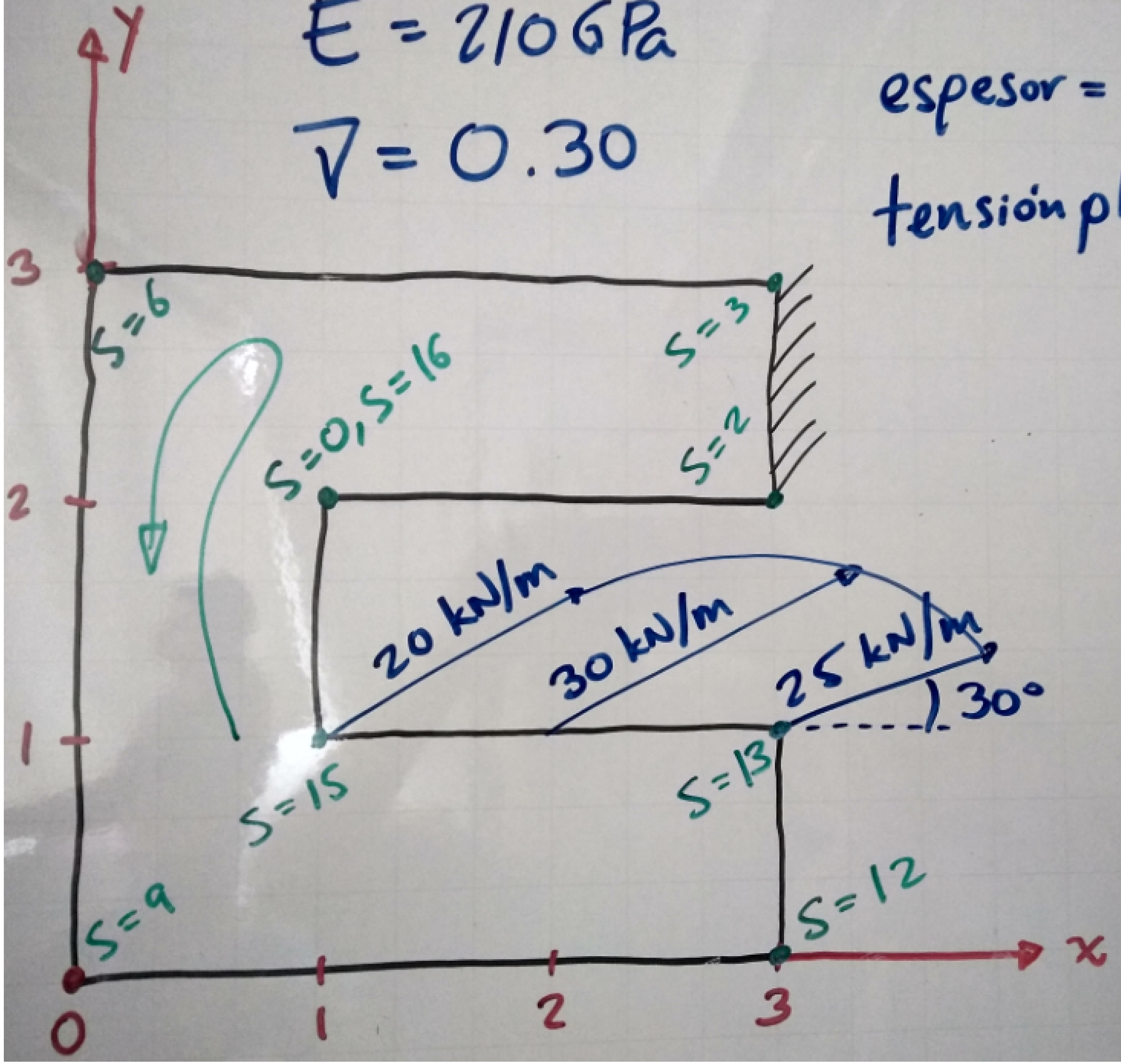
$E = 210\,GPa$

$\nu = 0.30$

espesor = t = 1m

tensión plana



$$\frac{\partial \sigma_x(x,y)}{\partial x} + \frac{\partial \tau_{xy}(x,y)}{\partial y} + X(x,y) = 0$$

$$\frac{\partial \tau_{xy}(x,y)}{\partial x} + \frac{\partial \sigma_y(x,y)}{\partial y} + Y(x,y) = 0$$

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} \longrightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\sigma_x + \sigma_y) = K_1 \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y}\right)$$

$$\begin{pmatrix} \bar{X}(s) \\ \bar{Y}(s) \end{pmatrix} = \begin{pmatrix} \sigma_x(s) & \tau_{xy}(s) \\ \tau_{xy}(s) & \sigma_y(s) \end{pmatrix} \begin{pmatrix} \alpha(s) \\ \beta(s) \end{pmatrix} \quad \hat{\boldsymbol{n}}(s) = \left[\frac{\mathrm{d}y(s)}{\mathrm{d}s}, -\frac{\mathrm{d}x(s)}{\mathrm{d}s}\right]^T$$

$$V = \rho g y$$

$$X(x,y) = -\frac{\partial V(x,y)}{\partial x}$$

$$Y(x,y) = -\frac{\partial V(x,y)}{\partial y}$$

$$\sigma_x(x,y) = \frac{\partial^2 \phi(x,y)}{\partial y^2} + V(x,y)$$

$$\sigma_y(x,y) = \frac{\partial^2 \phi(x,y)}{\partial x^2} + V(x,y)$$

$$\tau_{xy}(x,y) = -\frac{\partial^2 \phi(x,y)}{\partial x \partial y}$$

$$\underbrace{\frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4}}_{\nabla^4 \phi} = K_2 \underbrace{\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right)}_{\nabla^2 V}$$

$$K_2 = \begin{cases} \nu - 1 & \text{para el caso de tensión plana} \\ -\frac{1-2\nu}{1-\nu} & \text{para el caso de deformación plana} \end{cases}$$

$$\frac{\partial \phi\left(x(s), y(s)\right)}{\partial y} = \int \bar{X}(s) - V(s)\frac{\mathrm{d}y(s)}{\mathrm{d}s}\,\mathrm{d}s$$

$$\frac{\partial \phi\left(x(s), y(s)\right)}{\partial x} = -\int \bar{Y}(s) + V(s)\frac{\mathrm{d}x(s)}{\mathrm{d}s}\,\mathrm{d}s$$

$$\phi(s) = \int \frac{\partial \phi}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}s}\,\mathrm{d}s + \int \frac{\partial \phi}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}s}\,\mathrm{d}s$$

$$\iiint_{\Omega} \boldsymbol{b}(\boldsymbol{x})\,\mathrm{d}V + \oiint_{\partial\Omega} \boldsymbol{f}_{\Gamma}(\boldsymbol{x})\,\mathrm{d}S = 0$$

$$\iiint_{\Omega} \boldsymbol{x} \times \boldsymbol{b}(\boldsymbol{x})\,\mathrm{d}V + \oiint_{\partial\Omega} \boldsymbol{x} \times \boldsymbol{f}_{\Gamma}(\boldsymbol{x})\,\mathrm{d}S = 0,$$

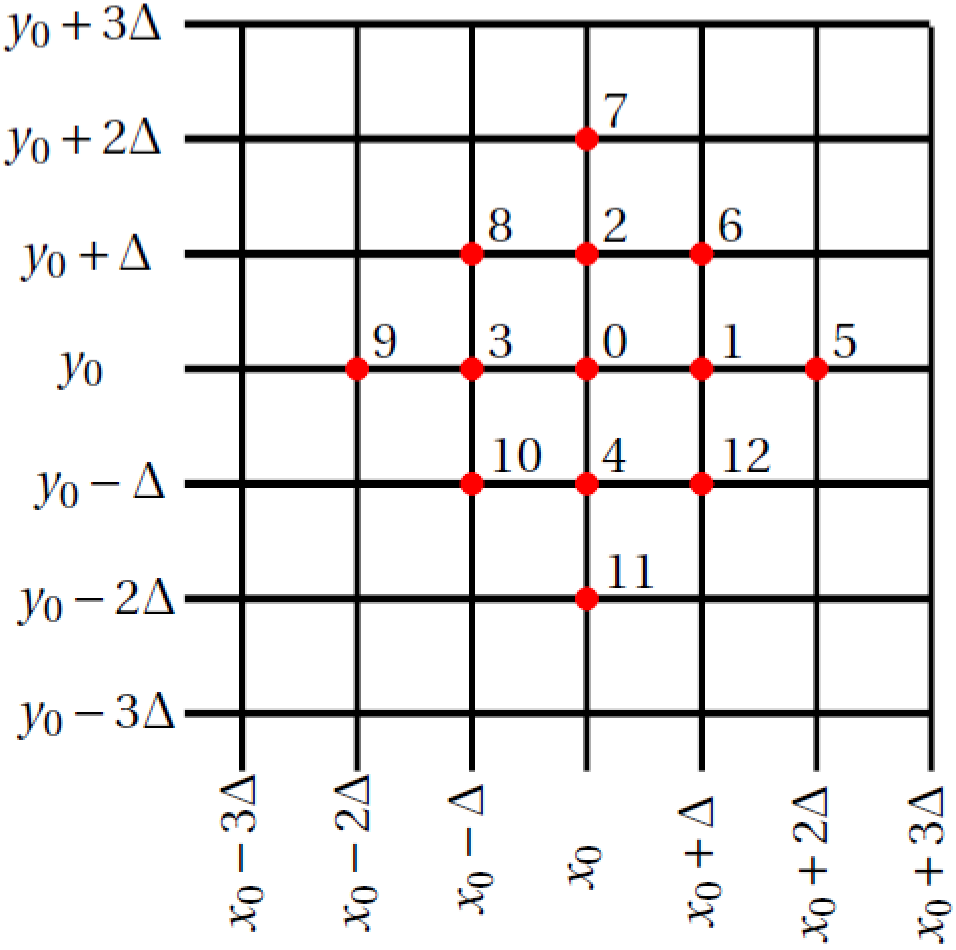
$$\iint_{\Omega} X(\boldsymbol{x})\,\mathrm{d}A + \oint_{\partial\Omega} \bar{X}(s)\,\mathrm{d}s = 0 \qquad \iint_{\Omega} Y(\boldsymbol{x})\,\mathrm{d}A + \oint_{\partial\Omega} \bar{Y}(s)\,\mathrm{d}s = 0$$

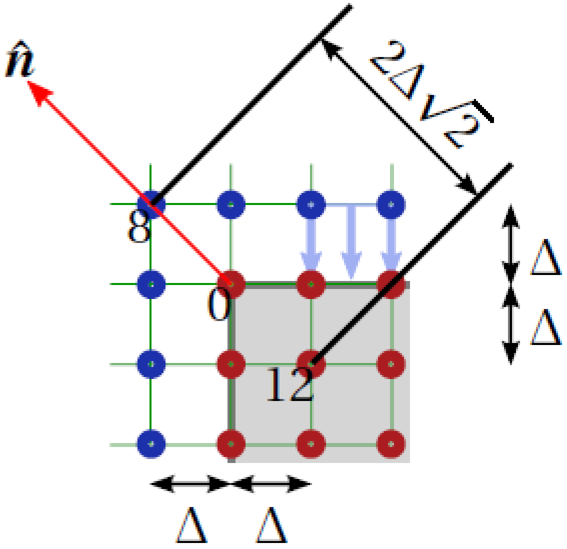
$$\iint_{\Omega} \big(xY(\boldsymbol{x}) - yX(\boldsymbol{x})\big)\,\mathrm{d}A + \oint_{\partial\Omega} \big(x(s)\bar{Y}(s) - y(s)\bar{X}(s)\big)\,\mathrm{d}s = 0.$$

$$\left.\frac{\partial^4 f}{\partial x^4}\right|_{\substack{x=x_0\\ y=y_0}} \approx \frac{1}{\Delta^4}\left(f_9 - 4f_3 + 6f_0 - 4f_1 + f_5\right)$$

$$\left.\frac{\partial^4 f}{\partial x^2 \partial y^2}\right|_0 \approx \frac{1}{\Delta^4}\left(4f_0 - 2\left(f_1 + f_2 + f_3 + f_4\right) + \left(f_6 + f_8 + f_{10} + f_{12}\right)\right)$$

$$\left.\frac{\partial^4 f}{\partial y^4}\right|_0 \approx \frac{1}{\Delta^4}\left(f_{11} - 4f_4 + 6f_0 - 4f_2 + f_7\right)$$





$$\phi_8 = \phi_{12} + 2\Delta\left(-\left.\frac{\partial\phi}{\partial x}\right|_0 + \left.\frac{\partial\phi}{\partial y}\right|_0\right)$$

$$\phi_6 = \phi_{10} + 2\Delta\left(+\left.\frac{\partial\phi}{\partial x}\right|_0 + \left.\frac{\partial\phi}{\partial y}\right|_0\right)$$

$$\phi_{10} = \phi_6 - 2\Delta\left(+\left.\frac{\partial\phi}{\partial x}\right|_0 + \left.\frac{\partial\phi}{\partial y}\right|_0\right)$$

$$\phi_{12} = \phi_8 + 2\Delta\left(+\left.\frac{\partial\phi}{\partial x}\right|_0 - \left.\frac{\partial\phi}{\partial y}\right|_0\right).$$