

Week 7 – Long Answer Question

Question 1: Forming 5-Digit Even Numbers

To form 5-digit even numbers using the digits 0-9 without repeating any digits:

1. Identify Possible Last Digits: The last digit must be even: {0, 2, 4, 6, 8}.

2. Case Analysis Based on Last Digit:

- Case 1: Last digit is 0.
- First digit can be any of {1-9} (9 choices).
- Remaining three digits can be chosen from the remaining 8 digits.
- Total for this case:

$$9 \times P(8,3) = 9 \times (8 \times 7 \times 6) = 3024$$

- Case 2: Last digit is 2, 4, 6, or 8 (4 cases).
- First digit can be any of {1, 3, 4, 5, 6, 7, 8, 9} (8 choices).
- Total for each case:

$$8 \times P(8,3) = 8 \times (8 \times 7 \times 6) = 2688$$

- Total for all four cases:

$$4 \times 2688 = 10752$$

3. Total Arrangements:

- Total = Case with last digit 0 + Cases with last digits 2, 4, 6, and 8:

$$\text{Total} = 3024 + 10752 = 13776$$

Final Answer for Question 1:

13776 ways to form a five-digit even number.

Question 2: Arranging Letters of "ACTION"

The word "ACTION" has six letters: A, C, T, I, O, N.

Part (a): All Vowels Together

1. Group Vowels (AIO) Together: Treat AIO as one unit.

- Units to arrange: {AIO}, C, T (3 units).
- Arrangements of these units:

$$3! = 3 \times 2 \times 1 = 6.$$

- Arrangements within the vowel group (AIO):

$$3! = 3 \times 2 \times 1 = 6.$$

2. Total Arrangements:

$$\text{Total} = (3!) \times (3!) = 6 \times 6 = 36. \text{Total} = (3!) \times (3!) = 6 \times 6 = 36.$$

Final Answer for Part (a):

36 ways to arrange "ACTION" with all vowels together.

Part (b): Vowels Never Together

1. Total Arrangements Without Restrictions:

$$6! = 720 \quad 6! = 720.$$

2. Arrangements with Vowels Together:

From part (a), we found this to be:

$$36 \quad 36.$$

3. Use Complement Principle:

Arrangements where vowels are never together = Total arrangements – Arrangements with vowels together = $720 - 36 = 684$. Arrangements where vowels are never together = Total arrangements – Arrangements with vowels together = $720 - 36 = 684$.

Final Answer for Part (b):

684 ways to arrange "ACTION" such that vowels are never together. If you need further clarification or have more questions, feel free to ask!