Ejercicio 80 Análisis

69

g) $f(z) = \frac{1}{z(z+2)}$ en $D = \{ z \,/\, 0 < |z| < 2 \}$

$$f(z) = \frac{1}{z(z+2)} \quad \text{en} \quad D: |z| < 2$$

$$\frac{1}{z} \cdot \frac{1}{z+2} = \frac{1}{z} \cdot \frac{1}{2} \frac{1}{\frac{z}{2}+1} = \frac{1}{z} \cdot \frac{1}{2} \cdot \frac{1}{1+\frac{z}{2}} = \frac{1}{z} \cdot \frac{1}{2} \cdot \sum_{0}^{\infty} (-1)^n \frac{z^n}{2^n}$$

$$\frac{z}{2} < 1 \qquad = \sum_{0}^{\infty} (-1)^n \frac{z^{n-1}}{2^{n+1}}$$

$$z < 2 \qquad = \frac{1}{2z} - \frac{1}{4} + \frac{z}{8} - \frac{z^2}{16}$$

h) $f(z) = \frac{e^z}{z^3}$ en $D = \{ z \,/\, |z| > 0 \}$

$$f(z) = \frac{e^z}{z^3} \quad \text{en} \quad D: |z| > 0$$

$$= \frac{1}{z^3} \cdot e^z = \frac{1}{z^3} \cdot \sum_{0}^{\infty} \frac{z^n}{n!} = \sum_{0}^{\infty} \frac{z^{n-3}}{n!}$$

$$= \frac{1}{z^3} + \frac{1}{z^2} + \frac{1}{2z} + \frac{1}{3!} + \frac{z}{4!} + \frac{z^2}{5!} + \dots$$

i) $f(z) = \dfrac{e^z - 1}{z^2}$ en $D = \{ z \,/\, |z| > 0\}$

$$f(z) = \frac{e^z - 1}{z^2} \quad \text{en} \quad D: |z| > 0$$

$$= \frac{1}{z^2} e^z - \frac{1}{z^2} = \frac{1}{z^2} \sum_0^\infty \frac{z^n}{n!} - \frac{1}{z^2} = \sum_0^\infty \frac{z^{n-2}}{n!} - \frac{1}{z^2}$$

$$= \frac{1}{z^2} - \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2} + \frac{z}{3!} + \frac{z^2}{4!} + \cdots$$

$$= \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \frac{z^2}{4!} \cdots$$

80- En cada caso encuentre la solución del problema con el valor inicial dado

a) $y' - y = 2x\, e^{2x}$ $\quad y(0) = 1$

b) $y' + 2y = x\, e^{-2x}$ $\quad y(1) = 0$

c) $x\, y' + 2\, y = x^2 - x + 1$ $\quad y(1) = ½, \quad x > 0$

d) $y' + \frac{2}{x}\, y = \frac{\cos x}{x^2}$ $\quad y(\pi) = 0, \quad x > 0$

e) $x\, y' + 2\, y = \text{sen}\, x$ $\quad y(\frac{\pi}{2}) = 1$

$y' - y = 2x e^{2x}$ $\quad P(x) = -1 \quad -\int P(x) = -\int -1\,dx$

$= x$

$Q(x) = 2x e^{2x}$

$\varphi(x) = e^{x} \int e^{-x} \cdot 2x e^{2x}\,dx + C$

$\varphi(x) = e^{x} \int \frac{2x e^{2x}}{\cancel{e^{x}}}\,dx + C$

$\varphi(x) = e^{x} \int 2x e^{x}\,dx + C$

$u = x \quad dv = 2e^{x}$

$du = 1 \quad v = 2e^{x}$

$\varphi(x) = e^{x} \left[x2e^{x} - \int 2e^{x}\,dx + C \right]$

$\varphi(x) = e^{x} \left[x2e^{x} - 2e^{x} + C \right]$

$1 = e^{0} \left[0 - 2 + C \right]$

$1 = -2 + C$

$-1 = C$

$\varphi(x) = e^{x} \left[x2e^{x} - 2e^{x} - 1 \right]$

$\varphi(x) = e^{x} x2e^{x} - e^{x}2e^{x} - e^{x}$

$\varphi(x) = 2x e^{2x} - 2e^{2x} - e^{x}$

$\varphi(x) = e^{2x}(x - 1)2 - e^{x}$

78- Encuentre la solución general de las siguientes ecuaciones, de variables separables y homogéneas:

a) $y' = \operatorname{sen} x + e^{x} - 5x$
b) $y' = -x/y$
c) $y' = 2x^2 y$
d) $y' = y/(1+x^2)$
e) $y' = (y-x)/(y+x)$
f) $y' = y/x + (x^2+y^2)/x^2$
g) $y' = (x^3+y^3)/(xy^2)$
h) $y' = x^2/(1-y^2)$
i) $y' = (x+3y)/(x-y)$
j) $y' = \cos^2 x \cos^2 y$
k) $y' = x^2/(1+y^2)$
l) $y' = (3x^2+4x+2)/2(y-1)$
m) $y' = (y\cos x)/(1+2y^2)$
n) $y' = 1+x^2+y^2+x^2y^2$
o) $y' + y^2 \operatorname{sen} x = 0$
p) $y' = x^2/y$
q) $y' = x^2/y(1+x^3)$
r) $y' = (x^2+3y^2)/(2xy)$

j) $y' = \cos^2 x \cos^2 y$

$$\frac{dy}{dx} = \cos^2 x \cos^2 y$$

$$\frac{1}{\cos^2 y}\,dy = \cos^2 x\,dx$$

$$\int \frac{1}{\cos^2 y}\,dy = \int \cos^2 x\,dx$$

$$\tan y + c = \frac{1}{2}x + \frac{\cos x \operatorname{sen} x}{2} + C$$

$$\tan y = \frac{1}{2}(x + \cos x \operatorname{sen} x) + c$$

r) $y' = (x^2+3y^2)/(2xy)$

$$\frac{2xy\frac{dy}{dx}}{x^2} = \frac{x^2}{x^2}+\frac{3y^2}{x^2}$$

$$2\frac{y}{x}\frac{dy}{dx} = 1+\frac{3y^2}{x^2} \qquad v=\frac{y}{x} \qquad v\cdot x = y$$

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

No Entiendo Porque no es v

Rta: Es Porque v no es Constante y depende de x tambien

$$2v\left(v + x\frac{dv}{dx}\right) = 1+3v^2$$

$$\cancel{2}v^2 + 2vx\frac{dv}{dx} = 1+\cancel{3}v^2$$

$$\frac{\cancel{2v}x\frac{dv}{dx}}{\cancel{2v}} = \frac{1+v^2}{2v} = x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\int \frac{2v}{1+v^2}dv = \ln|x|+c$$

$u = 1+v^2$

$du = 2v\,dv$

$$\int\frac{du}{u} = \ln|x|+c$$

$$\ln|1+v^2| = \ln|x|+c \qquad e^{a+b} = e^a\cdot e^b$$

$$e^{\ln|1+v^2|} = e^{\ln|x|+c} = 1+v^2 = e^{\ln(x)+c}$$

$$1+v^2 = e^{\ln|x|}\cdot e^c \qquad e^c = k$$

$$1+v^2 = x\cdot k \qquad v=\frac{y}{x}$$

$$1+\frac{y^2}{x^2} = x\cdot k$$

$$x^2+y^2 = x^3\cdot k$$

Raices

$$W_k = r^{1/n}\left(\cos\left(\frac{\theta+2\pi k}{n}\right) + i\,\text{sen}\left(\frac{\theta+2\pi k}{n}\right)\right)$$

Serie de Laurent

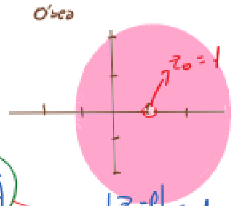
lunes, 17 de junio de 2024 15:40

1- Encuentre la serie de Laurent en una vecindad de z = 1 de $f(z)=\dfrac{e^z}{(z-1)}+\dfrac{1}{(4-z)}$. Indique el dominio de convergencia. Clasifique las singularidades de la función. Encuentre el residuo de *f* en las singularidades.

Desarrollo en $z_0 = 1$

ósea $|z-1|>0$

Ósea



$$f(z) = e\,\frac{e^{z}e^{-1}}{(z-1)} + \frac{1}{(3+1-z)}$$

$$= e\,\frac{e^{z-1}}{(z-1)} + \frac{1}{3}\cdot\frac{1}{1-\left(\frac{z-1}{3}\right)} \rightarrow \left|\frac{z-1}{3}\right|<1$$

Paraque quede en Potencias de $z-1$

$$f(z) = e\,\frac{1}{z-1}\sum_{0}^{\infty}\frac{(z-1)^k}{k!} + \frac{1}{3}\sum_{0}^{\infty}\left(\frac{z-1}{3}\right)^k$$

$$= e\sum_{0}^{\infty}\frac{(z-1)^{k-1}}{k!} + \sum_{0}^{\infty}\frac{(z-1)^k}{3^{k+1}} \qquad 0<|z-1|<3$$

Puede ser usada cualquier vecindad de $z=1$

Ecuaciones Diferenciales

lunes, 17 de junio de 2024 17:35

$y'\cos x - y\,\text{sen}\,x = \cos x$ Tal que $\varphi(0)=1$

$y' - y\dfrac{\text{sen}\,x}{\cos x} = 1$

$\varphi(x) = e^{-\int \frac{\text{sen}\,x}{\cos x}dx}\left[\int e^{\int \frac{\text{sen}\,x}{\cos x}dx}dx + C\right]$

$\varphi(x) = e^{-\ln(\cos(x))}\left[\int e^{\ln(\cos(x))}dx + C\right]$

$\varphi(x) = \dfrac{1}{e^{\ln(\cos x)}}\left[\int \cos(x)\,dx + C\right]$

$$\boxed{\varphi(x) = \frac{1}{\cos(x)}\left[\text{sen}(x) + C\right]}$$

$\varphi(0) = \dfrac{1}{\cos(0)}\left(\text{sen}(0) + C\right) = 1$

$\varphi(0) = 1(0 + C) = \boxed{C = 1}$

$\int \dfrac{\text{sen}(x)}{\cos(x)}dx =$

$\int \dfrac{1}{\cos(x)}\cdot \text{sen}(x)dx$

$u = \cos(x)$

$du = -\text{sen}(x)\,dx$

$-\int \dfrac{1}{u}\cdot du = -\ln(u)$

$= -\ln(\cos(x))$

$$y' = 2xe^{-y} + 3x^2 e^{-y}$$

$$y' = xe^{-y}(2 + 3x)$$

$$y' = x\frac{1}{e^y}(2 + 3x)$$

$$\frac{dy}{dx}e^y = x(2+3x)$$

$$\int e^y dy = \int x(2+3x)dx \Rightarrow \boxed{e^y = x^2 + x^3 + C}$$

f) $f(z) = \dfrac{z}{(z-1)(2-z)}$ en *i)* $D_1 = \{ z \,/\, 1 < |z| < 2 \}$ *ii)* $D_2 = \{ z \,/\, |z| > 2 \}$

iii) $D_3 = \{ z \,/\, |z-1| > 1 \}$ *iv)* $D_4 = \{ z \,/\, 0 < |z-2| < 1 \}$

v) $D_5 = \{ z \,/\, |z| < 1 \}$

$$f(z) = \frac{z}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2} = \frac{A(z-2) + B(z-1)}{(z-1)(z-2)}$$

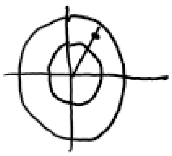
$$z = A(z-2) + B(z-1)$$

$$z = 2 \Rightarrow 2 = B \qquad z = 1 \Rightarrow 1 = -A \Rightarrow A = -1$$

$$f(z) = \frac{z}{(z-1)(z-2)} = -\frac{1}{z-1} + 2\frac{1}{z-2}$$

$$f(z) = -\frac{1}{(z-1)} + 2\frac{1}{(z-2)} \quad \text{en } D: 1 < |z| < 2$$

$$= -\frac{1}{z}\frac{1}{1-\frac{1}{z}} - \frac{1}{1-\frac{z}{2}} = -\sum_{0}^{\infty} \frac{1}{z^{n+1}} - \sum_{0}^{\infty} \frac{z^n}{2^n} =$$



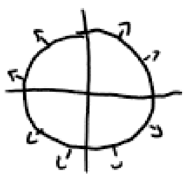
$\left|\frac{1}{z}\right| < 1$

$|z| > 1$

$|z| < 2$

$$-\frac{1}{z} - 1 - \frac{1}{z^2} - \frac{z}{2} - \frac{1}{z^3} - \frac{z^2}{4}$$

$$f(z) = \frac{z}{(z-1)(z-2)} = -\frac{1}{(z-1)} + 2\frac{1}{z-2} \quad \text{en } D: |z| > 2$$



$$-\frac{1}{z}\frac{1}{1-\frac{1}{z}} + 2\frac{1}{z}\frac{1}{1-\frac{2}{z}} = -\frac{1}{z}\sum_{0}^{\infty} \frac{1}{z^n} + \frac{2}{z}\sum_{0}^{\infty} \frac{2^n}{z^n}$$

$$-\sum_{0}^{\infty} \frac{1}{z^{n+1}} + \sum \frac{2^{n+1}}{z^{n+1}} = \sum_{0}^{\infty} \frac{1}{z^{n+1}}\left(2^{n+1} - 1\right)$$

$$f(z) = \frac{z}{(z-1)(z-2)} = -\frac{1}{z-1} + 2\,\frac{1}{z-2} \quad \text{en} \quad |z-1| > 1$$

$$-\frac{1}{z-1} + 2\,\frac{1}{(z-1)-1} = -\frac{1}{z-1} - 2\,\frac{1}{1-(z-1)} = -\frac{1}{z-1} + \frac{2}{z-1}\cdot\frac{1}{1-\frac{1}{z-1}} \qquad \left|\frac{1}{z-1}\right| < 1$$

$$|z-1| > 1$$

$$= -\frac{1}{z-1} + \frac{2}{z-1}\cdot\sum_0^\infty \frac{1}{(z-1)^n} = -\frac{1}{z-1} + \sum_0^\infty \frac{2}{(z-1)^{n+1}}$$

$$\frac{1}{z-1} + \frac{2}{(z-1)^2} + \frac{2}{(z-1)^3} + \dots$$

$$f(z) = \frac{z}{(z-1)(z-2)} = -\frac{1}{z-1} + \frac{2}{z-2} \quad \text{en} \quad D: 0 < |z-2| < 1$$

$$= \frac{1}{1-(z-1)-1} + \frac{2}{z-2} = \frac{1}{1+(z-2)} + \frac{2}{z-2} = \sum_0^\infty (-1)^n (z-2)^n + \frac{2}{z-2}$$

$$f(z) = \frac{z}{(z-1)(z-2)} = -\frac{1}{z-1} + \frac{2}{z-2} \quad \text{en} \quad D: |z| < 1$$

$$= \frac{1}{1-z} - \frac{1}{1-\frac{z}{2}} = \sum_0^\infty z^n - \sum_0^\infty \frac{z^n}{2^n} = \sum_0^\infty z^n\left(1 - \frac{1}{2^n}\right)$$

$$= \frac{1}{2}z + \frac{3}{4}z^2 + \frac{7}{8}z^3 \dots$$