FOCS HW11

(1)

$p \to q$
$p \to r$
prove $p \to (q \text{ AND } r)$

1. Assume
2. $p \to q$
3. $p \to r$
4. q (2, MP)
5. r (3, MP)
6. $q \wedge r$ (conjuction)

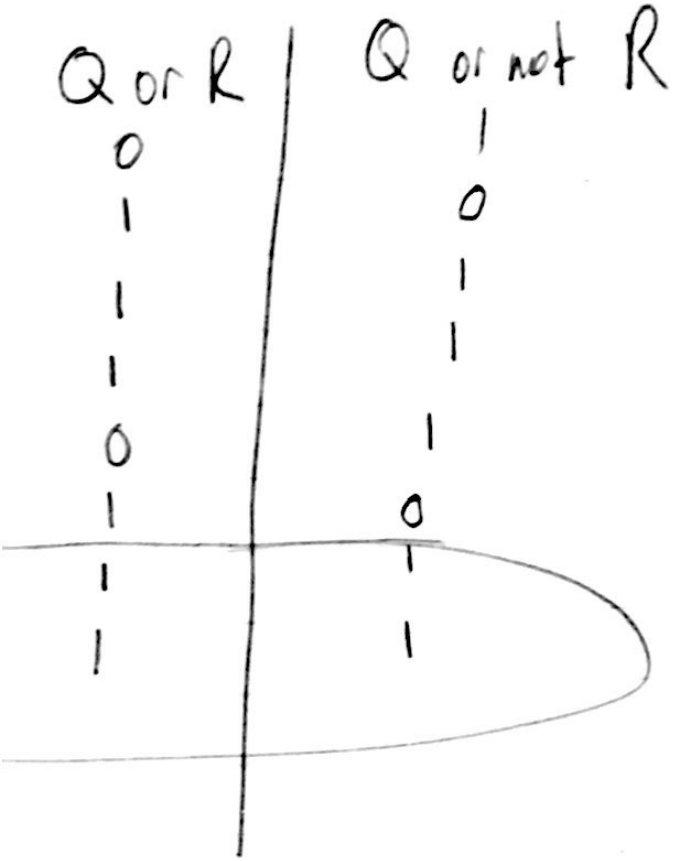
(2)

$p \to Q$ OR R
$p \to Q$ OR NOT R
Prove $p \to q$

1. Assume P
2. Q or R (MP)
3. Q or not R (MP)
4. (Q or R) $\wedge$ (Q or not R) (conjuctions)
5. Q or (R $\wedge$ not R) (4, Dist)
6. Q or F (5, negation)
7. Q (6, identity)

(3)

P	Q	R	Q AND R
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



q AND r → p OR q

Tautology

r	p OR q

q	r	(p → q) AND (q → r)	p → r	(p → q) AND (q → r) → p → r

③

p	q	p → q	(p → q) → p
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	1

Satisfiable

④

nd
- P or Q or R — cond 1
- ~P or ~Q or ~R — cond 2
- P or ~Q — cond 3
- Q or ~R — cond 4

r — r OR ~P — cond 5

P	Q	R	1	2	3	4	5	overall
0	0	0	0	1	1	1	1	1
0	0	1	1	1	1	0	1	1
0	1	0	1	1	0	1	1	1
0	1	1	1	1	0	1	1	1
1	0	0	1	1	1	1	0	1
1	0	1	1	1	1	0	1	1
1	1	0	1	1	1	1	0	1
1	1	1	1	0	1	1	1	1

tautology

④ $s \to t = (s \wedge t) \vee \sim s$

$\sim(r \wedge ((s \wedge t) \vee \sim s))$

$\sim r \vee \sim((s \wedge t) \vee \sim s)$

$\sim r \vee (\sim(s \wedge t) \wedge s)$

$\sim r \vee ((\sim s \vee \sim t) \wedge s)$

$p \vee (q \wedge (\sim r \vee ((\sim s \vee \sim t) \wedge s)))$

⑤ p, and any values for the rest, evaluates to True, ∴ Satisfiable