Classical Waves and Particles Worked Example

1. Suggested study steps, based on Feynman study model[(a)]

1. Read the statement of the problem
2. Read slowly through the solution to get the big picture.
3. With pencil, paper and calculator work through the self-explanation prompts.
4. Write a summary of the problem and the solution as you would explain it to a friend.

2. Statement of the problem

In quantum physics, it is not possible to separate the dual nature of quantum particles: particles and waves. To a better understanding of quantum particles is convenient to study the principal characteristics of classical waves and particles.

Make a parallel between classical particles and waves.

3. Strategy

To make the parallel between classical particles and waves it is necessary to describe simply the interaction mechanisms of particles and waves and the principal characteristics of the particles that made this difference that waves.

4. Implementation

4.1. Interaction in classical particles

The particle concept is related to a no shape and punctual representation of a material object. The Newton equations describe the classical non-relativistic particles; however, the Hamilton representation of classical physics let to explain the particles from an energy point of view. Hamilton canonical equations relate the momentum variation respect to time with the partial variation of energy function (Hamiltonian) respect to coordinates ($\frac{dp_i}{dt} = -\frac{\partial H}{\partial x_i}$) and the velocities with the partial variation of energy function respect to momentum ($\frac{dx_i}{dt} = \frac{\partial H}{\partial p_i}$).

A straightforward interpretation of Hamilton canonical equations is that particles are entities that interchange energy and momentum.

To a better understanding of particles as entities that interact by interchange of energy and momentum is proposed a simple model: assume a system conformed by N classical particles described by the energy function H. Suppose x_k and p_k are a coordinate and momentum of a α

particle in the system. The momentum variation of α particle in direction of $x_k$is given by the energy function partial variation respect the coordinate x_k, i.e. $\frac{dp_k}{dt} = -\frac{\partial H}{\partial x_k}$; this can be interpreted as an energy variation due to the particle interaction change the particle momentum, then if a particle interacts with others this particle change its momentum and energy. Furthermore, the velocity of the α particle in direction of $x_k$is given by the partial variation of the energy function as momentum component p_k, i.e. $\frac{dx_i}{dt} = \frac{\partial H}{\partial p_i}$; then to variate the position of a particle it is necessary a variation of the energy by change the momentum. The variation of the energy by change in the momentum can be interpreted as interchange of energy and momentum.

4.2. Interaction in classical waves

Classical waves are described by a function called wave function. It is impossible to discriminate between a wave and the wave function because the wave function represents the wave and the wave is represented by the wave function. In contrast, a particle cannot be represented by an function, it is possible to associate a position and other properties to a particle but the particle is not represented by a function.

The interaction between waves is given by superposition. To illustrate this suppose two waves $\psi_1(\vec{r},t)$ and $\psi_2(\vec{r},t)$; if this waves interact, then, the result of the interaction ψ_3 is the superposition of both waves, $\psi_3(\vec{r},t) = \psi_1(\vec{r},t) + \psi_2(\vec{r},t)$.

4.3. Comparison between classical particles and waves

Classical particle	Classical wave
Let to represent a material object by a no-shape and punctual representation.	Describe a physics phenomenon by a wave function.
It is not possible to write an function that represent the material object, only is possible associate to the particle some physical characteristics.	There is no difference between the physics wave and the wave function.
The particles interact by interchanging energy and momentum.	The waves interact by the superposition of their wave functions.
It is possible define univocally the position and momentum of the particle.	It is not possible to define a unique position to a wave because the wave not localized in the space.

5. Self-explanation prompts

5.1. If you produce a one-dimensional sinusoidal wave in a guitar, what are the differences between the physical wave (in the guitar) and the wave equation $\psi(\vec{r},t) = A_0 sin(kx - \omega t)$? Explain your answer.

5.2. Suppose you have a particle in the position $\vec{r} = 5\hat{i} + 7\hat{j} + 8\hat{k}[m]$, is $\vec{r}$ the particle?

5.3. Suppose you have a wave function $\Psi(\vec{r},t) = A_0 e^{-(\alpha_x x + \alpha_y y + \alpha_z z)} sin(k_x x + k_z z - \omega t)$, is $\Psi(\vec{r},t)$ the wave? Explain your answer.

5.4. Could two classical waves interchange momentum? Explain your answer.

5.5. Could two classical waves interchange energy? Explain your answer.

6. References and further reading

(a) https://mattyford.com/blog/2014/1/23/the-feynman-technique-model

This Worked Example was proposed by Prof. David A. Miranda at November 11, 2017