1 . Consider a large number N = 123456789101112131 4.........979899100. What is the remainder when first 100 digits of N is divided by 9?

Identify the First 100 Digits:

The number (N) is formed by concatenating consecutive integers starting from 1.

We need to determine how many digits are contributed by each integer until we reach 100 digits.

Count the Digits:

1-9: 9 numbers contribute 9 digits (1 digit each).

10-99: 90 numbers contribute 180 digits (2 digits each).

100: Contributes 3 digits.

We can see that:

From 1 to 9, we have 9 digits.

From 10 to 54, we have (45 \times 2 = 90) digits (totaling 99 digits).

The next number, 55, contributes 2 more digits, bringing the total to 101 digits. So, we only need the first digit of 55 to reach 100 digits.

Digits in the First 100 Digits:

The first 100 digits are:

The first digit of 55 is '5'.

Sum of the Digits:

Now, we sum the digits of the first 100 digits:

From 1 to 9: (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45)

From 10 to 54:

The tens place contributes: (10 \times 5 = 50) (for 10-19), (10 \times 4 = 40) (for 20-29), (10 \times 3 = 30) (for 30-39), (10 \times 2 = 20) (for 40-49), and (5 \times 1 = 5) (for 50-54).

The units place: (0 + 1 + 2 + ... + 9 + 0 + 1 + 2 + ... + 9 + 0 + 1 + 2 + 3 + 4) which sums to (45 + 45 + 15 = 105).

Total sum of digits from 1 to 54: (45 + (50 + 40 + 30 + 20 + 5) + 105 = 45 + 145 + 105 = 295).

Adding the first digit of 55: (295 + 5 = 300).

Finding the Remainder:

Finally, we find (300 \mod 9):

Sum of the digits of 300 is (3 + 0 + 0 = 3).

Therefore, (300 \mod 9 = 3).

Final Answer

The remainder when the first 100 digits of (N) is divided by 9 is 3.

2. what is the value of x if the number 78212x535 is divisible by 11?

To determine the value of (x) such that the number (78212x535) is divisible by 11, we can use the divisibility rule for 11. According to this rule, a number is divisible by 11 if the difference between the sum of its digits in odd positions and the sum of its digits in even positions is either 0 or divisible by 11.

Step-by-Step Solution

Identify the Positions:

The number (78212x535) has the following digit positions:

Odd positions: 7 (1st), 2 (3rd), x (5th), 5 (7th), 5 (9th)

Even positions: 8 (2nd), 1 (4th), 3 (6th), 3 (8th)

Calculate the Sums:

Sum of digits in odd positions: [7 + 2 + x + 5 + 5 = 19 + x]

Sum of digits in even positions: [8 + 1 + 3 + 3 = 15]

Find the Difference:

The difference (D) is given by: [D = (19 + x) - 15 = x + 4]

Set Up the Condition for Divisibility:

For (78212x535) to be divisible by 11, (D) must satisfy: [x + 4 \equiv 0 , (\text{mod} , 11)] This means (x + 4) should be either 0 or a multiple of 11.

Solve for (x):

Rearranging gives: [x \equiv -4 , (\text{mod} , 11)]

To find a positive value for (x), we can add 11 to -4: [x \equiv 7 , (\text{mod} , 11)]

Possible Values for (x):

Since (x) is a single digit (0-9), the only valid solution is: [x = 7]

Final Answer

The value of (x) that makes the number (78212x535) divisible by 11 is 7.