Ejercicio 80 Análisis

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g) $f(z) = \dfrac{1}{z(z+2)}$ en $D = \{ z \,/\, 0 < |z| < 2 \}$

$f(z) = \dfrac{1}{z(z+2)}$ en $D: |z| < 2$

$$\frac{1}{z} \cdot \frac{1}{z+2} = \frac{1}{z} \cdot \frac{1}{2} \cdot \frac{1}{\frac{z}{2}+1} = \frac{1}{z} \cdot \frac{1}{2} \cdot \frac{1}{1+\frac{z}{2}} = \frac{1}{z} \cdot \frac{1}{2} \cdot \sum_{0}^{\infty} (-1)^n \frac{z^n}{2^n}$$

$$\frac{z}{2} < 1 \qquad = \sum_{0}^{\infty} (-1)^n \frac{z^{n-1}}{2^{n+1}}$$

$$z < 2 \qquad = \frac{1}{2z} - \frac{1}{4} + \frac{z}{8} - \frac{z^2}{16}$$

h) $f(z) = \dfrac{e^z}{z^3}$ en $D = \{ z \,/\, |z| > 0 \}$

$f(z) = \dfrac{e^z}{z^3}$ en $D: |z| > 0$

$$= \frac{1}{z^3} \cdot e^z = \frac{1}{z^3} \cdot \sum_{0}^{\infty} \frac{z^n}{n!} = \sum_{0}^{\infty} \frac{z^{n-3}}{n!}$$

$$= \frac{1}{z^3} + \frac{1}{z^2} + \frac{1}{2z} + \frac{1}{3!} + \frac{z}{4!} + \frac{z^2}{5!} + \dots$$

i) $f(z) = \dfrac{e^z - 1}{z^2}$ en $D = \{ z \,/\, |z| > 0 \}$

$$f(z) = \frac{e^z - 1}{z^2} \quad \text{en} \quad D: |z| > 0$$

$$= \frac{1}{z^2} e^z - \frac{1}{z^2} = \frac{1}{z^2} \sum_0^\infty \frac{z^n}{n!} - \frac{1}{z^2} = \sum_0^\infty \frac{z^{n-2}}{n!} - \frac{1}{z^2}$$

$$= \frac{1}{z^2} - \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2} + \frac{z}{3!} + \frac{z^2}{4!} + \cdots$$

$$= \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \frac{z^2}{4!} \cdots$$

80- En cada caso encuentre la solución del problema con el valor inicial dado

a) $y' - y = 2x\, e^{2x}$	$y(0) = 1$
b) $y' + 2y = x\, e^{-2x}$	$y(1) = 0$
c) $x\,y' + 2\,y = x^2 - x + 1$	$y(1) = ½, \quad x > 0$
d) $y' + \frac{2}{x}\, y = \frac{\cos x}{x^2}$	$y(\pi) = 0, \quad x > 0$
e) $x\,y' + 2\,y = \text{sen}\, x$	$y(\frac{\pi}{2}) = 1$

$y' - y = 2xe^{2x}$ $\qquad P(x) = -1 \qquad -\int P(x) = -\int -1\,dx$

$= x$

$Q(x) = 2xe^{2x}$

$\varphi(x) = e^{x} \int e^{-x} \cdot 2xe^{2x}\,dx + C$

$\varphi(x) = e^{x} \int \frac{2xe^{2x}}{\cancel{e^{x}}}\,dx + C$

$\varphi(x) = e^{x} \int 2xe^{x}\,dx + C$

$u = x \quad dv = 2e^{x}$

$du = 1 \quad v = 2e^{x}$

$\varphi(x) = e^{x}\left[x2e^{x} - \int 2e^{x}\,dx + C\right]$

$\varphi(x) = e^{x}\left[x2e^{x} - 2e^{x} + C\right]$

$1 = e^{0}\left[0 - 2 + C\right]$

$1 = -2 + C$

$-1 = C$

$\varphi(x) = e^{x}\left[x2e^{x} - 2e^{x} - 1\right]$

$\varphi(x) = e^{x}x2e^{x} - e^{x}2e^{x} - e^{x}$

$\varphi(x) = 2xe^{2x} - 2e^{2x} - e^{x}$

$\varphi(x) = e^{2x}(x-1)2 - e^{x}$

78- Encuentre la solución general de las siguientes ecuaciones, de variables separables y homogéneas:

a) $y' = \operatorname{sen} x + e^{x} - 5x$
b) $y' = -x / y$
c) $y' = 2x^2 y$
d) $y' = y / (1 + x^2)$
e) $y' = (y - x) / (y + x)$
f) $y' = y / x + (x^2 + y^2) / x^2$
g) $y' = (x^3 + y^3) / (x y^2)$
h) $y' = x^2 / (1 - y^2)$
i) $y' = (x + 3y) / (x - y)$
j) $y' = \cos^2 x \cos^2 y$
k) $y' = x^2 / (1 + y^2)$
l) $y' = (3x^2 + 4x + 2) / 2(y - 1)$
m) $y' = (y \cos x) / (1 + 2y^2)$
n) $y' = 1 + x^2 + y^2 + x^2 y^2$
o) $y' + y^2 \operatorname{sen} x = 0$
p) $y' = x^2 / y$
q) $y' = x^2 / y(1 + x^3)$
r) $y' = (x^2 + 3y^2) / (2xy)$

j) $y' = \cos^2 x \cos^2 y$

$$\frac{dy}{dx} = \cos^2 x \cos^2 y$$

$$\frac{1}{\cos^2 y} dy = \cos^2 x \, dx$$

$$\int \frac{1}{\cos^2 y} dy = \int \cos^2 x \, dx$$

$$\tan y + c = \frac{1}{2} x + \frac{\cos x \operatorname{sen} x}{2} + C$$

$$\tan y = \frac{1}{2}(x + \cos x \operatorname{sen} x) + c$$

r) $y' = (x^2+3y^2)/(2xy)$

$$\frac{2xy\frac{dy}{dx}}{x^2} = \frac{x^2}{x^2}+\frac{3y^2}{x^2}$$

$$2\frac{y}{x}\frac{dy}{dx} = 1+\frac{3y^2}{x^2}$$

$v = \frac{y}{x}$ $\quad v \cdot x = y$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

No Entiendo Porque no es v

Rta: Es Porque v no es constante y depende de x tambien

$$2v\left(v+x\frac{dv}{dx}\right) = 1+3v^2$$

$$\cancel{2}v^2+2vx\frac{dv}{dx} = 1+\cancel{3}v^2$$

$$\frac{\cancel{2v}x\frac{dv}{dx}}{\cancel{2v}} = \frac{1+v^2}{2v} = x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\int\frac{2v}{1+v^2}dv = \ln|x|+c$$

$u = 1+v^2$

$du = 2v\,dv$

$$\int\frac{du}{u} = \ln|x|+c$$

$$\ln|1+v^2| = \ln|x|+c$$

$e^{a+b} = e^a \cdot e^b$

$$e^{\ln|1+v^2|} = e^{\ln|x|+c} = 1+v^2 = e^{\ln(x)+c}$$

$$1+v^2 = e^{\ln|x|}\cdot e^c$$

$e^c = k$

$$1+v^2 = x\cdot k$$

$v = \frac{y}{x}$

$$1+\frac{y^2}{x^2} = x\cdot k$$

$$x^2+y^2 = x^3\cdot k$$