FINDING THE FACTORS OF A GIVEN NUMBER:

Factors are positive or negative whole numbers (mind it, they cannot be fractions!) which divide the given number completely.

For example, the factors of 6 are 1, 2, 3 and 6. Clearly these four numbers divide6 completely.

You should note it here that every number will have 1 and itself as factors.

Let us now look into some methods of finding the factors. Here we go!

Method 1: Prime Factorization

- Prime factorization is a way of finding the prime factors of a number.
- The original number is evenly divisible by these factors.
- A *composite number* has more than two factors therefore, this method is applicable only for composite numbers and not for prime numbers.

For example, the prime factors of 126 will be 2, 3 and 7 as 2 × 3 × 3 × 7 = 126 and 2, 3, 7 are prime numbers.

- ***Example 1***: Find the Prime factorization of the following:

i) 12 is 2 x 2 x 3 = 2^2 x 3

ii) 18 is 2 x 3 x 3 = 2 x 3^2

iii) 24 is 2 x 2 x 2 x 3 = 2^3 x 3

iv) 20 is 2 x 2 x 5 = 2^2 x 5

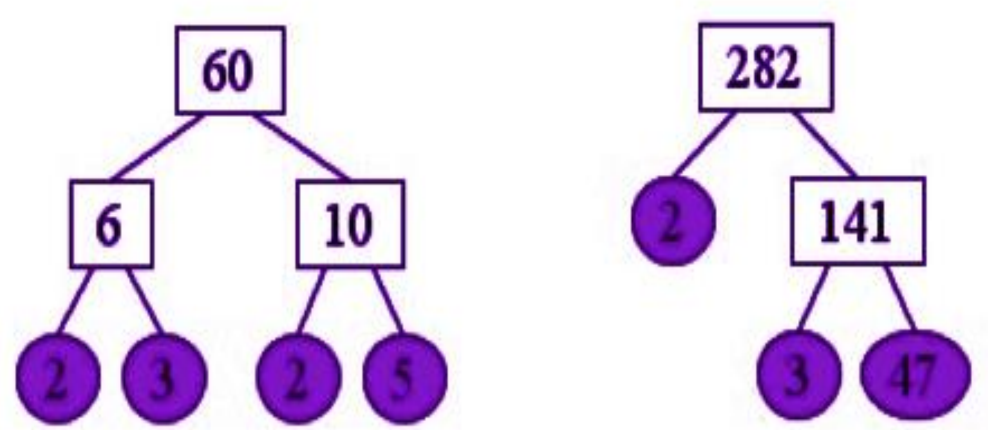
Method 2: Factor Tree Method

To find the prime factorization of the given number using factor tree method, follow the below steps:

- Step 1: Consider the given number as the root of the tree
- Step 2: Write down the pair of factors as the branches of a tree

- Step 3: Again factorize the composite factors, and write down the factors pairs as the branches
- Step 4: Repeat the step, until you find the prime factors of all the composite factors

In factor tree, the factors of a number are found and then those numbers are further factorized until we reach the closure. Suppose we have to find the factors of 60 and 282 using a factor tree. Then see the diagram given below to understand the concept.



In the above figure, we can see that 60 is first factorized into two numbers i.e 6 and 10. Again, 6 and 10 is factorized to get the prime factors.

60 = 6 x 10 = 2 x 3 x 2 x 5 and,

282 = 2 x 141 = 2 x 3 x 47

- ***Example 2*** **:** Find the prime factorization of 1936

    Solution:

    1936

    =11x11x2x2x2x2

    $=11^2x2^4$

- ***Example*** **3:** Find the prime factorization of 1240

    Solution:

1240

=124 x10

=4 x 31 x 5x 2

$=2^3$x5x31

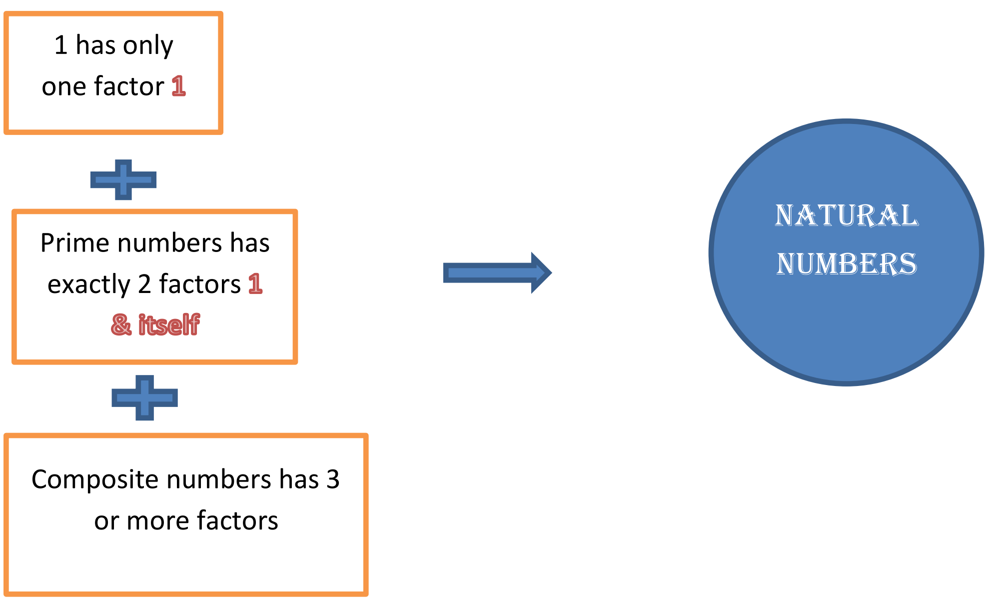
Now, let us look into finding the nature of these factors.

In ***number system*** the concept of **factors of numbers** is one of the most important sub-topic.

A simple diagram for your understanding of the number of factors:

Understand that the number line can be classified as 1(it is neither prime nor composite), prime numbers and, composite numbers

It is amazing that these are classified on the basis of their number of factors.



Let us now discuss on the formula for finding *number of factors, sum of factors, product of factors, even number of factors, odd number of factors, perfect square factors and perfect cube factors* for any number.

Let us follow the given rules to find the nature of factors:

- Take any number "N" and it is to be converted into **product of prime numbers** (*Prime factorization*)

  i.e $N = A^P \times B^Q \times C^R$ here A, B and C are prime numbers and P, Q and R are their respective powers.

- Total numbers of factors of N = (P + 1)(Q +1)(R +1)
- Total number of odd factors of N= (Q +1)(R +1) (assume A is 2)
- Total number of even factors of N =

  Total numbers of factors - Total number of odd factors
- Sum of all the factors =

  $(A^0+A^1+A^2\text{…….}A^p)x(B^0+B^1+B^2\text{…..}B^q)x(C^0+C^1+C^2+\text{…….}C^r)$

  or sum of all the factors of $N = A^P \times B^Q \times C^R$ is

  $$[A^{p+1} – 1] / (A-1) \;\; x \;\; [B^{q+1} – 1] / (B-1) \; x \;\; [C^{r+1} – 1] / (C-1)\text{……….}$$

- SUM OF ODD FACTORS

  $=(B^0+B^1+B^2\text{…..}B^Q)x(C^0+C^1+C^2+\text{…….}C^R)$ (assume A is 2)

  Or $[B^{q+1} – 1] / (B-1) \; x \;\; [C^{r+1} – 1] / (C-1)$ (assume A is 2)
- Sum of even factors

  =total sum of all the factors – sum of the odd factors

- ***Example 4***: Find number of all factors of 120

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Solution:

120 =4x3x5x2

$=2^3x3x5$

Number of all factors =(3+1) x (1+1) x (1+1)

=4x2x2

$=16$

➢ ***Example 5*: F**ind number of odd factors of 120

Solution:

120 $=4x3x5x2$

$=2^3x3x5$

Number of odd factors$=(1+1)$ x $(1+1)$

$=2x2$

$=4$

➢ ***Example 6*:** Find number of even factors of 120

Solution:

$120=4x3x5x2=2^3x3x5$

Number of even factors=total factors-odd factors=16-4-12

➢ ***Example 7*:** Find sum of all factors of 120

Solution:

120=

$4x3x5x2 = 2^3x3x5$

sum of all factors=

$= (2^0+2^1+2^2+2^3)$ x (3^0+3^1) x (5^0+5^1)

$= 15x4x6=360$

➢ ***Example 8*: F**ind sum of odd factors of 120

Solution:

Sum of odd factors

$= (3^0+3^1)x(5^0+5^1)$

$= 4x6=24$

- ***Example 9:*** Find sum of even factors of 120

Solution:

Sum of even factors = total sum of all factors –sum of odd factors=

= 360-24

=336

Finding the factors of can be dealt with a different method too…

Let us look into the method in detail now:

- ***Example 10:*** If Number N = $2^6 \times 5^5 \times 7^6 \times 10^7$; how many factors of N are even numbers?

Solution:

Given N $= 2^6 \times 5^5 \times 7^6 \times 10^7$

$= 2^{13} \times 5^{12} \times 7^6$

$= 2(2^{12} \times 5^{12} \times 7^6)$

Hence the number of even factors in N are

=(12+1) x (12 +1) x(6+1)

=1183

- ***Example 10:*** Find the number of factors of 243243 which are multiples of 21

Solution:

243243 = 243 x (1001)

$= 3^5 \times 7^1 \times 11^1 \times 13^1$

$= 3 \times 7(3^4 \times 11^1 \times 13^1)$

$= 21 (3^4 \times 11^1 \times 13^1)$

Hence (4+1) x (1 + 1) x (1 + 1) = 20

Therefore 20 multiples of 21 are there in the given product

- ***Example 11:*** How many factors of $2^4 \times 3^5 \times 10^4$ are perfect squares which are greater than 1?

  Solution:

  $2^4 \times 3^5 \times 10^4 = 2^4 \times 3^5 \times 2^4 \times 5^4$

  $= 2^8 \times 3^5 \times 5^4$

  $= (2^2)^4 \times (3^2)^2 \times (5^2)^2$

  (Considering only those factors which are perfect squares)

  Hence the number of perfect squares are

  $= (4 + 1) \times (2 + 1) \times (2 +1)$

  $= 5 \times 3 \times 3$

  $= 45$

But we need perfect squares which are greater than 1

Hence the perfect squares greater than 1 are 45 – 1 = 44

ARITHMETIC PROGRESSION

Have you come across a Number series?

What is a Number series?

Yes, ***It is nothing but numbers arranged in a particular sequence.***

For example: 2, 7, 12, 17 …is a series where these numbers are arranged following a pattern.

These types of sequences that are arranged following a uniform pattern is called as a progression.

There are 3 types of progressions. They are

1. Arithmetic Progression
2. Geometric Progression and
3. Harmonic Progression

Let us understand these progressions in detail.

- An **Arithmetic Progression** (AP) or an **arithmetic sequence** is a sequence of numbers such that the difference between the consecutive terms is constant.For instance, the sequence 3, 5, 7, 9, 11, 13, 15 … is an arithmetic progression with common difference of 2.

A finite progression of an arithmetic progression is called a **finite arithmetic progression** and sometimes just called an Arithmetic Progression. The sum of a finite arithmetic progression is called an **arithmetic sum**.

- **The behavior of the arithmetic progression depends on the common difference *d*. If the common difference is:**

  - Positive, the members (terms) will grow towards positive infinity.
  - Negative, the members (terms) will grow towards negative infinity.

- ***Example 1***:

  Check whether the given sequence is an A.P: 1, 3, 5, 7, 9, 11.

  Solution:

  To check if the given sequence is A.P or not, we must first prove that the difference between the consecutive terms is constant. So, $d = a_2 - a_1$ should be equal to $a_3 - a_2$ and so on…

  Here, d = 3 – 1 = 2 equal to 5 – 3 = 2

- ***Real-time Example***:

  Suppose while returning from school, you get into the taxi. Once you ride a taxi you will be charged an initial rate. But then the charge will be per mile or per kilometer. This show that the arithmetic sequence for every kilometer you will be charged a certain constant rate plus the initial rate. To understand this let us study the topic of arithmetic progression in detail.

- **Arithmetic Progression Formulas**

Here are some of the important Arithmetic Progression related formulas:

- The general form of an Arithmetic Progression is a, a + d, a + 2d, a + 3d and so on.
- The nth term of an Arithmetic Progression series is $A_n = a_1 + (n - 1)\,d$, where A_n = n^{th} term and a_1 = first term. Here d = common difference = $A_n - A_{n-1}$.
- The sum of the first n terms of an Arithmetic Progression series is S =(n/2)[2a + (n- 1)d]
- The sum of n terms can be calculated using the below given formula if the last term is given,

$$\frac{n(a_1 + a_n)}{2}$$

- Also, $A_n = S_n - S_{n-1}$, where $A_n = n^{th}$ term

- ***Few hints to solve problems on AP:***

To solve most of the problems related to AP

- To find 3 terms in AP take the terms as a-d, a, a+d
- To find 4 terms in AP take the terms as a-3d, a-d, a+d, a+3d
- To find 5 terms in AP take the terms as a-2d, a-d, a, a+d, a+2d

- ***Arithmetic Progression Problems***

- ***Example 2***:

Find the 15th term of the arithmetic progression 3, 9, 15, 21,....?

Solution:

In the given AP, we have a = 3, d = (9 - 3) = 6, n =15

t_{15} = a + (n - 1)d

= 3 + (15 -1)6

= 3 + 84 = 87

- ***Example 3:***

Find the sum of the series: 2, 5, 8, 11, …. , 95

Solution:

By formula

$$n = \frac{l - a}{d} + 1$$

$$= \frac{95 - 2}{3} + 1$$

= 32

Hence S_{32} =

32/2 (2 + 95)

= 16(97)

= 1552

- ***Example 4:***

Find the first term of an AP whose 8th and 12th terms are respectively 39 and 59.

Solution:

Given that $t_8 = 39$ and $t_{12} = 59$

⇨ $a + 7d = 39$ and $a + 11d = 59$

⇨ on solving these equations, we get $d = 5$ and $a = 4$.

- ***Example 5:***

Sum of 3 numbers in AP is 21 and their product is 231. Find the numbers.

Solution:

Let the 3 numbers in AP be $a – d, a, a+d$

⇨ Hence $a\text{-}d + a + a\text{+}d = 21$ ….(1)

and $(a\text{-}d)a(a\text{+}d) = 231$……(2)

⇨ From (1) $a = 7$ and substituting this in (2)

⇨ $7(49\text{-}d^2) = 231$

⇨ $49\text{-}d^2 = 33$

⇨ $d = \pm 4$

when $a = 7$, $d = 4$, the terms in AP are 3, 7, 11

and when $a = 7$, $d = -4$ the terms in AP are 11, 7, 3

- ***Example 6:***

If 4 times the 4th term of an A.P. is equal to 9 times the 9th term of the A.P., what is 13 times the 13th term of this A.P.

Solution:

Given, $4\, t_4 = 9\, t_9$

$4(a + 3d) = 9(a + 8d)$

⇨ $5a + 60d = 0$

⇨ $5(a + 12d) = 0$

⇨ $t_{13} = 0$

- **Example 7:**

In an A.P $t_9 : t_{12}$ is 4:5, find $t_{10} : t_{13}$.

Solution

Given $t_9 : t_{12} = 4 : 5$

i.e., $\dfrac{a + 8d}{a + 11d} = \dfrac{4}{5}$

➔ $5a + 40d = 4a + 44d$

➔ $a = 4d$

Hence $t_{10} : t_{13}$

➔ $\dfrac{a + 9d}{a + 12d} = \dfrac{4}{5}$

➔ $\dfrac{4d + 9d}{4d + 12d} = \dfrac{13}{16}$

➔ $t_{10} : t_{13} = 13 : 16$

Geometric Progression

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There are 3 types of progressions. They are

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5. Geometric Progression and
6. Harmonic Progression

Let us understand these progressions in detail.

- ***Geometric Progression***

A sequence in which the ratio of any two consecutive terms is constant is a Geometric progression.

In other words, a sequence of the form a, ar, ar^2, ar^3.... ar^{n-1}..... is said to be in Geometric Progression (GP), with

the first term, $t_1 = a$

the second term, $t_2 = ar$ and so on

and the nth term is $t_n = ar^{n-1}$

It is observed here that,

$$\frac{t2}{t1} = \frac{t3}{t2} = \frac{t4}{t3} = \ldots\ldots = r \text{ (common ratio)}$$

Important formulas to remember

- To find the nth term: $t_n = ar^{n-1}$
- To find the sum of n terms: $S_n = a\left(\frac{r^n-1}{r-1}\right)$, r>1

  $a\left(\frac{1-r^n}{1-r}\right)$, r<1
- Sum of infinite geometric series: $S_\infty = \frac{a}{1-r}$, clearly here r<1.

Remember these Properties

- If a, b, c are 3 terms in a GP, then b= √(a x c)

- The reciprocals of a GP will also form a GP
- If $a_1, a_2, a_3 \ldots a_n$ are in GP then $\log a_1, \log a_2 \ldots \log a_n$ are in AP and vice versa.

  (mind that $a_1, a_2 \ldots$ are non-zero, non-negative numbers)

(HP is only for your understanding)

- ***HARMONIC PROGRESSION:***

  A sequence of numbers whose reciprocals are in AP. is said to be in

  Harmonic Progression (HP)

  Ie, $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots\ldots \frac{1}{a+(n-1)d}$

- ***Few hints to solve problems on GP:***

  - To find 3 terms in GP take the terms as $\frac{a}{r}$, a, ar
  - To find 5 terms in GP take the terms as $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar^2

  Now, let us look into some examples.

- ***Example 1:***

  Find the 10 term of the GP 2, 4, 8,……

  Solution:

  Here $t_1 = a = 2$, r = 2*(4/2 = 8/4 =2)* and n=10

By formula $t_n = ar^{n-1}$

Hence, $t_{10} = 2^9 = 512$

- ***Example 2:***

Find the number of terms in the GP 4, 8, 16, …. 1024

Solution:

Here $t_1= 4$, $t_2 = 8$, $t_n =1024$, $r= 8/4 =2$

Now, $1024 = a\ r^{n-1}$

Substituting the values, we get

$1024 = 4 \times 2^{n-1}$

$2^{10} = 2^2 \times 2^{n-1}$

$2^8 = 2^{n-1}$

⇨ $n-1 = 8$

⇨ $n = 9$

- ***Example 3:***

Find the sum of the 9 terms in GP 5, -20, 80, -320

Solution:

Here, $t_1= 5$, $r = -20/5 = -4$, $n = 9$

By formula $S_n = a\left(\frac{1-r^n}{1-r}\right)$, r<1

⇨ $S_9 = 5\left(\frac{1-(-4)^9}{1+4}\right)$

⇨ $1 + 4^9$

- ***Example 4:***

Find b, such that 12, b, 3 are in GP

Solution:

By formula if a, b, c are in GP, then b=√ac

Hence b = √12 x 3 = 6

- ***Example 5:***

Find the sum of the 10 terms in GP √3, √9, √27, …..

Solution:

Here a = √3, r = √9/√3 = √3

By formula

$S_n = a\left(\frac{r^n-1}{r-1}\right)$

$S_{10} = \sqrt{3}\left(\frac{\sqrt{3}^{10}-1}{\sqrt{3}-1}\right)$

$$= \sqrt{3}\left(\frac{3^5-1}{\sqrt{3}-1}\right) \times \left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$$

$= (3 + \sqrt{3})\ 242/2$

$= 121(\sqrt{3} + 3)$

Example 6:

The 4th term of a GP is square of its 2nd term, and the first term is -3. Find the 7th term.

Solution:

Given $\mathbf{t_4 = (t_2)^2}$ **and**

$\mathbf{t_1 = -3}$ which means a=-3**.**

Now , $\mathbf{t_4 = (t_2)^2}$

➔ $-3 \times r^3 = (-3 \times r)^2$

➔ $3r^2 + r^3 = 0$

➔ $r^2(r +3) = 0$

➔ $r = -3$

➔ $t_7 = -3(-3)^6$

➔ $t_7 = -2187$