PROBABILITY

Introduction:

Probability has been used extensively in many areas such as biology, economics, genetics, physics and sociology etc.

In everyday life, we come across statements such as
(i) It will probably rain today.
(ii} I doubt that he will pass the test.

The words probably, doubt, most probably etc. involve an element of uncertainty. 'Probably rain' will mean it may rain or may not rain today. The uncertainty of probably can be measured numerically by
means of 'Probability' in many cases.

Random Experiment:
An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment. (e.g) Tossing a fair coin, drawing a card from a pack of
well-shuffled cards.

Sample Space:
The set of all possible outcomes of an experiment is called a Sample Space.

Event:
An event is a subset of a sample space.

Definition of Probability:
The probability of an event A is defined as the favorable number of cases of A, by the total number of cases. Thus,
$P(A) = \frac{n(A)}{n(S)}$

Results on Probability:

(1) $P(S) = 1$
(2) $0 \le P(A) \le 1$
(3) $P(\Phi) = 0$
(4) $P\overline{(A)} = 1 - P(A)$
(5) For any two events A and B, we have
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Mutually Exclusive Events:
If two or more events have no point in common (i.e) if they cannot occur simultaneously, the events are said to be mutually exclusive.

Note:
If A and Bare mutually exclusive events then $P(A \cup B) = P(A) + P(B)$

Independent Events:

Two events are said to be independent if the occurrence or non-occurrence of one event does not influence the occurrence **or** non-occurrence of the other event.

Dependent Events:

It implies that occurrence of one event affects the occurrence of the other event.

Example Problems:

1. If two dice are tossed simultaneously the number of elements in the resulting sample space is

A. 6 B. 8 C. 36 D. 24

Ans: C

Solution:

$$\text{The sample space } S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),\ (5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

Number of elements = 36 (or) n (S) = 6 × 6 = 36

2. What is the probability that a leap year selected at random will contain 53 Sundays?

A. 1/7 B. 1/3 C. 2/7 D. 4/7

Ans: C

Solution:

In a leap year, there are 366 days. 366 days= 52 weeks and 2 days.

The remaining 2 days can be

(i) Sunday, Monday (ii) Monday, Tuesday (iii) Tuesday, Wednesday

(iv) Wednesday, Thursday (v) Thursday, Friday (vi) Friday, Saturday (or)

(vii) Saturday, Sunday.

∴ Total no. of cases = 7 and favorable no. of cases = 2

∴ Required probability = 2/7

3. Four cards are drawn at random from a pack of 52 playing cards. Find the probability of getting all face cards.

A. $\frac{13c_4}{52c_4}$ B. $\frac{11c_4}{52c_4}$ C. $\frac{12c_4}{52c_4}$ D. $\frac{10c_4}{52c_4}$

Ans: C

Four cards can be drawn from a pack of 52 cards in 52C4 ways.

∴ n (S) = $52c_4$

There are 12 face cards (4 kings, 4 Queens and 4jacks) out of these 12 1ace cards, 4 cards can be selected in $12c_4$ ways.

∴ Favourable number of elementary events = $12c_4$

Required probability = $\frac{12c_4}{52c_4}$

4. A word consists of 9 letters; 5 consonants and 4 vowels. Three letters are chosen at random. What is the probability that more than one vowel will be selected?
A. 15/42 B. 16/42 C. 17/42 D. 19/42

Ans: C

Solution:
Three letters can be chosen out of 9 letters in $9C_3$ ways = 84 ways.
More than one vowel can be chosen in one of the following ways:
(i) 2 vowels and one consonant (or) (ii) 3 vowels
∴ Favourable number of elementary events = $4C_2 \times 5C_1 + 4C_3$
= 6 × 5 + 4 = 34

∴ Required probability = P(A) = $\frac{n(A)}{n(S)} = \frac{34}{84} = \frac{17}{42}$

5. A box contains 10 bulbs, of which just three are defective. If a random sample of five bulbs is drawn, find the probability that the sample contains no defective bulbs.
A. 1/6 B. 1/12 C. 1/10 D. 2/5

Ans: B

Out of 10 bulbs 5 can be chosen in $10C_5$ ways. No defective bulb means all non-defective bulbs.

No. of ways of selecting all 5 non-defective bulbs out of 7 is $7C_5$.

Required probability = $\frac{7c_4}{10c_4} = \frac{1}{12}$

6. Two players Reshma and Monisha ·play a tennis game. It is known that the probability of Reshma winning the match is 1/3 . What is the probability of Monisha winning the match?

A. 1/3 B. ½ C. 1/5 D. 2/3

Ans: D

Solution :

Let A and B denote the event of Reshma winning the match and Monisha winning the match.

Probability of Reshma winning = P(A) = 1/3

∴ . Probability of Monisha winning = P(B) = 1 - P (A) = $1 - \frac{1}{3} = \frac{2}{3}$

(∵ A and B are complementary events (i.e) if A wins B loose, and when B wins A loose)

7. An article manufactured by a company consists of two parts x and Y. In the process of manufacture of part X, 9 out of 104 parts may be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of part Y. The probability that the assembled product will not be defective is
A. 253/416 B. 361/416 C. 322/416 D. 251/416

Solution:
In the manufacture of part X, 9 out of 104 parts is defective.
⇒ 95 out of 104 parts is good. In the manufacture of part Y, 5 out of 100 parts is defective.
⇒ 95 out of 100 parts is good.

:. The probability that the assembled part will not be defective = $\frac{95}{104} \times \frac{95}{100} = \frac{361}{416}$

8. The odds in favour of an event are 3 : 5. Find the probability of occurrence of this event.
A. 3/8 B. 3/5 C. 5/3 D. 5/8

Ans: A

Solution:

Total no. of cases = 3 +5 = 8

Favourable number of cases = 3

:. Required probability = 3/8

9. A speaks truth in 60% cases and B speaks truth in 70% cases. The probability that they will say the same thing while describing single event is
A. 0.56 B. 0.68 C. 0.94 D. 0.54

Ans: D

Solution:
Let A and B be the events that A speaks truth and B speaks truth. Then,
P(A) = 60% = 60/100 = 3/5 and P(B) = 70% = 70/100 = 7/10
P(A') = $1 - \frac{1}{5} = \frac{2}{5}$ $and\ P(B') = 1 - \frac{7}{10} = \frac{3}{10}$

10. In a class, 30% of the students offered English, 20% offered Hindi and 10% offered both. If a student is selected at random, what is the probability that he has offered English or Hindi?
A. 2/5 B. 3/4 C. 3/5 D. 3/10

Ans: A

Solution:
Let A be the event that the student has offered English. B be the event that the student has offered Hindi.

P(A) = 30/100 = 3/10 ,
P(B) = 20/100 = 1/5
P(A⋂B) = 10/100=1/10
P(A orB) = P(A⋃B) = P(A) +P(B)-P(A⋂B)
= 3/10 + 1/5 – 1/10 = 4/10 = 2/5

11. If P(A) = 5/11, P(B) = 4/11 and P(A⋃B)=3/11, then P(A⋂B)=?
A. 1 B. 6/11 C. 5/11 D. 7/11

Ans: B

Solution:
P(AUB) = P(A)+ P(B) - P(A⋂B) = 5/11 + 4/11 – 3/11 = 6/11

12. A person throws a die, as many times till he gets 6. What is the probability that he will get 6 in the fourth throw?
A. 5/6 B. 5/36 C. $5^3 / 6^4$ D. $5^4 / 6^5$

Ans: C

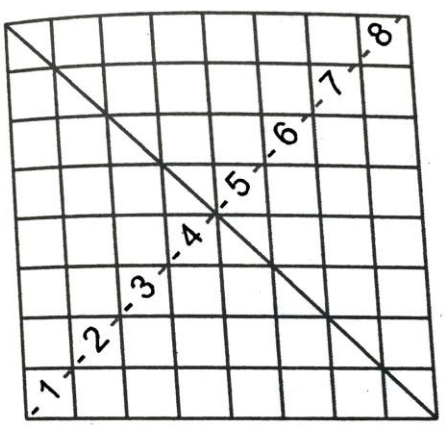
Solution:
The probability of not getting a six in the 1st throw = 5/6

The probability of not getting a six in the 2nd throw = 5/6 × 5/6 = $(5/6)^2$

The probability of getting a 6 in the 4^{th} throw = 5/6 × 5/6 × 5/6 × 1/6 = $5^3 / 6^4$

13. From a chessboard two squares are chosen at random. What is the probability that they have only one corner in common?
A. 17/36 B. 7/36 C. 7/144 D. 17/144

Ans: C
Solution:
Number of ways in which 2 squares can be selected out of the total of 64 squares is $64C_2$ = 64 × 63 / 2



Along the main diagonal running from left bottom to right top (i.e dotted lines) 7 pairs of squares which have only one corner in common. Parallel to this row on either side of this diagonal there are 6 + 5 + 4 + 3 + 2 + 1 or 21 pairs of squares.
These account for 7 + 21 + 21 = 49 pairs.

Similarly, considering the other diagonal running from bottom right to top left, there are 49 pairs of diagonals.
Hence required probability,
P(A) $\frac{n(A)}{n(S)} = \frac{2\times 49}{64C_2} = \frac{98\times 2}{64\times 63} = \frac{7}{144}$