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Innovative Applications of O.R.

Dynamic cruise ship revenue management

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ABSTRACT

Recently, it has been recognized that revenue management of cruise ships is different from that of airlines or hotels. Among the main differences is the presence of multiple capacity constraints in cruise ships, i.e., the number of cabins in different categories and the number of lifeboat seats, versus a single constraint in airlines and hotels (i.e., number of seats or rooms). We develop a discrete-time dynamic capacity control model for a cruise ship characterized by multiple constraints on cabin and lifeboat capacities. Customers (families) arrive sequentially according to a stochastic process and request one cabin of a certain category and one or more lifeboat seats. The cruise ship revenue manager decides which requests to accept based on the remaining cabin and lifeboat capacities at the time of an arrival as well as the type of the arrival. We show that the opportunity cost of accepting a customer is not always monotone in the reservation levels or time. This non-monotone behavior implies that "conventional" booking limits or critical time periods capacity control policies are not optimal. We provide analysis and insights justifying the non-monotone behavior in our cruise ship context. In the absence of monotonicity, and with the optimal solution requiring heavy storage for "large" (industry-size) problems, we develop several heuristics and thoroughly test their performance, via simulation, against the optimal solution, well-crafted upper bounds, and a first-come first-served lower bound. Our heuristics are based on rolling-up the multi-dimensional state space into one or two dimensions and solving the resulting dynamic program (DP). This is a strength of our approach since our DP-based heuristics are easy to understand, solve and analyze. We find that single-dimensional heuristics based on decoupling the cabins and lifeboat problems perform quite well in most cases.

1. Introduction

According to the Business Research and Economic Advisors report [5], the cruise industry in the US has experienced an average annual passenger growth of around 4.5% and totalled 9 million passengers in 2006. The cruise line industry represents one of the fastest growing segments within the US leisure market with an average passenger annual growth of 8.2% since 1980 (Cruise Lines International Association [8]). In the UK, the growth in cruise industry is also reported to be the fastest in the holiday market since 1996 with an estimated capacity increase of 37.8% between 2000 and 2003 (Hoseason [12]). Despite these facts, cruise lines revenue management (RM) has received limited research attention unlike the extensive research on airline passenger RM.

The limited research on cruise RM has identified key differences between cruise ship revenue management and the widely studied airlines and hotel revenue management. The following list, compiled from Biehn [4], Ji and Mazarella [13], and Talluri and van Ryzin [25] (pp. 560–561), briefly presents some of these differences.

- **Guest pricing**. Unlike hotels, cruise ships do not charge on the basis of cabins (rooms) but rather per guest. The base fare is for a double occupancy for a cabin occupied by two passengers (a couple). Additional guests in groups of three or more occupying a cabin (e.g. children in a family) pay a lower fare per person.
- **Multiple capacity constraints**. Cruise ships have two types of capacity constraints in terms of number of *cabins and lifeboats* seats versus a single type of constraint in airlines and hotels (number of seats or rooms). Biehn [4] argues that lifeboat capacity is a critical factor in cruise revenue management. For example, there can be a significant loss of revenue if too many large families reserve early, and the ship runs out of lifeboat capacity before it sells out cabins. Biehn [4] supports this argument by reporting that several cruise ships are being restricted to accept no more than 2.5 or 3 passengers per cabin,

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on average, due to lifeboat capacity limitations. He also quotes several cruise ship managers highlighting a recent market trend of having more large families on-board. In addition, the tangible distinction between cabin categories (see below) leads to a capacity constraint for each cabin.

- **Tangible distinction between cabin categories with limited pricing variation and demand substitution**. A cruise ship is divided into multiple cabin categories (e.g. Interior, Ocean View, Balcony, and Suite), which are differentiated based on tangible characteristics such as view and room space. In addition, the pricing variation between categories with close characteristics is limited. This may lead to demand substitution when a cabin category is sold out.
- **Leisure passengers with a high show-up rate**. Most cruise ship passengers are leisure customers who book early and are not allowed to cancel their reservation close to departure. Ji and Mazzarella [13] report that the deadline for getting a full-refund from a cancelation predates departure by two months at least. This leads to a high customer show-up rate (with a cancellation rate less than 1% according to Ji and Mazzarella [13]). (Recall that an important segment of airline and hotel customers are business passengers who pay higher fares, book late, and are not penalized heavily for cancellations and no-shows.) The high show-up rate and the difficulty to handle an overbooked passenger seem to be behind the practice of not allowing overbooking for the cruise ship as a whole (Talluri and van Ryzin [25], pp. 560–561, Phillips [22], p. 211). Rather overbooking at a cabin category level is practiced (Talluri and van Ryzin [25]), which relates to the demand substitution behavior discussed above.
- **On-board spending**. An important component of a cruise ship revenue is derived from on-board spending, e.g. in restaurants, bars, and casinos. For example, Ji and Mazzarella [13] report on-board revenues of around 25% of total revenue. It is therefore important to consider the total potential revenue (i.e., cruise fare and on-board spending) of a cruise ship customer.
- **Wave booking period**. The majority of the reservations for cruise ships are made in a 3-month time period between January and March. This creates unique challenges in demand forecasting (see Biehn [4] for details).

It is therefore important to make cruise ship RM decisions in the light of these distinctive characteristics of cruise ships. Biehn [4] addresses this issue through a network model assuming deterministic demand. However, the variability of the demand and of customer arrival patterns is obviously a factor that is worth considering in such a situation. In this paper, we develop a dynamic model for cruise ship capacity management which handles multi-dimensional cabin category and lifeboat capacity constraints, and captures the variability of demand and arrivals in a natural way by assuming that customers arrive according to a discrete-time stochastic process.

Specifically, our model is developed with the following assumptions. The time horizon is divided into a large number of discrete-time periods so that at most one customer can arrive in a time period. An arrival requests one cabin and, implicitly, one or more lifeboat seats depending on the number of passengers that will stay in the cabin. Customers, assumed to be leisure passengers, are segmented by their group (family) size and the cabin category that they request[1]. The probability that a customer of a given class arrives (i.e., a customer of a given family size requesting a given cabin type) is known, and to simplify the presentation, time-independent. The time independence assumption holds if arrivals from different customer classes follow time homogeneous Poisson processes. The cruise ship revenue manager decides whether to accept a request in a time period based on the reservation levels of cabin categories and lifeboat. No further reservations will be accepted once the lifeboat capacity has been reached.

While we are aware of the potential demand substitution behavior and overbooking practices (see discussion above), we do not consider these factors in this paper. We make this modeling choice since this paper is the first in its class on cruise ship RM with stochastic demand. Therefore, a basic dynamic capacity control model not cluttered with the complexities of demand substitution and overbooking is appropriate to (i) gain sharp insights into the main trade-offs involved in dynamic cruise RM, and (ii) to lay a firm foundation that future research can build upon to incorporate detailed factors of possible practical importance such as substitution and overbooking. Another factor that justifies our modeling approach is that dynamic models are flexible and allow including factors such as overbooking, cancellations, no-shows and substitution easily (e.g. Subramanian et al. [24]).

For large industry-size problems, our resulting multi-dimensional dynamic capacity control model cannot be solved optimally via the classical DP recursive scheme due to excessive storage requirement which cannot be handled even with the ample computing powers currently available. This is a manifestation of the well-known "curse of dimensionality" which hinders the applications of DP in practice. Moreover, we demonstrate that the value function does not exhibit any particular monotonicity properties that allow obtaining an optimal solution with manageable storage. (Our analysis indicates that this non-monotone behavior is not due to our assumption of ignoring overbooking and demand substitution.) Therefore, our basic model's direct applicability is devising an optimal capacity control policy when the remaining cabin and lifeboat capacities are tight,[2] occurring usually close to the cruise ship departure. The optimal close to-departure RM offered by our model can be critical to profitability. To expand the scope of our model's applicability, we devise heuristics that give near-optimal solutions for large problems. Our heuristics are based on collapsing the multi-dimension state space into one or two dimensions, and solving the resulting DP optimally via the classic recursive approach. We also develop upper bounds on the DP value function. Extensive testing of the heuristics via simulation against the optimal solution, the upper bounds, and the first-come first-served policy, indicates that a class of one-dimensional heuristics based on decoupling the cabins and lifeboat problems perform well in most cases.

The rest of this paper is organized as follows. In Section 2, we briefly review the related literature. In Section 3 we present our model and assumptions by formulating a DP for cruise ship capacity control. In Section 4, we show that our DP model does not exhibit the monotonicity properties in time and reservation level which are typically observed in airline RM. In the absence of monotonicity, we propose several heuristics in Section 5 that give near-optimal solution and develop upper bounds on the value function. Detailed testing of the proposed heuristics via simulation is presented in Section 6. Finally, Section 7 concludes the paper and suggests areas for future research.

2. Literature review

For a recent survey on the development of RM in different industries including airlines, cruise lines, hotel and cargo, one

[1] Our model could allow for the conventional leisure/business passenger segmentation but this does not seem to be a common practice in cruise ships with a majority of leisure passengers.

[2] E.g. with three-cabin categories, the largest problem we can solve to optimality involves around 40 cabins and 100 lifeboat seats.

can refer to Chiang et al. [7]. Other surveys are by Weatherford and Bodily [26] and McGill and van Ryzin [19]. These surveys reveal that cruise RM has received little attention in the academic literature. We next present a brief survey of related research.

Biehn [4] argues that cruise ships are not floating hotels (contrary to what is stated in Talluri and van Ryzin [25]). He illustrates the similarities in RM practices between hotels and cruise ships and highlights some of the key differences which make the RM practices in the hotel industry not appropriate for cruise ships, concluding that separate models targeted uniquely for cruise ships need to be developed. He formulates a deterministic linear program for maximizing revenues for a cruise ship subject to two capacity limitations: the number of cabins and the number lifeboat seats. In addition, he discusses passenger behavior and related management policies such as buy up/buy down, overbooking, and cancelations.

Ji and Mazarella [13] investigate the unique characteristics of cruise line inventory and discuss how RM practices can be adapted to cruise inventory. Specifically, they propose a nested class allocation of the cruise inventory adapted from Belobaba [3] and a dynamic class allocation adapted from Lee and Hersh [17]. Based on simulations using actual booking data, they obtain an average of 4.2%–6.3% increase in revenue. Ji and Mazarella [13], however, do not consider lifeboat capacity constraint in their work as we do in this paper.

This paper presents a dynamic programming model for cruise ship capacity control with multiple fare classes and capacity constraints. To the best of our knowledge, such treatment of cruise ship RM has not been considered before in the literature. However, in the airline passenger seat inventory control, dynamic models are proposed by Alstrup [1], Chatwin [6], El-Haber and El-Taha [9], Lautenbacher and Stidham [16], Lee and Hersh [17], and Subramanian et al. [24], among others. Our heuristics are based on solving approximate decoupled DP problems with small state space. In the literature, other DP approximation techniques have been used for RM problems such as the reinforcement learning technique of Gosavi et al. [10,11].

Our work is close to that of Lee and Hersh [17] who develop a discrete-time dynamic model with multiple fare classes for a single-leg airline capacity control problem with multiple seat bookings (i.e., group arrivals). They argue that the model exhibits a time monotonicity property and propose an optimal booking policy which consists of a set of critical decision periods. While the lifeboat RM can be seen as a group arrival problem, we further link this problem to cabin RM via an integrated multi-dimensional capacity model. We find that Lee and Hersh's [17] time monotonicity does not hold in our cruise context. In fact, we find that this property is not valid in its original single-dimensional airline (with group arrival) context as we show via a special case of our model.

Some of the other related dynamic capacity control works are as follows. Lautenbacher and Stidham [16] consider a dynamic model similar to Lee and Hersh [17] (but without group arrivals) and obtain an optimal booking limit policy by borrowing results from queueing control theory. Subramanian et al. [24] develop a dynamic model and optimal booking policy similar to that of Lautenbacher and Stidham [16] and further allow for cancellations, overbooking, and no-shows. They establish the optimality of a policy characterized by a set of critical booking limits. El-Haber and El-Taha [9] develop a dynamic model and critical booking limits policy for a two-leg airline seat inventory control and account for cancellations, no-shows, and overbooking as in Subramanian et al. [24].

The dynamic multi-capacity control approach in cruise RM problems is also applicable to cargo RM problems. A cargo capacity is characterized by at least two dimensions, weight and volume (e.g. Kasilingam [15]). Amaruchkul et al. [2] develop a dynamic model for the cargo capacity control problem. They propose heuristics that are shown to perform better than the first-come first-served trivial policy. Other works on multi-dimensional cargo RM are by Karaesmen [14] who develops a deterministic bid prices method for capacity control and Moussawi and Çakanyildirim [21] who focus on developing an effective overbooking policy.

3. Model and assumptions

Our model is formulated as follows. We divide the total planning horizon into T decision periods, small enough so that no more than one request arrives per period. That is, the discretization of time is sufficiently fine so that the probability of more than one request is negligible, and that a single request cannot make multiple reservations. We then number the decision periods in reverse order. Thus, time is counted backwards, with $t = 1$ referring to the last period before the cruise ship departs, where t represents a point t periods from the end of the horizon. We also define $t = 0$ as the time of cruise ship departure where no decision is made, but boundary conditions apply. The cruise ship type to be used is known, i.e., the total number of cabin categories, cabins in each category, and the total number of lifeboat seats available are known in advance. Denote by n the number of cabin categories, C_i the capacity of cabin category $i, i = 1, \ldots, n$, and L the lifeboat capacity.

Fare prices are assumed to be known. Furthermore, there are m classes of requests for each cabin category, with a request of class $j, j = 1, \ldots, m$, requiring one cabin and j lifeboat seats, (e.g. a family of four would require one cabin and four lifeboat seats). The revenue from arrival of class ij, i.e., an arrival requesting a cabin in cabin category i and requiring j lifeboats seats, $i = 1, \ldots, n, j = 1, \ldots, m$, is r_{ij}. We assume that the probability of an arrival of class ij in a period is known, and is independent of time and the booking levels at the time of the arrival. This probability is denoted by p_{ij}. The probability of no-arrival in a period, p_0, is also assumed to be known and independent of time and booking levels. Note that with the assumption that there is at most one arrival per period, we have

$$p_0 + \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij} = 1.$$

We formulate a finite-horizon discrete-time Markov decision process[3] in which the state variables are the total number of cabins booked in different cabin categories and the total number of lifeboat seats reserved. Specifically, the state of the reservations system is given by $(\mathbf{x}, y)$ where $\mathbf{x} = (x_1, \ldots, x_n), x_i$ being the number of cabins reserved in cabin category i, and y is the number of lifeboat seats already reserved. We assume that all customers will show-up and claim their cabins, and that overbooking any of the cabin categories or the lifeboat is not allowed.

At each stage t, the booking agent decides whether or not to accept a request, based on the fare class of the customer and the current state $(\mathbf{x}, y)$. If there is no arrival, then no decision needs to be made. However, if the system controller accepts a request for a seat in fare class ij, the cruise ship earns r_{ij}. Our objective is to maximize the expected total net benefit over the horizon from period T to period 1, starting from state $(\mathbf{x}, y) = (\mathbf{0}, 0)$ (no cabins/seats booked) at the beginning of period T.

Denote by $V_t(\mathbf{x}, y)$ the maximal expected net controllable benefit of operating the system over periods t to 1. The optimal value functions, $V_t(\cdot)$, are determined recursively as we explain next. First, we

[3] Our problem is a MDP under the approximating assumption of at most one arrival per decision period, which holds if the number of periods is large enough. Without this assumption, our problem is a semi-MDP.

introduce additional notation. Let $A = \{1,2,\ldots, n\}$ be the set of cabin categories, $A_r(\mathbf{x}) = \{i \in A — x_i < C_i\}$ be the set of cabin categories which are not sold out (i.e., remaining) when the cabin reservation level is $\mathbf{x}$, and $\mathbf{X} = \{\mathbf{x} \in \mathbf{R}^n — x_i \leqslant C_i, i \in A, \text{and } A_r(\mathbf{x}) \neq \emptyset\}$ be the set of admissible cabin category states when a request can be accepted (provided that lifeboat capacity allows it). In addition, let $\mathbf{e}^i$ be a vector having n entries all equal to zero except the i^{th} entry which is equal to 1 and $\mathbf{c} = (C_1, C_2, \ldots, C_n)$ be the vector representing the state where all cabins are full. Then, the $V_t(\cdot)$ value functions are evaluated as follows:

$$V_t(\mathbf{x},y) = \sum_{i\in A_r(\mathbf{x})} \sum_{j=1}^{\min(m,L-y)} p_{ij} \max\{r_{ij} + V_{t-1}(\mathbf{x}+\mathbf{e}^i, y+j),\ V_{t-1}(\mathbf{x},y)\} + \left(1 - \sum_{i\in A_r(\mathbf{x})} \sum_{j=1}^{\min(m,L-y)} p_{ij}\right) V_{t-1}(\mathbf{x},y),$$
$$\mathbf{x}\in\mathbf{X}, \quad 0 \leqslant y \leqslant L-1, \tag{1}$$

where the boundary conditions are

$$\begin{aligned} &V_0(\mathbf{x},y) = 0, \quad \mathbf{x}\in\mathbf{X}, \quad 0\leqslant y\leqslant L-1,\\ &V_t(\mathbf{c},y) = 0, \quad 0\leqslant y\leqslant L, \quad 0\leqslant t\leqslant T,\\ &V_t(\mathbf{x},L) = 0, \quad \mathbf{x}\in\mathbf{X}, \quad 0\leqslant t\leqslant T. \end{aligned} \tag{2}$$

If a booking request of class ij is accepted in period t when the system state is $(\mathbf{x}, y)$, the maximum profit is $r_{ij} + V_{t-1}(\mathbf{x} + e_i, y + j)$. If the request is rejected, the maximum profit is $V_{t-1}(\mathbf{x}, y)$. Therefore, a request of class ij arriving in period t when the system state is $(\mathbf{x}, y)$ is accepted if and only if

$$r_{ij} > V_{t-1}(\mathbf{x},y) - V_{t-1}(\mathbf{x}+\mathbf{e}_i, y+j). \tag{3}$$

It is useful to interpret the right hand side of (3) as the opportunity cost of accepting a booking of class ij, given that the reservation level is $(\mathbf{x}, y)$. Formally, define this opportunity cost as

$$\begin{aligned} &u_t^{ij}(\mathbf{x},y) \equiv V_{t-1}(\mathbf{x},y) - V_{t-1}(\mathbf{x}+\mathbf{e}^i, y+j),\\ &\mathbf{x}\in\mathbf{X}, \quad i\in A_r(\mathbf{x}), \quad 0\leqslant y\leqslant L-j,\ 1\leqslant t\leqslant T. \end{aligned} \tag{4}$$

Then, a booking request for class ij in period t when the reservation state is $(\mathbf{x}, y)$ is accepted if and only if $r_{ij} > u_t^{ij}(\mathbf{x}, y)$.

Once the optimal value functions, $V_t(\cdot)$, are determined, the cruise ship revenue manager can decide which requests to accept. However, storing and retrieving the $V_t(\cdot)$'s is not possible in practice when the number of remaining time periods, t, is large and the number of remaining accessible states is large (i.e., when x_i and y are small). Specifically, the storage requirement for the $V_t(\cdot)$'s is $T(\Pi_{i=1}^n C_i)L$ integers. For example, for a problem of reasonable size with $T = 1000$ time periods, three cabin categories with capacities $C_1 = C_2 = 350$ and $C_3 = 300$, and a lifeboat capacity $L = 2{,}700$, the storage requirement is 9.922×10^{13} integers, which is equivalent (at approximately 4 bites per integer) around 40,000 GB of computer memory (RAM).

Our optimal solution methodology based on (4) is therefore applicable when the remaining time periods and accessible states are small, which typically occurs close to the departure of the cruise ship. Given the non-monotone behavior discussed in Section 4, we focus in Section 5 on devising effective DP-based heuristics and bounds for large problems.

4. Non-monotone behavior

In this section we present results demonstrating that the opportunity cost, $u_t^{ij}(\mathbf{x}, y)$, is *not* monotone in the reservation level, $(\mathbf{x}, y)$, and time, t. This implies that control policies based on critical booking levels or time periods (e.g. Lautenbacher and Stidham [16], Lee and Hersh [17], and Subramanian et al. [24]) are not optimal in the context of cruise ship RM. Our objective in this section is to demonstrate the non-monotone behavior and to gain insight on the particular ingredients of the cruise problem that lead to such a behavior. An additional result of this section is proving that the time monotonicity property for the airline problem with batch arrivals given by Lee and Hersh [17] is not valid.

To simplify the analysis, we present our results within the context of a "small cruise ship" with a single cabin category with capacity C_1, a lifeboat with capacity L, and two types of customers: Type 1 (singles) requiring one cabin and one lifeboat seat and type 2 (couples) requiring one cabin and two lifeboat seats. We simplify the notation by writing the fare prices and arrival probabilities for the two types as $r_j \equiv r_{1j}$ and $p_j \equiv p_{1j}$, $j = 1,2$. The following is a counter-example to booking level monotonicity.

Example 1. In this example, we show that for a small cruise ship DP model, with $C_1 = 4, L = 6$, and $T = 3$, a control policy based on critical booking levels is not optimal if $r_1 < p_2r_2$. Specifically, we show that when $r_1 < p_2r_2$ the decision to accept/reject a type 1 arrival in period 2 (i.e., two periods before departure) cannot be made based on a booking limit type of control. Denote by (x^2, y^2) the cabin and lifeboat booking levels in period 2. We show that it is optimal to accept a type 1 request in period 2 when $(x^2, y^2) = (2,3)$ or $(x^2, y^2) = (2,5)$, but not when $(x^2, y^2) = (2,4)$. This shows that there is *no* optimal lifeboat booking level for type 1 in period 2 which we denote by y_2^1, such that it is optimal to accept a type 1 request in period 2 if $y^2 \leqslant y_2^1$. The details of our argument are as follows. When $(x^2, y^2) = (2,3)$, it is obvious that it is optimal to accept a type 1 request in period 2, as there is enough lifeboat and cabin capacity to accommodate any type of arrivals in periods 1 and 2. Likewise, an acceptance decision for a type 1 arrival in period 2 is obvious when $(x^2, y^2) = (2,5)$, as there is lifeboat capacity to accommodate only one arrival of type 1. However, when $(x^2, y^2) = (2,4)$, it is optimal to *reject* a type 1 request in period 2 if $r_1 + V_1(3,5) < V_1(2,4)$, where $V_1(3,5)$ and $V_1(2,4)$ are given by (1). It follows that it is optimal to reject a type 1 arrival in period 2 if and only if $r_1 + p_1r_1 < p_1r_1 + p_2r_2$, which is equivalent to $r_1 < p_2r_2$.

Example 1 indicates that a control policy based on booking limits is not optimal for small families (the singles in Example 1). In fact, it is optimal to accept such requests when there is ample capacity (allowing accommodation of all types of customers) or limited capacity (sufficient for small families only), but not when such types compete on scarce capacity with high-paying larger families having high arrival probabilities. In our example, the sufficient condition $r_1 < p_2r_2$, indicates that the fare of type 1 is less than the expected payoff from a future type 2 arrival. This gives a simple criterion for rejecting singles on a small cruise ship when they are competing with couples on scarce capacity.

Note also that the absence of a booking limit policy can also be seen from the non-monotone behavior in y of type 1 opportunity cost, which we denote by $u_t^1(x, y)$ as indicated in Fig. 1. The parameters in Fig. 1 are chosen so that the sufficient condition of Example 1, $r_1 < p_2r_2$, is satisfied. Fig. 1 indicates that in period 2, type 1 requests should be accepted if the booking level is (2,3) since the opportunity cost $u_2^1(2,3)$ is less than r_1. Such requests should be *rejected* if the booking level is (2,4) as $u_2^1(2,4) > r_1$, and accepted when this level is (2,5).

The following example demonstrates that control based on critical time periods is also not always optimal.

Example 2. In this example, we show that for a small cruise ship, with $C_1 = 4$, $L = 6$, a control policy based on critical time periods is not optimal if

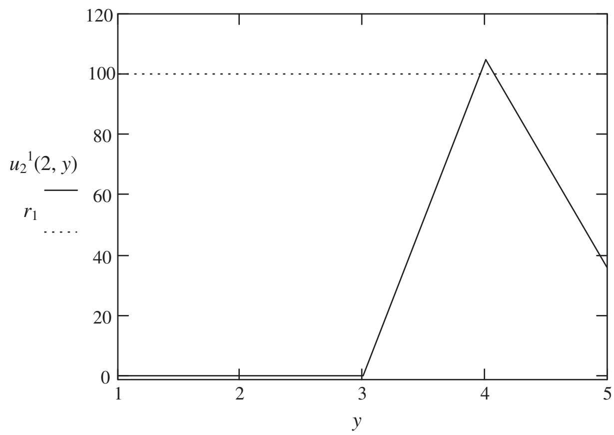

Fig. 1. Example of non-monotonicity in reservation level for a small cruise ship, $C_1 = 4, L=6, p_1 = 0.35, r_1 = 100, p_2 = 0.55$, and $r_2 = 190$.

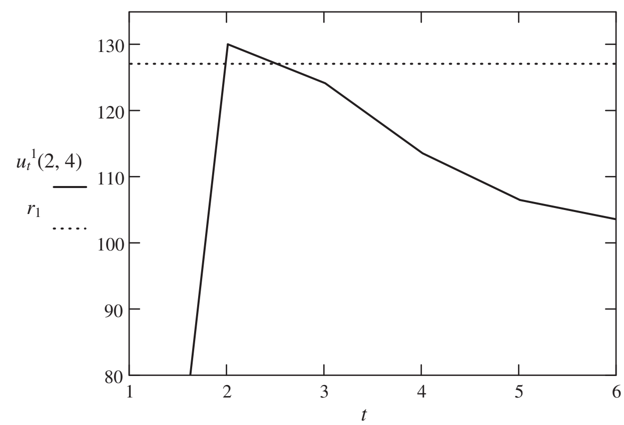

Fig. 2. Example of non-monotonicity in reservation level for a small cruise ship, $C_1 = 4, L=6, p_1 = 0.3, r_1 = 127, p_2 = 0.65$, and $r_2 = 200$.

$$\frac{1+p_1+p_0}{1+p_1+p_1p_2-p_1^2}p_2r_2 < r_1 < p_2r_2. \tag{5}$$

Specifically, we show that, under (5), it is optimal to accept a type 1 request in periods 1 and 3, but not in period 2. This implies that there is no critical time period, $t_1(2,4)$, such that it is optimal to accept a type 1 request, if the time $t > t_1(2,4)$. The details of this argument are as follows. In period 1, it is obvious that a type 1 request should be accepted if the reservation level is (2,4), as period 1 is the last period before departure, so rejecting type 1 will only lead to spoiling revenue. In period 2, a type 1 request should be rejected at the reservation level (2,4) if and only if $r_1 + V_1(3,5) < V_1(2,4)$, where $V_1(3,5)$ and $V_1(2,4)$ are given by (1), which is equivalent to $r_1 < p_2r_2$, the second inequality in (5), similar to Example 1. Finally, in period 3, a type 1 request should be accepted at the reservation level (2,4) if and only if $r_1 + V_2(3,5) > V_2(2,4)$, where from (1), $V_2(3,5) = p_1\max\{r_1 + V_1(4,6),V_1(3,5)\} + (p_2 + p_0)V_1(3,5) = p_1r_1(1 + p_2 + p_0)$. Similarly, $V_2(2,4) = p_2r_2 + (p_1 + p_0)(p_1r_1 + p_2r_2)$. It follows that in period 3, a type 1 request should be accepted at the reservation level (2,4) if and only if

$$r_1 > p_2r_2\frac{1+p_1+p_0}{1+p_1+p_1p_2-p_1^2}, \tag{6}$$

which is the first inequality in (5).

While (5) is a sufficient[4] condition for the non-monotone behavior in this example, it is not clear how to come up with parameter values (i.e., values of p_i and r_i) that satisfy it (if any). In the following, we derive conditions which are required for (5) to hold. This also provides a simple scheme for generating a set of parameters that satisfy (5). Note that (5) holds only if $\frac{1+p_1+p_0}{1+p_1+p_1p_2-p_1^2} < 1$, which is equivalent to $-p_1^2 + p_1p_2 - p_0 > 0$. This last quadratic is positive if and only if $p_2^2 - 4p_0 > 0$, i.e., $p_2 > 2\sqrt{p_0}$, and

$$\frac{p_2-\sqrt{p_2^2-4p_0}}{2} < p_1 < \frac{p_2+\sqrt{p_2^2-4p_0}}{2}. \tag{7}$$

One can then easily pick parameter values that satisfy (5). First, pick p_2 and p_0 such that $p_2 > 2\sqrt{p_0}$. Then, pick p_1 such that (7) holds. Finally, pick r_1 such that (5) holds. For example, the parameters in Fig. 2 were chosen in this manner.

Example 2 indicates that a control policy based on critical time periods is not always optimal for small families. In fact, it is optimal to accept such requests close to departure time when the expected future revenue from large families is small to insure better utilization of the remaining capacity. This is the case of type 1 in period 1 in Example 2. In addition, requests from small families may be optimal to accept earlier in the booking process where there is a high chance of a more profitable filling of capacity with smaller families. This is the case of type 1 arrival in period 3 where a type 1 request is accepted with the expectation that another type 1 request will arrive in the next two periods. However, small families are rejected as we come closer to the departure and the remaining time is not long enough to allow having more arrivals of these types. This is the case of type 1 in period 2 in Example 2. These observations imply that non-monotonicity in time is expected when the fare price and the arrival probabilities of small families are neither too small nor too large, which is reflected by the bounds on p_1 and r_1 in (5) and (7).

The absence of a control policy based on critical time periods can be also deduced from the non-monotone behavior of $u_t^{ij}(x,y)$ in t, as indicated in Fig. 2.

Finally, we use the result of Example 2 to address a well-known problem in the airline revenue management involving capacity control of a single-leg flight with batch arrivals, without overbooking and no-shows. For this problem, Lee and Hersh [17] give a result (Theorem 3, p. 258) indicating time monotonicity of the opportunity cost. The proof of this result ignores the trade-offs resulting from the different ways capacity can be filled by arrivals of different sizes, which we highlight in Example 2. The following example contradicts Lee and Hersh's result.

Example 3. In this example, we show that for the single-leg airline dynamic capacity control problem with batch arrivals, the opportunity cost is not always increasing in time, t. This follows directly from Example 2 by noting that the argument there holds if one sets $C_1 = 0$ and considers state (0,4). Here we assume that both types of customer arrivals require only lifeboat seats. This is equivalent to an airline dynamic control problem with batch arrivals.

5. Heuristics and bounds

Optimal policies are too complex to store and compute for large problems. In this section, we describe five heuristics which are based on reducing the multi-dimensional state space of the original DP into one- or two-dimensional space, thus reducing storage and computational requirements. We also derive upper bounds on the value function which are useful in assessing the heuristics performance as well as quantifying an upper limit on the profitability of cruise RM. In the following, we describe the five heuristics in Sections 5.1–5.5, then we derive two upper bounds

[4] Condition (5) is not a sufficient and necessary condition for non-monotonicity in time as one could establish other sufficient non-monotonicity conditions by analyzing reservation levels other than (2,4) for a small cruise ship or by considering a cruise ship of a different size.

on the value function in Section 5.6. The performance of these heuristics is tested in Section 6.

5.1. Aggregate cabin (AC) heuristic

This heuristic works by aggregating all the cabin categories into one category with capacity $C = \sum_{i=1}^{n} C_i$. The state of the system consists of the total number of cabins booked, x, with $0 \leqslant x \leqslant C$ and the number of lifeboat seats reserved $y, 0 \leqslant y \leqslant L$. Denote the corresponding value function by $V_t^{AC}(x,y)$, $V_t^{AC}(x,y) = \sum_{j=1}^{\min(m,L-y)} p_j \max\{r_j + V_{t-1}^{AC}(x+1,y+j), V_{t-1}^{AC}(x,y)\} + \left(1 - \sum_{j=1}^{\min(m,L-y)} p_j\right) V_{t-1}^{AC}(x,y)$, where the boundary conditions are similar to those in (1),[5] p_j is the probability that a family of size j arrives in a period, $p_j = \sum_{i=1}^{n} p_{ij}$, and r_j is the associated expected revenue, $r_j = \left[\sum_{i=1}^{n} p_{ij} r_{ij}\right] / \left[\sum_{i=1}^{n} p_{ij}\right]$. Define the opportunity cost of accepting a request of type j as $u_t^{AC,j}(x,y) = V_{t-1}^{AC}(x,y) - V_{t-1}^{AC}(x+1,y+j), 1 \leqslant t \leqslant T, 0 \leqslant x \leqslant C-1, 0 \leqslant y \leqslant L-j$. To apply the AC heuristic, we need to keep track of the number of reservations made in cabin category i, x_i. Then, the AC heuristic is as follows: Accept a class ij-booking request in period t when the state is (x,y) if and only if $r_j > u_t^{AC,j}(x,y)$ and $x_i < C_i$.

The storage requirement for the AC heuristic is $T(\sum_{i=1}^{n} C_i)L$. This represents a major reduction in storage relative to the $T(\Pi_{i=1}^{n} C_i)L$ requirement of the optimal approach defined in Section 3. For example, for the three-cabin example (with $T = 1000, C_1 = C_2 = 350, C_3 = 300$, and $L = 2{,}700$) given at the end of Section 3, the storage requirement is 2.7×10^9 integers which is approximately 11 GB of computer memory (down from 40,000 GB for the optimal approach).

5.2. Divide lifeboat (DL) heuristic

The second heuristic divides the lifeboat capacity among the n categories based on the fraction of the total expected revenue per period contributed by a category (assuming all categories are available). Specifically, the lifeboat capacity allocated to category i is $L_i = L \sum_{j=1}^{m} r_{ij} p_{ij} / \left(\sum_{k=1}^{n} \sum_{j=1}^{m} r_{kj} p_{kj}\right)$. Then, n two-dimensional subproblems are formulated, with subproblem i (corresponding to cabin category i) having a single cabin category with capacity C_i and a lifeboat with capacity L_i. The state variables for subproblem i are (x_i, y_i), the reservation level of cabin i and its corresponding share of the lifeboat, where $0 \leqslant x_i \leqslant C_i$ and $0 \leqslant y_i \leqslant L_i$. Let $V_t^{DL,i}(x_i, y_i)$ be the corresponding value function in period $t, V_t^{DL,i}(x_i, y_i) = \sum_{j=1}^{\min(m, L_i - y_i)} p_{ij} \max\{r_{ij} + V_{t-1}^{DL,i}(x_i+1, y_i+j), V_{t-1}^{DL,i}(x_i, y_i)\} + (1 - \sum_{j=1}^{\min(m,L_i-y_i)} p_{ij}) V_{t-1}^{DL,i}(x_i, y_i)$, where the boundary conditions are similar to those in (1). Then, if a booking request is accepted, the maximum profit is $r_{ij} + V_{t-1}^{DL,i}(x_i+1, y_i+j)$, otherwise, it is $V_{t-1}^{DL,i}(x_i, y_i)$. Define the opportunity cost $u_t^{DL,i}(x_i, y_i) = V_{t-1}^{DL,i}(x_i, y_i) - V_{t-1}^{DL,i}(x_i+1, y_i+j), 1 \leqslant t \leqslant T, 0 \leqslant x_i \leqslant C_i - 1, 0 \leqslant y \leqslant L_i - j$. Then, the DL heuristic is as follows: Accept a type ij-booking request in period t when the state of subproblem i is (x_i, y_i) if and only if $r_{ij} > u_t^{DL,i}(x_i, y_i)$.

The storage requirement for the DL heuristic is $T(\sum_{i=1}^{n} C_i L_i)$. This represents a further reduction in storage relative to the optimal approach. For our three-cabin example (with $T = 1000$, $C_1 = C_2 = 350$, $C_3 = 300$, and $L = 2{,}700$), suppose that L_i are such that $L_i / \sum L_i = C_i / \sum C_i$ (which gives $L_1 = L_2 = 945$ and $L_3 = 810$). Then, the storage requirement is 9.045×10^8 integers which is approximately 4 GB of computer memory.

[5] See the unabridged version of this paper [20] for the complete and precise formulation corresponding to AC and other heuristics in this section.

5.3. Nested divide lifeboat (NDL) heuristic

In the DL heuristic, the lifeboat booking limit for a category i is L_i and all the booking limits sum up to the total lifeboat capacity L. We now introduce a nested reservation system in which the lifeboat booking limits are structured such that a fare request for a category will not be refused as long as there are remaining lifeboat seats available in categories with lower expected fare. Let r_i be the expected fare from category i, $r_i = \left[\sum_{j=1}^{m} p_{ij} r_{ij}\right] / \left[\sum_{j=1}^{m} p_{ij}\right]$, and assume without loss of generality that $r_1 \geqslant r_2 \geqslant \cdots \geqslant r_n$. Also, let $p_i = \sum_{j=1}^{m} p_{ij}$ be the probability than an arrival requests a cabin in category i. Then, the lifeboat seats are divided in a nested structure as follows:

$$\begin{aligned} &L_1 = L, \quad \text{and} \\ &L_i = L_{i-1}(r_i / r_{i-1}), \quad i = 2, \ldots, n. \end{aligned} \tag{8}$$

Once the nested lifeboat capacities are determined from (8), the NDL heuristic proceeds in exactly the same manner as the DL heuristic described in Section 5.2.

The storage requirement for the NDL heuristic is $T(\sum_{i=1}^{n} C_i L_i)$ similar to DL. However, L_i is larger here. For our three-cabin example (with $T = 1000, C_1 = C_2 = 350, C_3 = 300$, and $L = 2{,}700$) assume that $(r_i)/(r_{i-1}) = 0.9, i = 2,3$ (which gives $L_1 = 2{,}700, L_2 = 2{,}430$ and $L_3 = 2{,}187$), the storage requirement is 2.45×10^9 integers which is approximately 10 GB of computer memory.

5.4. Decoupling based on marginal revenue (DCM) heuristic

The idea behind this heuristic is to reduce the high-dimensional MDP by decoupling the original problem into $n+1$ one-dimensional subproblems, a lifeboat subproblem and n cabin subproblems. We allocate the revenue from a request of type kj among the $n+1$ subproblems as follows. The revenue from a request of type kj allocated to cabin category i subproblem is

$$r_{kj}^{i} = \begin{cases} r_{i2} & \text{if } k = i, \\ 0 & \text{if } k \neq i. \end{cases} \tag{9}$$

For the lifeboat subproblem, the allocated revenue from a request of type kj is

$$r_{kj}^{l} = r_{kj} - r_{k2}. \tag{10}$$

The rational behind this distribution of revenues among subproblems is that lifeboat capacity drives the marginal revenue from additional third, fourth and more passengers in large families[6]. The allocation of revenues as in (9) and (10) implies that $\sum_{i=1}^{n} r_{kj}^{i} + r_{kj}^{l} = r_{ij}$.

The lifeboat subproblem is a one-dimensional subproblem with the state being the number of lifeboat seats already reserved y. Its value function is:

$$\begin{aligned} V_t^l(y) = &\sum_{k=1}^{n} \sum_{j=1}^{\min(m,L-y)} p_{kj} \max\{r_{kj}^l + V_{t-1}^l(y+j), V_{t-1}^l(y)\} \\ &+ \left(1 - \sum_{k=1}^{n} \sum_{j=1}^{\min(m,L-y)} p_{kj}\right) V_{t-1}^l(y), \quad 0 \leqslant y \leqslant L-1, \end{aligned} \tag{11}$$

where the boundary conditions are

$$\begin{aligned} &V_0^l(y) = 0, \quad 0 \leqslant y \leqslant L-1, \\ &V_t^l(L) = 0, \quad 0 \leqslant t \leqslant T. \end{aligned} \tag{12}$$

[6] Note that, in practice, the allocation of revenues in (9) and (10) works for customers who are singles as these usually pay a "single surcharge", which is most likely related to on-board spending, (e.g. Biehn [4]), which implies that $r_{i1} > r_{i2}, i \in A$.

Next, we define the cabin subproblem for a particular category $i, i \in A$, with the state being the number of cabins booked in category i, x_i, as follows:

$$V_t^i(x_i) = \sum_{k=1}^{n} \sum_{j=1}^{\min(m,L-y)} p_{kj} \max\{r_{kj}^i + V_{t-1}^i(x_i + e_{ik}), V_{t-1}^i(x_i)\} + \left(1 - \sum_{k=1}^{n} \sum_{j=1}^{\min(m,L-y)} p_{kj}\right) V_{t-1}^i(x_i), \quad 0 \leqslant x_i \leqslant C_i - 1, \tag{13}$$

where $e_{ik} = 1$ if $k = i$ and $e_{ik} = 0$ if $k \neq i$. The boundary conditions are

$$V_0^i(x_i) = 0, \quad 0 \leqslant x_i \leqslant C_i - 1, \\ V_t^i(C_i) = 0, \quad 0 \leqslant t \leqslant T. \tag{14}$$

Note that since $r_{kj}^i = 0$ and $e_{ik} = 0$ if $k \neq i$, the value function (13) of the cabin subproblem can also be written in the following equivalent form:

$$V_t^i(x_i) = \left(\sum_{j=1}^{\min(m,L-y)} p_{ij}\right) \max\{r_{i2} + V_{t-1}^i(x_i + 1), V_{t-1}^i(x_i)\} + \left(1 - \sum_{j=1}^{\min(m,L-y)} p_{ij}\right) V_{t-1}^i(x_i), \quad 0 \leqslant x_i \leqslant C_i - 1.$$

Define the opportunity cost $u_t^{ij}(\mathbf{x}, y) = \left[\sum_{k=1}^{n} V_{t-1}^k(x_k) + V_{t-1}^l(y)\right] - \left[\sum_{k=1}^{n} V_{t-1}^k(x_k + e_{ik}) + V_{t-1}^l(y + j)\right]$, $\mathbf{x} \in \mathbf{X}$, $0 \leqslant y \leqslant L - 1, 0 \leqslant t \leqslant T$. Then the decision rule under the DC heuristic becomes: In period t, accept a request of type ij if and only if $r_{ij} > u_t^{ij}(\mathbf{x}, y)$.

The storage requirement for the DCM heuristic is $T(L + \sum_{i=1}^{n} C_i)$. This represents a significant improvement in storage relative to the optimal approach and the two-dimensional heuristics. Given today's computing power, any large industry-size problem can be solved with this storage requirement. For our three-cabin example (with $T = 1000$, $C_1 = C_2 = 350, C_3 = 300$, and $L = 2{,}700$), the storage requirement is 3.7×10^8 integers which is approximately 15 MB of computer memory.

5.5. Decoupling based on average revenue (DCA) heuristic

This heuristic is similar to DCM but the way the fare price is allocated among different subproblems is different. In DCA, the allocation is based on a simplified "average" revenue from cabins and the lifeboat. Specifically, let $\overline{R}_{i,\infty} = C_i \sum_{j=1}^{m} p_{ij} r_{ij}$ be the expected revenue from cabin categories assuming that the lifeboat capacity is infinite and all requests are accepted on a FCFS basis. Then, $\overline{R}_{C,\infty} = \sum_{i=1}^{n} \overline{R}_{i,\infty}$. Similarly, we define $\overline{R}_{\infty,L} = L \sum_{i=1}^{n} \left[\sum_{j=1}^{m} p_{ij} r_{ij} / j\right]$, as the expected revenue from the lifeboat assuming infinite cabin capacities and a FCFS admission policy. Let $\alpha = \overline{R}_{C,\infty} / [\overline{R}_{C,\infty} + \overline{R}_{\infty,L}]$. Then, the revenue from a request of type kj allocated to cabin category i subproblem is

$$r_{kj}^i = \begin{cases} \alpha r_{ij} & \text{if } k = i, \\ 0 & \text{if } k \neq i. \end{cases}$$

For the lifeboat subproblem, the allocated revenue from a request of type kj is

$$r_{kj}^l = (1 - \alpha) r_{kj}.$$

Once the revenues are allocated to different subproblems, the DCM heuristic proceeds in exactly the same manner as the DCA heuristic utilizing the formulation in (13) and (14). Obviously, the storage requirements for DCA is exactly the same as DCM (see Section 5.4).

5.6. Upper bounds

The value function of our original problem in (1) is bounded above by the sum of the value functions of all one-dimensional subproblems of the DC heuristics in Sections 5.4 and 5.5. This is proven in the following theorem.

Theorem 1

$$V_t(\mathbf{x}, y) \leqslant \sum_{i=1}^{n} V_t^i(x_i) + V_t^l(y), \quad \mathbf{x} \in \mathbf{X}, \quad 0 \leqslant y \leqslant L - 1, 0 \leqslant t \leqslant T.$$

The proof is available in the unabridged version of this paper [20].

Theorem 1 gives the following upper bound on the total expected cruise ship revenue.

$$\overline{V} = \sum_{i=1}^{n} V_T^i(0) + V_T^l(0). \tag{15}$$

This has two important implications. First, it allows quantifying an upper bound on the maximum revenue that could be generated from optimal capacity control. This is especially useful since evaluating $\overline{V}$ requires solving single-dimensional DPs which can be done basically for any practical problem size (where optimal solutions are often not possible due to storage requirements). Second, it can be used to evaluate the performance of heuristics presented in this section by comparing its estimated expected revenue (obtained from simulation) to the upper bound. We utilize the bound in Theorem 1 for testing the heuristics in the next section.

6. Numerical results

In this section, we present numerical results to illustrate the application of our model and to test the performance of the proposed heuristics. As discussed in Section 3, only "small-size" problems can be solved optimally due to storage and computational requirements. Accordingly, Section 6.1 presents numerical results using the optimal solution, heuristics, and upper bounds for such small problems. In addition, we find that the two-dimensional heuristics presented in Section 5 are hindered by storage requirements. We refer to problems that can be solved by these heuristics but not optimally as "medium-size" problems, and we present results for these problems in Section 6.2. The remaining "large problems" that can be only solved by one-dimensional heuristics are analyzed in Section 6.3. We conclude in Section 6.4 with a summary of the results and recommendations on solution approaches to be applied in practice.

6.1. Results for small-size problems

All of our results consider a cruise ship with three-cabin categories and lifeboat. Due to excessive storage requirements, the largest problem we are able solve optimally has the following cabin and lifeboat capacities: $\mathbf{c} = (15,15,10)$, and $L = 108$[7]. We decreased these capacities by around 10% and used $\mathbf{c} = (13,13,9)$ and $L = 98$ for our "base case". In terms of arrival probabilities, we use the values in Table 1 for all of our numerical results. The values of cabin capacities in the base case and the arrival probabilities in Table 1 assign high demand to capacity ratios for the low-fare cabins. (The cabins are such that cabin 1 has the highest fares followed by cabins 2 and then 3, see Tables 2 and 3). Such a high demand for low-price products represents a common situation. In terms

[7] The computer we used has 1 GB RAM and 1.83 GHz processor. More powerful machines could allow increasing these limits.

Table 1
Arrival probabilities, p_{ij}.

		Lifeboat seats, j			Total
		2	3	4	
Cabin category, i	1	0.060	0.036	0.012	0.108
	2	0.084	0.096	0.048	0.228
	3	0.096	0.108	0.06	0.264
	Total	0.240	0.240	0.120	0.600

Table 2
First set of fare prices, r_{ij}.

		Lifeboat seats, j			Mean, r_i
		2	3	4	
Cabin category, i	1	\$2,080	\$2,560	\$3,040	\$2,347
	2	\$1,800	\$2,250	\$2,700	\$1,793
	3	\$1,700	\$2,100	\$2,500	\$ 2045
	Mean, r_j	\$1,830	\$2,229	\$2,634	

Table 3
Second set of fare prices, r_{ij}.

		Lifeboat seats, j			Mean, r_i
		2	3	4	
Cabin category, i	1	\$2,080	\$2,560	\$3,040	\$2,347
	2	\$1,500	\$1,850	\$2,190	\$2,179
	3	\$1,000	\$1,230	\$1,460	\$1,199
	Mean, r_j	\$1,445	\$1,678	\$1,910	

of family size, we do not consider singles. We assign equal high proportions to couples and families of 3, with each representing, on average, 40% of the total number of arrivals. The remainder 20% are families of size 4.

We associate the arrival probabilities in Table 1 with two sets of fare prices presented in Tables 2 and 3. The values in Table 2 were compiled from a cruise travel website based on fares for Oceanview (category 1), Balcony (category 2), and Inside (category 3). In Table 3, these values were modified to increase the heterogeneity of fare prices between cabin categories (since RM is typically recommended for heterogeneous products). In Table 2, the mean revenue per cabin varies by around 10% between two adjacent cabin categories (e.g. cabins 1 and 2). Concerning the family size, the marginal fare for the third and fourth passenger is around 20% of the base double-occupancy rate. These values are in-line with our discussions of cruise fare products and guest pricing in Section 1. In Table 3, the variation between adjacent cabin categories is 25% for cabin categories 1 and 2 and 35% for categories 2 and 3, and the marginal fare is around 16% of the double-occupancy rate. We consider a booking period length $T = 70$ for all the results in this section. This represents an average demand to capacity ratios, for the base case, of $(70)(0.108)/13 = 58\%$, $(70)(0.228)/13 = 123\%$, and $(70)(0.264)/9 = 205\%$ for cabin categories 1,2,3, respectively, reflecting the high demand for low-fare categories. For the lifeboat, the average demand to capacity ratio is $(70)(2 \times 0.24 + 3 \times 0.24 + 4 \times 0.12)/98 = 120\%$.

Table 4 presents a summary of our numerical study based on the arrival probabilities in Table 1 and the fare prices in Table 2. It gives the expected revenue for each of the heuristics, $\Pi^J, J \in \{AC, DL, NDL, DCM, DCA\}$, and the upper bounds, $\Pi^{UJ}, J \in \{DCM, DCA\}$, presented in Section 5.6, as a percentage of the optimal expected profit.

Table 4 also presents similar results from a trivial FCFS admission policy where all reservation requests are accepted while cabin and lifeboat capacities allow it. This provides the lowest acceptable bound on the heuristics performance. These results were obtained as follows. We first solve the MDPs corresponding to the optimal and heuristic solutions in Sections 3 and 5, respectively, and then store the corresponding opportunity costs for the 70 time periods we consider. To obtain the upper bounds in Section 5.6 we utilize the value functions at time $t = 0$ for the DCM and DCA heuristics as indicated in (15). Then, we generate sample paths of arrivals based on the arrival probabilities in Table 1 and obtain the expected profit from the optimal solution and heuristics by taking the mean over all the generated samples. In the simulation, arrivals of type ij were generated in a discrete fashion in every period. Specifically, in every period, a potential arrival of type ij is generated via a Bernoulli random variate with parameter p_{ij}[8]. The number of sample paths utilized in each simulation is 1,000.

The first row in Table 4 gives the results for our base case. The results in the other rows are based on changing the parameters as indicated in the "Change" column. We consider changes in the capacities of the cabin categories and the lifeboat to test the effect of demand to capacity ratio (since demand given by the arrival probabilities in Table 1 is unchanged). The last row in Table 4 shows the mean performance of the nine cases in the table.

Table 4 indicates the following insights.

- Heuristics NDL and DCM perform the best among the five heuristics proposed with an average optimality gap of 0.4%.
- On average, the expected revenue of the two leading heuristics, NDL and DCA, is 6% better than the trivial FCFS benchmark. In addition, FCFS does not beat either of two heuristics in any of the nine cases considered in Table 4 with different demand to capacity ratios. This indicates a significant benefit from adopting RM through these heuristics.
- Among the three remaining heuristics, the other decoupling heuristic, DCA, performs close to DCM and NDL, on average. However, DCA performs slightly better than DCM in only one out of the nine cases (case 8 where lifeboat capacity is low at an average demand to capacity ratio of 134%).
- Among the two leading heuristics NDL and DCM, Table 4 indicates that NDL performs better when lifeboat capacity is high relative to cabin capacity (e.g. cases 4 and 7) and nesting is useful. DCM performs better in opposite cases when lifeboat capacity is scarce (e.g. cases 3 and 8).
- Among the upper bounds Π^{UDCM} and Π^{UDCA}, the one based on DCM is the better one by not exceeding 3% of the optimal solution in all the cases, with a mean value exceeding the optimal solution by 1.3%. This encourages using this bound as a benchmark for large problems where the optimal solution cannot be obtained.

Table 5 gives results similar to those in Table 4, but with the fare prices presented in Table 3. (Recall that Table 3 fares are more heterogeneous among different customer segments than those in Table 2.)

Table 5 reveals similar insights as Table 4 in the sense that heuristics NDL and DCM are the best of the five heuristics with average optimality gaps of 0.5% and 0.2% respectively and with a significant improvement over FCFS exceeding 6% by both heuristics. Similar to the cases in Table 4, NDL (DCM) performs better when the lifeboat capacity is ample (scarce) as in case 4 (case 8). In addition, DCA continues to rank third. However, it is dominated by DCM in all cases of Table 5, except in case 8 where the lifeboat capacity is tight and the two heuristics exhibit similar performance. Finally,

[8] This represents a discrete approximation to Poisson arrivals for each type of customers. This approximation is valid with a large number of arrival periods, i.e., a small period size (see, for example, Theorem 5.1, p. 291, in Ross [23]).

Table 4
Expected revenue as percentage of optimal. Base case: $(\mathbf{c},L) = (13,13,9,98), T = 70$, p_{ij} in Table 1, and r_{ij} in Table 2.

Case	Change	Π^{AC}	Π^{DL}	Π^{NDL}	Π^{DCM}	Π^{DCA}	Π^{FCFS}	Π^{UDCM}	Π^{UDCA}
1	None	96.3	94.6	99.8	99.8	98.8	93.4	100.9	105.6
2	$C_1 = 15$	95.3	96.3	99.7	99.7	99.0	93.4	100.9	105.3
3	$C_3 = 10$	96.2	94.8	99.6	99.8	99.1	94.0	100.9	104.3
4	$L = 108$	94.9	97.9	100.0	99.6	97.9	93.1	100.8	110.2
5	$(\mathbf{c},L) = (15,15,10,108)$	97.1	93.4	99.9	99.8	98.7	93.5	101.0	105.1
6	$C_1 = 11$	94.8	95.5	99.8	99.8	98.6	93.5	101.0	106.1
7	$C_3 = 8$	95.8	95.2	99.9	99.7	98.7	93.0	101.2	107.3
8	$L = 88$	93.6	93.1	98.3	99.1	99.6	95.1	103.0	103.3
9	$(\mathbf{c},L) = (11,11,8,88)$	93.9	96.3	99.5	99.6	98.7	93.0	101.9	106.2
	Mean	95.4	95.3	99.6	99.6	98.8	93.6	101.3	106.0

Table 5
Expected revenue as percentage of optimal. Base case: $(\mathbf{c},L) = (13,13,9,98), T = 70, p_{ij}$ in Table 1, and r_{ij} in Table 3.

Case	Change	Π^{AC}	Π^{DL}	Π^{NDL}	Π^{DCM}	Π^{DCA}	Π^{FCFS}	Π^{UDCM}	Π^{UDCA}
1	None	96.8	92.7	99.7	99.9	98.7	93.0	101.1	103.1
2	$C_1 = 15$	95.7	91.9	99.7	99.9	98.9	93.0	101.1	102.8
3	$C_3 = 10$	96.5	91.4	99.5	99.9	98.9	93.5	101.3	102.4
4	$L = 108$	94.8	96.5	100.0	99.7	98.4	92.8	101.0	106.4
5	$(\mathbf{c},L) = (15,15,10,108)$	97.6	91.6	99.9	99.9	99.0	93.0	100.1	102.2
6	$C_1 = 11$	94.7	92.1	99.7	99.9	98.4	93.2	101.3	103.7
7	$C_3 = 8$	96.3	93.2	99.8	99.8	98.7	92.7	101.3	104.1
8	$L = 88$	93.6	89.9	97.8	99.1	99.1	94.2	101.1	101.8
9	$(\mathbf{c},L) = (11,11,8,88)$	93.8	94.7	99.5	99.8	97.3	92.7	100.1	104.4
	Mean	95.5	92.7	99.5	99.8	98.6	93.1	101.0	103.4

the upper bound based on DCM continues to be of excellent quality by not exceeding the optimal solution by more than 1.3% in all the cases of Table 5.

6.2. Results for medium-size problems

The largest three-cabin problem we could solve with our two-dimensional heuristics has the following parameters: $\mathbf{c} = (75,75,52)$, $L = 560$ and $T = 400$. We decreased the capacities in this problem by around 10% and used $\mathbf{c} = (67,67,47)$ and $L = 504$ for our base case. We utilize the arrival probabilities in Table 1. These give average demand to capacity ratios of 58%, 122%, 203% and 133% for the three-cabins and the lifeboat similar to our small-size problems in Section 6.1. Table 6 presents results similar to those in Table 4 for our medium-size problem. Since the optimal solution could not be obtained here, we utilize the FCFS policy benchmark, and we report our results in Table 6 as a percentage of this benchmark.

Table 6 reveals similar insights to those observed for small-size problems in Section 6.1. Specifically, the two leading heuristics continue to be DCM and NDL achieving an improvement of 4.5% and 5% in expected revenue over the FCFS policy. It is to be noted here that NDL dominates DCM in eight out of the nine cases (except in case 8). However, this dominance is based on a small margin which might not be statistically significant. Case 8 is unusual as both DCM and NDL perform below FCFS while DCA performs marginally better at 101.1% of FCFS. This could be explained by the tight lifeboat capacity in case 8 where the average lifeboat demand to capacity ratio is 148%. Recall that in Case 8 of Table 4 for the small-size problems DCA exhibited a similar marginally superior performance. This implies that one may consider utilizing DCA when lifeboat capacity is low. Finally, Table 6 also indicates that the upper bound based on DCM, Π^{UDCM}, is of better quality than that based on DCA, Π^{UDCA}.

Table 7 presents results similar to those in Table 6 but with fare prices given by Table 3 rather than Table 2. Insights similar to those in Table 6 are observed in Table 7, with DCM and NDL being the best heuristics in most cases, the upper bound based on DCM being better than that based on DCA, and DCA performing well when the lifeboat capacity is tight (case 8).

6.3. Results for large-size problems

In this section, we are concerned with large problems that cannot be solved optimally or with two-dimensional heuristics. These problems can be only solved with the single-dimensional decoupling heuristics proposed in Section 5. We choose a base case having a realistic size with $\mathbf{c} = (650,650,450), L = 4{,}900$ and $T = 3500$.

Table 6
Expected revenue as percentage of FCFS. Base case: $(\mathbf{c}, L) = (67,67,47,504), T = 400, p_{ij}$ in Table 1, and r_{ij} in Table 2.

Case	Change	Π^{AC}	Π^{DL}	Π^{NDL}	Π^{DCM}	Π^{DCA}	Π^{UDCM}	Π^{UDCA}
1	Base	98.9	99.9	105.9	105.2	105.0	108.6	112.1
2	$C_1 = 75$	97.6	100.1	105.9	105.2	105.1	108.6	112.0
3	$C_3 = 52$	98.0	98.4	104.9	104.6	104.7	107.8	110.3
4	$L = 560$	98.0	103.4	106.2	105.1	103.2	108.7	118.2
5	$(\mathbf{c},L) = (75,75,52,560)$	100.3	99.3	105.0	104.4	103.8	106.8	112.2
6	$C_1 = 60$	98.1	100.0	105.9	105.2	104.9	108.6	112.3
7	$C_3 = 42$	99.1	100.6	106.3	105.2	104.9	109.3	114.1
8	$L = 454$	93.7	95.0	99.6	99.9	101.1	108.0	107.5
9	$(\mathbf{c},L) = (60,60,42,454)$	96.2	100.5	105.5	105.3	105.4	109.5	112.1
	Mean	97.8	99.7	105.0	104.5	104.2	108.4	112.3

Table 7
Expected revenue as percentage of FCFS. Base case: $(\mathbf{c}, L) = (67, 67, 47, 504), T = 400$, p_{ij} in Table 1, and r_{ij} in Table 3.

Case	Change	Π^{AC}	Π^{DL}	Π^{NDL}	Π^{DCM}	$\boldsymbol{\Pi^{DCA}}$	Π^{UDCM}	Π^{UDCA}
1	Base	100.0	96.1	105.0	104.6	102.8	106.8	110.8
2	$C_1 = 75$	97.6	96.3	105.0	104.6	102.9	106.8	110.6
3	$C_3 = 52$	98.5	94.8	104.0	104.0	102.6	106.3	109.5
4	$L = 560$	96.5	99.9	105.3	104.5	102.8	107.0	115.3
5	$(\mathbf{c}, L) = (75, 75, 52, 560)$	100.4	95.3	104.3	103.8	102.9	105.4	110.1
6	$C_1 = 60$	98.2	96.3	105.0	104.6	102.7	106.8	111.0
7	$C_3 = 42$	99.4	97.4	105.4	104.6	103.0	107.3	112.1
8	$L = 454$	93.8	92.1	98.7	99.7	101.6	106.6	107.6
9	$(\mathbf{c}, L) = (60, 60, 42, 454)$	99.3	96.8	104.4	104.6	102.1	107.7	111.5
	Mean	98.2	96.1	104.1	103.9	102.6	106.7	110.9

Table 8
Expected revenue as percentage of FCFS. Base case: $(\mathbf{c}, L) = (650, 650, 450, 4,900), T = 3500$, p_{ij} in Table 1, and r_{ij} in Table 2.

Case	Change	Π^{DCM}	Π^{DCA}	Π^{UDCM}	Π^{UDCA}
1	Base	104.1	103.7	106.8	113.0
2	$C_1 = 715$	104.1	103.8	106.8	112.8
3	$C_3 = 495$	104.4	104.2	106.2	111.2
4	$L = 5390$	101.2	101.2	106.8	118.5
5	$(\mathbf{c}, L) = (715, 715, 495, 5390)$	101.1	101.1	105.2	112.9
6	$C_1 = 585$	104.1	103.6	106.8	113.1
7	$C_3 = 405$	103.8	103.3	107.5	114.9
8	$L = 4410$	103.0	103.4	106.8	107.7
9	$(\mathbf{c}, L) = (585, 585, 405, 4410)$	105.2	105.5	108.7	112.7
	Mean	103.4	103.3	106.9	113.0

With the arrival probabilities given in Table 1, the average demand to capacity ratios for this base case are similar to those for small and medium problems in Sections 6.1 and 6.2. Table 8 presents results similar to those in Table 6 for our large-size problem. Results in Table 8 are also presented as a percentage of the FCFS benchmark.

Table 8 indicates that both the DCM and DCA heuristic perform well with average revenues of 3.4% and 3.3% of FCFS, with DCM performing marginally better. This table also indicates that DCM (DCA) does marginally better when the lifeboat capacity is high (low) as in case 4 (case 8). Note also that the upper bound based on DCM continues to be tighter. Table 9 presents results similar to those in Table 8 but with fare prices given by Table 3 rather than Table 2. Insights similar to those in Table 8 are observed in Table 9.

6.4. Summary and recommendations

The classification of problems by their sizes in the numerical/simulation analysis above is useful in deciding when to apply the optimal solution and each heuristic over the booking period of a real cruise ship. At the beginning of the booking period, the problem at hand is large, and only single-dimensional heuristics can be applied. In this case, the results in this section suggest that the DCM heuristic is a good alternative with modest storage requirement that can be applied for any problem size. When the lifeboat capacity is low and the fare prices are not too heterogeneous, then our analysis indicates that the DCA heuristic may offer marginal improvements.

Towards the middle of the booking period, the problem becomes of medium size, and one could consider switching from single-dimensional heuristics to two-dimensional ones. Our analysis indicates that only the NDL heuristic is promising in this regard. However, NDL appears to offer only a marginal improvement over DCM, and may offer no improvement when lifeboat capacity is low and fare prices are not too heterogeneous. Therefore, unless one is willing to carry over a careful "offline" analysis (similar to what we do above) comparing NDL, DCM, and DCA, we recommend using DCM for middle size problems.

Finally, close to the cruise ship departure when most of the capacity is sold, one may switch to the more storage intensive optimal approach. Nonetheless, our numerical results indicate that adopting the DCM heuristic leads to a minor loss of expected revenue at the benefit of much less storage (and perhaps speed as retrieving data from large databases can be time consuming).

To summarize, the DCM heuristic appears to offer a good alternative that can be applied at all stages of the booking period when revenue managing a cruise ship. However, one may consider switching to other heuristics in some extreme cases where lifeboat capacity is low.

Table 9
Expected revenue as percentage of FCFS. Base case: $(\mathbf{c}, L) = (650, 650, 450, 4,900)$, $T = 3500$, p_{ij} in Table 1, and r_{ij} in Table 3.

Case	Change	Π^{DCM}	Π^{DCA}	Π^{UDCM}	Π^{UDCA}
1	Base	101.6	100.9	104.0	110.0
2	$C_1 = 715$	103.3	102.4	105.3	110.8
3	$C_3 = 495$	103.3	102.5	105.3	110.6
4	$L = 5390$	103.6	102.5	104.9	109.6
5	$(\mathbf{c}, L) = (715, 715, 495, 5390)$	102.0	100.9	105.3	114.7
6	$C_1 = 585$	103.3	102.4	105.3	110.9
7	$C_3 = 405$	103.1	102.2	105.8	112.1
8	$L = 4410$	102.1	102.5	105.3	106.9
9	$(\mathbf{c}, L) = (585, 585, 405, 4410)$	104.4	102.6	106.9	111.3
	Mean	103.0	102.1	105.3	110.8

7. Conclusion

In this paper, we present an original model and analysis for cruise ship revenue management via dynamic capacity control. Our model is the first to offer a comprehensive analysis of cruise ship RM with stochastic demand and multi-dimensional capacity constraints accounting for both cabin categories and lifeboat capacity. We formulate the problem as a Markov decision process that gives an optimal control policy of the cruise ship. We illustrate that the optimal policy is not of the threshold-type (defined, for example, by optimal booking limits or critical time periods) widely observed in airline RM due to non-monotone behavior of the problem. The non-monotone behavior seems to be related to two factors: (i) The multi-dimensional capacity nature of the problem, and (ii) the uneven utilization of capacity by different customers (here families of different sizes). The multi-dimensional nature of the problem and the lack of optimal threshold-type optimal policies hinders the development of an optimal solution for large-size problems that may arise in practice. We, therefore, focus on developing several efficient heuristics and bounds that work by aggregating the state space into a reasonable number of single or two-dimensional states. Our careful numerical analysis indicates that a single-dimensional heuristic, that works by decoupling the cabin and lifeboat control problems based on the marginal revenue beyond the double-occupancy rate, presents a control policy and an upper bound on the optimal revenue of high quality in most cases. This heuristic shows some weakness in certain extreme cases with very low lifeboat capacity, where we recommend alternate approaches.

Our analysis of the cruise problem offers a basic model that can be extended in several directions by incorporating practical factors that may enhance capacity control. Two factors that we are currently working on are the consumer behavior of demand substitution and the management practice of cabin category overbooking. Other special features of cruise ship RM such as on-board spending may be addressed in future research.

Our work opens the door for analyzing several multi-dimensional revenue management problems via MDPs. Recent literature has already picked-up on a related problem in cargo RM (e.g. Amaruchkul et al. [2] and Levin et al. [18]). There remain several interesting problems arising in several areas such as multi-leg (network) airline and hotel systems, airline passenger luggage, and commuter trains and ferries (where passengers may bring their cars on-board). The non-monotone behavior demonstrated in this paper implies that no unified capacity control policies can be applied for these multi-dimensional problems. Instead, research needs to develop well-crafted ad hoc heuristics and bounds that are based on understanding the distinctive features of every problem, as we do in this paper for the cruise problem.

References

[1] J. Alstrup, S. Boas, O.B.G. Masden, R. Vidal, V. Victor, Booking policy for flights with two types of passengers, European Journal of Operational Research 27 (1986) 274–288.
[2] K. Amaruchkul, W.L. Cooper, D. Gupta, Single-leg air-cargo revenue management, Transportation Science 41 (2007) 457–469.
[3] P.P. Belobaba, Application of a probabilistic decision model to airline seat inventory control, Operations Research 37 (1989) 183–197.
[4] N. Biehn, A cruise ship is not a floating hotel, Journal of Revenue and Pricing Management 5 (2006) 135–142.
[5] Business Research and Economic Advisors, The Contribution of the North American Cruise Industry to the U.S. Economy in 2006. Available online at <http://www.cruising.org/press/research/U.S.CLIA.Economic.Study.2006.pdf>.
[6] R.E. Chatwin, Multiperiod airline overbooking with a single fare class, Operations Research 46 (1998) 805–819.
[7] W.C. Chiang, J.C.H. Chen, X. Xu, An overview of research on revenue management: Current issues and future research, International Journal of Revenue Management 1 (2007) 97–128.
[8] Cruise Lines International Association, The 2006 Overview, Available online at <http://cruising.org/press/overview%202006/2006OV.pdf>.
[9] S. El-Haber, M. El-Taha, Dynamic two leg airline seat inventory control with overbooking, cancellations and no-shows, Journal of Revenue and Pricing Management 3 (2004) 142–170.
[10] A. Gosavi, N. Bandla, T.K. Das, A reinforcement learning approach to a single-leg airline revenue management problem with multiple fare classes and overbooking, IIE Transactions 34 (2002) 729–742.
[11] A. Gosavi, A reinforcement learning algorithm based on policy iteration for average reward: Empirical results with yield management and convergence analysis, Machine Learning 55 (2004) 5–29.
[12] J. Hoseason, Capacity management in the cruise industry, in: A. Ingold, I. Yeoman, U. McMahon-Beattie (Eds.), Yield Management: Strategies for the Service Industries, second ed., Thomson, London, 2000.
[13] L. Ji, J. Mazzarella, Application of modified nested and dynamic class allocation models for cruise line revenue management, Journal of Revenue and Pricing Management 6 (2007) 19–32.
[14] I. Karaesmen. Three essays on revenue management, Ph.D. Thesis, Columbia University, 2001.
[15] R.G. Kasilingam, Air cargo revenue management: Characteristics and complexities, European Journal of Operational Research 96 (1996) 36–44.
[16] C.J. Lautenbacher, S. Stidham, The underlying Markov decision process in the single-leg airline yields management problem, Transportation science 33 (1999) 136–146.
[17] T.C. Lee, M. Hersh, A model for dynamic airline seat inventory control with multiple seat bookings, Transportation science 27 (1993) 252–265.
[18] Y. Levin, M. Nediak, H. Topaloglu, Cargo capacity management with allotments and spot market demand, Working paper, Queen's University, 2008.
[19] J.I. McGill, G.J. van Ryzin, Revenue management: Research overview and prospects, Transportation Science 33 (1999) 233–256.
[20] B. Maddah, L. Moussawi, M. El-Taha, H. Rida, Dynamic Cruise Ship Revenue Management - Unabridged, Working Paper, Engineering Management Program, American University of Beirut, Beirut, Lebanon, 2009. Available online at staff.aub.edu.lb/ ~ bm05/research/cruise-full.pdf.
[21] L. Moussawi, M. Çakanyildirim, Profit maximization in air-cargo overbooking, Working Paper, School of Management, University of Texas at Dallas, 2006.
[22] R. Phillips, Pricing and Revenue Optimization, Stanford University Press, 2005.
[23] S.M. Ross, Introduction to Probability Models, eighth ed., Academic Press, Amesterdam, 2003.
[24] J. Subramanian, C.J. Lautenbacher, S. Stidham, Airline yield management with overbooking, cancellations and no-shows, Transportation science 33 (1999) 147–167.
[25] K.T. Talluri, G.J. van Ryzin, The Theory and Practice of Revenue Management, Springer, New York, 2005.
[26] L.R. Weatherford, S.E. Bodily, Taxonomy and research overview of perishable asset revenue management: Yield management, overbooking, and pricing, Operations Research 40 (1992) 831–844.