LEAST COMMON MULTIPLE (LCM)

After learning the concepts of factors, we are going to learn one more important concept of finding the least common multiple and the highest common factors

A common multiple is a number that is a multiple of two or more numbers.

The common multiples of 3 and 4 are 0, 12, 24, ...

The least common multiple (LCM) of two numbers is the smallest number (not zero) that is a multiple of both.

Let us now look into the different ways of finding the LCM

- ***Method 1 –***

  Simply list the multiples of each number (multiply by 2, 3, 4, etc.) then look for the smallest number that appears in each list.

- ***Example1:***

  Find the least common multiple for 5, 6, and 15.

  Solution:

  Multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, etc.

  Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, etc.

  Multiples of 15 are 15, 30, 45, 60, 75, 90, etc.

Now, when you look at the list of multiples, you can see that 30 is the smallest number that appears in each list.

Therefore, the least common multiple of 5, 6 and 15 is 30.

- ***Method 2-***

  To use this method factor each of the numbers into primes. Then for each different prime number in all of the factorizations, do the following: (For Prime factorisation refer week 4 contents)

- ***Step 1***:

  Count the number of times each prime number appears in each of the factorizations.

- ***Step 2:***

  For each prime number, take the largest of these counts.

- ***Step 3:***

  Write down that prime number as many times as you counted for it in step 2.

- ***Step 4:***

  The least common multiple is the product of all the prime numbers

- ***Example 2*** : Find the least common multiple of 5, 6 and 15.

Solution : Prime factorization of 5 is 5 , 6 is 2 x 3 and 15 is 3 x 5

(* Note that the different primes are 2, 3 and 5)

Now, Count the number of times each prime number appears in each of the factorizations...

The count of primes in 5 is one 5 , 6 is one 2 and one 3 and 15 is one 3 and one 5

The largest count of 2s is one ,3s is one and 5s is one

So, the least common multiple is the product of all the prime numbers

2 x 3 x 5 = 30

Therefore, the least common multiple of 5, 6 and 15 is 30.

So, there you have it. A quick and easy method for finding least common multiples.

NOTE: LCM of a set of numbers cannot be less than the largest of the numbers.

- ***Example 3*:**

  Find the LCM of 10,15 and 25.

  ***Solution*:** Using Prime factorization,

  $10 = 2*5$, $15 = 3*5$, $25 = 5^2$

5 is the common prime and maximum power of 5 is 2. And the remaining prime numbers are 2 and 3.

So, the LCM for the given number is $5^2*2*3 = 150$

- **Example 4:**
- Find the least common multiple of 70 and 90

Solution:

$70 = 7*2*5$

$90 = 3^2*2*5$

LCM = $3^2*2*5*7 = 630$

- **Example 5:**

Find the least common multiple for 120, 100, 150

Solution:

$120 = 2^3*3*5$

$100 = 2^2*5^2$

$150 = 5^2*2*3$

LCM = $5^2*2^3*3 = 600$

- **Example 6:**

Find the least number which is exactly divisible by 75,125,200

Solution:

The least ***number divisible*** by 75, 125, 200 should be the LCM of the given numbers.

$75 = 5^2*3$

$125 = 5^3$

$200 = 2^3*5^2$

$LCM = 5^3*2^3*3 = 125*8*3 = 3000$

Example 7: (Short cut method) : Find the L.C.M. of 12, 15, 18, 27.

Solution.

2	12	15	18	27
2	6	15	9	27
3	3	15	9	27
3	1	5	3	9
3	1	5	1	3
5	1	5	1	1
	1	1	1	1

LCM = 2 × 2 × 3 × 3 × 3 × 5

LCM = 540

- **Example 8:**

Bus A leaves the bus station for every 8 minutes. Bus B leaves the bus station for every 12 minutes. The two buses leave the bus station together at 9 am. When is the next time the two buses leave the bus station together?

Solution:

Bus A leaves for every 8 minutes. Bus B for every 12 minutes.

So, the LCM of 8 and 12,

$8 = 2^3$

$12 = 2^2*3$

LCM = 2^3*3 = 24 minutes. Both the buses leave the bus station together at 9:24 am.

<u>To find the LCM of fractions:</u>

LCM of fraction = $\dfrac{\textbf{LCM of the numerator}}{\textbf{HCF of the denominator}}$

- **Example 9:**

Find the H.C.F. and L.C.M of $\dfrac{8}{9}, \dfrac{32}{81}$ $\boldsymbol{and}$ $\dfrac{10}{27}$

<u>Solution:</u>

L.C.M. = $\dfrac{L.C.M.of\ 8,32,10}{H.C.F..of\ 9,81,27} = \dfrac{160}{9}$

Relationship between the LCM HCF and the numbers is   ***The product of the numbers = LCM X HCF***

- **Example 10:**

The H.C.F. of two numbers is 14 and their L.C.M. is 11592. If one of the numbers is 504. Find the other.

<u>Solution:</u>

Other number = $\frac{H.C.F. \times L.C.M}{Given\ Number}$

$$= \frac{14 \times 11592}{504}$$

$$= 322$$

- **Example 11:**

Find the least number which when divided by 6, 7, 8, 9 and 12 leaves the same remainder 2 in each case.

<u>Solution:</u>

Required number =

= (L.C.M. of 6, 7, 8, 9, 12) + 2

= 206

- **Example 12:**

Find the greatest number of 4 digits which is exactly divisible by

12, 15, 20 and 35.

Solution:

LCM of 12, 15, 20, 35 = 420

The greatest 4 digit number is 9999.

Now dividing this by 420, we get

420) 9999 (23

<u>9660</u>

<u>339</u>

Hence the required number is

9999 – 339 = 9663

Hence the greatest 4 digit number divisible by 12, 15, 20 and 35

Is **9663.**

Example 13: Find the smallest number of 4 digits which is exactly

divisible by 12, 15, 20 and 35.

Solution:

Proceeding in the same way LCM of 12, 15, 20 and 35 is 420.

On dividing the least 4 digit number 1000 by 420, we get

420) 1000 (2

<u>840</u>

160

Hence the required number is 1000 + (420 – 160) = 1260.

Hence the smallest 4 digit number divisible by 12, 15, 20 and 35

Is **1260.**

- **Example 14:**

Find the LCM of 2.2, 540, 1.08

Solution:

LCM of 22, 540 and 108 is 5940

Hence the LCM of 2.2, 540, 1.08 = 594

(Because the least decimal place in the given numbers is 1.

HIGHEST COMMON FACTOR (HCF)

H.C.F. of two natural numbers is the largest common factor (or divisor) of the given natural numbers.

In other words, H.C.F. is the greatest element of the set of common factors of the given numbers.

H.C.F. is also called Greatest Common Divisor (abbreviated as G.C.D.)

To find the HCF of the given numbers:

- Factorize each of the given numbers into prime factors and their powers there of
- Take the common prime factors that contain the minimum power available and multiply.
- The product is known as the HCF or GCD of the given numbers

HCF can be obtained by different methods.

They are:

- Prime factorisation method
- Long division method
- Division method or Short cut method

Let us see these methods in detail with some examples

- **Example :1**

Find the H.C.F. of 72, 126 and 270.

Solution: Let us solve this by using ***Prime factorization method***

72 $= 2 \times 2 \times 2 \times 3 \times 3$

$= 2^3 \times 3^2$

126 $= 2 \times 3 \times 3 \times 7$

$= 2 \times 3^2 \times 7$

270 $= 2 \times 3 \times 3 \times 3 \times 5$

$= 2 \times 3^3 \times 5$

H.C.F. of the given numbers =

= the product of common factors with least index

$= 2 \times 3^2$

- **Example :2**

First find H.C.F. of 72 and 126

Solution:

Let us solve this by using **long-Division method**

$$
\begin{array}{r|l|l}
72 & \overline{126} & 1 \\
 & \underline{72} & \\
54 & \overline{72} & 1 \\
 & \underline{54} & \\
18 & \overline{54} & 3 \\
 & \underline{54} & \\
 & 0 &
\end{array}
$$

H.C.F. of 72 and 126 = 18
Similarly, calculate H.C.F. of 18 and 270 as 18
Hence H.C.F. of the given three numbers = 18

- ***Example 3*:**

  Find the H.C.F. of 12, 15, 18, 27.

Solution:

This is solved by division or short cut method.

$$
\begin{array}{r|l}
3 & \underline{12,\ 15,\ 18,27} \\
 & 4,\ \ 5,\ \ 6,\ \ 9
\end{array}
$$

All these numbers i.e., 4, 5, 6, 9 do not have any common factor except 1, we conclude that the HCF of these numbers is 3.

NOTE: HCF of a set of numbers cannot be greater than the smallest of the numbers.

Let us look into some examples to justify our understanding.

***Example 4*:**

Find the HCF of 8 and 14 by prime factorization method.

***Solution*:**

$8 = 2^3$ and $14 = 2*7$

Since, 2 is the only common prime and the least power of 2 is 1.

HCF of the given numbers is 2.

- ***Example 5:***
  Which greatest possible length can be used to measure exactly 15 cm, 25 cm and 30 cm.

  Solution:

  The greatest possible length should be the HCF of the given numbers.

  $15 = 5*3$, $25 = 5^2$, $30 = 5*2*3$

  Hence, the HCF of the given numbers is 5.

- ***Example 6***:
  Find the greatest number that will divide 427 and 899 leaving the remainders 3 and 7 respectively?

  Solution:

  Since, the respective remainders are 3 and 8.

  HCF of [427 – 3, 899 – 7]

  HCF of [424, 892]

  $424 = 2^3*53$, $892 = 2^2*223$

  The greatest number is 4

- ***Example 7***:
  What is the greatest number which divides 639, 1065 and 1491 exactly?

  Solution:

  HCF of 639, 1065 and 1491

  639 = 9*71, 1065 = 15*71, 1491 = 71*3*7

  HCF = 71*3 = 213

- ***Example* 8:**
  In a store there are 345 l mustard oil, 120 l sunflower oil and 225 l soyabean oil. What will be the capacity of the largest container to measure the above three types of oil?

  Solution:

  Capacity of the largest container will be HCF of 345, 120 and 225

  $345 = 5*3*23$, $120 = 2^3*3*5$, $225 = 5^2*3^2$

HCF is 5*3 = 15.

- ***Example* 9:**

Find the largest number which can exactly divide 513, 783 and 110

Solution:

Required number =

=H.C.F. of 513, 783 and 1107

= 27

- ***Example* 10:**

The greatest possible length which can be used to measure exactly the

lengths 7m, 3m 85cm and 12m 95cm is:

Solution:

The greatest length which can measure these lengths will be their HCF.

Now	7m	= 700cm
	3m85cm	= 385 cm
	12m95cm	= 1295cm

Their HCF is 35.

Hence the possible length which can be used to measure exactly the

lengths 7m, 3m 85cm and 12m 95cm is 35cm

- ***Example* 11:**

Three piece of timber 42m, 49m and 63m long have to be divided into planks of the same length. What is the greatest possible length of each plank?

Solution:

Similar to the previous question the HCF of 42, 49 and 63 is 7m

Hence the greatest possible length is 7m

To find the HCF of fractions:

HCF of fraction = $\dfrac{\textbf{HCF of the numerator}}{\textbf{LCM of the denominator}}$

- **Example 12:**

  Find the HCF of $\frac{2}{3}, \frac{8}{7}, \frac{4}{9}$

  Solution:

  By formula : $\frac{HCF\ of\ (2,8,7)}{LCM\ of\ (3,7,9)}$

  = 2/6

- **Example 13:**

  Find the HCF of 2^{25} , 4^{20} , 2^{10} and 8

  Solution:

  To find the HCF bring the base number the same and proceed as in the previous problems.

  These numbers can be written as 2^{25} , 2^{40} , 2^{10} and 2^{3}

  Hence their HCF is 2^{3}

- **Example 14:**

  HCF of two numbers is 12 and the sum of the numbers is 156.

  How many pairs of such numbers are possible?

  Solution:

  Let the 2 numbers be x and y.

  Given x = 12a and y = 12b

  Since the numbers common divisor is 12.

  Also given x + y = 156

  → 12a + 12b = 156

  → a + b = 13

  → for a + b to be 13, **6 pairs** are possible

  → they are (1,12), (2,11), (3,10)

To find the HCF of decimals:

To find the HCF of decimals follow the steps explained:

- *Find the HCF of numbers given by ignoring the decimals*
  For example if the question has numbers of the form a, 0.b, 0.0cde.
  Then consider the numbers as a, b, cde
  Then find the HCF of a, b, cde.

- *The maximum decimal places included in the numbers given should be used in the HCF .*
  i.e., if the HCF of a, b, cde is X, then the HCF of a, 0.b, 0.0cde is 0.000X

- **Example 15:**

Find the HCF of 11, 0.121 and 0.1331

Solution:

- To find the HCF of decimals find the HCF of numbers given by ignoring the decimals
- HCF of 11, 121 and 1331 is 11
- So the HCF of 11, 0.121 and 0.1331 is 0.0011

VBODMAS Rule-Simplification Tricks – Order of Mathematical Operation in Simplification

Each letter in this word indicates a specific symbol or mathematical operation to be performed.

V– Vinculum or Bar (–) i.e first you have to perform the mathematical operation under a bar, then go for removing of brackets

B– Brackets (order to remove brackets for simplification: 1st **( )** circular / smaller brackets, 2nd **{ }** Curly or flower brackets and finally **[ ]** square brackets.

O– Of (After removing brackets, then go for operation having 'Of".in complex expression

D– Next do Division

M-Multiplication

A – then Addition follows finally

S– Subtraction.

Let us use this rule to solve the problems given below:

EXAMPLE 1: Simplify:

$$8\frac{1}{2}-\left[3\frac{1}{5}\div 4\frac{1}{2}\text{ of }5\frac{1}{3}+\left\{11-\left(3-1\overline{\frac{1}{4}-\frac{5}{8}}\right)\right\}\right]$$

Solution Given expression

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\text{ of }\frac{16}{3}+\left\{11-\left(3-\overline{\frac{5}{4}-\frac{5}{8}}\right)\right\}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\text{ of }\frac{16}{3}+\left\{11-\left(3-\frac{5}{8}\right)\right\}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\text{ of }\frac{16}{3}+\left\{11-\left(\frac{19}{8}\right)\right\}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\text{ of }\frac{16}{3}+\frac{16}{8}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\times\frac{16}{3}+\frac{69}{8}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{24}{1}+\frac{69}{8}\right]$$

$$= \frac{17}{2} - \left[\frac{16}{5} \times \frac{1}{24} + \frac{69}{8}\right]$$

$$= \frac{17}{2} - \left[\frac{16}{120} + \frac{69}{8}\right]$$

$$= \frac{17}{2} - \left[\frac{16 + 1035}{120}\right]$$

$$= \frac{17}{2} - \frac{1051}{120}$$

$$= \frac{1020 - 1051}{120} = \frac{31}{120}$$

EXAMPLE 2: Simplify:

$$5\frac{1}{3} - \left[4\frac{1}{3}\left(3\frac{1}{3} - 2\overline{\frac{1}{3} - \frac{1}{3}}\right)\right]$$

Solution : Given expression

$$= \frac{16}{3} - \left[\frac{13}{3} - \left(\frac{10}{3} - \frac{7}{3} - \frac{1}{3}\right)\right]$$

$$= \frac{16}{3} - \left[\frac{13}{3} - \left(\frac{10}{3} - \frac{6}{3}\right)\right]$$

$$= \frac{16}{3} - \left[\frac{13}{3} - \frac{4}{3}\right] = \frac{16}{3} - \left[\frac{9}{3}\right] = \frac{16}{3} - \frac{9}{3} = \frac{7}{3} = 2\frac{1}{3}$$

EXAMPLE 3:

3. Simplify: $\left[3\frac{1}{4} + \left\{1\frac{1}{4} - \frac{1}{2}\left(2\frac{1}{2} - \overline{\frac{1}{4} - \frac{1}{6}}\right)\right\}\right]$

Sol. Given exp. $= \left[\frac{13}{4} + \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{5}{2} - \frac{3-2}{12}\right)\right\}\right] = \left[\frac{13}{4} \div \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{5}{2} - \frac{1}{12}\right)\right\}\right]$

$$= \left[\frac{13}{4} \div \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{30-1}{12}\right)\right\}\right] = \left[\frac{13}{4} \div \left\{\frac{5}{4} - \frac{29}{24}\right\}\right]$$

$$= \left[\frac{13}{4} \div \left\{\frac{30-29}{24}\right\}\right] = \left[\frac{13}{4} \div \frac{1}{24}\right] = \left[\frac{13}{4} \times 24\right] = 78$$

EXAMPLE 4:

Simplify:

$1 - [2 - \{5 - (4 - \overline{3 - 2})\}]$

Solution:

= 1-[2-{5-(4-1}]

$=1-[2-\{5-3\}]$

$= 1-[2-2]=1-0=1$

EXAMPLE 5:

5. Simplify: b- $\left[b-(a+b)-\left\{b-\left(b-\overline{a-b})\right)\right\}+2a\right]$

Sol. Given Exp. $= b-\left[b-a(+b)-\left\{b-(b-a+b)\right\}+2a\right]$

$= b-\left[b-a-b-\left\{b-(2b-a)\right\}+2a\right]$

$= b-\left[-a-\left\{b-2b+a\right\}+2a\right]$

$= b-\left[-a-\left\{-b+a\right\}+2a\right]$

$= b-\left[-a+b-a+2a\right]=b-b=0$