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AGING AND MEMORY EFFECTS IN SOCIAL AND ECONOMIC DYNAMICS

David Abella Bujalance

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A en Manuel Miranda
pel seu suport i ajuda
durant tots aquests anys.
Sempre estaràs amb mi.
i recordare sempre
el que em vas ensenyar.

Dr José Javier Ramasco of the Consejo Superior de Investigaciones Científicas (CSIC) and Dr Maxi San Miguel of the Universitat de les Illes Balears (UIB)

WE DECLARE:

That the thesis titles *Dynamics of social interactions*, presented by David Abella Bujalance to obtain a doctoral degree, has been completed under my supervision and meets the requirements to opt for an International Doctorate.

For all intents and purposes, I hereby sign this document.

Signature

Dr. José Javier Ramasco Sukia	Dr. Maxi San Miguel
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Tambe afegir que aquest projecte no hagués estat possible sense l'ajuda de tots els companys que han fet possible que aquest projecte sigui una realitat. Jo que soc un dels que ha fet possible que aquest projecte sigui una realitat, vull agrair a tots els companys que han fet possible que aquest projecte sigui una realitat, per tot el suport que m'han donat durant tot el projecte.

Resum

En els sistemes complexos distribuïts, els sistemes de memòria transaccional distribuïda (DTM) són una eina molt útil per a la programació concurrent. Aquests sistemes permeten als desenvolupadors de software escriure codi concurrent sense haver de preocupar-se per la gestió de la memòria compartida. A més, els DTM ofereixen una interfície molt senzilla per a la programació concurrent, ja que permeten als desenvolupadors de software escriure codi concurrent de forma semblant a com ho farien si el codi fos seqüencial. Tot i això, els DTM no són una eina perfecta, ja que tenen un rendiment molt inferior al de les estructures de dades distribuïdes. A més, els DTM no són capaços de gestionar estructures de dades distribuïdes de forma eficient. Per aquest motiu, els DTM no són una eina adequada per a la programació de sistemes distribuïts.

Resumen

En los sistemas complejos distribuidos, los sistemas de memoria transaccional distribuida (DTM) son una herramienta muy útil para la programación concurrente. Estos sistemas permiten a los desarrolladores de software escribir código concurrente sin tener que preocuparse por la gestión de la memoria compartida. Además, los DTM ofrecen una interfaz muy sencilla para la programación concurrente, ya que permiten a los desarrolladores de software escribir código concurrente de forma similar a como lo harían si el código fuera secuencial. Sin embargo, los DTM no son una herramienta perfecta, ya que tienen un rendimiento muy inferior al de las estructuras de datos distribuidas. Además, los DTM no son capaces de gestionar estructuras de datos distribuidas de forma eficiente. Por este motivo, los DTM no son una herramienta adecuada para la programación de sistemas distribuidos.

Abstract

In complex systems distributed transactional memory (DTM) systems are a very useful tool for concurrent programming. These systems allow software developers to write concurrent code without having to worry about managing shared memory. In addition, DTM systems offer a very simple interface for concurrent programming, as they allow software developers to write concurrent code in a similar way to how they would if the code were sequential. However, DTM systems are not a perfect tool, as they have a much lower performance than distributed data structures. In addition, DTM systems are not able to manage distributed data structures efficiently. For this reason, DTM systems are not a suitable tool for programming distributed systems.

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List of publications

The list of articles detailed below, in chronological order by date of publication, form the basis of the present thesis.

1. David Abella, Maxi San Miguel, and José J. Ramasco. "Aging effects in Schelling segregation model". In: *Scientific Reports* 12.1 (Nov. 2022). ISSN: 2045-2322. DOI: 10.1038/s41598-022-23224-7. URL: http://dx.doi.org/10.1038/s41598-022-23224-7

2. David Abella, Maxi San Miguel, and José J. Ramasco. "Aging in binary-state models: The Threshold model for complex contagion". In: *Phys. Rev. E* 107 (2 Feb. 2023), page 024101. DOI: 10.1103/PhysRevE.107.024101. URL: https://link.aps.org/doi/10.1103/PhysRevE.107.024101

3. David Abella et al. "Ordering dynamics and aging in the symmetrical threshold model". In: *New Journal of Physics* 26.1 (Jan. 2024), page 013033. DOI: 10.1088/1367-2630/ad1ad4. URL: https://dx.doi.org/10.1088/1367-2630/ad1ad4

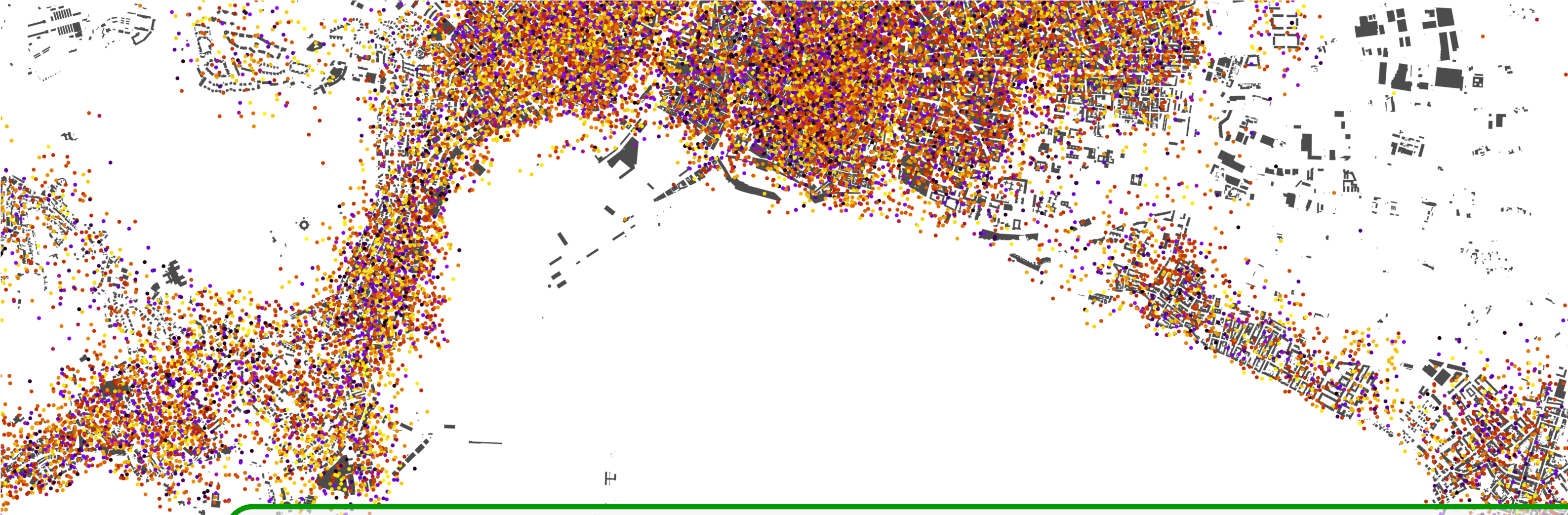
4. Idealista model for complex systems housing

5. Idealista spatial segmentation of the real state market

Other publications published during the PhD period are also included in the following list.

- David Abella, Giancarlo Franzese, and Javier Hernández-Rojas. "Many-Body Contributions in Water Nanoclusters". In: *ACS Nano* 17.3 (Jan. 2023), pages 1959–1964. ISSN: 1936-086X. DOI: 10.1021/acsnano.2c06077. URL: http://dx.doi.org/10.1021/acsnano.2c06077

- David Abella et al. "Unraveling higher-order dynamics in collaboration networks". In: *arXiv preprint arXiv:2306.17521* (2023)



1. Introduction and positioning

This thesis provides a general overview of the research that I have been developing since the beginning of my PhD studies in September, 2021. I could define myself as a curious, creative and open-minded person, following the so called *IFISC attitude*, which means that I am always willing to learn new methods and address new problems, even though they are not directly related to my field of expertise. That is why, through this thesis many topics will be covered, from the study of human behavior and social systems, to the study of complex systems and network theory........

1.1 Scientific Landscape

This thesis adress the study of human behavior and social systems from a *complex systems* perspective, which studies the emergence of collective phenomena that arise from the interactions of many individuals, and that cannot be understood by studying the behavior of individual agents in isolation (the so-called *reductionist* approach) (7). The study of collective phenomena has a long history in the natural sciences, specially in the branch of statistical physics (118). This branch traditionally studies the emergence of collective phenomena in physical systems, such as the phase transitions in magnetic materials via spin models (94), the turbulence in fluids (50), the synchronization in oscillatory systems (100), or percolation (123). However, in recent years, the study of complex systems has evolved into the study of emergent phenomena beyond physical systems, such as biological (101), ecological (85), economic (10), and social systems (27). From the migration of birds (106) to the spreading of a fake news through social media (130), there are many examples of collective phenomena at which the study of complex systems can be applied.

The cascade of failures in power grids (41), the spread of a disease in a population (8), the consensus in political elections (6), the emergence of social norms (45) are some examples of social collective behavior in which the global phenomena cannot be understood by looking at a single individual. Social and economic collective phenomena has been studied from a variety of perspectives (sociology, psychology, economics, political sciences...), which often relies on qualitative methods, such as interviews, surveys, or ethnographic studies (26). However, the study of social systems from a complex systems perspective aims to provide a quantitative framework to understand the collective behavior, based on methodologies from statistical mechanics and network theory (17, 92). Nevertheless, this approach needs for a large amount of detailed data to validate theories and develop models, which historically has been a limitation for the study of social systems. It is in fact surprising how other branches of physics, where the typical scale of the phenomena is very large, as astrophysics, or very small, as particle physics, do not suffer from a lack of data, while the study of social systems, where the typical scale is human, has been historically limited by the lack of data.

Thankfully, the digital revolution has changed this picture, allowing the storage of large amounts of data generated by human activities, such as social media, mobile phones, or online platforms. Nowadays, every two year, more human socio-economic data is produced than during all the preceding years of human history together (74). This data, often referred to as *Big data*, has opened a new era for the study of social systems at a large scale, together with a paradigm shift in the way we understand human behavior (84). Nevertheless, this new era comes with an awareness, as the use of big data for the study of human behavior raises important ethical and privacy concerns, which need to be addressed in order to ensure the responsible use of data for the study of social systems (25). Moreover, from the technical point of view, this huge amount of data needs for a set of computational and mathematical resources to be analyzed and modeled.From this demand, the field of *Computational Social Science* has emerged, with the aim to develop new methods to study human behavior (79). This branch of the complex systems science was born as a combination of methodologies borrowed from social sciences, such as sociology, psychology, or economics, with computational methods from computer science, such as machine learning, data mining, or network theory (134). This interdisciplinary approach has allowed to develop new methods for forecasting social phenomena and understanding the basic mechanism behind human interactions.

One can differentiate two main approaches to build a representation of the reality from the data source. The first one is to focus on the prediction and forecasting of a certain social phenomena, such as the spread of a disease or the price of a stock. In this approach, the data is seen as a necessary input to our methodology to make quantitative predictions (79). However, in this approach, the mechanisms behind the phenomena are often hidden in the data, and the model is seen as a black box that provides accurate predictions (109). In this context, the use of machine learning (90) and deep learning (58) models are very popular, as they are able to capture complex patterns in the data. The second approach is to focus on the understanding of the mechanisms behind the phenomena. In this approach, the data is seen as a problem to be understood, an observation from which we can extract qualitative behaviors and patterns (14). In this context, the aim is to develop very simple models that are able to reproduce the main features of the data, and to extract the basic mechanisms behind the phenomena.

Following the later approach, network science has a critical role in the study of socio-economic systems, as it provides a natural framework to study the interactions between individuals. A network, or graph, is a mathematical representation of a set of nodes (individuals) connected by links (interactions), which allows to study the structure of the interactions and the dynamics of the system. The study of networks has a long history in the natural sciences, from the neurons network in the brain () to food webs in an ecosystem (). However, in recent years, new data sources lead to the discovery that complex properties and heterogeneities, present in most social systems, need for a topological descrition in terms of a complex network (). A complex network can be defined is a network that exhibits non-trivial topological properties, which we will explain later on this thesis. These properties are often found in social networks, such as a social media (), the collaboration network of scientists (), or the trade network of countries (). In particular, the study of information spreading as a dynamical system on networks has allowed to understand how information spreads through a social system and how consensus emerges ().

Contagion of information has been a topic of interest for social scientist. Early theoretical frameworks,influenced by psychological and sociological theories, show how individuals in a crowd lose their sense of self and are more susceptible to the ideas and emotions of the crowd (). Social imitation of behaviors and ideas was proposed as a mechanism for social change, facilitated by close contact and communication among individuals ()......................

- In particular, human interactions exhibit complex activity patterns that are difficult to understand and to model, and that are not present in the study of physical systems.

1.2 Challenges of Computational Social Science

- The study of human behavior and social systems is a complex problem that requires the use of computational methods to study human behavior and social systems.

- There are some challenges that are unique to the study of human behavior and social systems, and that are not present in the study of physical systems.

1.2.1 Data availability

- The main problem is the data availability, and the fact that the data that is generated by human activities is not always available for study.

- Notice that the data sources typically used for the study of human behavior does not come from controlled experiments, but from the digital traces that are generated by human activities.

1.2.2 Data analysis

- The second problem is the data analysis, and the fact that the data that is generated by human activities is not always easy to analyze.

- The data source to analyze usually is a piece of a larger dataset, so we need to be careful to avoid biases in the analysis driven by the data size.

- Temporal windows are also a problem, because when we analyze the dynamics of a system, we need to be careful to avoid biases in the analysis driven by the temporal window.

1.2.3 Modeling

- The third problem is the modeling, and the fact that the data that is generated by human activities is not always easy to model.

- Deterministic models are not always useful to model human behavior, and we need to use stochastic models to model human behavior.

- Also, mechanistic models and data driven models is something that we need to consider when we model human behavior.

- Another possibility is to use agent-based models to model human behavior. With the advent of computational methods in the latter half of the 20th century, researchers gained powerful tools to simulate and analyze complex social systems. Agent-based modeling (ABM) emerged as a particularly influential approach, enabling scientists to create and study systems of interacting agents (individuals or collective entities) and observe emergent behaviors from simple rules of interaction.

1.2.4 Applications

- Computational social science has many applications, and it is being used to study human behavior and social systems.

- Sociotechnical systems, social networks, and human dynamics are some of the applications of computational social science.

- fake news detection, information spreading, and social influence are some of the applications of computational social science.

1.3 Terminology and general concepts

- In this section, we introduce some terminology and general concepts that are used in the study of human behavior and social systems.

Complex networks, interface density, and community structure are some of the concepts that are used in the study of human behavior and social systems.

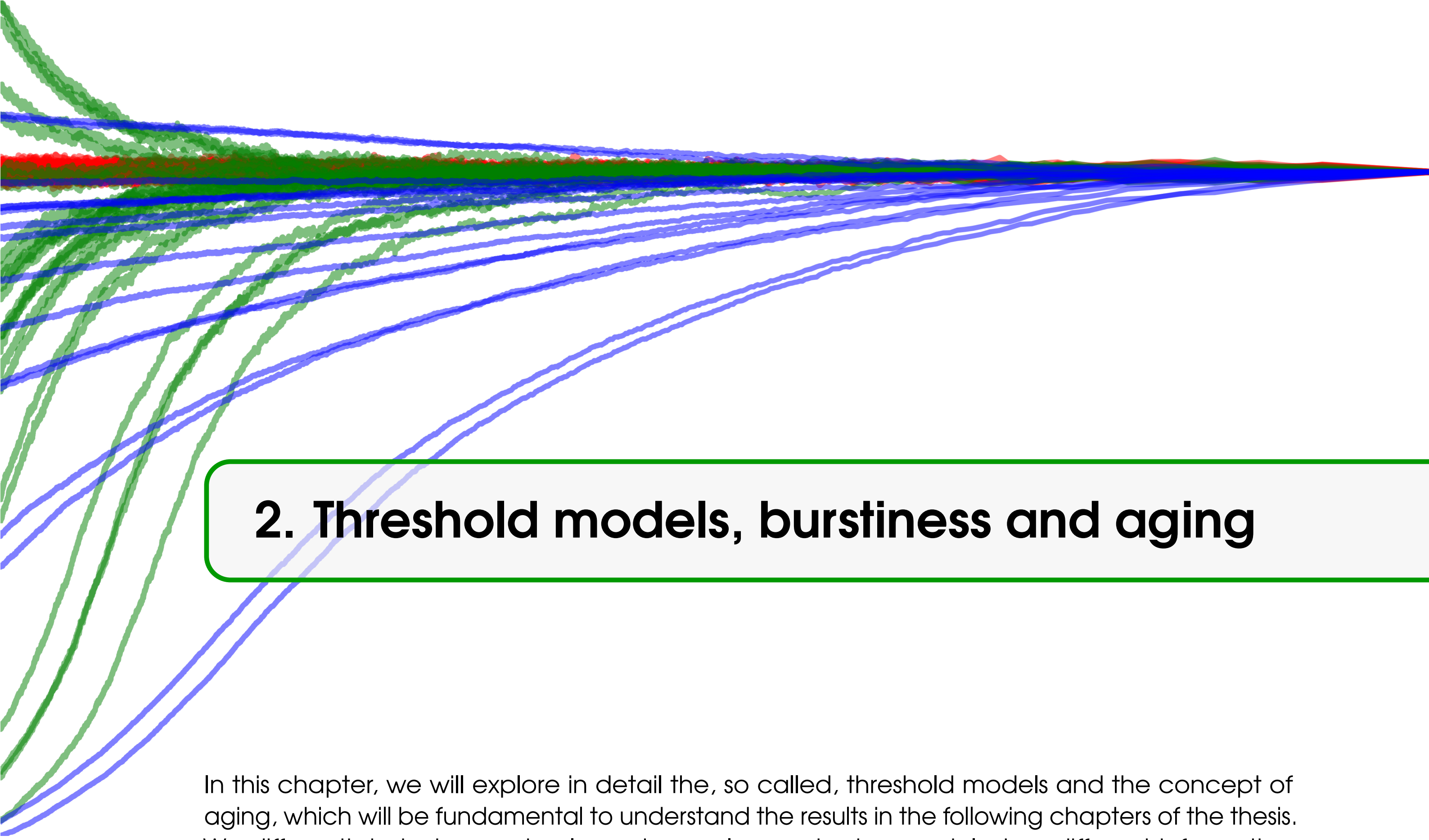
binary state models, random networks, configuration models, and preferential attachment are some of the models that are used in the study of human behavior and social systems.

1.4 Datasets

- We used the idealista dataset

- The strong point of the idealista dataset is that it contains information about the real estate market in Spain, and that it is a large dataset that contains information about the real estate market in Spain.

- The missing point of the idealista dataset is that it contains information about the real estate market in Spain, and that it is a large dataset that contains information about the real estate market in Spain.



2. Threshold models, burstiness and aging

In this chapter, we will explore in detail the, so called, threshold models and the concept of aging, which will be fundamental to understand the results in the following chapters of the thesis. We differentiate between simple and complex contagion models, two different information transmission mechanisms that have been widely studied in the literature. We introduce two well-known threshold models: the Granovetter-Watts threshold model, a fundamental model for understanding the dynamics of complex contagion in social networks, and the Sakoda-Schelling model, a segregation model that was a precursor of the nowadays agent-based simulations. Furthermore, we also introduce topics such as the bursty dynamics in human interactions and the role of aging, a non-Markovian mechanism that captures the tendency of individuals to stick to their previous beliefs or habits. It is shown how incorporating this factors to traditional Markovian models dramatically changes the phase diagram and the dynamics.

2.1 Introduction

Social contagion is a process that has been studied for many years and is present in many social systems, ranging from small groups and communities to large networks and societies at a global scale. This process, often referred to as social contagion (33), involves the spread of ideas, behaviors, innovations, and emotions (spread of "information") among individuals and groups through various forms of social interaction. The metaphor of contagion highlights the similarities between the spread of infectious diseases and the transmission of information, where a single "infected" individual can influence multiple others, leading to widespread information.

In this context, binary-state models have emerged as a versatile tool to describe a variety of natural and social phenomena in systems formed by many interacting agents. Each agent is considered to be in one of two possible states: susceptible/infected, adopters/non-adopters, favor/against, etc., depending on the context of the model. In all cases, one of the states represent the presence/spreading of information and the other the absence of it. The interaction among agents is determined by the underlying network and the update rules of the model. Examples of binary-state models include processes of opinion formation and consensus (47, 81, 104, 117), disease or social contagion (59, 96), among others.

With the advent of network theory and the increasing availability of large-scale data from online platforms, researchers have been able to study the contagion of ideas with unprecedented precision and detail. Duncan Watts and Steven Strogatz's small-world model (133) and Albert-László Barabási and Réka Albert's work on scale-free networks (15) provided foundational insights into the structure of social networks and their role in facilitating or hindering the spread

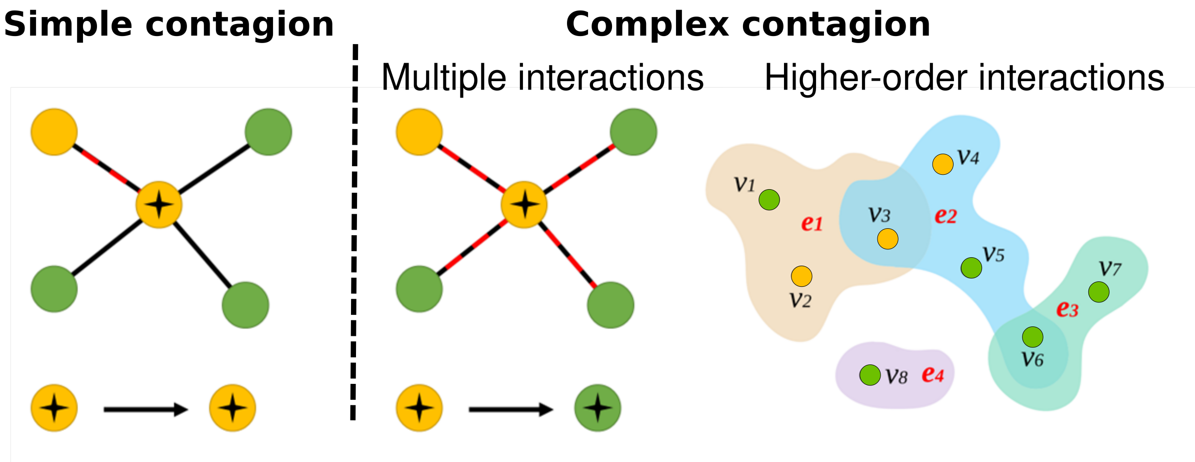


Figure 2.1: Comparison between the different types of social interaction. **Simple contagion**, where the agent considers just the pairwise interaction with one social contact (interaction highlighted with a dashed red line) and **Complex contagion**, where the agent considers the interaction with multiple social contacts. There are two distinguishable types of Complex contagion: **Multiple pairwise interactions**, where the agent considers the interaction with all social contacts (interactions highlighted with dashed red lines) and **Higher-order interactions**, where the agent considers the interaction with a group of social contacts, all at once, in a single interaction (not pairwise). The green, yellow colors represent the state (idea, position, political party...). (The hypergraph representation is from Ref. (9)).

of information and ideas.

On the other hand, the desicion-making process in social systems is influenced by a variety of factors, including social media influence (36, 93), peer pressure (70), emotional engagement (48, 124) and individual preferences. Peer effects and social influence have been shown to play a significant role in the adoption of new technologies, with individuals more likely to adopt new products or services if they see others in their social network doing the same (24, 107, 126).

In this chapter, we introduce the terms simple and complex contagions, two different mechanisms that describe how information spreads through social networks. Once we have defined these concepts, we move forward to introduce the Granovetter-Watts and the Sakoda-Schelling models, two fundamental models which update rules are based on a threshold mechanism, a particular case of complex interactions. We will introduce a theoretical framework useful to treat threshold models in complex networks and finally, we will introduce the concepts of bursty human dynamics and aging mechanism, which show that the Markovian assumption is not always valid in the study of social dynamics.

2.2 Simple and Complex Contagion

In the study of social contagion, researchers distinguish between two main types of contagion processes: simple contagion and complex contagion (30). Simple contagion refers to the spread of ideas, behaviors, or innovations primarily through single exposures or interactions, much like the transmission of infectious diseases. This process is characterized by the principle that an individual's likelihood of adopting a new idea or behavior increases with each additional exposure to that idea or behavior within their social network (34, 49). In contrast, complex contagion involves multiple exposures or reinforcements from different sources within the network, often requiring a critical mass of adopters before an individual is influenced to adopt the idea or behavior (29, 30, 59).

Simple contagion is often described as a process that involves only dyadic interactions, where the adoption of an idea or behavior is facilitated by direct contact between two individuals (see Fig. 2.1). This type of contagion is fundamental to understanding how information, beliefs, or diseases spread through populations via direct, pairwise connections (91, 95). Features of simple contagion include the rapid dissemination of information, continuous phase transition between full adoption - no adoption states, and the efficient spread of both beneficial and detrimental behaviors across social ties (34, 49).

In contrast, Complex contagion takes place in scenarios where adoption is not merely a result of dyadic interactions but also involves group dynamics and/or the reinforcement from multiple sources within the network. This type of contagion often requires a critical mass or threshold of adopters at the individual's surroundings to trigger the adoption of information (29, 30). This condition that characterizes complex contagion can be understood in two ways: (i) as a reinforcement of the idea or behavior from multiple pairwise (dyadic) interactions (29, 30), or (ii) as a reinforcement from multiple sources in a single group interaction (higher-order interaction) (9, 20, 67). In the first case, the peer pressure, characteristic of complex contagion processes, is included into the model, which is designed to be used a simple network of dyadic social contacts. In the second case, the group interaction is included in a higher-order network or hypergraph (21), which is a more general representation of the social contacts where the interactions are not restricted to dyads. In this case, the complex contagion process takes place via a single group interaction. See Fig. 2.1 for a graphical representation of the different examples of complex contagion. Features of complex contagion include global cascades of adoption, discontinuous phase transitions, and the emergence of echo chambers and polarization in social networks (30, 40).

Moreover, real-world processes are influenced not solely by either simple or complex contagion mechanisms but by a complex interaction between the two (Hybrid contagion) (40, 86). Such multifaceted interactions give rise to varied outcomes, including phenomena like discontinuous transitions, tricriticality, and echo chambers emergence , all of which profoundly affect how information is spread, how behaviors are adopted, and how collective actions are formed.

There have been attempts to extract the simple/complex nature of a process from real data. For example, by analyzing the correlation between the infection order of network nodes and their local topology, it is possible to infer the type of contagion process that is taking place (28). Nevertheless, the classification of contagion processes remains a challenging task, as the dynamics of social contagion are influenced by a multitude of factors and high-quality data related to the infection process is often scarce.

2.3 Granovetter-Watts threshold model

In this thesis, we are interested in the dynamics of complex contagion driven by multiple interactions in a network of dyadic social contacts. In particular, we focus on a particular category of models called **threshold models**.

Threshold models represent a critical conceptual framework in understanding how individual behaviors aggregate to produce collective outcomes, especially in contexts where decisions are influenced by the actions of others (59, 61). By defining a "threshold" — the point at which an individual's perception of the collective behavior of others prompts them to act — these models offer insights into the pivotal role of social influence and network structure in driving large-scale changes from small initial actions (43). Rooted in the interdisciplinary nexus of sociology, economics, and network theory, these models illuminate the mechanics behind phenomena as diverse as social movements, technological adoption, market dynamics, and even cascading failures within infrastructures. All these phenomena share a common thread: the need for a critical mass of adopters to trigger a response, a threshold that must be crossed to initiate a cascade of adoption (29, 30).

When we talk about threshold models, the model that comes to our minds is the threshold model introduced by Mark Granovetter in 1978 (59), exploring how individual adoptions depend

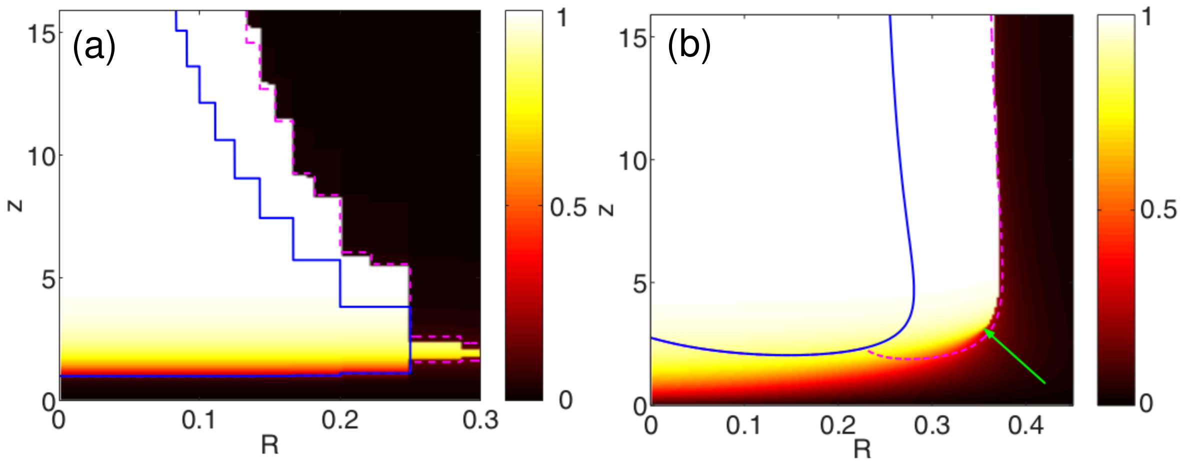


Figure 2.2: Average density n of nodes adopting as a heatmap for the Granovetter-Watts model. The simulations run in aER graph of mean degree z and uniform threshold value R **(a)** and threshold distributed is Gaussian with mean R and standard deviation 0.2 **(b)**. Initial adopters seed is set $n_0 = 0.01$. Lines show a first (blue) and a second (purple) order approximations to the cascade condition. The phase transition is discontinuous. Image from Ref. (56).

on the proportion of others adopting the behavior, highlighting the nonlinear nature of social influence and the importance of group interaction in complex contagion processes. In this model, each individual has a threshold that determines the number of neighbors they need to observe adopting a behavior before they themselves adopt it. This threshold can be interpreted as a measure of an individual's susceptibility to social influence, capturing the idea that some people are more likely to adopt a behavior if they see few other people doing the same, while others may require more convincing or reinforcement before they act. This concept of threshold was was inspired from Thomas C. Schelling work on segregation (114), in which the threshold is understood as the maximum tolerance of different neighbors that an individual can withstand before changing their behavior. Duncan J. Watts, in 2002, built upon Granovetter's concept, applying mathematical analysis to explore the model within complex networks (132) . His work, particularly on how minor initial actions can lead to large cascades, further elucidated the cascade condition dependence on both individual thresholds and network structures. This model, named as the Granovetter-Watts threshold model, has since become a cornerstone of research on complex contagion and collective behavior, offering a powerful lens through which to study the cascade dynamics in complex networks.

Update rules — Granovetter-Watts model. An individual time step of the model is defined as follows:

1. Each node i has a threshold R_i.
2. At each time step, a node i is selected at random.
3. If the fraction of active neighbors of i is greater than R_i, then i becomes active.

The Granovetter-Watts model exhibits a phase transition from a regime where the adoption is rare, where there are only small cascades of adoption and none of them is global, to a regime where the adoption is widespread, where there are large cascades that reach all the system. This phase transition is discontinuous (56, 132), and it is characterized by a critical threshold value R_c that separates the two regimes (refer to Fig. 2.2). The regime where the global cascades are rare,small and localized is a supercritical regime $R > R_c$ while the regime where cascades are fast and global is subcritical $R \leq R_c$. The discontinuous transition between the two regimes is driven by the interplay between the individual thresholds and the network structure, and it is a result of the collective dynamics of the system (see dependence of R_c with the average degree in Fig. 2.2).

The exploration of this model has been widespread, encompassing studies on various types of networks including regular lattices and small-world networks (30), as well as on random graphs (56). It has also been examined within the contexts of networks with modular and community structures (53), networks that exhibit clustering (63, 64), hypergraphs (9), and networks characterized by homophily (40), among others. In addition, the literature has expanded to cover the effects of varying the rules for adoption, such as incorporating social reinforcement across multiple layers (32), examining the influence of opinion leaders and initial seed size on the process (82, 116), the introduction of on-off thresholds (42), and analyzing the dynamics when simple contagions compete with complex ones (38, 40, 86). Further, empirical data have been used to test the predictions of the Granovetter-Watts model, demonstrating its applicability across a wide range of real-world situations (29, 62, 72, 76, 77, 88, 108, 125).

2.4 The Sakoda-Schelling model

Thomas C. Schelling's segregation model (114), illustrates how individual preferences regarding neighbors can lead to significant segregation in urban areas, even when these preferences are relatively mild. The model utilizes a checkerboard setup where each agent (representing a household) prefers to live in a neighborhood where at least a certain percentage of neighbors are of the same type (see Fig. 2.3). There are locations in the checkerboard that do not have an agent, these will be called vacancies. Agents move to a new vacancy location if their tolerance threshold is not met [1]. This simple rule leads to complex patterns, showing that even a slight preference for similar neighbors can result in highly segregated communities, an insight that has profound implications for understanding social dynamics and urban planning.

Update rules — Schelling's model. An individual time step of the model is defined as follows:

1. Each node i has a tolerance threshold T_i.
2. At each time step, a node i is selected at random.
3. If the fraction of different kind neighbors of i is greater than T_i, then i moves to a neighboring location where the fraction of different kind neighbors is less than T_i.
    - If there is no available location, then i remains in the same location.

On the other hand, James M. Sakoda's model, initially conceptualized in his 1949 dissertation and fully introduced in Ref. (111), offers a more nuanced approach to modeling social interactions using a similar checkerboard framework. Unlike Schelling's, Sakoda's model incorporates a broader range of social interactions by allowing agents to exhibit positive, neutral, or negative attitudes towards their neighbors. These attitudes influence the agents' movements across the board, aiming to optimize their local environment according to specific utility functions that aggregate the effects of surrounding agents. Sakoda's model is capable of simulating a variety of social phenomena beyond segregation, such as the formation of stable social clusters and the dynamics of group interactions (65).

An important contribution associated to both models is the use of a checkerboard setup as the computational space where agents (or tokens) reside and interact according to predefined rules. The "hand-made" simulations performed by Sakoda and Schelling using this discrete spatial representation has since become a standard framework for studying agent-based models in social systems (65). This structure allows for the exploration of local interactions, emergence of global patterns and interface analysis, providing a powerful tool for understanding the dynamics of social systems.

[1] Note that this tolerance threshold acts as a measure of a critical mass of different neighbors that the individual tolerates (large tolerance allows for more diverse neighborhood while small tolerance requires several similar neighbors to be satisfied). This threshold definition is complementary to the one in the Granovetter-Watts model, where it is a measure of the number of similar neighbors required to adopt (small threshold allows to adopt with a more diverse neighborhood while large threshold requires several similar neighbors to adopt).

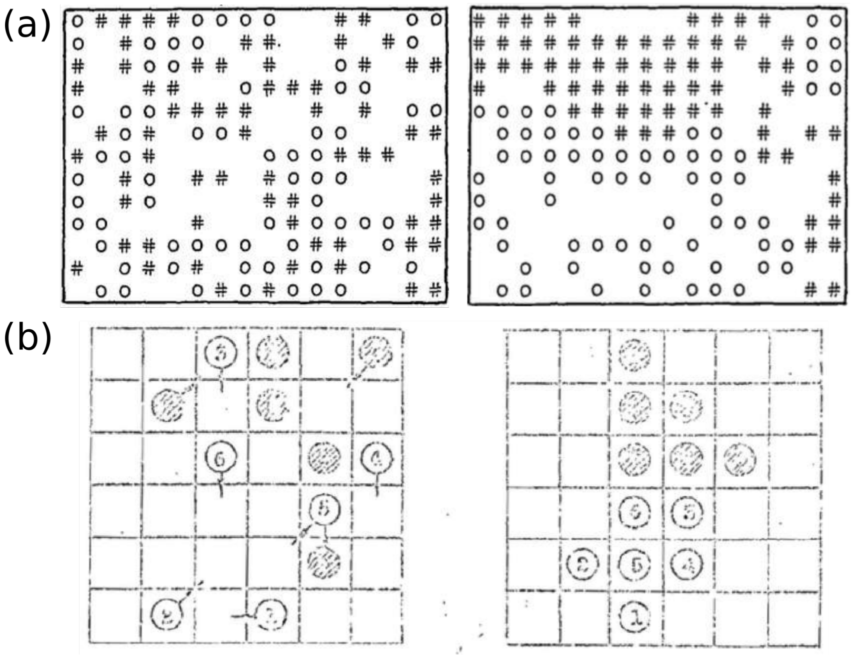


Figure 2.3: (a) Examples of the dynamics in Schelling's Segregation Model (from Schelling's original work (115)). **(b)** Examples of the dynamics in Sakoda's Checkerboard Conceptual Model (from Sakoda's original work (112)). In both cases, the author show at the left the initial configuration of the board and at the right the final segregated configuration after several iterations.

Update rules — Sakoda's model. An individual time step of the model is defined as follows:

1. Each node i has an attitude matrix A_i, that defines the attitudes of i towards all other agents on the board.
2. At each time step, a node i is selected at random.
3. i evaluates the total utility for each neighboring location based on the sum of influences (according to the attitudes) from all other agents on the board, weighted by distance.
4. i moves to the available location with the highest utility.

The results of the Schelling's model demonstrated how even mild personal preferences can unexpectedly lead to significant spatial segregation. Its insights have been applied across economics, sociology, urban planning, and complexity science, profoundly influencing both academic research and practical policy discussions. Schelling's model became a foundational example in agent-based modeling, helping to educate countless researchers and practitioners about the impact of individual actions on broader social patterns. Nevertheless, when we check the update rules, we observe that the Schelling's model is a particular case of the previous Sakoda's model, where agents have a negative attitude towards different-kind agents and a fixed tolerance threshold T_i. To honor the original contributions of both authors, we refer to this model through the thesis as the Sakoda-Schelling model.

In particular, the Sakoda-Schelling model has been studied from a Statistical Physics point of view due to its close relation to different forms of Kinetic Ising-like models (121, 122), and also addressing general questions of clustering and domain growth phenomena, as well as for the existence of phase transitions from segregated to non-segregated phases. For example, the relation with phase separation in binary mixtures has been considered (39, 129), as well as the connection with the phase diagram of spin-1 Hamiltonians (22, 51, 52, 113). In this context, a useful classification of models is to distinguish between two possible types of dynamics (39): **"constrained"**, where unsatisfied agents just move to satisfying vacancies (if possible), and **"unconstrained"**, where each agent try to improve its satisfaction, but after a move they may remain unsatisfied. In addition, the motion can be short-range (only to neighboring sites, as in the original model) or long-range. Constrained motion has been named "solid-like" because it generally leads to frozen small clusters, while unconstrained motion has been considered "liquid-like" because it allows for large growing clusters (129). Including the motion of satisfied agents leads to a noisy effect playing the role of temperature in a statistical physics approach (51, 52).

Despite there has been many attempt in the literature, the description of the phase diagram and the transitions in the Sakoda-Schelling model is a difficult task. In Ref. (83), the authors need to reduce the model to a simpler binary-state version that allows to obtain the phase transition from a segregated to a mixed phase.

2.5 Theoretical Framework

To explain the emergent properties exhibited by the agent based models and its computer simulations, we need to develop a theoretical framework that captures the essential features of the system. This framework should provide a mathematical description of the dynamics, allowing us to analyze the system's behavior and predict its evolution over time. In the context of social contagion and collective behavior, the theoretical framework typically involves a set of differential equations or master equations that describe the evolution of the system's state variables.

The theoretical framework for agent-based models running in complex networks can be broadly classified into two main categories: mean-field approaches and network-based approaches (19). The so-called mean-field[2] approaches treat the system as a homogeneous entity, where each agent interacts with the average behavior of the entire population. These approaches are well-suited for capturing the macroscopic dynamics of the system and are particularly useful for understanding the collective behavior that emerges from individual interactions. Network-based approaches, on the other hand, explicitly model the interactions between agents as a network structure, where nodes represent agents and edges represent interactions between them. These approaches are valuable for capturing the influence of the underlying network structure on the system's dynamics and for studying the impact of network properties on the spread of information and ideas.

2.5.1 Approximate Master Equation

A general framework for binary-state models in complex networks was developed by J. P. Gleeson (54, 55), which provides a general set of differential equations, the Approximate Master Equation (AME), to describe the dynamics of any Markovian binary-state model on a generic network. This framework has been widely used to study the dynamics of social contagion, opinion formation, consensus problems, and other collective behaviors in complex networks. The framework is particularly very useful in the context of thresholds models, which are not well suited for a mean-field approach (56), allowing us to identify phase transitions, compute critical thresholds, and predict the final state of the system. The full AME description and derivation can be found in Ref. (55), but we will provide a brief summary of the main concepts here.

Consider a system of N nodes in a network, where each node can be in one of two states: $+1$ or -1. The state of each node evolves over time according to a set of rules that depend on the states of its neighbors. Let us consider a node i, with a degree k (i.e., k connections to other nodes) and m neighbors of i in state -1 (e.g., "adopter"). If node i is in state $+1$ (e.g., "non-adopter"), the rate $T^{+}_{k,m}$ defines the probability per unit time that i will switch to state -1. Similarly, $T^{-}_{k,m}$ defines the probability per unit time for i, in state -1, to switch to state $+1$. These rates are, in general, functions of both the degree k and the number m of neighbors in state -1,

[2]Mean-field is a term borrowed from statistical physics, where it refers to the approximation of the interactions between particles by an average field. In the context of networks, mean-field is related to the approximations of ignoring the local effects and assuming all-to-all connections and infinite system size.

reflecting how the local network configuration influences state transitions.

Taking into account this framework, the AME can be written as:

$$\frac{d}{dt}x^{\pm}_{k,m} = -T^{\pm}_{k,m}x^{\pm}_{k,m} + T^{\mp}_{k,m}x^{\mp}_{k,m} - (k-m)\beta^{\pm}x^{\pm}_{k,m} + (k-m+1)\beta^{\pm}x^{\pm}_{k,m-1} - m\gamma^{\pm}x^{\pm}_{k,m} + (m+1)\gamma^{\pm}x^{\pm}_{k,m+1} \quad (2.1)$$

Here, $x^{+}_{k,m}$ and $x^{-}_{k,m}$ represent the fractions of nodes with degree k and m infected neighbors that are in state $+1$ and -1, respectively. $\beta^{\pm}$ and $\gamma^{\pm}$ are time-dependent rates that describe how the adoption process spreads and recedes across the edges of the network, encapsulating the network's dynamic connectivity and its influence on the spread of states.

A key advantage of the AME is its ability to capture the complex dynamics of networks by considering the interactions between neighboring nodes, making it more accurate than simpler models like the mean-field theory, which assumes independence between nodes. Moreover, from the AME, one can approximate the shape of the solutions $x^{\pm}_{k,m}(t)$ to reduce the number of differential equations, recovering the pair approximation and the heterogeneous mean field (54, 55).

On the other hand, the AME assumes a tree-like structure with negligible levels of clustering. This assumption implies that there are very few short loops in the network. This tree-like assumption simplifies the calculation and application of the AME by reducing the network's complexity, and becomes very useful for networks generated with the configuration model (92), with any given degree distribution, at the limit $N \to \infty$. Another limitation of the AME is based on the formulation itself, since framework is built assuming binary-state Markovian dynamics, which may not always accurately capture the real-world dynamics of social contagion. In fact, in next section, we will introduce the concept of bursty human dynamics, which is is an empirical evidence of the presence of non-Markovian effects. This effects can significantly impact the dynamics of social contagion processes.

2.6 Bursty Human Dynamics

Bursty behavior refers to the irregular and sporadically temporal patterns of interactions that include natural phenomena, like earthquakes and neuron firing, as well as human activities, such as communication, mobility, and social dynamics. This section delves into the characteristics of bursty behavior, highlighting by empirical evidence, and discusses its significant implications for modeling human behavior.

Human activities often exhibit complex temporal patterns characterized by bursts—short periods of high activity interspersed with longer periods of inactivity (see examples Fig. 2.4(a-c)). This non-Poissonian behavior, referred to as burstiness, manifests across diverse human-driven processes and is extensively documented in communication dynamics, web browsing habits, and social interactions (16, 128). The seminal work, by A. L. Barabási (16), shown from email communication's data that activity periods do not follow a regular pattern but are clustered in bursts. This phenomenon has since been observed universally across various platforms such as mobile phone calls, emails, tweets, text messaging and social media (11, 68, 69, 73, 78, 87, 110, 135). Further research has analyzed burstiness, focusing on the persistence and periodicity of human interactions (35) or the effects of circadian rhythms (71), and has extended these analyses to web activity to predict behaviors across different online platforms (102).

An important evidence of bursty dynamics is the heavy-tailed distribution of inter-event times, indicating that the probability of short inter-event times is higher than expected from a Poisson process (see Fig. 2.4(c)). As a result of this bursty human behavior, there is an emergence of heterogenous degree distributions (89), which have been observed in many social systems (15). Further insights into the impact of burstiness on system dynamics come from studies linking it to memory and the structured nature of human dialogues, enhancing our understanding of how past interactions influence future activities (44, 57, 75).

Traditional models based usually rely on numerical simulations to update agents following

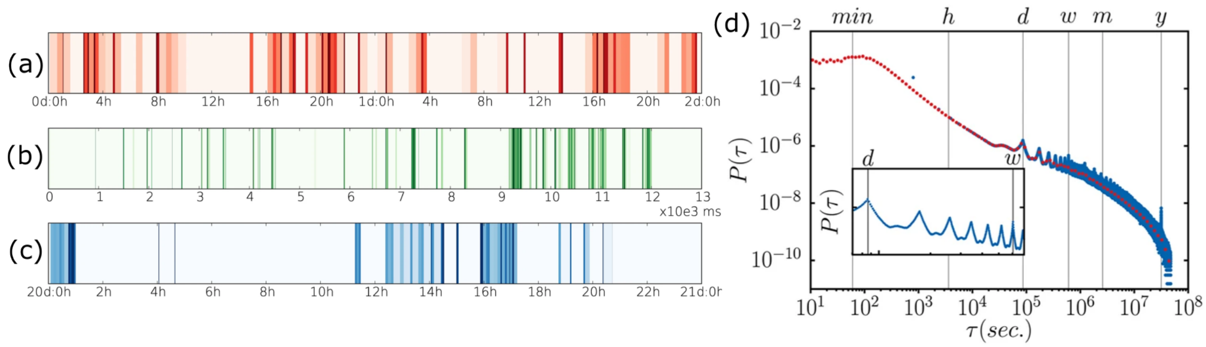


Figure 2.4: (a) Sequence of earthquakes with magnitude larger than two at a single location (South of Chishima Island, 8th–9th October 1994). **(b)** Firing sequence of a single neuron (from rat's hippocampal). **(c)** Outgoing mobile phone call sequence of an individual. Shorter the time between the consecutive events darker the color (a,b,c from Ref. (75)). **(d)** Distribution of inter-event times between tweets for several users. Blue and red dots represent the lin- and log-binned scales in the τ axis. The localized maxima in the tail of the distribution correspond to circadian rhythms, as shown in the bottom inset (from Ref. (11)). The distribution is heavy-tailed, indicating bursty behavior.

independent random Poisson processes, called Random Asyncronous Update (RAU), in which the characteristic time a node is updated is the Monte Carlo step, in which all agents has been updated once on average (46). In this process, an exponential interevent time distribution is expected, which does not capture the bursty nature of human dynamics. To address this phenomenon, non-Poissonian models are necessary to provide a better fit for empirical observations (128). We differentiate two main approaches to include bursty human dynamics in our models:

- **Activity-driven models (nodes get activated):** These models incorporate the temporal aspects of human activity by assigning activity potentials to nodes within a network, dictating the likelihood of interactions based on observed human activity patterns (119, 120, 127).
- **Temporal networks (links get activated):** These models incorporate time-stamped interactions, such that at each time step our interaction network changes (66, 99).

While both approaches have been successful to include bursty human dynamics, they offer different perspectives on the underlying mechanisms driving these behaviors: activity-driven models emphasize the burstiness of individual attempts to interact with others, while temporal networks focus on the burstiness of the interactions themselves. The choice of model depends on the specific research question and the level of detail required to capture the dynamics of interest.

It has been shown that implications of bursty behavior are dramatic, influencing the dynamics of network processes such as the spread of epidemics and information diffusion (105, 131). Understanding the mechanism behind this burstiness allows us to improve our predictions, aligning them more closely with natural human activity patterns.

2.7 Aging mechanism

'The concept of "Aging" is understood in many different ways in the literature. For us, aging is one form of memory effect on which the rate of interactions depends on the persistence time of an agent in a state, modifying the transition to a different state (23, 46, 98).

This concept of aging was introduced in (119) taking as inspiration the non-equilibrium dynamics of spin glasses, where the effective temperature of the system changes with the time since a given perturbation was applied (37). In a context of social systems, this resistance to change can be interpreted as conformism or laziness. In models of species competition (103), this would imply that neighboring species are less likely to be displaced at a later stage of growth

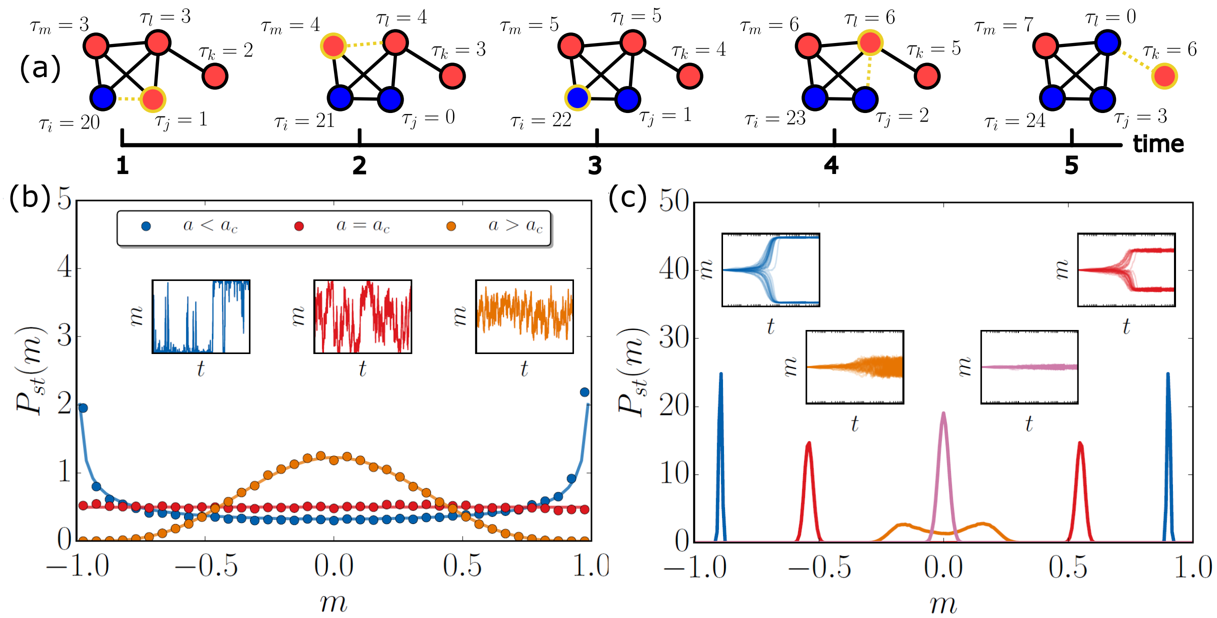


Figure 2.5: (a) Schematic representation of the evolution of the Voter model with aging (given an initial configuration for both states and internal times): At 1, node j activates, copies i and resets age. At 2, node m activates and copies m (same state). At 3, i does not activate due to the age. At 4, node l copies j and resets age. At 5, k activates, copies l and resets age. **(b)** Stationary probability density function (pdf) of the magnetization in the three different regimes. Points come from simulations, solid lines are the theoretical curves. The insets show one typical trajectory of the dynamics, in each of the regimes. **(c)** Stationary pdf for the noisy voter model with aging, in the different regimes. The insets show 50 trajectories of the magnetization. (b-c from Ref. (11)).

(119). In our words, the motivation behind the aging mechanism is to capture the tendency of individuals to stick to their previous beliefs or habits, a common feature in human behavior (61). In other words, the longer an individual holds a particular habit, the more he/she will accumulate experience, leading to a higher self-involvement and resistance to change (80). Moreover, this emotional attachment balances the memory-less and purely rational considerations of traditional models (60).

The aging mechanism is a non-Markovian effect that can be included in a model via an activation function that modifies the transition rates between states (activity driven model). This function depends on the time since the last transition, allowing us to include bursty dynamics in the individuals' attempts to interact with others (given a proper choice of the activation function (46)). This activation function is build such that probability of an individual to interact with another individual decreases with the persistence time in a given state, even though there are studies that also account for anti-aging mechanisms (probability to interact increases) (31, 97).

2.7.1 Aging in pairwise interactions

Aging effects have been already shown to modify drastically the dynamics in the Voter model (81), a popular framework for exploring consensus formation in statistical physics and social dynamic. The Voter model is a simple model of opinion dynamics, where agents update their state by copying the state of a randomly selected neighbor. This rules lead to dynamically active state that reaches consensus as a finite-size effect in a finite time, but consensus is not reached at the thermodynamic limit (81). It is shown that, while aging can decelerate microdynamics by making state changes less frequent as agents' states age, it can accelerate macrodynamics, leading the dynamics to a well-defined coarsening process, allowing the system to reach consensus even in the thermodynamic limit citeStark2008,fernandez-gracia-2011, perez-2016, boguna-2014, perez-2016, peralta-2020C. This effect is shown in Fig. 2.5(a), where the aging mechanism is able to modify the dynamics of the Voter model, leading to a faster consensus of the minority state.

In terms of stability, in Voter-like models, incorporating aging exhibit a higher tendency toward reaching consensus than their non-aging counterparts. The persistence of the majority state, reinforced by aging, contributes to this stabilization, making aging a significant factor in determining which is the consensus state (13, 18, 97). As an example, aging modifies the nature of the noise-driven phase transition in the noisy Voter model. Specifically, it transforms a finite-size discontinuous transition between ordered and disordered phases into a continuous transition that falls into Ising universality class (12) (see discontinuous transition for noisy Voter model in Fig. 2.5(b) and the continuous transition when aging is included in Fig. 2.5(c)).

Regarding models of multiple pairwise interactions or higher-order interactions, the aging implications are still an open question and it is a topic of current research. Just for the specific case of the noisy majority vote model (31), it is known that aging mechanism is able to modify the critical point of the noise-driven disordered-ordered phase transition [3]. Further research is needed to understand the joint effect of aging and multiple interactions.

[3] The noisy majority voter model exhibits a continuous phase transition from an ordered phase, where the system reaches consensus according to the majority in the system, to a disordered phase, where consensus is not reached.

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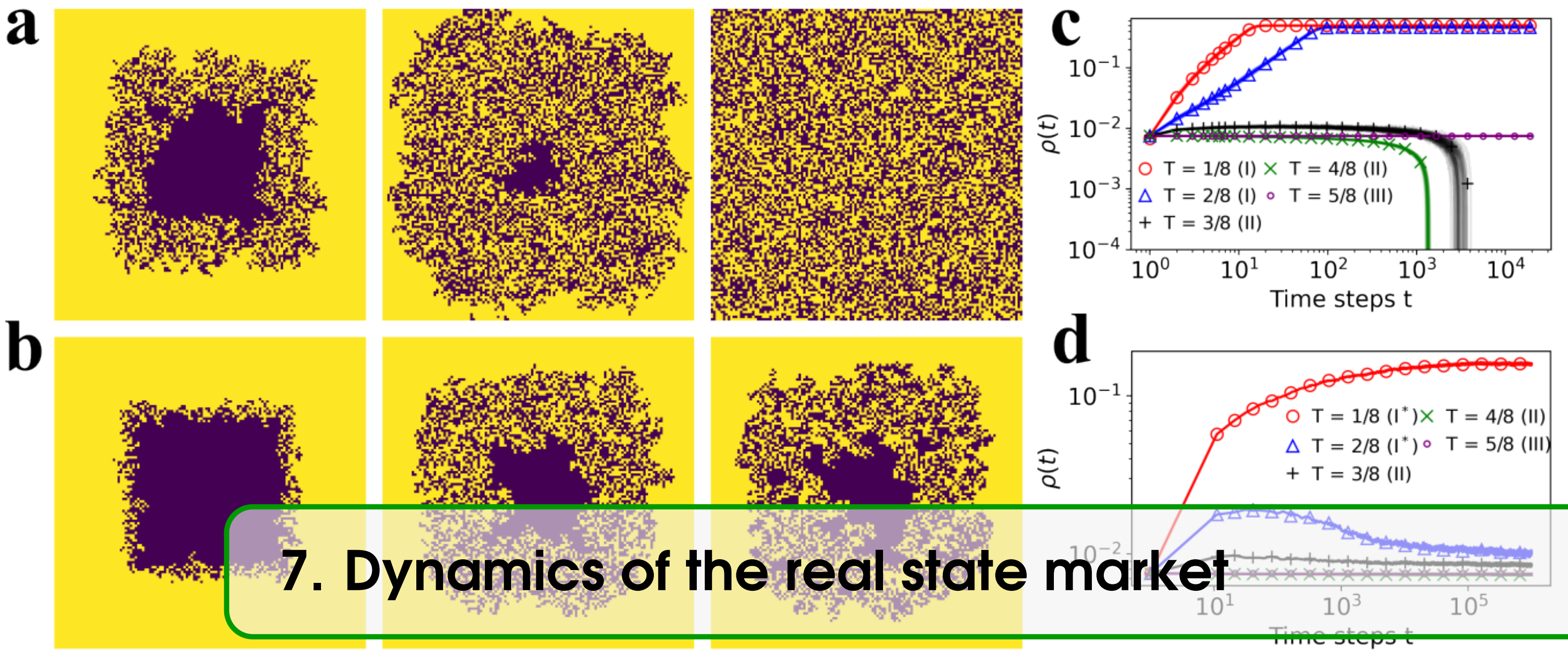

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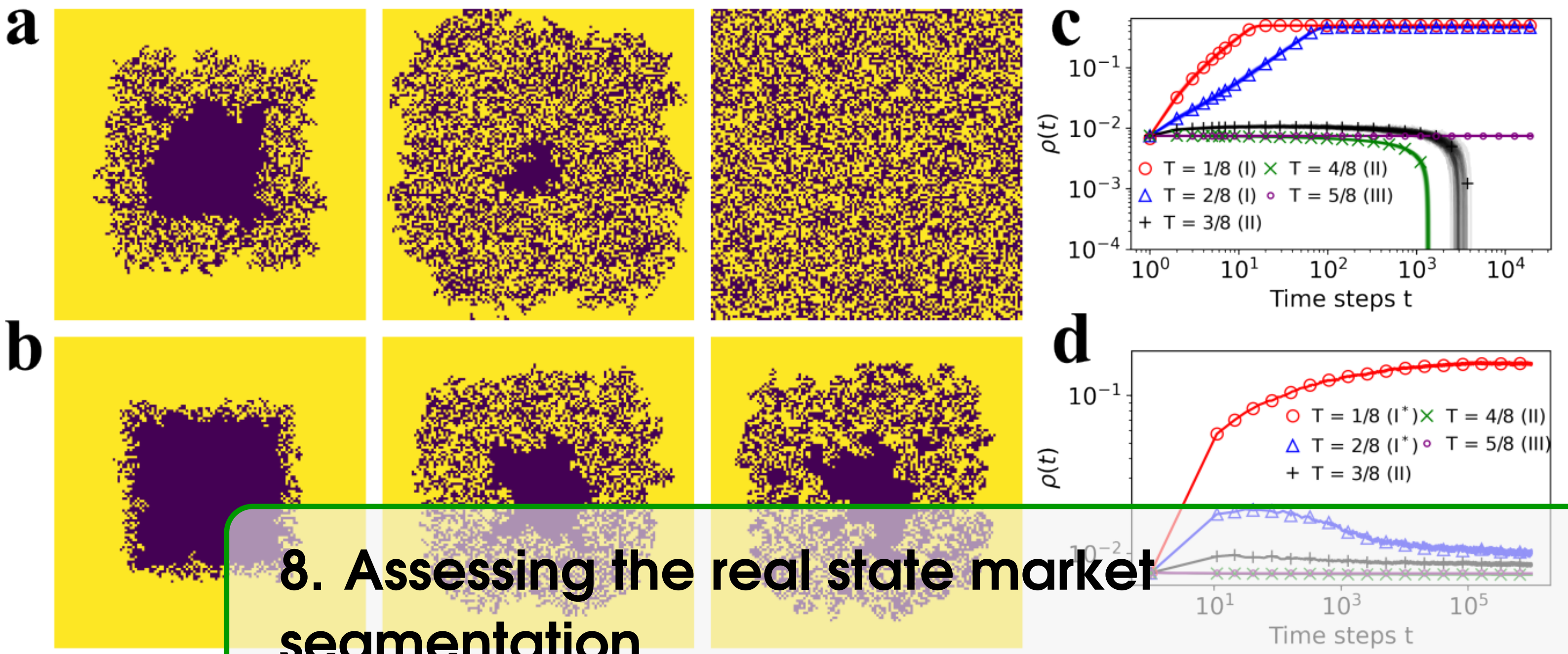

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Bibliography

Bibliography

[1] David Abella, Giancarlo Franzese, and Javier Hernández-Rojas. "Many-Body Contributions in Water Nanoclusters". In: *ACS Nano* 17.3 (Jan. 2023), pages 1959–1964. ISSN: 1936-086X. DOI: 10.1021/acsnano.2c06077. URL: http://dx.doi.org/10.1021/acsnano.2c06077 (cited on page 17).

[2] David Abella, Maxi San Miguel, and José J. Ramasco. "Aging effects in Schelling segregation model". In: *Scientific Reports* 12.1 (Nov. 2022). ISSN: 2045-2322. DOI: 10.1038/s41598-022-23224-7. URL: http://dx.doi.org/10.1038/s41598-022-23224-7 (cited on page 17).

[3] David Abella, Maxi San Miguel, and José J. Ramasco. "Aging in binary-state models: The Threshold model for complex contagion". In: *Phys. Rev. E* 107 (2 Feb. 2023), page 024101. DOI: 10.1103/PhysRevE.107.024101. URL: https://link.aps.org/doi/10.1103/PhysRevE.107.024101 (cited on page 17).

[4] David Abella et al. "Unraveling higher-order dynamics in collaboration networks". In: *arXiv preprint arXiv:2306.17521* (2023) (cited on page 17).

[5] David Abella et al. "Ordering dynamics and aging in the symmetrical threshold model". In: *New Journal of Physics* 26.1 (Jan. 2024), page 013033. DOI: 10.1088/1367-2630/ad1ad4. URL: https://dx.doi.org/10.1088/1367-2630/ad1ad4 (cited on page 17).

[6] Jeffrey J. Anderson and Arend Lijphart. "Patterns of Democracy: Government Forms and Performance in Thirty-Six Countries". In: *CrossRef Listing of Deleted DOIs* 30.2 (Jan. 2000), page 117. DOI: 10.2307/3331092. URL: https://doi.org/10.2307/3331092 (cited on page 1).

[7] Philip W Anderson. "More Is Different: Broken symmetry and the nature of the hierarchical structure of science." In: *Science* 177.4047 (1972), pages 393–396 (cited on page 1).

[8] Roy M Anderson and Robert M May. *Infectious diseases of humans: dynamics and control*. Oxford university press, 1991 (cited on page 1).

[9] Guilherme Ferraz de Arruda, Giovanni Petri, and Yamir Moreno. "Social contagion models on hypergraphs". In: *Physical Review Research* 2.2 (2020). DOI: 10.1103/physrevresearch.2.023032 (cited on pages 6, 7, 9).

[10] W. Brian Arthur. *Increasing returns and path dependence in the economy*. Jan. 1994. DOI: 10.3998/mpub.10029. URL: https://doi.org/10.3998/mpub.10029 (cited on page 1).

[11] Oriol Artime, José J. Ramasco, and Maxi San Miguel. "Dynamics on networks: competition of temporal and topological correlations". In: *Scientific Reports* 7.1 (2017). DOI: 10.1038/srep41627 (cited on pages 12–14).

[12] Oriol Artime et al. "Aging-induced continuous phase transition". In: *Physical Review E* 98.3 (2018). DOI: 10.1103/physreve.98.032104 (cited on page 15).

[13] Oriol Artime et al. "Herding and idiosyncratic choices: Nonlinearity and aging-induced transitions in the noisy voter model". In: *Comptes Rendus. Physique* 20.4 (2019), pages 262–274 (cited on page 15).

[14] Robert Axelrod. *The Complexity of Cooperation: Agent-Based models of competition and collaboration*. Dec. 1997. DOI: 10.1515/9781400822300. URL: https://doi.org/10.1515/9781400822300 (cited on page 2).

[15] Albert-László Barabási. "Scale-free networks: a decade and beyond". In: *Science* 325.5939 (2009), pages 412–413 (cited on pages 5, 12).

[16] Albert-László Barabási. "The origin of bursts and heavy tails in human dynamics". In: *Nature* 435.7039 (May 2005), pages 207–211. DOI: 10.1038/nature03459. URL: https://nature.com/articles/nature03459 (cited on page 12).

[17] Albert-László Barabási. “Network science”. In: *Philosophical transactions - Royal Society. Mathematical, Physical and engineering sciences/Philosophical transactions - Royal Society. Mathematical, physical and engineering sciences* 371.1987 (Mar. 2013), page 20120375. DOI: 10.1098/rsta.2012.0375. URL: https://doi.org/10.1098/rsta.2012.0375 (cited on page 1).

[18] Joseph W Baron et al. “Analytical and numerical treatment of continuous ageing in the voter model”. In: *Entropy* 24.10 (2022), page 1331 (cited on page 15).

[19] Alain Barrat, Marc Barthelemy, and Alessandro Vespignani. *Dynamical processes on complex networks*. Cambridge university press, 2008 (cited on page 11).

[20] Federico Battiston et al. “The physics of higher-order interactions in complex systems”. In: *Nature Physics* 17.10 (Oct. 2021), pages 1093–1098. DOI: 10.1038/s41567-021-01371-4. URL: https://doi.org/10.1038/s41567-021-01371-4 (cited on page 7).

[21] Claude Berge. *Hypergraphs: combinatorics of finite sets*. Volume 45. Elsevier, 1984 (cited on page 7).

[22] M. Blume, V.J. Emery, and Robert B. Griffiths. “Ising Model for the Lambda Transition and Phase Separation in He3-He4 Mixtures”. In: *Phys. Rev. A* 4 (1971), page 1071. DOI: https://doi.org/10.1103/PhysRevA.4.1071 (cited on page 10).

[23] Marian Boguñá et al. “Simulating non-Markovian stochastic processes”. In: *Physical Review E* 90.4 (2014). DOI: 10.1103/physreve.90.042108 (cited on page 13).

[24] Bryan Bollinger and Kenneth Gillingham. “Peer effects in the diffusion of solar photovoltaic panels”. In: *Marketing science (Providence, R.I.)* 31.6 (Nov. 2012), pages 900–912. DOI: 10.1287/mksc.1120.0727. URL: https://doi.org/10.1287/mksc.1120.0727 (cited on page 6).

[25] Danah Boyd and Kate Crawford. “CRITICAL QUESTIONS FOR BIG DATA”. In: *Information, communication & society* 15.5 (June 2012), pages 662–679. DOI: 10.1080/1369118x.2012.678878. URL: https://doi.org/10.1080/1369118x.2012.678878 (cited on page 2).

[26] Alan Bryman. *Social research methods*. Feb. 2010, pages 157–184. DOI: 10.4324/9780203381175\{_}chapter\{_}9. URL: https://doi.org/10.4324/9780203381175_chapter_9 (cited on page 1).

[27] Claudio Castellano, Miguel A. Muñoz, and Romualdo Pastor-Satorras. “Nonlinearq-Voter model”. In: *Physical Review E* 80.4 (2009). DOI: 10.1103/physreve.80.041129 (cited on page 1).

[28] Giulia Cencetti et al. “Distinguishing Simple and Complex Contagion Processes on Networks”. In: *Phys. Rev. Lett.* 130 (24 June 2023), page 247401. DOI: 10.1103/PhysRevLett.130.247401. URL: https://link.aps.org/doi/10.1103/PhysRevLett.130.247401 (cited on page 7).

[29] Damon Centola. “The Spread of Behavior in an Online Social Network Experiment”. In: *Science* 329.5996 (2010), pages 1194–1197. DOI: 10.1126/science.1185231 (cited on pages 6, 7, 9).

[30] Damon Centola, Víctor M. Eguíluz, and Michael W. Macy. “Cascade dynamics of complex propagation”. In: *Physica A: Statistical Mechanics and its Applications* 374.1 (2007), pages 449–456. DOI: 10.1016/j.physa.2006.06.018 (cited on pages 6, 7, 9).

[31] Hanshuang Chen et al. “Non-Markovian majority-vote model”. In: *Physical Review E* 102.6 (2020). DOI: 10.1103/physreve.102.062311 (cited on pages 14, 15, 47).

[32] Ling-Jiao Chen et al. “Complex contagions with social reinforcement from different layers and neighbors”. In: *Physica A: Statistical Mechanics and its Applications* 503 (2018), pages 516–525. DOI: 10.1016/j.physa.2018.03.017 (cited on page 9).

[33] Nicholas A Christakis and James H Fowler. “Social contagion theory: examining dynamic social networks and human behavior”. In: *Statistics in medicine* 32.4 (2013), pages 556–577 (cited on page 5).

[34] Nicholas A. Christakis and James H. Fowler. “The Spread of Obesity in a Large Social Network over 32 Years”. In: *The New England Journal of Medicine* 357.4 (July 2007), pages 370–379. DOI: 10.1056/nejmsa066082. URL: https://doi.org/10.1056/nejmsa066082 (cited on pages 6, 7).

[35] A. Clauset and N. Eagle. *Persistence and periodicity in a dynamic proximity network*. 2012. URL: https://www.semanticscholar.org/paper/Persistence-and-periodicity-in-a-dynamic-proximity-Clauset-Eagle/3a3238793f27ba421f9fb258dd0e1887aacf2128 (cited on page 12).

[36] "Creating social contagion through viral product design: a randomized trial of peer influence in networks on JSTOR". In: *www.jstor.org* (). URL: https://www.jstor.org/stable/41261920 (cited on page 6).

[37] Leticia F Cugliandolo and Jorge Kurchan. "Analytical solution of the off-equilibrium dynamics of a long-range spin-glass model". In: *Physical Review Letters* 71.1 (1993), page 173 (cited on page 13).

[38] Agnieszka Czaplicka, Raul Toral, and Maxi San Miguel. "Competition of simple and complex adoption on interdependent networks". In: *Physical Review E* 94.6 (2016). DOI: 10.1103/physreve.94.062301 (cited on page 9).

[39] L Dall'Asta, C Castellano, and M Marsili. "Statistical physics of the Schelling model of segregation". In: *Journal of Statistical Mechanics: Theory and Experiment* 2008.07 (July 2008), page L07002. ISSN: 1742-5468. DOI: 10.1088/1742-5468/2008/07/l07002. URL: http://dx.doi.org/10.1088/1742-5468/2008/07/L07002 (cited on page 10).

[40] Fernando Diaz-Diaz, Maxi San Miguel, and Sandro Meloni. "Echo chambers and information transmission biases in homophilic and heterophilic networks". In: *Scientific Reports* 12.1 (2022). DOI: 10.1038/s41598-022-13343-6 (cited on pages 7, 9).

[41] Ian Dobson et al. "Complex systems analysis of series of blackouts: Cascading failure, critical points, and self-organization". In: *Chaos* 17.2 (June 2007). DOI: 10.1063/1.2737822. URL: https://doi.org/10.1063/1.2737822 (cited on page 1).

[42] Peter Sheridan Dodds, Kameron Decker Harris, and Christopher M. Danforth. "Limited Imitation Contagion on Random Networks: Chaos, Universality, and Unpredictability". In: *Physical Review Letters* 110.15 (2013). DOI: 10.1103/physrevlett.110.158701 (cited on page 9).

[43] Peter Sheridan Dodds and Duncan J. Watts. "Universal Behavior in a Generalized Model of Contagion". In: *Physical Review Letters* 92.21 (2004). DOI: 10.1103/physrevlett.92.218701 (cited on page 7).

[44] Jean-Pierre Eckmann, Elisha Moses, and Danilo Sergi. "Entropy of dialogues creates coherent structures in e-mail traffic". In: *Proceedings of the National Academy of Sciences of the United States of America* 101.40 (Sept. 2004), pages 14333–14337. DOI: 10.1073/pnas.0405728101. URL: https://doi.org/10.1073/pnas.0405728101 (cited on page 12).

[45] Robert C. Ellickson. "The Evolution of Social Norms: A Perspective from the Legal Academy". In: *Social Science Research Network* (Jan. 1999). DOI: 10.2139/ssrn.191392. URL: https://doi.org/10.2139/ssrn.191392 (cited on page 1).

[46] J. Fernández-Gracia, V. M. Eguíluz, and M. San Miguel. "Update rules and interevent time distributions: Slow ordering versus no ordering in the voter model". In: *Physical Review E* 84.1 (2011). DOI: 10.1103/physreve.84.015103 (cited on pages 13, 14).

[47] Juan Fernández-Gracia et al. "Is the Voter Model a Model for Voters?" In: *Physical Review Letters* 112.15 (2014). DOI: 10.1103/physrevlett.112.158701 (cited on page 5).

[48] Emilio Ferrara and Zeyao Yang. "Measuring emotional contagion in social media". In: *PLOS ONE* 10.11 (Nov. 2015), e0142390. DOI: 10.1371/journal.pone.0142390. URL: https://doi.org/10.1371/journal.pone.0142390 (cited on page 6).

[49] James H. Fowler and Nicholas A. Christakis. "Cooperative behavior cascades in human social networks". In: *Proceedings of the National Academy of Sciences of the United States of America* 107.12 (Mar. 2010), pages 5334–5338. DOI: 10.1073/pnas.0913149107. URL: https://doi.org/10.1073/pnas.0913149107 (cited on pages 6, 7).

[50] Uriel Frisch. *Turbulence: the legacy of AN Kolmogorov*. Cambridge university press, 1995 (cited on page 1).

[51] L. Gauvin, J. Vannimenus, and J.-P. Nadal. "Phase diagram of a Schelling segregation model". In: *The European Physical Journal B* 70.2 (July 2009), pages 293–304. ISSN: 1434-6036. DOI: 10.1140/epjb/e2009-00234-0. URL: http://dx.doi.org/10.1140/epjb/e2009-00234-0 (cited on pages 10, 11).

[52] Laetitia Gauvin, Jean-Pierre Nadal, and Jean Vannimenus. "Schelling segregation in an open city: A kinetically constrained Blume-Emery-Griffiths spin-1 system". In: *Physical Review E* 81.6 (June 2010). ISSN: 1550-2376. DOI: 10.1103/physreve.81.066120. URL: http://dx.doi.org/10.1103/PhysRevE.81.066120 (cited on pages 10, 11).

[53] James P. Gleeson. "Cascades on correlated and modular random networks". In: *Physical Review E* 77.4 (2008). DOI: 10.1103/physreve.77.046117 (cited on page 9).

[54] James P. Gleeson. "High-Accuracy Approximation of Binary-State Dynamics on Networks". In: *Physical Review Letters* 107.6 (2011). DOI: 10.1103/physrevlett.107.068701 (cited on pages 11, 12).

[55] James P. Gleeson. "Binary-State Dynamics on Complex Networks: Pair Approximation and Beyond". In: *Physical Review X* 3.2 (2013). DOI: 10.1103/physrevx.3.021004 (cited on pages 11, 12).

[56] James P. Gleeson and Diarmuid J. Cahalane. "Seed size strongly affects cascades on random networks". In: *Physical Review E* 75.5 (2007). DOI: 10.1103/physreve.75.056103 (cited on pages 8, 9, 11).

[57] K.-i. Goh and Albert-László Barabási. "Burstiness and memory in complex systems". In: *Europhysics letters* 81.4 (Jan. 2008), page 48002. DOI: 10.1209/0295-5075/81/48002. URL: https://doi.org/10.1209/0295-5075/81/48002 (cited on page 12).

[58] Ian Goodfellow, Yoshua Bengio, and Aaron Courville. *Deep learning*. Nov. 2016. URL: https://dl.acm.org/citation.cfm?id=3086952 (cited on page 2).

[59] Mark Granovetter. "Threshold Models of Collective Behavior". In: *American Journal of Sociology* 83.6 (1978), pages 1420–1443. DOI: 10.1086/226707 (cited on pages 5–7).

[60] Mark Granovetter. "Economic Action and Social Structure: The Problem of Embeddedness". In: *American Journal of Sociology* 91.3 (Nov. 1985), pages 481–510. DOI: 10.1086/228311. URL: http://dx.doi.org/10.1086/228311 (cited on page 14).

[61] Mark S. Granovetter. "The Strength of Weak Ties". In: *American Journal of Sociology* 78.6 (1973), pages 1360–1380. DOI: 10.1086/225469 (cited on pages 7, 14).

[62] Douglas Guilbeault and Damon Centola. "Topological measures for identifying and predicting the spread of complex contagions". In: *Nature Communications* 12 (2021), page 4430. DOI: 10.1038/s41467-021-24704-6 (cited on page 9).

[63] Adam Hackett and James P. Gleeson. "Cascades on clique-based graphs". In: *Physical Review E* 87.6 (2013). DOI: 10.1103/physreve.87.062801 (cited on page 9).

[64] Adam Hackett, Sergey Melnik, and James P. Gleeson. "Cascades on a class of clustered random networks". In: *Physical Review E* 83.5 (2011). DOI: 10.1103/physreve.83.056107 (cited on page 9).

[65] Rainer Hegselmann. "Thomas C. Schelling and James M. Sakoda: The Intellectual, Technical,and Social History of a Model". In: *Journal of Artificial Societies and Social Simulation* 20.3 (2017), page 15. DOI: 10.18564/jasss.3511 (cited on page 9).

[66] Petter Holme and Jari Saramäki. "Temporal networks". In: *Physics reports* 519.3 (Oct. 2012), pages 97–125. DOI: 10.1016/j.physrep.2012.03.001. URL: https://www.sciencedirect.com/science/article/abs/pii/S0370157312000841 (cited on page 13).

[67] Iacopo Iacopini et al. "Simplicial models of social contagion". In: *Nature Communications* 10.1 (2019). DOI: 10.1038/s41467-019-10431-6 (cited on page 7).

[68] J. L. Iribarren and E. Moro. "Impact of human activity patterns on the dynamics of information diffusion." In: *Physical Review Letters* 103 (2009), page 038702 (cited on page 12).

[69] José Luis Iribarren and Esteban Moro. "Impact of Human Activity Patterns on the Dynamics of Information Diffusion". In: *Physical Review Letters* 103.3 (2009). DOI: 10.1103/physrevlett.103.038702 (cited on page 12).

[70] Michaeline Jensen and Thomas J. Dishion. "Mechanisms and processes of peer contagion". In: *Psychology* (Jan. 2015). DOI: 10.1093/obo/9780199828340-0165. URL: https://doi.org/10.1093/obo/9780199828340-0165 (cited on page 6).

[71] Hang-Hyun Jo et al. “Circadian pattern and burstiness in mobile phone communication”. In: *New journal of physics* 14.1 (Jan. 2012), page 013055. DOI: 10.1088/1367-2630/14/1/013055. URL: https://doi.org/10.1088/1367-2630/14/1/013055 (cited on page 12).

[72] Fariba Karimi and Petter Holme. “Threshold model of cascades in empirical temporal networks”. In: *Physica A: Statistical Mechanics and its Applications* 392.16 (2013), pages 3476–3483. DOI: 10.1016/j.physa.2013.03.050 (cited on page 9).

[73] M. Karsai et al. “Small but slow world: How network topology and burstiness slow down spreading”. In: *Physical Review E* 83.2 (2011). DOI: 10.1103/physreve.83.025102 (cited on page 12).

[74] Márton Karsai. “Computational human dynamics”. In: *arXiv preprint arXiv:1907.07475* (2019) (cited on page 2).

[75] Márton Karsai et al. “Universal features of correlated bursty behaviour”. In: *Scientific reports* 2.1 (May 2012). DOI: 10.1038/srep00397. URL: https://www.nature.com/articles/srep00397 (cited on pages 12, 13).

[76] Márton Karsai et al. “Complex contagion process in spreading of online innovation”. In: *Journal of The Royal Society Interface* 11.101 (2014), page 20140694. DOI: 10.1098/rsif.2014.0694 (cited on page 9).

[77] Márton Karsai et al. “Local cascades induced global contagion: How heterogeneous thresholds, exogenous effects, and unconcerned behaviour govern online adoption spreading”. In: *Scientific Reports* 6.1 (2016). DOI: 10.1038/srep27178 (cited on page 9).

[78] Pinaki Kumar et al. “On interevent time distributions of avalanche dynamics”. In: *Scientific Reports* 10.1 (2020). DOI: 10.1038/s41598-019-56764-6 (cited on page 12).

[79] David Lazer et al. “Computational Social Science”. In: *Science* 323.5915 (Feb. 2009), pages 721–723. DOI: 10.1126/science.1167742. URL: https://doi.org/10.1126/science.1167742 (cited on page 2).

[80] Jose Lejarraga, Tomas Lejarraga, and Charlotte Gaston-Breton. “Let me handle this, I’ve done it before: Experience and self-involvement in superstitious learning”. In: *Academy of Management Proceedings*. Volume 2011. 1. Academy of Management Briarcliff Manor, NY 10510. 2011, pages 1–6 (cited on page 14).

[81] Thomas M Liggett et al. *Stochastic interacting systems: contact, Voter and exclusion processes*. Volume 324. springer science & Business Media, 1999 (cited on pages 5, 14).

[82] Quan-Hui Liu et al. “Impacts of opinion leaders on social contagions”. In: *Chaos: An Interdisciplinary Journal of Nonlinear Science* 28.5 (2018), page 053103. DOI: 10.1063/1.5017515 (cited on page 9).

[83] Antoine Lucquiaud. “Modéliser l’émergence et détecter les structures de la ségrégation sociospatiale”. PhD thesis. Université Paris 1-Panthéon Sorbonne, 2022 (cited on page 11).

[84] James Manyika. *Big data: The next frontier for innovation, competition, and productivity*. May 2011. URL: http://abesit.in/wp-content/uploads/2014/07/big-data-frontier.pdf (cited on page 2).

[85] Robert M. May. *Stability and complexity in model ecosystems*. Dec. 2001. DOI: 10.1515/9780691206912. URL: https://doi.org/10.1515/9780691206912 (cited on page 1).

[86] Byungjoon Min and Maxi San Miguel. “Competing contagion processes: Complex contagion triggered by simple contagion”. In: *Scientific Reports* 8.1 (2018). DOI: 10.1038/s41598-018-28615-3 (cited on pages 7, 9).

[87] Giovanna Miritello et al. “Limited communication capacity unveils strategies for human interaction”. In: *Scientific reports* 3.1 (June 2013). DOI: 10.1038/srep01950. URL: https://www.nature.com/articles/srep01950 (cited on page 12).

[88] Bjarke Mønsted et al. “Evidence of complex contagion of information in social media: An experiment using Twitter bots”. In: *PLOS ONE* 12.9 (2017), e0184148. DOI: 10.1371/journal.pone.0184148 (cited on page 9).

[89] Lev Muchnik et al. “Origins of power-law degree distribution in the heterogeneity of human activity in social networks”. In: *Scientific reports* 3.1 (May 2013). DOI: 10.1038/srep01783. URL: https://www.nature.com/articles/srep01783 (cited on page 12).

[90] Kevin Murphy. *Machine Learning : A Probabilistic Perspective*. Aug. 2012. URL: http://cds.cern.ch/record/1981503 (cited on page 2).

[91] M. E. J. Newman. "Spread of epidemic disease on networks". In: *Phys. Rev. E* 66 (1 July 2002), page 016128. DOI: 10.1103/PhysRevE.66.016128. URL: https://link.aps.org/doi/10.1103/PhysRevE.66.016128 (cited on page 7).

[92] Mark Newman. *Networks: An Introduction*. Oxford University Press, Mar. 2010. ISBN: 9780199206650. DOI: 10.1093/acprof:oso/9780199206650.001.0001. URL: https://doi.org/10.1093/acprof:oso/9780199206650.001.0001 (cited on pages 1, 12).

[93] *Online platforms: Economic and societal effects | Panel for the Future of Science and Technology (STOA) | European Parliament*. URL: https://www.europarl.europa.eu/stoa/en/document/EPRS_STU%282021%29656336 (cited on page 6).

[94] Lars Onsager. "Crystal Statistics. I. A Two-Dimensional Model with an Order-Disorder Transition". In: *Physical review* 65.3-4 (Feb. 1944), pages 117–149. DOI: 10.1103/physrev.65.117. URL: https://doi.org/10.1103/physrev.65.117 (cited on page 1).

[95] Romualdo Pastor-Satorras and Alessandro Vespignani. "Epidemic Spreading in Scale-Free Networks". In: *Phys. Rev. Lett.* 86 (14 Apr. 2001), pages 3200–3203. DOI: 10.1103/PhysRevLett.86.3200. URL: https://link.aps.org/doi/10.1103/PhysRevLett.86.3200 (cited on page 7).

[96] Romualdo Pastor-Satorras et al. "Epidemic processes in complex networks". In: *Reviews of Modern Physics* 87.3 (2015), pages 925–979. DOI: 10.1103/revmodphys.87.925 (cited on page 5).

[97] Antonio F. Peralta, Nagi Khalil, and Raúl Toral. "Ordering dynamics in the voter model with aging". In: *Physica A: Statistical Mechanics and its Applications* 552 (2020), page 122475. DOI: 10.1016/j.physa.2019.122475 (cited on pages 14, 15).

[98] Toni Pérez, Konstantin Klemm, and Víctor M. Eguíluz. "Competition in the presence of aging: dominance, coexistence, and alternation between states". In: *Scientific Reports* 6.1 (2016). DOI: 10.1038/srep21128 (cited on page 13).

[99] Nicola Perra et al. "Activity driven modeling of time varying networks". In: *Scientific reports* 2.1 (June 2012). DOI: 10.1038/srep00469. URL: https://www.nature.com/articles/srep00469 (cited on page 13).

[100] Arkady Pikovsky et al. "A universal concept in nonlinear sciences". In: *Self* 2 (2001), page 3 (cited on page 1).

[101] Ilya Prigogine and Gregoire Nicolis. "Self-organization in Non equilibrium Systems". In: *Non-equilibrium system* (1977) (cited on page 1).

[102] Filippo Radicchi. "Human activity in the web". In: *Physical review. E, Statistical, nonlinear and soft matter physics* 80.2 (Aug. 2009). DOI: 10.1103/physreve.80.026118. URL: https://doi.org/10.1103/physreve.80.026118 (cited on page 12).

[103] Mária Ravasz, György Szabó, and Attila Szolnoki. "Spreading of families in cyclic predator-prey models". In: *Physical Review E* 70.1 (2004), page 012901 (cited on page 13).

[104] Sidney Redner. "Reality-inspired Voter models: A mini-review". In: *Comptes Rendus Physique* 20.4 (2019), pages 275–292. DOI: 10.1016/j.crhy.2019.05.004 (cited on page 5).

[105] Luis E. C. Rocha and Vincent D. Blondel. "Bursts of vertex activation and epidemics in evolving networks". In: *PLOS computational biology/PLoS computational biology* 9.3 (Mar. 2013), e1002974. DOI: 10.1371/journal.pcbi.1002974. URL: https://doi.org/10.1371/journal.pcbi.1002974 (cited on page 13).

[106] John P. Roche and John H. Rappole. "The Ecology of Migrant Birds: A Neotropical Perspective." In: *Ecology* 78.1 (Jan. 1997), page 328. DOI: 10.2307/2266005. URL: https://doi.org/10.2307/2266005 (cited on page 1).

[107] Everett M Rogers, Arvind Singhal, and Margaret M Quinlan. "Diffusion of innovations". In: *An integrated approach to communication theory and research*. Routledge, 2014, pages 432–448 (cited on page 6).

[108] Sara Brin Rosenthal et al. "Revealing the hidden networks of interaction in mobile animal groups allows prediction of complex behavioral contagion". In: *Proceedings of the National Academy of Sciences* 112.15 (2015), pages 4690–4695. DOI: 10.1073/pnas.1420068112 (cited on page 9).

[109] Cynthia Rudin. "Stop explaining black box machine learning models for high stakes decisions and use interpretable models instead". In: *Nature machine intelligence* 1.5 (May 2019), pages 206–215. DOI: 10.1038/s42256-019-0048-x. URL: https://doi.org/10.1038/s42256-019-0048-x (cited on page 2).

[110] Diego Rybski et al. "Communication activity in a social network: relation between long-term correlations and inter-event clustering". In: *Scientific Reports* 2.1 (2012). DOI: 10.1038/srep00560 (cited on page 12).

[111] James M Sakoda. "The checkerboard model of social interaction". In: *The Journal of Mathematical Sociology* 1.1 (1971), pages 119–132 (cited on page 9).

[112] JM Sakoda. "Minidoka: An analysis of changing patterns of social behavior". PhD thesis. PhD thesis, University of California, 1949 (cited on page 10).

[113] D. M. Saul, Michael Wortis, and D. Stauffer. "Tricritical behavior of the Blume-Capel model". In: *Phys. Rev. B* 9 (11 June 1974), pages 4964–4980. DOI: 10.1103/PhysRevB.9.4964. URL: https://link.aps.org/doi/10.1103/PhysRevB.9.4964 (cited on page 10).

[114] Thomas Schelling. "Models of Segregation". In: *American Economic Review* 59.2 (1969), page 488 (cited on pages 8, 9).

[115] Thomas C Schelling. "Dynamic models of segregation". In: *The Journal of Mathematical Sociology* 1.2 (1971), pages 143–186. DOI: 10.1080/0022250X.1971.9989794. URL: https://doi.org/10.1080/0022250X.1971.9989794 (cited on page 10).

[116] P. Singh et al. "Threshold-limited spreading in social networks with multiple initiators". In: *Scientific Reports* 3.1 (2013). DOI: 10.1038/srep02330 (cited on page 9).

[117] V. Sood and S. Redner. "Voter Model on Heterogeneous Graphs". In: *Physical Review Letters* 94.17 (2005). DOI: 10.1103/physrevlett.94.178701 (cited on page 5).

[118] H Eugene Stanley. *Phase transitions and critical phenomena*. Volume 7. Clarendon Press, Oxford, 1971 (cited on page 1).

[119] Hans-Ulrich Stark, Claudio J. Tessone, and Frank Schweitzer. "Decelerating Microdynamics Can Accelerate Macrodynamics in the Voter Model". In: *Phys. Rev. Lett.* 101 (1 June 2008), page 018701. DOI: 10.1103/PhysRevLett.101.018701. URL: https://link.aps.org/doi/10.1103/PhysRevLett.101.018701 (cited on pages 13, 14).

[120] Michele Starnini, James P. Gleeson, and Marián Boguñá. "Equivalence between Non-Markovian and Markovian Dynamics in Epidemic Spreading Processes". In: *Physical Review Letters* 118.12 (2017). DOI: 10.1103/physrevlett.118.128301 (cited on page 13).

[121] D. Stauffer and S. Solomon. "Ising, Schelling and self-organising segregation". In: *The European Physical Journal B* 57.4 (2007), pages 473–479. DOI: 10.1140/epjb/e2007-00181-8 (cited on page 10).

[122] Dietrich Stauffer. "A Biased Review of Sociophysics". In: *Journal of Statistical Physics* 151.1-2 (2013), pages 9–20. DOI: 10.1007/s10955-012-0604-9 (cited on page 10).

[123] Dietrich Stauffer and Amnon Aharony. *Introduction to percolation Theory*. Jan. 1985. DOI: 10.4324/9780203211595. URL: https://doi.org/10.4324/9780203211595 (cited on page 1).

[124] Steffen Steinert and Matthew Dennis. "Emotions and Digital Well-Being: on Social Media's Emotional Affordances". In: *Philosophy and Technology* 35.2 (Apr. 2022). DOI: 10.1007/s13347-022-00530-6. URL: https://doi.org/10.1007/s13347-022-00530-6 (cited on page 6).

[125] Samuel Unicomb, Gerardo Iñiguez, and Márton Karsai. "Threshold driven contagion on weighted networks". In: *Scientific Reports* 8.1 (2018). DOI: 10.1038/s41598-018-21261-9 (cited on page 9).

[126] Thomas W. Valente. “Social network thresholds in the diffusion of innovations”. In: *Social Networks* 18.1 (1996), pages 69–89. ISSN: 0378-8733. DOI: https://doi.org/10.1016/0378-8733(95)00256-1. URL: https://www.sciencedirect.com/science/article/pii/0378873395002561 (cited on page 6).

[127] P. Van Mieghem and R. van de Bovenkamp. “Non-Markovian Infection Spread Dramatically Alters the Susceptible-Infected-Susceptible Epidemic Threshold in Networks”. In: *Physical Review Letters* 110.10 (2013). DOI: 10.1103/physrevlett.110.108701 (cited on page 13).

[128] Alexei Vázquez et al. “Modeling bursts and heavy tails in human dynamics”. In: *Physical review. E, Statistical, nonlinear and soft matter physics* 73.3 (Mar. 2006). DOI: 10.1103/physreve.73.036127. URL: https://doi.org/10.1103/physreve.73.036127 (cited on pages 12, 13).

[129] D. Vinkovic and A. Kirman. “A physical analogue of the Schelling model”. In: *Proceedings of the National Academy of Sciences* 103.51 (2006), pages 19261–19265. DOI: 10.1073/pnas.0609371103 (cited on pages 10, 11).

[130] Soroush Vosoughi, Deb Roy, and Sinan Aral. “The spread of true and false news online”. In: *Science* 359.6380 (Mar. 2018), pages 1146–1151. DOI: 10.1126/science.aap9559. URL: https://doi.org/10.1126/science.aap9559 (cited on page 1).

[131] Pu Wang et al. “Understanding the spreading patterns of mobile phone viruses”. In: *Science* 324.5930 (May 2009), pages 1071–1076. DOI: 10.1126/science.1167053. URL: https://doi.org/10.1126/science.1167053 (cited on page 13).

[132] D. J. Watts. “A simple model of global cascades on random networks”. In: *Proceedings of the National Academy of Sciences* 99.9 (2002), pages 5766–5771. DOI: 10.1073/pnas.082090499 (cited on page 8).

[133] Duncan J Watts and Steven H Strogatz. “Collective dynamics of ‘small-world’ networks”. In: *nature* 393.6684 (1998), pages 440–442 (cited on page 5).

[134] Duncan J. Watts. “A twenty-first century science”. In: *Nature* 445.7127 (Jan. 2007), page 489. DOI: 10.1038/445489a. URL: https://doi.org/10.1038/445489a (cited on page 2).

[135] Matteo Zignani et al. “Walls-in-one: usage and temporal patterns in a social media aggregator”. In: *Applied Network Science* 1.1 (2016). DOI: 10.1007/s41109-016-0009-9 (cited on page 12).

A. Vacancy density effect on the Schelling model dynamics

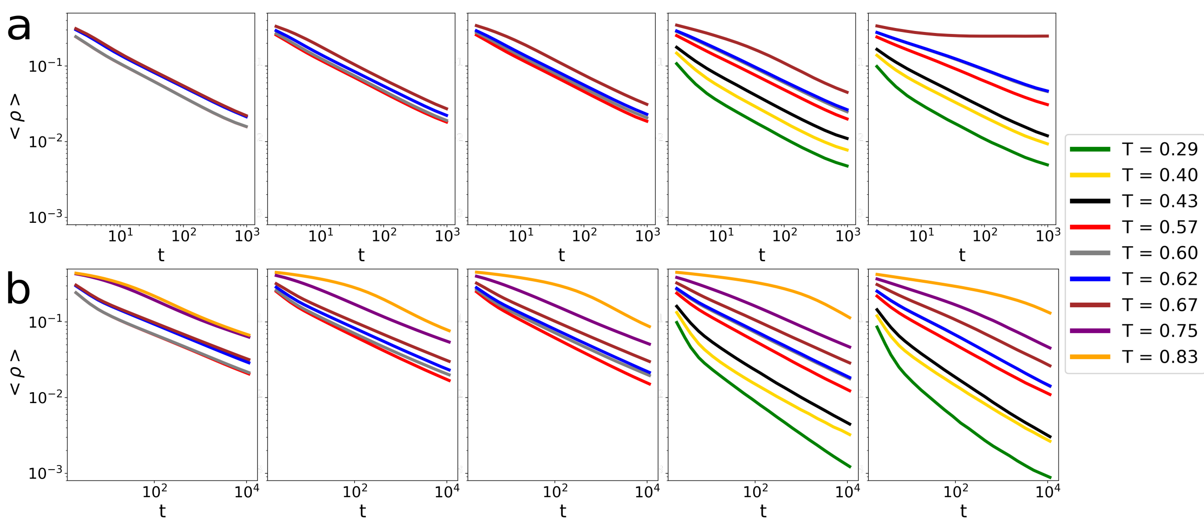

Figure A.1: Average interface density $\langle \rho(t) \rangle$ as a function of time steps for different values of the tolerance parameter T fr the Schelling model (**a**) and the version with aging (**b**). The different plots show a the evolution at a different value of the vacancy density, increasing from left to right $\rho_v = 0.005, 0.15, 0.2, 0.3$ and 0.45. Average performed over 10^3 realisations with system size $100\ x\ 100$.

Since we restrain ourselves to the region $\rho_v < 0.5$, the increase/decrease of the number of vacancies does not change dramatically the behaviour. Above this value, we approach the segregated-dilute transition ($\rho_v \sim 0.62$). Nevertheless, it is worth to mention a few features we observe on the coarsening dynamics. Essentially, when we set a higher vacancy density, the number of agents which see vacancies at their surroundings increases. This results in a family of similar power-law decays towards the segregated state for every meaningful value of T (see Fig. A.1).

Moreover, a higher ρ_v allows us to study the coarsening phenomena for lower values of T according to the phase diagram for the original Schelling model. For those particular cases, when the aging is introduced, we observe a power law decay faster than without aging (Fig. A.1b). Therefore, the aging effect accelerates segregation in this region of the phase diagram, contrary as for lower values of ρ_v. This acceleration is not caused by reaching the 2-clusters state in less time. Since there is a large presence of vacancies, aging causes a formation of vacancy

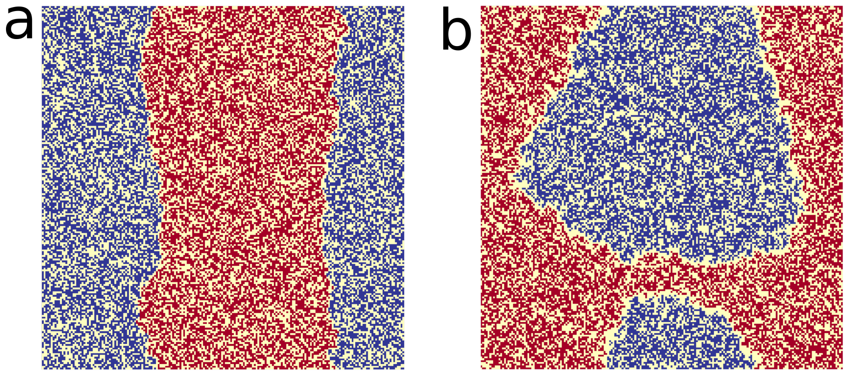

Figure A.2: Snapshots of the system at the final segregated state (after 10^6 MC steps) for the Schelling model (**a**) and the version with (**b**). System size $200\ x\ 200$ with $\rho_v = 0.45$ and $T = 0.29$.

clusters at the interface. Fig. A.2 shows the final segregated state with and without aging. This spontaneous behaviour is result of the low tolerance combined with the persistence of clusters (once formed) due to aging effect and the large number of vacancies that allows the possibility of the formation of clusters at the interface.

In order to quantify this vacancy cluster formation, we define a measure inspired in the segregation coefficient:

$$s_v = \frac{1}{(L^2 \rho_v)^2} \sum_{\{c\}} n_c^2 \tag{A.1}$$

where c is the size of a vacancy cluster and n_c is the number of clusters with size c. The sample average of s_v after reaching equilibrium is called the cluster coefficient of vacancies $\langle s_v \rangle$.

The results of this measure as a function of ρ_v for a few values of T are represented in Fig.A.3 for the Schelling model with and without aging. We observe an increasing dependence of $\langle s_v \rangle$ with ρ_v for both models, but the effect reducing tolerance changes dramatically the behaviour for the case with aging, highlighting the vacancy cluster formation.

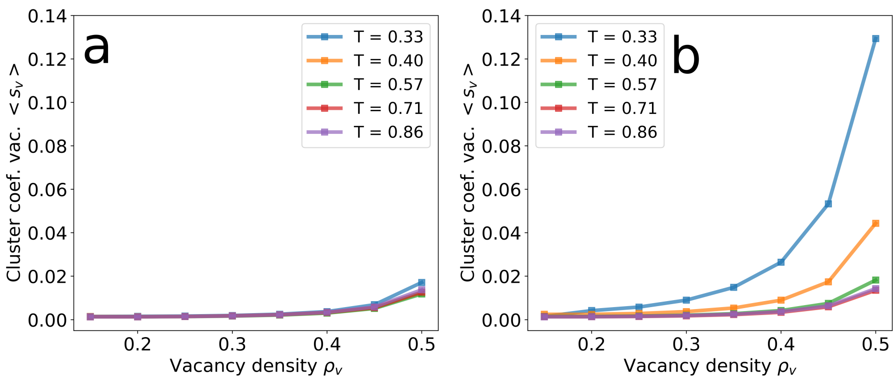

Figure A.3: Cluster coefficient of vacancies as a function of the vacancy density ρ_v for the Schelling model (**a**) and the version with (**b**) for different values of the tolerance T.

B. Heterogeneous mean-field taking into account aging (HMFA)

Setting the time derivatives to 0 in Eqs. (**??**), we obtain the relations for the stationary state:

$$x_{k,0}^{\pm} = \sum_{j=0}^{\infty} x_{k,j}^{\mp}\, \omega_{k,j}^{\mp}, \qquad x_{k,j}^{\pm} = x_{k,j-1}^{\pm}\,(1-\omega_{k,j-1}^{\pm}) \qquad j>0, \tag{B.1}$$

from where we extract the stationary condition $x_{k,0}^{-} = x_{k,0}^{+}$, as in Ref. (31). Notice that by setting $p_A(j) = 1$ and summing over all ages j, we recover the HMF approximation (Eq. **??**) for the model without aging. Defining $x_j^{\pm}(t)$ as the fraction of agents in state ± 1 with age j:

$$x_j^{\pm} = \sum_k p_k\, x_{k,j}^{\pm}, \tag{B.2}$$

and using the degree distribution of a complete graph $p_k = \delta(k-N+1)$ (where $\delta(\cdot)$ is the Dirac delta), we sum over the variable k and rewrite Eq. (B.1) in terms of $x_j^{\pm}$:

$$x_0^{\pm} = \sum_{j=0}^{\infty} x_j^{\mp}\, \omega_j^{\mp}, \qquad x_j^{\pm} = x_{j-1}^{\pm}\,(1-\omega_{j-1}^{\pm}) \qquad j>0, \tag{B.3}$$

where $\omega_j^{\pm} \equiv \omega_{N-1,j}^{\pm}$. Note that the stationary condition $x_0^{-} = x_0^{+}$ remains valid after summing over the degree variable. We compute the solution $x_j^{\pm}$ recursively as a function of $x_0^{\pm}$:

$$x_j^{\pm} = x_0^{\pm}\, F_j^{\pm} \qquad \text{where} \qquad F_j^{\pm} = \prod_{a=0}^{j-1}(1-\omega_a^{\pm}), \tag{B.4}$$

and summing all j,

$$x^{\pm} = x_0^{\pm}\, F^{\pm} \qquad \text{where} \qquad F^{\pm} = 1 + \sum_{j=1}^{\infty} F_j^{\pm}. \tag{B.5}$$

Using the stationary condition $x_0^{-} = x_0^{+}$, we reach:

$$\frac{x^{+}}{x^{-}} = \frac{F^{+}}{F^{-}}. \tag{B.6}$$

Notice that, for the complete graph, $\tilde{x}^{+} = x$, $\tilde{x}^{-} = 1-x$. Therefore, $F^{\pm}$ is a function of the variable $x^{\mp}$ ($F^{+} = F(1-x)$). Thus, we rewrite the previous expression just in terms of the variable x:

$$\frac{x}{1-x} = \frac{F(1-x)}{F(x)}. \tag{B.7}$$

C. Internal time recursive relation in Phase I/I^*

In Phase I and I^*, the exceeding threshold condition ($m/k > T$) is full-filled for almost all agents in the system. Thus, agents will change their state and reset the internal time once activated. For the original model, all agents are activated once in a time step on average, but for the model with aging, the activation probability plays an important role. We consider here a set of N agents that are activated randomly with an activation probability $p_A(j)$ and, once activated, they reset their internal time. Being $n_i(t)$ the fraction of agents with internal time i at the time step t, we build a recursive relation for the previously described dynamics in terms of variables i and t:

$$n_1(t) = \sum_{i=1}^{t-1} p_A(i) n_i(t-1) \qquad n_i(t) = (1 - p_A(i-1)) n_{i-1}(t-1) \qquad i > 1. \tag{C.1}$$

This recursion relation can be solved numerically from the initial condition ($n_1(0) = 1$, $n_i(0) = 0$ for $i > 1$). To obtain the mean internal time at time t, we just need to compute the following:

$$\bar{\tau}(t) = \sum_{i=1}^{t} i n_i(t). \tag{C.2}$$

The solution from this recursive relation describes the mean internal time dynamics with great agreement with the numerical simulations performed at Phase I (for the complete graph) and Phase I^* (for the Erdős-Rényi and Moore lattice).