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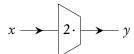
How I Think About Math Part I: Linear Algebra

David Dalrymple davidad@alum.mit.edu

March 6, 2014

Chapter 1: Relations 1 Relations Labels Composing Joining Inverting Commuting Subspaces Image & Coimage Decomposition Kernel & Cokernel 4日) 4周) 4日) 4日)

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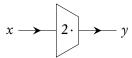
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Relations are a generalization of functions; they're actually more like constraints. Here's an example:



$$y(x) = 2 \cdot x$$

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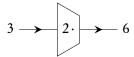
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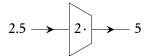
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$$6 = 2 \cdot 3$$

Relations are a generalization of functions; they're actually more like constraints. Here's an example:



This might be more familiar to you as the equation:

$$5 = 2 \cdot 2.5$$

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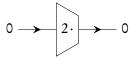
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$$0 = 2 \cdot 0$$

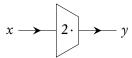
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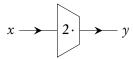


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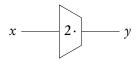


Really, the directional annotations on the arrows are just that: annotations.

$$y(x) = 2 \cdot x$$

Relations

Relations are a generalization of functions; they're actually more like constraints. Here's an example:



Really, the directional annotations on the arrows are just that: annotations. Only the directionality of the operator "2." is significant.

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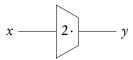


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Analogously, writing y(x) is just politics: "x gets to tell y what to do!"

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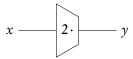
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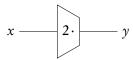
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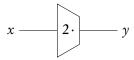
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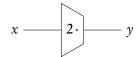
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• Like the arguments of a subroutine, the labels of a relation are just a convenient "interface" for connecting it to a context or environment.

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- Like the arguments of a subroutine, the labels of a relation are just a convenient "interface" for connecting it to a context or environment.
- If a label isn't serving that purpose, we can remove it.

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This is way easier than composing functions.

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This is way easier than composing functions.

We just stick them together.

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2. 1+

This is way easier than composing functions.

We just stick them together.

Sticking relations together like this will always give you a relation.

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What does this mean?

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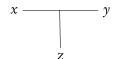
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What does this mean?



You could think of it as:

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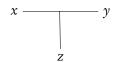
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What does this mean?



You could think of it as:

$$x = y$$

$$x = z$$

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What does this mean?



You could think of it as:

$$x = y$$

$$y = x$$

or

$$x = z$$

$$y = z$$

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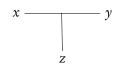
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What does this mean?



You could think of it as:

$$x = y$$

or

$$x = z$$

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What does this mean?



You could think of it as:

$$x = y$$
 $y = x$ $x = z$ or $z = y$

They're all the same! But with complex joins, this is easier to see in pictures.

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You could think of it as:

$$x = y$$
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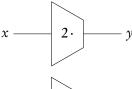
They're all the same! But with complex joins, this is easier to see in pictures.

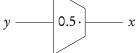
Relations with more than two "sides" (like this) are sometimes called

systems of equations.

But I find a single 3-sided relation more intuitive than a "system" of two equations.

Let's write "multiplication by 0.5 is the inverse of multiplication by 2."





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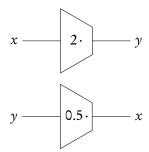
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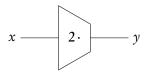


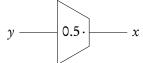
Note: This is like the system of equations

$$y = 2 \cdot x$$

$$x = 0.5 \cdot y$$

Let's write "multiplication by 0.5 is the inverse of multiplication by 2."





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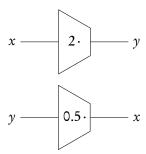
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We can turn the bottom diagram around,

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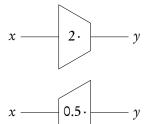
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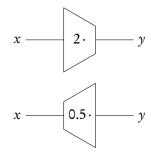
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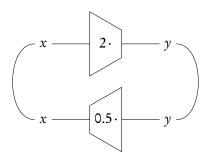
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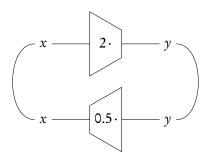
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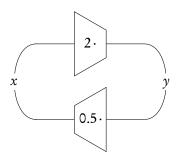
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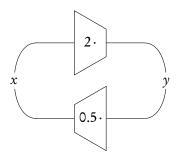
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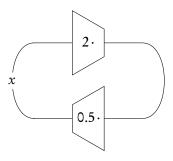
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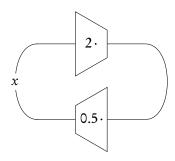
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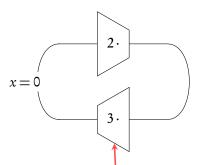
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- The meaning is still imprecise. Even this is valid if *x* happens to be 0.

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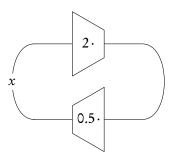
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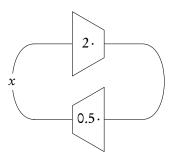
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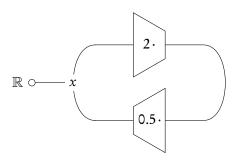
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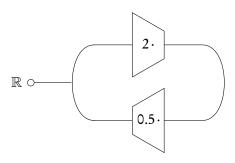
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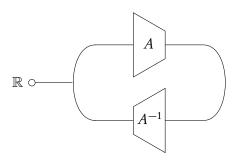
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Let's write " A^{-1} is the inverse of A over \mathbb{R} ."



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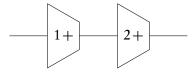
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CP decomposition

If we can reverse the order of two operators and get equal results, we say that they **commute**.



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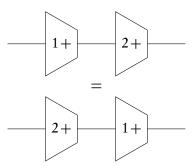
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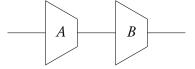
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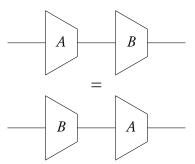
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If we can reverse the order of two operators *A* and *B* and get equal results, we say that *A* and *B* **commute**.



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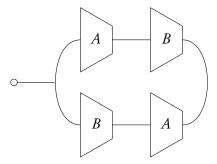
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If we can reverse the order of two operators A and B and get equal results, we say that *A* and *B* commute. We can express "*A* and *B* commute" like this:



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