to be related to the dynamical correlation length (0, 0). In our case, we use $h(t) = t$ to scale the results in Fig. **??**b (see inset). This scaling is valid for values of $T \in [0.5, 0.75)$. Nevertheless, higher values of T do not hold a linear scaling, and we need to turn to other functional forms as the normalization factor $u(\tau, t_w) = \log(\tau + t_w)/\log(t_w) - 1$ used in Fig. **??**c. This indicates that for $T > 0.75$, the dynamical correlation length evolves in a different and slower way.

3.5 Summary and discussion

We have studied the effect of aging on a 3-state threshold model (with two symmetrical states $\sigma_i = \pm 1$), which combines long-range mobility with local short-range interactions. Specifically, taking as basis the noisy constrained Sakoda-Schelling model, we assign to the agents an internal clock counting the time spent in the same satisfying location. The probability of changing state decreases then inversely proportional to this time. Therefore, older satisfied agents are less prone to update resident locations. The original model displays a transition between a segregated phase and a mixed one as the tolerance control parameter T increases. This transition disappears when aging is introduced into the system, the mixed phase is replaced by a segregated phase even for high values of the tolerance parameter T. As a result, the model with aging presents a higher global satisfaction than without this effect for all values of the tolerance.

On the dynamical perspective, the relaxation towards the segregated phase features a coarsening phenomena characterized by a power-law decay of the average interface density with time $\langle \rho \rangle \sim t^{-\alpha}$. For the original model in the limit of low vacancy density, the exponent is around $\alpha = 1/2$. This exponent is also reported in other variants of the Sakoda-Schelling model (0, 0). Aging gives rise to long-lived small clusters and a slower coarsening, reducing the exponent to $\alpha \simeq 0.2$. We investigated the autocorrelation functions in the segregated phase and found that aging breaks the asymptotic time-translational invariance of the dynamics. This result, along with a nontrivial scaling of the autocorrelation functions, establish close similarities with low-coarsening systems, such as glassy systems, and our Sakoda-Schelling model with aging for high values of the tolerance parameter. Moreover, this work studies the case for equal size populations ignores effects arising from the competition between different population sizes. Further work would be to study a joint effect of minority population and aging.

As for the implications of our results from a social perspective, we must note that the fact that aging favors segregation, inhibiting the segregation-mixed phase transition, is rather counter-intuitive, but gives support to the argument that segregation is a stochastically stable state and may prevail in an all-integrationist world (0). Our model predicts the appearance of segregation even for tolerance values close to one. Additionally, the model relaxation time multiplies manifold, which implies that if aging is present the natural state of this system seems to be generically out of equilibrium.

The results in this chapter are published as:

David Abella et al. "Ordering dynamics and aging in the symmetrical threshold model". In: *New Journal of Physics* 26.1 (Jan. 2024), page 013033. DOI: 10.1088/1367-2630/ad1ad4. URL: https://dx.doi.org/10.1088/1367-2630/ad1ad4

7.1 Theorems

7.1.1 Several equations

This is a theorem consisting of several equations.

Update rules — Name of the theorem. In $E = \mathbb{R}^n$ all norms are equivalent. It has the properties:

$$\big| ||\mathbf{x}|| - ||\mathbf{y}|| \big| \leq ||\mathbf{x} - \mathbf{y}|| \tag{7.1}$$

$$|| \sum_{i=1}^{n} \mathbf{x}_i || \leq \sum_{i=1}^{n} ||\mathbf{x}_i|| \quad \text{where } n \text{ is a finite integer} \tag{7.2}$$

7.1.2 Single Line

This is a theorem consisting of just one line.

Update rules A set $\mathscr{D}(G)$ in dense in $L^2(G)$, $|\cdot|_0$.

7.2 Definitions

A definition can be mathematical or it could define a concept.

Definition 7.1 — Definition name. Given a vector space E, a norm on E is an application, denoted $||\cdot||$, E in $\mathbb{R}^+ = [0, +\infty[$ such that:

$$||\mathbf{x}|| = 0 \;\Rightarrow\; \mathbf{x} = \mathbf{0} \tag{7.3}$$

$$||\lambda \mathbf{x}|| = |\lambda| \cdot ||\mathbf{x}|| \tag{7.4}$$

$$||\mathbf{x} + \mathbf{y}|| \leq ||\mathbf{x}|| + ||\mathbf{y}|| \tag{7.5}$$

7.3 Notations

▪ **Notation 7.1** Given an open subset G of $\mathbb{R}^n$, the set of functions φ are:

1. Bounded support G;

2. Infinitely differentiable;

a vector space is denoted by $\mathscr{D}(G)$.

7.4 Remarks

This is an example of a remark.

R The concepts presented here are now in conventional employment in mathematics. Vector spaces are taken over the field $\mathbb{K}=\mathbb{R}$, however, established properties are easily extended to $\mathbb{K}=\mathbb{C}$.

7.5 Corollaries

Corollary 7.1 — Corollary name. The concepts presented here are now in conventional employment in mathematics. Vector spaces are taken over the field $\mathbb{K}=\mathbb{R}$, however, established properties are easily extended to $\mathbb{K}=\mathbb{C}$.

7.6 Propositions

7.6.1 Several equations

Proposition 7.1 — Proposition name. It has the properties:

$$\big| ||\mathbf{x}|| - ||\mathbf{y}|| \big| \leq ||\mathbf{x}-\mathbf{y}|| \tag{7.6}$$

$$||\sum_{i=1}^{n} \mathbf{x}_i|| \leq \sum_{i=1}^{n} ||\mathbf{x}_i|| \quad \text{where } n \text{ is a finite integer} \tag{7.7}$$

7.6.2 Single Line

Proposition 7.2 Let $f,g \in L^2(G)$; if $\forall \varphi \in \mathscr{D}(G)$, $(f,\varphi)_0 = (g,\varphi)_0$ then $f = g$.

7.7 Examples

7.7.1 Equation Example

■ **Example 7.1** Let $G = \{x \in \mathbb{R}^2 : |x| < 3\}$ and denoted by: $x^0 = (1,1)$; consider the function:

$$f(x) = \begin{cases} e^{|x|} & \text{si } |x-x^0| \leq 1/2 \\ 0 & \text{si } |x-x^0| > 1/2 \end{cases} \tag{7.8}$$

The function f has bounded support, we can take $A = \{x \in \mathbb{R}^2 : |x-x^0| \leq 1/2+\varepsilon\}$ for all $\varepsilon \in]0;5/2-\sqrt{2}[$. ■

7.7.2 Text Example

■ **Example 7.2 — Example name.** Aliquam arcu turpis, ultrices sed luctus ac, vehicula id metus. Morbi eu feugiat velit, et tempus augue. Proin ac mattis tortor. Donec tincidunt, ante rhoncus luctus semper, arcu lorem lobortis justo, nec convallis ante quam quis lectus. Aenean tincidunt sodales massa, et hendrerit tellus mattis ac. Sed non pretium nibh. Donec cursus maximus luctus. Vivamus lobortis eros et massa porta porttitor. ■

7.8 Exercises

Exercise 7.1 This is a good place to ask a question to test learning progress or further cement ideas into students' minds. ■

7.9 Problems

Problem 7.1 What is the average airspeed velocity of an unladen swallow?

7.10 Vocabulary

Define a word to improve a students' vocabulary.

- **Vocabulary 7.1 — Word.** Definition of word.

The results in this chapter are published as:

David Abella et al. "Ordering dynamics and aging in the symmetrical threshold model". In: *New Journal of Physics* 26.1 (Jan. 2024), page 013033. DOI: 10.1088/1367-2630/ad1ad4. URL: https://dx.doi.org/10.1088/1367-2630/ad1ad4

8.1 Table

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8.2 Figure

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Figure 8.1: Figure caption.

Referencing **??** in-text using its label.

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Figure 8.2: Floating figure.

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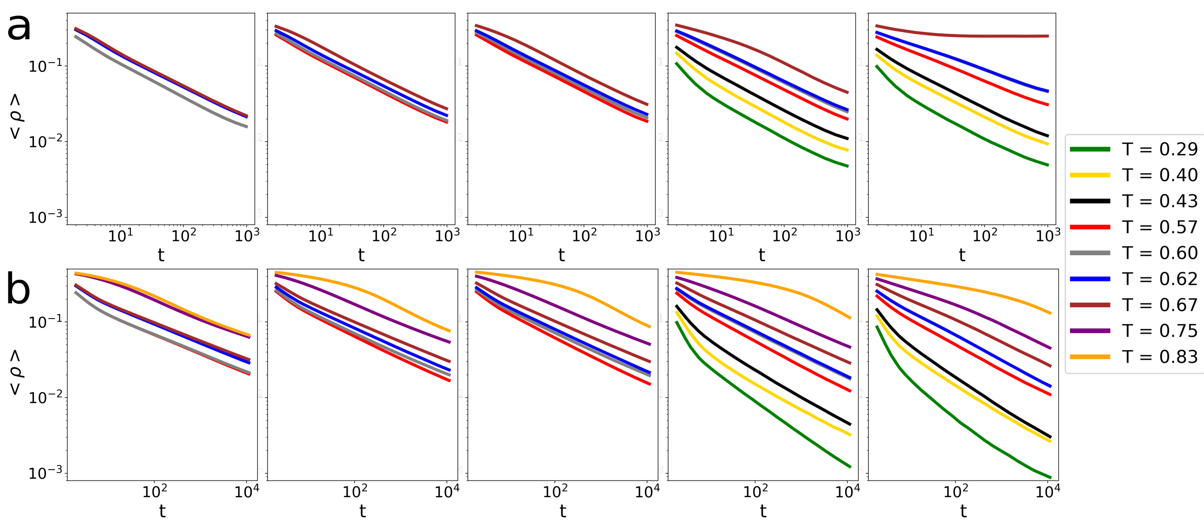

Figure A.1: Average interface density $\langle\rho(t)\rangle$ as a function of time steps for different values of the tolerance parameter T fr the Schelling model (**a**) and the version with aging (**b**). The different plots show a the evolution at a different value of the vacancy density, increasing from left to right $\rho_v = 0.005$, 0.15, 0.2, 0.3 and 0.45. Average performed over 10^3 realisations with system size $100\ x\ 100$.

Since we restrain ourselves to the region $\rho_v < 0.5$, the increase/decrease of the number of vacancies does not change dramatically the behaviour. Above this value, we approach the segregated-dilute transition ($\rho_v \sim 0.62$). Nevertheless, it is worth to mention a few features we observe on the coarsening dynamics. Essentially, when we set a higher vacancy density, the number of agents which see vacancies at their surroundings increases. This results in a family of similar power-law decays towards the segregated state for every meaningful value of T (see Fig. **??**).

Moreover, a higher ρ_v allows us to study the coarsening phenomena for lower values of T according to the phase diagram for the original Schelling model. For those particular cases, when the aging is introduced, we observe a power law decay faster than without aging (Fig. **??**b). Therefore, the aging effect accelerates segregation in this region of the phase diagram, contrary as for lower values of ρ_v. This acceleration is not caused by reaching the 2-clusters state in less time. Since there is a large presence of vacancies, aging causes a formation of vacancy

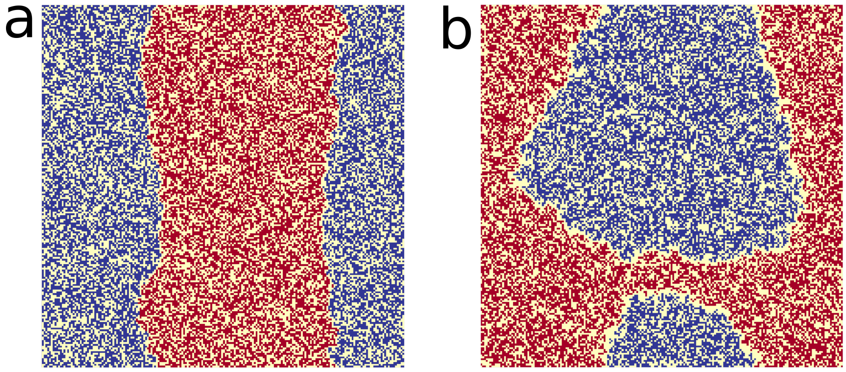

Figure A.2: Snapshots of the system at the final segregated state (after 10^6 MC steps) for the Schelling model (**a**) and the version with (**b**). System size $200\ x\ 200$ with $\rho_v = 0.45$ and $T = 0.29$.

clusters at the interface. Fig. **??** shows the final segregated state with and without aging. This spontaneous behaviour is result of the low tolerance combined with the persistence of clusters (once formed) due to aging effect and the large number of vacancies that allows the possibility of the formation of clusters at the interface.

In order to quantify this vacancy cluster formation, we define a measure inspired in the segregation coefficient:

$$s_v = \frac{1}{(L^2\rho_v)^2}\sum_{\{c\}} n_c^2 \tag{A.1}$$

where c is the size of a vacancy cluster and n_c is the number of clusters with size c. The sample average of s_v after reaching equilibrium is called the cluster coefficient of vacancies $\langle s_v \rangle$.

The results of this measure as a function of ρ_v for a few values of T are represented in Fig.**??** for the Schelling model with and without aging. We observe an increasing dependence of $\langle s_v \rangle$ with ρ_v for both models, but the effect reducing tolerance changes dramatically the behaviour for the case with aging, highlighting the vacancy cluster formation.

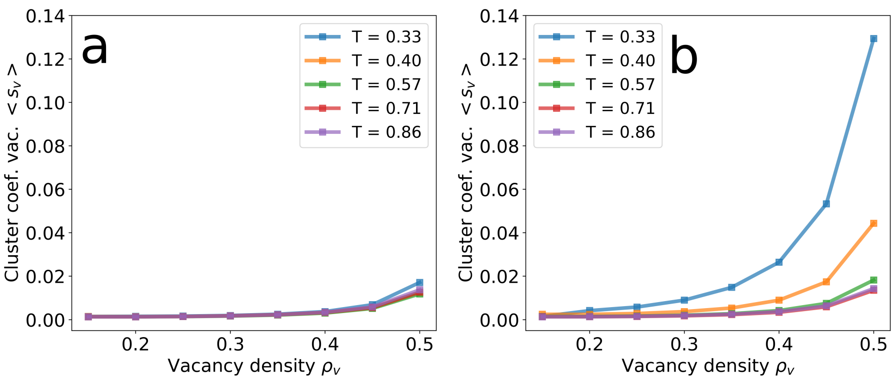

Figure A.3: Cluster coefficient of vacancies as a function of the vacancy density ρ_v for the Schelling model (**a**) and the version with (**b**) for different values of the tolerance T.

Setting the time derivatives to 0 in Eqs. (**??**), we obtain the relations for the stationary state:

$$x_{k,0}^{\pm} = \sum_{j=0}^{\infty} x_{k,j}^{\mp}\,\omega_{k,j}^{\mp}, \qquad x_{k,j}^{\pm} = x_{k,j-1}^{\pm}\,(1-\omega_{k,j-1}^{\pm}) \qquad j>0, \tag{B.1}$$

from where we extract the stationary condition $x_{k,0}^{-} = x_{k,0}^{+}$, as in Ref. (0). Notice that by setting $p_A(j) = 1$ and summing over all ages j, we recover the HMF approximation (Eq. **??**) for the model without aging. Defining $x_j^{\pm}(t)$ as the fraction of agents in state ± 1 with age j:

$$x_j^{\pm} = \sum_k p_k\, x_{k,j}^{\pm}, \tag{B.2}$$

and using the degree distribution of a complete graph $p_k = \delta(k-N+1)$ (where $\delta(\cdot)$ is the Dirac delta), we sum over the variable k and rewrite Eq. (**??**) in terms of $x_j^{\pm}$:

$$x_0^{\pm} = \sum_{j=0}^{\infty} x_j^{\mp}\,\omega_j^{\mp}, \qquad x_j^{\pm} = x_{j-1}^{\pm}\,(1-\omega_{j-1}^{\pm}) \qquad j>0, \tag{B.3}$$

where $\omega_j^{\pm} \equiv \omega_{N-1,j}^{\pm}$. Note that the stationary condition $x_0^{-} = x_0^{+}$ remains valid after summing over the degree variable. We compute the solution $x_j^{\pm}$ recursively as a function of $x_0^{\pm}$:

$$x_j^{\pm} = x_0^{\pm}\,F_j^{\pm} \qquad \text{where} \qquad F_j^{\pm} = \prod_{a=0}^{j-1}(1-\omega_a^{\pm}), \tag{B.4}$$

and summing all j,

$$x^{\pm} = x_0^{\pm}\,F^{\pm} \qquad \text{where} \qquad F^{\pm} = 1 + \sum_{j=1}^{\infty} F_j^{\pm}. \tag{B.5}$$

Using the stationary condition $x_0^{-} = x_0^{+}$, we reach:

$$\frac{x^{+}}{x^{-}} = \frac{F^{+}}{F^{-}}. \tag{B.6}$$

Notice that, for the complete graph, $\tilde{x}^{+} = x$, $\tilde{x}^{-} = 1-x$. Therefore, $F^{\pm}$ is a function of the variable $x^{\mp}$ ($F^{+} = F(1-x)$). Thus, we rewrite the previous expression just in terms of the variable x:

$$\frac{x}{1-x} = \frac{F(1-x)}{F(x)}. \tag{B.7}$$

In Phase I and I^*, the exceeding threshold condition ($m/k > T$) is full-filled for almost all agents in the system. Thus, agents will change their state and reset the internal time once activated. For the original model, all agents are activated once in a time step on average, but for the model with aging, the activation probability plays an important role. We consider here a set of N agents that are activated randomly with an activation probability $p_A(j)$ and, once activated, they reset their internal time. Being $n_i(t)$ the fraction of agents with internal time i at the time step t, we build a recursive relation for the previously described dynamics in terms of variables i and t:

$$n_1(t) = \sum_{i=1}^{t-1} p_A(i)\, n_i(t-1) \qquad n_i(t) = (1 - p_A(i-1))\, n_{i-1}(t-1) \qquad i > 1. \tag{C.1}$$

This recursion relation can be solved numerically from the initial condition ($n_1(0) = 1$, $n_i(0) = 0$ for $i > 1$). To obtain the mean internal time at time t, we just need to compute the following:

$$\bar{\tau}(t) = \sum_{i=1}^{t} i\, n_i(t). \tag{C.2}$$

The solution from this recursive relation describes the mean internal time dynamics with great agreement with the numerical simulations performed at Phase I (for the complete graph) and Phase I^* (for the Erdős-Rényi and Moore lattice).