AVERAGE

- ***DEFINITION:***

Mean or what is popularly known as ***average*** represents the central value of a set of values or observations.

Whenever the observations about a group is mentioned, it is generally represented in terms of average.

For example, marks of 50 students of a class will be easier to understand if it is given in terms of average than their individual marks.

- ***FORMULA 1:***

Suppose if we have to find the average age of men or women in a group or average male height in India, then we calculate it by adding all the values and dividing it by the number of values. Below is the formula to evaluate the average of a given set of numbers.

$$\textit{AVERAGE} = \frac{\textbf{\textit{SUM OF OBSERVATIONS}}}{\textbf{\textit{NUMBER OF OBSERVATIONS}}}$$

- ***FORMULA 2:***

AVERAGE OF TWO OR MORE GROUPS TAKEN TOGETHER:

If the number of quantities in two groups are n_1 and n_2 and their average is x and y respectively, then the combined average or weighted average is

$$\frac{n_1x + n_2y}{n_1 + n_2}$$

- ***FORMULA 3:***

TO FIND THE AVERAGE SPEED

If a particular distance is covered with a speed of x kmph and the same distance is covered at y kmph then the average speed throughout the journey is

$$\frac{2xy}{x+y}\ \textit{kmph}$$

FORMULA 4:

If 3 equal distances are covered at 3 different speeds x kmph, y kmph and z kmph respectively, then the average speed throughout the journey is

$$\frac{3xyz}{xy+yz+zx}\ \textit{kmph}$$

SYMBOL:

The average is basically mean of the values which are represented by $\bar{x}$. It is also denoted by the symbol 'μ'.

PROPERTIES OF MEAN:

1. The average of $x_1, x_2, \ldots. x_n$ is x, then the average of $x_1 + a, x_2 + a, \ldots. x_n + a$ Is x +a
2. The average of $x_1, x_2, \ldots. x_n$ is x, then the average of $x_1 - a, x_2 - a, \ldots. x_n - a$ is x -a
3. The average of $x_1, x_2, \ldots. x_n$ is x, then the average of $ax_1, ax_2, \ldots. ax_n$ is ax
4. The average of $x_1, x_2, \ldots. x_n$ is x, then the average of $x_1/a, x_2/a, \ldots. x_n/a$ is x/a
5. Hence, If all values are increased or decreased by a certain quantity, the mean also increases or decreases by the same quantity.
6. If all values are multiplied or divided by a certain quantity, the mean also gets multiplied or divided by the same quantity.
7.

EXAMPLE 1:

If there are a group of numbers say, 20, 21, 23, 22, 21, 20, 23. Then find the average of these values.

By average formula, we know,

$$\textit{AVERAGE} = \frac{\textit{SUM OF OBSERVATIONS}}{\textit{NUMBER OF OBSERVATIONS}}$$

$$= \frac{20+21+23+22+21+20+23}{7}$$

$= \frac{150}{7}$

= 21.42

- ***EXAMPLE 2:***

Find the average of 3, −7, 6, 12, −2.

AVERAGE = $\frac{SUM\ OF\ NUMERS}{NUMBER\ OF\ UNITS}$

$= \frac{3-7+6+12-2}{5}$

$= \frac{12}{5}$ = 2.4

- **EXAMPLE 3:**

A grocer has a sale of Rs.6435, Rs.6927, Rs.6855, Rs.7230 and Rs.6562 for 5 consecutive months. How much sale must he have in the sixth month so that he gets an average sale of Rs.6500?

SOLUTION:

Total sale for 5 months = (6435 + 6927 + 6855 + 7230 + 6562) = Rs.34009.

Total sale for 6 months = 6500*6 = Rs.39000

Sale on sixth month = 39000 – 34009 = Rs.4991.

- **EXAMPLE 4**:

Sanjay's average score was 20. Last innings he took 50 runs and his average became 25. How many runs should he take in his next inning to make his average 30?

Let's consider Sanjay has played 'n' number of innings.

AVERAGE = $\frac{SUM\ OF\ NUMERS}{NUMBER\ OF\ UNITS}$

20 = $\frac{SUM\ OF\ NUMERS}{NUMBER\ OF\ UNITS}$

Sum = 20*n

Last innings he scored 50 runs and his average became 25. So,

$25 = \frac{Sum+50}{n+1}$

25(n+1) = Sum +50

25n + 25 = 20n + 50   [Sum = 20n]

5n = 25

N = 5

To make his average 30 in next inning,

Total runs for 5 matches is 100 and for 6 matches would be 150.

Now the number of innings will be 7.

Sum = 7*30 = 210 runs.

210 – 150 = 60. He needs 60 runs in 7^{th} innings to make his average as 30.

- **EXAMPLE 5**:

Dhoni's average in first 30 innings was 50. After the 31^{st} inning his average was 49. How many runs did he make in the 31^{st} inning?

Total runs in 30 innings = 30*50 = 1500

Total runs in 31 innings = 31*49 = 1519

Runs he made in the 31^{st} inning = 1519 – 1500 = 19 runs.

SHORTCUT:

Since average represents the central value of a set of observations.

Then in every 30 innings his central value is 50. After the 31^{st} inning, his average reduces to 49. Then average of all the 31^{st} inning will be 49. Previously, 30 innings average was 50. Now it becomes 49. So, the runs which he made in 31^{st} inning is 49 – 30 = 19 runs.

- **EXAMPLE 6:**

The average weight of A, B and C is 45 kg. If the average weight of A and B be 40 kg and that of B and C be 43 kg, then the weight of B is:

Let A, B, C represent their weights. Then, A+B+C = 45*3 = 135 kg …….(1)

A+B = 40*2 = 80 kg …………(2)

B+C = 43*2 = 86 kg ………(3)

Adding (1) and (2),

A+B+B+C = 80+86

A+2B+C = 166…….(4)

Subtracting (4) and (1),

A+2B+C – (A+B+C) = 166 – 135

B = 31 kg.

- **<u>EXAMPLE 7</u>**:

The average weight of 16 boys in a class is 50.25 kg and that of the remaining 8 boys is 45.15 kg. Find the average weights of all the boys in the class.

Total weight of 16 boys = 50.25*16

= 804 kg

Total weight of 8 boys = 45.15*8

= 361.2 kg

Average Weight of all the boys =

$$= \frac{804+361.2}{24}$$

$$= \frac{1165.2}{24}$$

= 48.55 kg

- **<u>EXAMPLE 8:</u>**

A pupil's marks were wrongly entered as 83 instead of 63. Due to that the average marks for the class got increased by half (1/2). The number of pupils in the class is:

Let there be x pupil in the class.

Total increase in the marks = x*$\frac{1}{2}$

$\frac{x}{2}$ = 83 – 63

$\frac{x}{2}$ = 20

X = 40.

- ***EXAMPLE 9:***

If half of the journey is travelled at a speed of 15kmph and the other half is travelled at a speed pf 12kmph, then find the average speed during the entire journey.

By formula:

Average speed = $\frac{2xy}{x+y}$ *kmph*

➔ $\frac{2 \; X \; 15 \; X \; 12}{15+12}$

➔ 360/27

➔ 13 1/3 kmph

- ***EXAMPLE* 10 :**

Find the average of the of first n natural numbers

Average = $\frac{SUM \; OF \; NUMERS}{NUMBER \; OF \; UNITS}$

={ n(n+1)/2} / n

= n+1/2

STATISTICS

Statistics deals with the collection, classification, presentation, analysis and interpretation of numeric data.

Why Statistics:

After collection of data, they are classified, analysed and interpreted. To analyse the data various quantitative measures are used. These are classified as

1. ***Measures of central tendency and***
2. ***Measures of dispersion***

Measures of Central Tendency

The measures of central tendency are

- **Arithmetic mean**
- **Geometric mean**
- **Harmonic mean**
- **Median and**
- **Mode**

MEAN VALUE:

Mean value refers to the average of a set of values. The simplest way to find the mean is sum of all the values in the set divided by total number of values in the set.

Given $x_1, x_2, x_3, \ldots. x_n$ are n observations then

$$A.M = \bar{x} = \frac{x1 \ + x2 + x3 + \ldots xn}{n}$$

i.e., Mean = Sum of all values/total number of values

Example 1:

Find the mean of set S = { 5,10,15,20,30}

Solution:

Mean of set S = 5+10+15+20+30/5

= 80/5

= 16

MEDIAN:

Median signifies the central value or the middle value in a sorted list of numbers. To calculate median, the data has to be sorted in ascending or descending order.

Example 2:

Find the median of the set = { 2,4,4,3,8,67,23 }

Solution:

As we can see the list is not arranged in any order.

The sorted list in ascending order = {2,3,4,4,8,23,67}.

The list contains 7 terms, thus 4^{th} term of the list will be the median, so the median is 4.

If the list contains 'n' terms (n is an odd number), the median will be the (n+1)/2 term.

In case the list consists of even number of terms, the median will be the average of n^{th} and $(n+1)^{th}$ term.

Example 3:

Find the median of the set = { 11,22,33,55,66,99 }

Solution:

As we can see the list is already in ascending order and the list contains 6 terms, hence the average of the third and fourth term will be the median.

Median = (33+55)/2 = 44.

Example 4:

Find the median of a series of all the even terms from 4 to 296.

Solution:

The given sequence is 4,6,8,10,12,14….296.

As we can see, the given sequence is an Arithmetic progression (An arithmetic progression is a sequence of terms where any two consecutive terms differ by a constant difference).

To find out the median, we need to know the number of terms.

We will use the n^{th} term of an arithmetic progression formula

($a_n = a_1 + (n-1)d$) to calculate the number of terms. Then, depending on whether n is odd or even we can find out the median.

Now, 296=4+(n-1)2

- ⇨ N-1=(296-4)/2
- ⇨ N = 147

Median = (147+1)/2 th term

= 74^{th} term

T_{74} = 4+(73*2) = 150

So, median is 150

MODE:

For a given sequence, the value with the highest frequency is known as the mode. In other words in the item which is found most often in the given set of observations, i.e., ***the value occurring the highest number of times***.

Example 5:

Find the mode of the Set = {1,3,3,6,9}

Solution:

In the sequence, the value ‘3’ occurs maximum number of times, hence the mode is 3.

Remember: There can be more than one mode in a series.

For example, in the set = {2,4,4,6,8,9,9}, both 4 and 9 are the Modes as their frequency of occurrence is more than other values.

RANGE:

The range for any distribution is given by = Highest value – Lowest value.

Example 6:

Find the range of the set ={7,2,3,8,1,7,3,2,9}

Solution:

Highest value = 9

Lowest value = 1

Range = 9 – 1=8

Example 7:

Find the mean, median, mode, and range for the following list of values: 1, 2, 4, 7

Solution:

The mean is the usual average:

(1 + 2 + 4 + 7) ÷ 4 = 14 ÷ 4 = 3.5

The median is the middle number. In this example, the numbers are already listed in numerical order, so we don’t have to rewrite the list. But there is no “middle” number, because there are even number of numbers. Because of this, the median of the list will be the mean (that is, the usual average) of the middle two values within the list. The middle two numbers are 2 and 4, so:

(2 + 4) ÷ 2 = 6 ÷ 2 = 3

So, the median of this list is 3, a value that isn’t in the list at all.

The mode is the number that is repeated most often, but all the numbers in this list appear only once, so there is no mode.

The largest value in the list is 7, the smallest is 1, and their difference is 6, so the range is 6.

Mean: 3.5 | median: 3

mode: none |range: 6

Example 8:

Find the mean, median, mode, and range for the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13

Solution:

The mean is the usual average, so we'll add and then divide:
(13 + 18 + 13 + 14 + 13 + 16 + 14 + 21 + 13) ÷ 9 = 15

Note that the mean, in this case, isn't a value from the original list. This is a common result. You should not assume that your mean will be one of your original numbers.

The median is the middle value, so first we'll have to rewrite the list in numerical order:

13, 13, 13, 13, 14, 14, 16, 18, 21

There are nine numbers in the list, so the middle one will be

(9 + 1) ÷ 2 = 10 ÷ 2 = 5th number:

13, 13, 13, 13, **14**, 14, 16, 18, 21

So the median is 14.

The mode is the number that is repeated more often than any other, so 13 is the mode, since 13 is being repeated 4 times.

The largest value in the list is 21, and the smallest is 13,

so the range is 21 – 13 = 8.

Mean: 15 |median: 14

mode: 13 |range: 8

Example 9:

A sequence consists of 7 terms arranged in descending order. The mean value of the sequence is 70. If 30 is added to each term, and then each term is divided by 2 to get the new mean as 'K'. Find the difference between K and the original mean.

Solution:

Let us say the seven terms are a1, a2, a3, a4, a5, a6, a7. As the mean is 70,

a1 + a2 + a3 + a4 + a5 + a6 + a7 = 70×7 = 490

After adding 30 to each term, the new sum becomes,

(a1+30) + (a2+30) + (a3+30) + (a4+30) + (a5+30) + (a6+30) + (a7+30) = 70×7 + 30×7 = 700

The mean becomes 700/7 = 100.

After dividing each term by 2, the sum becomes,

New mean = 350/7 = 50

Difference between new mean and original mean = 70 – 50 = 20.

As you might have noticed in this example that the mean value is directly changed by any operation done on the values of the sequence.

Example 10:

What is the median in the following set of numbers?

16, 19, 16, 7, 2, 20, 9, 5

Solution:

16, 19, 16, 7, 2, 20, 9, 5

Order the numbers from smallest to largest.

2,5,7,9,16,16,19,20

The median is the number in the middle.

In this case, there is a 9 and 16 in the middle.

When that happens, take the average of the two numbers.

Median = (9+16)/2 = 12.5

Example11:

Find the median of the set 4,6,12,9,12,90,12,18,12,12,12,4,4,4,9,7,76

Solution:

To find the median, arrange the numbers from smallest to largest:

4,4,4,4,6,7,9,9,12,12,12,12,12,12,18,76,90

There are 17 numbers in total. Since 17 is an odd number, the median will be the middle number of the set. In this case, it is the 9th number, which is 12.

Example 12:

8,12,9,8,7,11,10,6

In the set above, which is larger: the median, the mean, or the mode?

Solution:

Begin by ordering the set from smallest to largest:

6,7,8,8,9,10,11,12

Already, we see that the mode is 8.

Find the median by taking the average of the two middle numbers:

(8+9)/2=8.5

Find the mean by adding all numbers and dividing by the total number of terms:

(6+7+8+8+9+10+11+12)/8=8.875

Of the three, the mean of the set is the largest.

Example 13:

If the average of 5 positive integers is 40 and the difference between the largest and the smallest of these 5 numbers is 10, what is the maximum value possible for the largest of these 5 integers?

Solution:

The average of 5 positive integers is 40.

i.e., the sum of these integers = 5 × 40 = 200

Let the least of these 5 numbers be x.

Because the range of the set is 10, the largest of these 5 numbers will be x + 10.

If we have to maximize the largest of these numbers, we have to minimize all the other numbers.

That is 4 of these numbers are all at the least value possible = x.

So, x + x + x + x + x + 10 = 200

Or x = 38.

So, the maximum value possible for the largest of these 5 integers is 48.

Example 13:

Find the mean, median, mode and range of the data:

Five fives, seven sevens, nine nines, eleven eights

Solution:

Mean=(5*5+7*7+9*9+11*8)/32

= 243/32

= 7.59

Median

Arrange the data in ascending order.

x	f	Cum. freq
5	5	5
7	7	12
8	11	23
9	9	32

There are 32 terms. So, median = mean of 16 and 17^{th} terms.

= (8+8)/2

=8

Mode = 8 (Value 8 occurs 11 times)

Range = Largest value – Smallest value

= 9-5 =4

Example 14:

Find the range of the data:

9,8,12,80,75,45,36 and 72

Solution:

Range = Largest value – Smallest value

= 80-8 = 72

Example 15:

The mean, median and mode of 30 numbers is 27. If each value is increased by 2, what is the new mean, median and mode?

Solution:

If each value increased by 2, then mean, median and mode each increases by 2. So, Mean=Median=Mode=27+2=29