Practice Article

Modeling a cruise line revenue management problem

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ABSTRACT The success of cruise line revenue management at the company which is studied in this article depends on pricing and assignment of room capacity. Market demand is typically defined by such criteria as view requirements, bedding requirements, room locations and amenities. The essence is to price each market segment and allocate cruise room capacity at the same time in order to maximize the total revenue potential. Side constraints include capacity limits on lifeboats and personnel requirements, for example, the number of baby counselors (required by law). It is also important to follow the business rules such as those pertaining to refundable tickets. However, this article's focus and the key priority is to develop a linear model that can facilitate demand forecasting, which not only considers pricing and room assignment, but also considers potential onboard expenses of customers. A numerical test is conducted based on production data. The results indicate that the solution obtained through this model is valid in practice and shows significant annual revenue increase compared with the previous system.

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INTRODUCTION

Cruise is a special tourism industry that combines travel, resort and hospitality. In the United States, three dominant cruise lines, Norwegian Cruise Line (NCL), Royal Caribbean International (RCI) and Carnival Cruise Lines, were founded in 1966, 1968 and 1972, respectively. Over a period of about 30 years, cruise lines have been one of the fastest growing segments of the tourism industry. Customer numbers, cruise line revenues, the fleet size of cruises and their total capacities have been increasing enormously since the late 1970s. According to Cruise Lines International Association (CLIA), the annual passenger demand has grown at a rate of 7.6 per cent and capacity utilization is consistently over 100 per cent. In 2010 the cruise line industry occupancy rate was 103.2 per cent. (CLIA, 2011). Judged by the published occupancy rates of the major cruise lines, demand has been keeping up with the supply of capacity (Vogel *et al*, 2012).

The cruise line industry, in spite of its flourish, has received limited attention in the community of research on advanced revenue management. In fact, the average price of the passenger tickets has declined significantly in the last decade partially because of the capacity expansion, indicating a fierce competition between cruise lines in today's market. This not only requires cruise lines to provide better services, but also presses for adaption of better revenue management strategies to remain competitive.

Thanks partially to its unique services, the company which is studied in this article (simply referred to as the company later) has made a great success since it entered the market in August of 1999 by focusing on a special market segment: kids and families. The company had two ships prior to 2010, offering 3, 4, 7, 10 and 14 night trips, to the Bahamas and the Caribbean. To increase the overall profit, the company's department of revenue and profit management has been looking for a better commercial software package to expedite pricing and room capacity assignment decisions since 2007. Several prototypes were delivered prior to our work, but none of them have met the needs of the company's complicated practical concerns such as the capacity constraints and business rules. With current market expansion, especially with two new ships added in the last 3 years, such a software becomes much more important in optimizing the revenue. Nowadays, the company's cruise capacity includes 8508 lower berths, up from a total of 3508 in year 2010 (CLIA, 2012).

LITERATURE REVIEW

The cruise line business is a special application of revenue management theories. It sells perishable products and services over a finite time period and shares a number of common characteristics with airlines and hotel industries such as constrained capacity, seasonally varying demands (Lieberman and Dieck, 2001) and highly segmented markets (Dickinson and Vladimir, 1989). The objective is to maximize the revenue received by the cruise line from the sales of rooms on each sailing. However, according to a survey by Chiang *et al* (2007), there has been a lack of successful revenue management practice for the cruise industry. Literature in this area is sparse and has received limited attention compared with the extensive research on airline and hotel revenue management, largely because of its smaller scale of operations compared with airlines.

Early studies (Kimes, 1989; Ladany and Arbel, 1991) consider cruise ships as floating hotels. Gibson (2006) provides a review in detail for these studies. Although this assumption appears insightful and sheds light on the nature of the revenue management problem, recent studies (Biehn, 2006; Maddah *et al*, 2010) identify a list of key features that differentiate cruise ships from floating hotels. These include guest pricing, multiple capacity constraints, leisure customers with a high show-up rate, wave booking period, on-board spending and high load factor, which are sequentially explained below.

Somewhat different from guest pricing in the hotel industry, cruise line prices appear to be more sensitive to the mix of each traveling party. Hotels prefer to price on a per room basis because the cost of a double room is not much more than a single room. The cost of servicing a hotel room is almost invariant to the number of occupants (Toh *et al*, 2005). In the cruise industry, however, the cost of a room depends much more on the characteristics of a traveling party – an additional person means substantial additional cost due to food, beverages and other on-board consumptions/costs and consumption. In addition, the cruise lines often incorporate stops at resorts and entertainment in the cruise tour package, where the size of a traveling party has a non-trivial (highly important) cost to the operations. For example, some cruises include a visit to Castaway Cay, a 1000-acre private Bahamian island. As a result, in the measurement of occupancy rates in the cruise industry, the number of people and rooms should both be considered, which explains the current pricing practices in the cruise industry. In the current practice of the company that is studied here, all cruise rooms are priced based on at least two adults in each room. For example, when one adult travels alone or with a child, the company still charges the same room rate as for two adults. On top of the price for two adults, the third and fourth passengers in the room are each charged at an incrementally reduced marginal rate.

Regarding the capacity, Biehn (2006) argues that the limit number of lifeboats is a critical factor in cruise revenue management. For example, there can be a significant loss of revenue if too many large families reserve early, and the ship runs out of lifeboat capacity faster than room capacity. Likely because of this reason, several cruise lines accept no more than 2.5 or 3 customers per room. By formulating a deterministic linear program, Biehn (2006) introduces a method to maximize the cruise revenues subject to two capacity limitations: the number of rooms according to room category and the number of lifeboat seats in total. In most cases, the number of rooms in each category can slightly vary while the total number of rooms is capped at a fixed number.

For better applications of revenue management, theories suggest the presence of high fixed costs and low variable costs with product sales (Lieberman and Dieck, 2001). One may conclude that it is sufficient to seek booking policies that maximize revenue in the cruise line industry. Typical overbooking calculation depends on the prediction of the probability distribution of the close-to-departure cancellations and no-shows (McGill and van Ryzin, 1999). In the airline industry, revenue passengers are sometimes denied boarding with compensations because of overbooking and more-than-expected final show-up. Thus, overbooking control is an important research topic. Descriptions of statistical models toward overbooking calculations can be found in Beckmann and Bobkowski (1958). However, traditional overbooking management in the airline and hotel cannot be simply transplanted to the cruise lines because of the differences of travel characteristics between industries. Toh *et al* (2005) find that the common practice of overbooking in the case of a whole ship would be unrealistic due to the high show-up rate and the difficulty of handling the subsequent high percentage of overbooked customers. In the cruise industry, it is almost impossible to deny passenger boarding because of overbooking. The reason is that there is no alternative cruise in a short period. Talluri and van Ryzin (2004), however, argue that it is still possible to overbook the low-fare rooms because of the possible upgraded services in practice. The high show-up rate is attributed not only to leisure travelers who usually plan their trips well before the departure date, but also to the well-publicized deadlines before which the payments must be made in full. For example, the company regulates that payment for the cruises that sail for less than 10 days must be made no later than 75 days before departure. Late cancellations are penalized according to the remaining time before departure. This kind of partially refundable fares can be viewed as a risk management strategy that appeals to customers with high uncertainty in travel schedules (Gallego and Sahin, 2010). As a result, price risk protection is necessary to prevent price dilution.

It is also encouraging to observe that cruise lines do not have to worry early departures and holdovers as hotels do. According to Smith Travel Research (2004), the occupancy rate for the non-resort hotel industry is only about 60 per cent. This rate is very low when compared with hotel resorts and cruises. Li *et al* (2013) recently published an article how to optimally assign rooms in major resort hotels. Disney Corporation (2012) reports a total occupancy rate of 82 per cent considering all its hotels in fiscal year 2012. And this number is in contrast with the cruise lines' almost 100 per cent rate as reported in CLIA (2013). Owing to the nature of the cruise business, cruise passengers are almost always loyal to their scheduled cruise trips, which is an important factor in developing the revenue management models and practices for the cruise industry.

Demand forecasting is particularly critical in revenue management. One major issue with it is that the past demand in historical booking records is censored by the available capacity. Swan (1993) developed a formula to address the bias due to this truncated data. This method provides a simple statistical remedy that is used by practitioners for many years to patch the demand. McGill (1995) develops a multivariate multiple regression analysis to remove the effects of censorship in multiple booking categories. Sun *et al* (2011) apply a variety of forecasting methods using data from a major North American cruise company to generate final bookings; they find that classical pickup (CP) methods perform the best, followed by advanced pickup (AP) methods and non-pickup methods. One cruise industry study from Ji and Mazzarella (2007) investigates the industry's unique demand patterns and proposes a dynamic room allocation model to help ease the stress in demand forecasting. Based on simulations using actual booking data, they obtain an average of 4.2–6.3 per cent revenue increase. Unfortunately, their model does not consider lifeboat capacity constraint and other side constraints mentioned above. Later work of Maddah *et al* (2010) develops an approach to consider lifeboat capacity while looking into the stochastic nature of customer bookings. In order to obtain reasonable solutions, a heuristic algorithm is proposed in their work as the model itself is too complicated to achieve an optimal solution.

High load factor is the second primary objective in cruise line operations. Unlike the hotel industry where staffing can be adjusted according to occupancy, cruise lines cannot easily change staffing from cruise to cruise (Lieberman and Dieck, 2001). Hence, ships with many unfilled rooms may have serious problems because staff depend on gratuities for their income. This is particularly true for the company that is studied in this article. For example, some of the international waitresses on their ships are salaried very low and they heavily rely on tips for their living. And, a high load factor is necessary for many activities/events that need a large audience to participate. Of course, a large load factor contributes directly to revenue. It is reported that for an average cruise, the onboard revenue generated from casino operations, sales of alcohol, on-shore tours, spas, salon care, and other discretionary goods and services such as photographs and souvenirs during the cruise can be about 25 per cent of the total revenue (Cruise Market Watch, 2013).

Overall, there is an information gap concerning cruise line revenue management that addresses the distinct characteristics of this tourism industry. Although there is an academic need for research on optimal room booking policies and their structural properties, a need to develop practical and implementable approximations is imminent for the interest of practitioners (McGill and van Ryzin, 1999). This article finds its niche by presenting a practical research within the studied company on how the rooms/cabins should be managed and priced at the same time through innovative mathematical modeling and algorithm development.

PROBLEM DEFINITION

This article is based on a research project within one of the major cruise line to develop mathematical models and algorithms to improve the profitability of its revenue management. The details of this problem may be described as follows. Each cruise ship has around 800 guestrooms called staterooms (cabins), classified into 12 categories according to size, location, view and other amenities. Based on their size, each room can lodge from two to eight guests. In each room category, guests are charged according to three types of fares: double occupancy (DO), extra adults (XA) and extra children (XC), with each type of fare having 18 price tiers. The company charges at least the rate of two persons (DO) for each room. Table 1 explains how to identify the party (passenger) mix.

The price tier descends from a full fare called *brochure*, indicating published rate in a public brochure or T12 – all the way to T0 numerically, and continues to TA, TB, TC and TD alphabetically as shown in Table 2. The problem is to make pricing and room assignment decisions for the rooms according to these price tiers in order to maximize the total revenue. In other words, given the market demand and its elasticity, the decision is to simultaneously decide how much to charge for each room category/fare type and how many rooms should be assigned to that category given the limited number of rooms in total. Note that we forecast demand based on historical data according to customers' requirements on room amenities and their affordable prices for each price tier. Therefore, the demand for each price tier is assumed to be known – a common practice in airline and hotel industries. A fare tier is the combination of a fare type and a price tier. For example, room category two with a DO fare in the price tier TC is called a fare tier. When a non-zero room capacity is assigned to a fare tier for sale, the fare and all tiers above for that specific room type are all open, which mimics a nesting structure fare and is popular in the service industry. When no capacity is assigned to a fare tier – in other words, a fare tier is closed for a specific room-fare type – all tiers below for the same room-fare type are not for sale.

There are unique characteristics in this cruise revenue management study, making this

Table 1: Example party mix

Party mix	*Fare charge*
1 adult	2 DO
2 adults	2 DO
1 adult+1 child	2 DO
3 adults	2 DO+1 XA
2 adults+1 child	2 DO+1 XC
1 adults+2 children	2 DO+1 XC
3 adults+1 child	2 DO+1 XA+1 XC
4 adults	2 DO+2 XA

Abbreviations: DO, double occupancy; XA, extra adults; XC, extra children.

Table 2: Parameters

Parameters	*Description*
i	Stateroom category i
j	Fare type j
k	Price tier k
R	Set of stateroom category, $R = \{1, 2, \ldots, 12\}$
T	Set of fare type, $T = \{0(\text{DO}), 1(\text{XA}), 2(\text{XC})\}$
P	Set of price tier, $P = \{1, 2, \ldots, 18\}$
d_{ijk}	Forecasted maximum demand associated with i, j, k
a	The number of available staterooms on the ship
b	Total capacity of lifeboat seats
c	Total availability of baby counselors
θ	The percentage of babies that need counselors among extra children
p_{ijk}	Price of room category i, fare type j under price tier k
t_{ij}	Previous price for room category i and fare type j
e_{ijk}	Average expense on board per customer for room category i, fare type j and price tier k

Abbreviations: DO, double occupancy; XA, extra adults; XC, extra children.

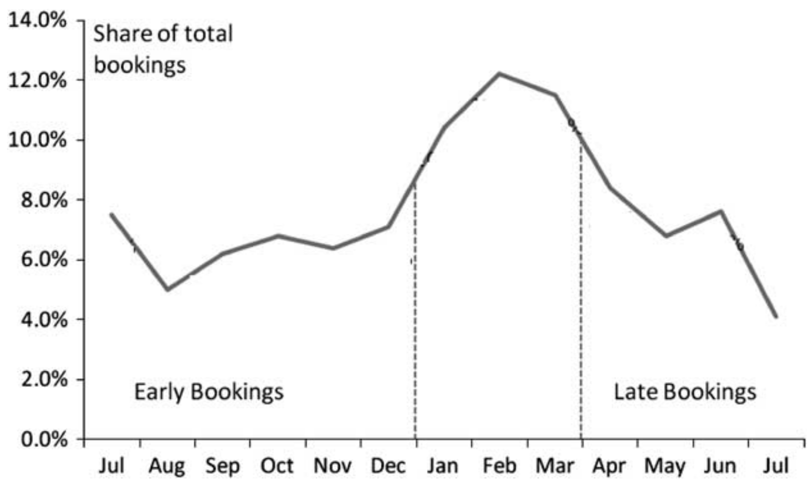

Figure 1: An example booking trend of a cruise departing in late July.

seemingly straightforward problem harder to tackle. Different from typical airlines and hotel room revenue management, this problem not only has to do with the number of room requests, party size, party mix and room-related inventories, but also needs to consider lifeboat seats and the availability of required personnel (for example, baby counselors). By law, each guest on board must have a lifeboat seat. Infants under 2 years old must have a baby counselor. These counselors are trained by the company to take care of the children while their parents take part in more grown-up adventures on and off the ship. For information, the older ships have about 2700 lifeboat seats on each ship (about 2400 for passengers and 300 for crews), while the number of baby counselors is limited as well. The youngest ship has a larger capacity of 4000. In addition, due to the fact that the company is targeting relatively young customers and catering to family travels, business rules regulate that the price to each person in DO must be greater than that of XA, and the price to XA must be greater than that of XC. This ensures the larger group has a lower average price per person. Needless to say, a better room category must have a higher price according to marketing strategy. In addition, the lowest available fare tier posted for the same room type must be non-decreasing over the time of the entire booking horizon. This strict rule in practice is intended to give incentives to early bookings. Because the company is committed to unconditional ticket cancellation and refunds with the only exception for ships that are within few days of departure, a non-decreasing fare structure is also needed to maintain the value of early bookings. Figure 1 illustrates an example booking curve of a cruise departing in late July.

This cruise line revenue management problem of pricing and room assignment directly determines the cruise line profitability. The authors develop an integer programming model to solve this problem. The model has a linear objective function and linear constraints to provide quality solutions in order to maximize total revenue. Numerical tests with production data are conducted at the end of this article. The results indicate that the solution obtained through our model is valid in practice and shows significant annual revenue increase when compared with prior practice.

SELLING UNCERTAINTY TO THE CUSTOMER

Before presenting our formulation, it is necessary to provide information on how we address demand forecasting and uncertainty. Demand forecasting is based on the historical data, which is the actual demand in the past. However, past demand records are censored by the available capacity. Even if we could fix this bias through statistical remedies, there is no guarantee that the forecasted demand would work for overbooking, cancellation and purchasing behavior such as the spiral-down effect.

The spiral-down effect occurs when incorrect assumptions about customer behavior cause high-fare ticket sales, protection levels and revenues to systemically decrease over time (Cooper *et al*, 2006). For example, the availability of low-fare tickets may reduce high-fare sales, resulting in lower future estimates of high-fare demand. Therefore, one simply cannot forecast demand for each price tier independently in practice. This phenomena, associated with problems caused by cancellation, creates not only difficulty in forecasting demand but also in determining ticket revenues and pricing policy.

On the other hand, there is no doubt that overbooking can be applied to cruise lines since it is a popular way to address uncertainty. When a no-show or cancellation occurs, a reasonable level of overbooking can still guarantee a high load factor on the ship. The problem is that if everyone shows up, it is impossible to increase the capacity to board everyone or call for a backup ship like the majority of the airlines would do. Therefore, the overbooking in our studied company is set to a very low level and cannot deal with a massive cancellation and rebooking caused by price dilution. In the work of Gallego and Talebian (2010), the authors note that although most economists assume that the provider is aware of the demand function, this is rarely the case in practice; indeed, market uncertainty is prevalent in most cases. The solution here for the company is to sell uncertainty to the customers.

First of all, the company promises and imposes a non-decreasing structure for ticket selling. That is to say, the price becomes only higher when the departure date approaches. In this case, the customers who bought tickets early will not complain about their lowered ticket price in the future, which prevents them from cancelling the purchased tickets or rebooking for cheaper ones. This not only encourages early booking but also facilitates future demand forecasting and pricing because the uncertainty is, therefore, transferred to the customers with minimized cancellation. Note that the starting price of early booking has to be set relatively low to attract customers. Otherwise, very few people would like to book a ticket that is one year ahead of the sailing date. Typically, the huge discount is provided for early booking.

Second, the company distributes the unsold rooms to its employers as internal benefit. No matter how we forecast the demand, it is still possible that the actual demand is lower than the capacity. Suppose that due to a large group booking, the availability of staterooms is reduced today and optimization model recommends closing all lower price tiers, that is, raising the price. However, a few months later, this group booking is cancelled and the availability of staterooms increases. Given that company is not allowed to open a lower price tier again to the public, it will offer these rooms to its employers at low prices, considering the on aboard non-ticket revenue is still the significant. This kind of ticket selling is for internal purpose only, it serves as employer benefit and therefore does not create any problem with the non-decreasing ticket structure promised to the public. Different than any other cruise lines, the company studied here is an entertainment complex that houses 24 themed resorts, four theme parks, two water parks and several additional recreational venues. It also shines in the film industry. In fact, 58 000 employees are employed by the company as of 2006, which provides a huge potential for internal ticket selling.

INTEGER PROGRAMMING FOR CAPACITY ALLOCATION AND PRICING

The cruise line typically has a large booking window (one year or longer) and, therefore, requires constant updating of bookings.

Although it might be possible to develop sophisticated real-time booking models through dynamic programming (DP), it is unknown to what extent the performance of such a system depends on frequency and accuracy of the updates. In this case, DP is not only time consuming, but also makes it hard to manage multiple resource constraints as the number of stages grow rapidly. Of course, sophisticated systems by themselves are costly. Enlightened by the work of Gallego and van Ryzin (1994), where a simple deterministic model is used for approximate optimal pricing and market segmentation, this article presents a static model that formulates an integer programming model with linear constraints and a linear objective function. The model is repeatedly applied during the booking period with updated remaining capacities and adjusted demand forecasting each time. x_{ijk} is a binary decision variable. It equals to 1 if stateroom (cabin) category i in fare type j is selected at price tier k; it is 0 otherwise. y_{ijk} is an integer decision variable that represents accepted demand for room category i in fare type j under price tier k. The goal is to make room assignment and pricing decisions simultaneously. Table 2 summarizes all the parameters.

Then we have:

$$\text{Maximize} \sum_{i\in R}\sum_{j\in T}\sum_{k\in P}\left(p_{ijk}\,y_{ijk}+e_{ijk}\,y_{ijk}\right) \qquad (1)$$

subject to:

$$\sum_{k\in P} x_{ijk}=1 \qquad \forall i\in R,\ \forall j\in T \qquad (2)$$

$$y_{ijk}\leqslant d_{ijk}x_{ijk} \qquad \forall i\in R,\ \forall j\in T,\ \forall k\in P \qquad (3)$$

$$\sum_{i\in R}\sum_{k\in P} y_{i0k}\leqslant 2a \qquad (4)$$

$$\sum_{i\in R}\sum_{J\in T}\sum_{k\in P} y_{ijk}\leqslant b \qquad (5)$$

$$\sum_{i\in R}\sum_{k\in P}\theta y_{i2k}\leqslant c \qquad (6)$$

$$\sum_{k\in P} p_{ijk}\,x_{ijk}\geqslant\sum_{k\in P} p_{i+1jk}\,x_{i+1jk} \qquad \forall i\in R/\{12\},\ \forall j\in T \qquad (7)$$

$$\sum_{k\in P} p_{ijk}\,x_{ijk}>\sum_{k\in P} p_{ij+1k}\,x_{ij+1k} \qquad \forall i\in R,\ \forall j\in T/\{2\} \qquad (8)$$

$$\sum p_{ijk}\,x_{ijk}\geqslant\omega\cdot t_{ij} \qquad \forall i\in R,\ \forall j\in T \qquad (9)$$

$$x_{ijk}\ \text{binary},\ y_{ijk}\ \text{integer} \quad \forall i\in R, \forall j\in T, \forall k\in P \qquad (10)$$

The first term $\sum_{i\in R}\sum_{j\in T}\sum_{k\in P}p_{ijk}y_{ijk}$ of the objective function (1) is for the ticket revenue; the second term $\sum_{i\in R}\sum_{j\in T}\sum_{k\in P}e_{ijk}y_{ijk}$ is for the non-ticket revenue. As mentioned earlier, staterooms (cabins) are priced differentially. The non-ticket revenue e_{ij} is also assumed to be different because a different party mix and a different ticket price would indicate a different consumption level on board, such as drinking, eating, souvenirs, tips and gambling. Owing to the existence of non-ticket revenue, it is important to optimize the number of customers as well as room utilizations. In an extreme case, a stateroom that contains eight customers may generate more tip revenue (\$80–\$100 per customer) than the room charge itself. Constraint (2) means each stateroom can be assigned to at least and at most one price tier. Constraint (3) puts the capacity on y_{ijk}, which is defined as the decision variable for the actual demand accepted for room category i and fare type j under price tier k. If price tier k is not selected for category i and fare type j, we have $x_{ijk}=0$, therefore $y_{ijk}=0$. If price tier k is chosen for category i fare type j, y_{ijk} then is limited by the maximum demand d_{ijk}. Note that d_{ijk} is a parameter obtained from a forecasted price-demand curve, which is tabulated before the optimization. We assume the demands across fare type are independent, that is to say, the party mix (that is, two adults, three adults or two adults with a child) is independent (see Table 1). Constraint (4) is for the capacity limit of the staterooms. The only consideration is DO because XA and

XC are associated with each pair of double occupancies (DOs). In other words, DO is the number of the parties regardless of their mix (see Table 1). It is worth noting that some rooms have flexibility to be equipped and be classified into different room categories while the total number of the available rooms on a ship remains unchanged. Constraint (5) is availability of life-boat seats, which puts a limit on how many customers can be accepted. Constraint (6) represents the availability of baby counselors. Constraint (7) and (8) together provide the requirement when pricing different room categories and fare types. Constraint (7) simply ensures that better rooms are priced higher for the same fare type (that is, DO). To ensure the larger group has a lower average price per person, the price for each person in DO (double occupancy) must be greater than XA (extra adults), and the price to XA is greater than XC (extra children). This is expressed in constraint (8). Table 3 illustrates this structure. The highest fare, or full fare in other words, is called 'brochure' fare, referring to the published fare.

Table 3: An example of price tiers (in dollars)

Room category	*Fare type*	*Price tier (in dollars)*									
		TD	*TC*	*TB*	*TA*	*T0*	*T1*	*T2*	…	*T12*	*Brochure*
1	DO	330	340	350	360	370	380	390	—	490	500
	XA	320	330	340	350	360	370	380	—	480	490
	XC	310	320	330	340	350	360	370	—	470	480
2	DO	320	330	340	350	360	370	380	—	480	490
	XA	310	320	330	340	350	360	370	—	470	480
	XC	300	310	320	330	340	350	360	—	460	470
3	DO	310	320	330	340	350	360	370	—	470	480
	XA	300	310	320	330	340	350	360	—	460	470
	XC	290	300	310	320	330	340	350	—	450	460
4	DO	300	310	320	330	340	350	360	—	460	470
	XA	290	300	310	320	330	340	350	—	450	460
	XC	280	270	260	250	240	230	220	—	440	450
5	DO	290	300	310	320	330	340	350	—	450	460
	XA	280	270	260	250	240	230	220	—	440	450
	XC	270	260	250	240	230	220	210	—	430	440
.	DO	—	—	—	—	—	—	—	—	—	—
.	XA	—	—	—	—	—	—	—	—	—	—
.	XC	—	—	—	—	—	—	—	—	—	—
12	DO	220	230	240	250	260	270	280	—	380	390
	XA	210	220	230	240	250	260	270	—	370	380
	XC	200	210	220	230	240	250	260	—	360	370

Abbreviations: DO, double occupancy; XA, extra adults; XC, extra children.
Note that each room category can have up to three different price tiers associated with three fare types respectively, as long as the constraints are satisfied.

Table 4: An example of stateroom categories

Category	1: Concierge royal suite with verandah	2: Concierge 1-bedroom suite with verandah	…	9: Deluxe family oceanview stateroom	10: Deluxe oceanview stateroom	11: Deluxe inside stateroom	12: Standard inside stateroom
Sleeps	5	5	…	3–5	3–4	3–4	3–4
Size:	1781 sq. ft. including verandah	614 sq. ft. including verandah	…	241 sq. ft.	204 sq. ft.	200 sq. ft.	204 sq. ft.
Deck	11, 12	11, 12	…	7, 8	9	5, 6, 7, 8, 9	5, 6, 7, 8, 9
Feature	Private verandah	Private verandah	…	One large porthole window with seating	One porthole with obstructed view	Magical porthole with real-time views	One porthole without exterior view

All the other 17 fares that go from T12 down to T0 and continue to go down to TA, TB, TC and TD represent larger discount in fares, for example, they go from 100 to 90 per cent and then to 80 per cent, and so on. Littlewood's (1972) rule sets up the principle: A discount fare class remains open as long as the revenue of a marginal seat exceeds the expected revenue of future full fare bookings. Note that during the implementation, the price for a stateroom remains at least the rate of two DO passengers. Constraint (9) is a business rule for preventing revenue dilution; when $\omega = 1$, the current price tier should be higher than or equal to earlier ones (that is, the price for same room same fare type weeks ago). Otherwise, but more generally, this business rule could be released by setting $\omega = 0$ for other applications, that is, non-decreasing or non-increasing is only a special case of this model. Slightly different from other major cruise lines, the company adopts a non-decreasing structure to facilitate early booking and demand forecasting given the unique characteristic that the remaining capacities can be utilized as an internal employee benefit.

The first row in Table 4 shows the categories of rooms. The second and third rows represent bed settings and room size, respectively. The fourth row lists the available room locations for customers to choose from. The last row provides additional information such as amenities. There are actually 12 categories on each ship. Owing to the space limit of this article, only six of them are presented here in decreasing order according to price.

ENHANCING THE ROBUSTNESS OF THE MODEL

When the booking window opens, the periodically updated demand information allows us to apply this deterministic model over a stochastic model for the sake of practical and also reliable solutions. Actually, one can only solve a real-world stochastic combinatory optimization problem heuristically, which is an approximation equivalent to solving a deterministic problem repeatedly and each time optimally. Some evidence is seen in Gallego and van Ryzin (1994).

If there is a unique, deterministic demand for each class, the proposed programming model with equations from (1) through (10) is ideal for application. However, demand in each class is random and can be typically characterized by a probability distribution. Therefore, there is a concern about how to choose a single deterministic number for demand to use in the deterministic math programming model (1) through (10). The question that needs to be answered is: Is it possible to have such a deterministic number to represent a random demand without significantly compromising the profitability or optimality of the model? In determining such a constant, a trade-off must be considered. If too low a number is chosen to represent the demand and when demand turns out to be high, which is possible for random demand although at a low likelihood, demand would not be fully satisfied. This would be problematic especially for high revenue demand. In other words, the cost of bumping off a high revenue customer is significant. On the other hand, if too high a number is used to represent the demand for a class and if the demand turns out to be low, a high revenue room would be left unfilled. By balancing the probabilities of both outcomes, truncating customers and leaving rooms unfilled, and their respective profits, one can decide an adequate scalar number to represent the demand to be used in the programming model proposed earlier. Again, this may be conducted in a context of a known demand probability distribution. This calculation is straightforward. Therefore, detailed calculation of this scalar number is skipped for the sake of saving space here.

In addition, although it is not a focus and is not elaborated in this article, estimating the true demand using historical data can be a theoretically challenging task due to the fact that historical booking was constrained and truncated by room capacities as well as whether price tiers were open or not and how long they were open. The truncated demand cannot be tracked. The lower the price tier, the more significant this issue becomes. Practitioners cannot observe the true demand at all, especially when capacity is highly constraining for some select price tiers according to historical data. To deal with this issue in demand forecasting, standard unconstraining methods such as expectation maximization can be applied (Queenan *et al*, 2007). One may argue for this assumption by noting a relatively stable economic structure and consumption ability. Overall, a hybrid (mixed priceable and yieldable) forecasting of unconstrained demand given the market, departure date, booking time window and price tier is required to deal with this less restrictive fare structure.

Another observation is that non-ticket revenue is significant and comparable with the ticket revenue in the company we studied here. Children are obsessed with costumes, toys and many other derivative products from fairy tales or films. For example, Cinderella's glass slipper is usually sold for \$125. Some products even have limited editions that are sold between \$500 and \$1000. During the cruise trip, the majority of the parents would like to purchase as many products as possible to make their children happy. This contributes to about 40 per cent of the non-ticket revenue. Therefore, it is important to consider the number of children on board in the model, which is y_{i2k} and its associated non-ticket revenue $e_{i2k}y_{i2k}$. Alcohol is another major contributor. Different than other cruises where the liquor can be only purchased at bar, the company provides a duty-free liquor shop onboard. The liquor purchased at the duty-free shop is held until the last night of the cruise and cannot be opened as long as the customer is on the ship. Without jeopardizing the revenue at the bar, this alcohol policy would actually encourage the customer to purchase extra wines and bring them home simply because they are cheap. Having the decision variable y_{i0k} and y_{i1k}, the total number of adults can be approximated by $y_{i0k}+y_{i1k}$ with its associated non-ticket revenue $e_{i0k}y_{i0k}+e_{i1k}y_{i1k}$.

COMPUTATIONAL RESULT

The problem formulation with equations (1)–(10) has 648 binary variables and 648 non-negative integer variables along with 852 constraints. Therefore, commercial software such as Cplex 12.1™ using a branch and bound algorithm (B&B) is able to solve the problem to optimality. An example of optimal solution can be seen in Table 5 for illustration purposes. The column 'price' shows the actual pricing (p_{ijk}) associated with optimized pricing tier variable x_{ijk}. The column 'Assigned Capacity' shows the number of rooms ($\sum_{k\in K}(y_{i0k}/2)$) assigned to category i.

Table 5: Sample solutions

Room category	*Assigned capacity (in rooms)*	*Fare type*	*Price (in dollars)*
1	2	DO	500(Brochure)
	—	XA	490(Brochure)
	—	XC	470(T12)
2	3	DO	490(Brochure)
	—	XA	470(T12)
	—	XC	460(T12)
3	14	DO	470(T12)
	—	XA	360(T2)
	—	XC	350(T2)
4	39	DO	460(T12)
	—	XA	350(T2)
	—	XC	230(T1)
5	52	DO	350(T2)
	—	XA	230(T1)
	—	XC	230(TO)
⋮	⋮	⋮	⋮
12	10	DO	240(TB)
	—	XA	220(TC)
	—	XC	200(TD)

Abbreviations: DO, double occupancy; XA, extra adults; XC, extra children.

Note that the optimality of our integer programming model does not indicate that the real problem of pricing and capacity allocation is solved to the optimal. Our model, which is subjected to the non-decreasing fare policy, is only an approximation to the original problem, which makes demand forecasting easier and the problem solvable. In fact, both stochastic and deterministic models are simplifications of real-world problems based on a set of assumptions. For instance, the arrival is assumed to be Poisson distributed, and the demand for each of the three fare types are independent of each other, which are absolutely simplifications and approximations. The company spent 5 years developing a real-time stochastic booking system, which is supposed to closely reflect the original problem. However, such a system still cannot be implemented because of the complexity of the nonlinear dynamic model and the extensive requirement on the frequency and the accuracy of demand forecasting. Therefore, the current practice within the company is based on the linear model presented in this article.

Figure 2 shows the test with historical production data. The data is gathered in each season from the beginning of 2008 to the beginning of 2010. The problem is solved to optimality and compared with the revenue obtained from the real-time booking system because we simply cannot implement two models at the same time in reality. During the test, the booking prior to the testing period (before 2008) is known to both systems; however, the booking during the testing period only reveals itself when the demand actually occurs. The then-future demand is forecasted by two systems separately and is considered separately within their own models. Stochastic systems update their demand on a real-time basis and our deterministic system updates only weekly. The horizontal axis represents cruises according to the departure date; the vertical axis stands for revenue of that cruise (in thousand dollars).

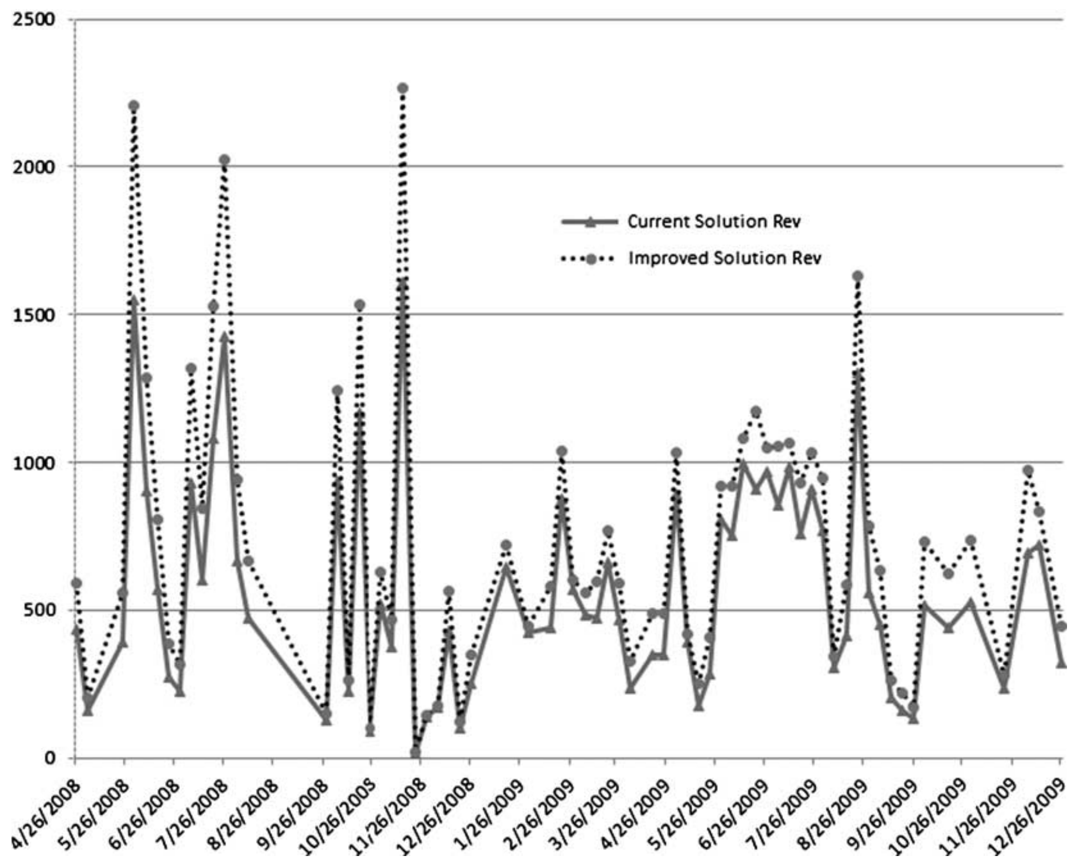

Figure 2: The full size production data for one ship from year 2008 to 2010.

The solid line represents the revenue from the real-time system. The dashed line is computational revenue utilizing our model. The results indicate that given the same historical data, our solution yields a total of 50.44 million dollars, which represent an increase of about 28 per cent when compared with the current solution from real-time booking system. Again, given the limited computational time and the available accuracy of the forecasted price-demand elasticity, the expected average in a deterministic model gives more stable and reliable solutions than those generated by a stochastic model.

CONCLUSIONS

In this article, we have studied a cruise line pricing problem for a large company having cruise line operation. Multiple characteristics make this problem unique compared with airlines and hotel room revenue management. The unique constraints include but are not only limited to room category, fare type, economical concern and several restrictions from both law enforcement and business implementation. An integer programming model is developed with linear constraints and objective function to solve this pricing problem efficiently. Numerical tests using operational data show a great revenue improvement potential (up to 28 per cent) using our model when compared with the current stochastic model.

A distinct feature at this company is that retail prices of rooms do not go down as it approaches the time of departure, a practice referred to as non-decreasing policy or price protection in other industries. The reason is that the company allows early bookings to be cancelled for a time period prior to departure at a zero cost. Since the booking window is very large, deeper discount at later time would cause early bookings to cancel and re-book at lower prices. The loss in such a situation cannot be nearly compensated by the additional customers solicited at deeper discount. The non-decreasing fare policy, however, is not equal to the elimination of discounts at the studied cruise line. The way the company offers discount pricing is through making rooms available to each tier of price. As Table 3 shows, only the last tier is a full fare. All the rest represent varying levels of discounting. Therefore, the graduate closing of low tier fares with time to departure is just a practice of offering discounts to customers. Note that the customary practice here, when rooms are left unfilled, would be to fill these rooms with their own special customers, typically company employees with a deep discount for a smaller profit.

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