PERMUTATION

The word *Permutation* means *Arrangement*. It is in other words an orderly arrangement.

The arrangement of n objects taken r at a time is nPr, and

$$nPr = \frac{n!}{(n-r)!}$$

To continue on permutations let us understand fundamental principle of counting.

FUNDAMENTAL PRINCIPLE OF COUNTING:

If a job is done in ***m*** *ways and the other job is done in* ***n*** *ways, then*

i)Both the jobs together will be done in ***m x n ways***

ii)Either the first job or the second will be done in ***m + n ways.***

Let us understand this with an example:

There are 10 boys and 15 girls in a class. Find in how many ways a teacher can select

i)a boy and a girl

ii)a boy or a girl to represent the class?

One boy out of 10 can be selected in 10 ways and one girl out of 15 will be selected in 15 ways

Hence i) a boy and a girl will be selected in 10 x 15 = 150 ways

ii)a boy or a girl will be selected in 10 + 15 = 25 ways.

Factorial Notation:

n! = n. (n-1) . (n-2) …….3.2.1, i.e. the continued product of the first n natural numbers.

$n! = n\,(n-1)! = n.\,(n-1)\,(n-2)!$ And so on…

$0! = 1$

1. How many numbers of 5 digits can be formed with the digits 0,2,3,4 and 5 if the digits may repeat?

   Ans: To form a 5-digit number let us consider 5 blocks

   _ _ _ _ _

The number of ways in which the unit place will be filled is 5 ways

In a similar way the tens, hundredth, thousandth place will be filled each in 5 ways

and ten thousandth place will be filled in 4 ways(except zero)

Hence the total ways = 4X5X5X5X5=2500

1. Find the number of three-digit numbers such that at least one of its digits as 9 and repetitions of the digits are allowed

   Answer:

   Number of three-digit numbers is 9X10X10=900

   Three-digit numbers not having 9 as one of its digit is

   8X9X9=648

   In above multiplication 1st place can fill in 8 ways other than 0 and 9

Three digits numbers having at least one of its digits is 9 = Number of three-digit numbers – three-digit numbers not having 9

So, three digits numbers having at least one digit 9 is =900-648=252

2. How many 4-digit numbers can be formed using the digits 0 to 9, which are divisible by 5 (repetition of the digits allowed)?

   Ans:

   Any number is divisible by 5 if its unit digit is 0 or 5

   Since 4 digit numbers are required 1000^{th} place cannot be 0

   Hence, 9X10X10X2=1800 numbers

3. How many 4-digit numbers can be formed by using the digits 0 to 9, which are all divisible by 5 without any repetition of the digits?

   Ans:

   Any number is divisible by 5 if its unit digit is 0 or 5

   Since 4-digit numbers are required 1000^{th} place cannot be 0

   But digits cannot be repeated more than one time so we can do this by taking case 1 and case2 separately

   Case1: last digit is 0

9x8x7x1=504 ways

Case2; last digit is 5

8x8x7x1=448 ways

Therefore the total number ways 504+448=952

4. How many natural numbers not exceeding 4321 can be formed using the digits 1,2,3,4, if repetition is allowed?

   Ans:

   Single digit numbers and two-digit numbers and three-digit numbers and four digits numbers not exceed 4321

   Total one-digit numbers =4

   Total two-digit numbers =4x4=16

   Total three-digit numbers =4x4x4=64

   Total 4-digit numbers starting with 1 =1x4x4x4=64

   Total 4-digit numbers starting with 2=1x4x4x4=64

   Total 4-digit numbers starting with 3=1x4x4x4=64

   Total 4-digit numbers starting with 41=1x1x4x4=16

Total 4-digit numbers starting with 42=1x1x4x4=16

Total 4-digit numbers starting with 431=1x1x1x4=4

Total 4-digit numbers starting with 432=1x1x1x4=4

Hence total natural numbers not exceed 4321 is

4+16+64+64+64+64+16+16+4+4=313

5. How many 5 digit even numbers can be formed using the digits from 0 to 9?

   I)with repetition of the digits

   ii)without repetition of the digits

Ans:

With repetition of the digits

9x10x10x10x5=4500

Without repetition of the digits

Case1: last digit is 0

9 ways (Except 0)	8 ways	7 ways	6 ways	1 way (Only 0)

9x8x7x6x1=3024

Case2: last digit is 2,4 ,6 or 8

8 ways (Except 0 and an even number)	8 ways	7 ways	6 ways	4 ways (even number except 0)

8x8x7x6x4=10752

Total possibilities **3024+10752=13776**

6. How many 4-digit numbers can be formed by using the digits 0 to 8, which are all divisible by 4, digits can repeat more than once?

Ans:

Any number is divisible by 4, if the last two digits of the number is a multiple of 4. The possibilities are 04,08,12,16,20,24,28,32,36,40,44,48,52,56,60,64,68,72,76,80,84,88

So, there are 22 possibilities for last two digits

First two digits can fill in 9 and 10 ways respectively.

9*10*1*1=90 ways for one group and total number of ways **90X22=1980**

PERMUTATIONS a ^{r} formula questions

1. In how many ways 5 letters be posted in 4 letter boxes and each box contain any number of letters?

   Ans:

   By using the multiplication rule

   First letter can post in 4 ways and 2nd letter can post in 4 ways like all 5 letters can post in 4 ways each

   So, answer is 4x4x4x4x4=4^5 ways

2. A man has 7 friends. In how many ways can he invite one or more of them to party ?

   Ans:

   This question can answer by using multiplication rule

   First friend can deal in two ways invite or not invite like all 7 friends can deal in 2 ways each

So, answer is

2x2x2x2x2x2x2=2^7

But in question given that at least he invite one so answer is 2^7-1 because he is not invite any one that also included in that so in final we subtracted one

3. There are total 5 matches between team A and B. In how many ways results be declared?

   Ans:

   First match can deal in 3 ways A win B win and drawn between two teams so answer is

   3x3x3x3x3=3^5

4. A question paper had 10 questions. Each question could only be answered as true (T) of False (F). Each candidate answered all the questions, yet no two candidates wrote the answers in an identical sequence. How many different sequences of answers are possible?

   Ans:

   Each question can be answered in 2 ways.

   10 Questions can be answered = 2^{10}= 1024 ways.

<u>PERMUTATIONS RANK problems</u>

1. Find the rank of the word ‘TIME’?

ANS:

Rank means the word TIME write in all possible ways by randomly change letters i.e 24 ways because 4!=24and arranged in dictionary ways TIME in which position

Two ways we can solve this one normal method and other one is short cut method

TIME first arranged in alphabetical order EIMT

First count staring with E -*-*-=3!=6

starting with I= I -*-*-=3!=6

starting with M=M*-*-*-=3!=6

starting with TE=TE*-*-=2!=2

starting with TIE=TIE*-=1

last one TIME i.e 1

so rank is 6+6+6+2+1+1=22^{nd}

so rank of the word is 22^{nd}

Permutations

1) Permutations Formula: Permutations can also be termed as ordered choices or arrangements. Each of the arrangements that can be made by selecting ‘r’ things out of ‘n’ things can be termed as a permutation. In permutations, the order in which the items are arranged is significant.

Consider arranging r things selected from n things. There are n possibilities for the first choice, (n – 1) possibilities for the second choice, (n – 2) possibilities for the third choice and so on. In other words, the available choices reduce by 1 after every selection.

- Therefore, ways to arrange r things selected out of n things are **n x (n-1) x (n-2) .... r terms = n x (n-1) x (n-2)... (n-r+1) = n!/(n-r)!**

- Hence the generalize formula for permutations is

$$nP_r = \frac{n!}{(n-r)!}$$

2) Similar items in permutations: While arranging 'n' things from which 'p' things are of one kind and 'q' things are of the second kind, with the rest of the things being distinct, the number of different arrangements will be

$$\frac{n!}{p!q!}$$

3) The total number of ways in which 'n' things can be arranged in 'r' ways with repetition allowed is equal to $\mathbf{n^r}$ ways.

4) Circular Arrangements:

- If 'n' objects are arranged in a circular way and if the clockwise and anti-clockwise arrangement is different, then the formula is **(n-1)!** ways.

- When there is no difference between clockwise and anticlockwise arrangements. In those cases, the total possible arrangements are half of the original ways of arrangements, i.e **(n-1)!/2**

Example problems

1. Write down all the permutations of the set of four letters E,O,N,T, taken two at a time .
   Ans:
   EO
   OE

EN
NE
ET
TE
ON
NO
OT
TO
NT
TN
SO TOAL 12 PERMUTATIONS
I.E 4P2=4*3/2*1=12

2. In how many ways can the letters of the word 'LEADER' be arranged?

Ans:

The word 'LEADER' contains 6 letters, namely 1L, 2E, 1A, 1D and 1R.

$$\therefore \text{ Required number of ways} = \frac{6!}{(1!)(2!)(1!)(1!)(1!)} = 360.$$

3. How many three letter words are formed using the letters of the word TIME?
   Ans:
   The number of letters in the given word is four.
   The number of three letter words that can be formed using these four letters is ${}^4P_3 = 4 * 3 * 2 = 24$.
4. Using all the letters of the word "THURSDAY", how many different words can be formed?
   Ans:
   Total number of letters = 8
   Using these letters the number of 8 letters words formed is ${}^8P_8 = 8!$.

5. Using all the letters of the word "NOKIA", how many words can be formed, which begin with N and end with A?
Ans:
here are five letters in the given word.
Consider 5 blanks ....
The first blank and last blank must be filled with N and A all the remaining three blanks can be filled with the remaining 3 letters in 3! ways.
The number of words = 3! = 6.

6. The number of arrangements that can be made with the letters of the word MEADOWS so that the vowels occupy the even places?
Ans:
The word MEADOWS has 7 letters of which 3 are vowels.
-V-V-V-
As the vowels have to occupy even places, they can be arranged in the 3 even places in 3! i.e., 6 ways. While the consonants can be arranged among themselves in the remaining 4 places in 4! i.e., 24 ways.
Hence the total ways are 24 * 6 = 144.

7. How many four digit even numbers can be formed using the digits {2, 3, 5, 1, 7, 9}
Ans:
The given digits are 1, 2, 3, 5, 7, 9
A number is even when its units digit is even. Of the given digits, two is the only even digit.
Units place is filled with only '2' and the remaining three places can be filled in ${}^5P_3$ ways.
Number of even numbers = ${}^5P_3$ = 60.

8. In how many ways can five boys and three girls sit in a row such that all boys sit together?
Ans:
Treat all boys as one unit. Now there are four students and they can be arranged in 4! ways. Again five boys can be arranged among themselves in 5! ways.
Required number of arrangements = 4! * 5! = 24 * 120 = 2880.

9. The number of sequences in which 7 players can throw a ball, so that the youngest player may not be the last is -.
Ans:
x Not younger________ ↑
The last ball can be thrown by any of the remaining 6 players. The first 6 players can throw the ball in ${}^6P_6$ ways.
The required number of ways = 6(6!) = 4320

10. Find the number of ways of arranging the letters of the word "MATERIAL" such that all the vowels in the word are to come together?
Ans:
In the word, "MATERIAL" there are three vowels A, I, E.
If all the vowels are together, the arrangement is MTRL'AAEI'.
Consider AAEI as one unit. The arrangement is as follows.
M T R L **A A E I**
The above 5 items can be arranged in 5! ways and AAEI can be arranged among themselves in 4!/2! ways.
Number of required ways of arranging the above letters = 5! * 4!/2!
= (120 * 24)/2 = 1440 ways.

11. In how many different ways can six players be arranged in a line such that two of them, Ajeet and Mukherjee are never together?

Ans:
As there are six players, So total ways in which they can be arranged = 6!ways =720.
A number of ways in which Ajeet and Mukherjee are together = 5!x2 = 240.
Therefore, Number of ways when they don’t remain together = 720 -240 =480.

12. In how many different ways can four books A, B, C and D be arranged one above another in a vertical order such that the books A and B are never in continuous position?
Ans:
The number of arrangement in which A and B are not together

= Total number of arrangements

= Number of arrangements in which A and B are together
=4!-3!x2! = 24-12 =12.

13. In how many different ways can the letters of the word 'MATHEMATICS' be arranged so that the vowels always come together?
Ans:
In MATHEMATICS total AAEI 4 vowels here question is all vowels together so treat this as one unit reaming 7 and this 1 total 8 units can arranged in 8! And vowels unit 4 can arranged in 4!
So answer is
8!*4!/2!*2!*2! = 120960

14. In how many ways 10 members can set in a circular table?
Ans:
N objects can arranged in circle in (N-1)! Ways

In same way 10 members can set in 9! Ways

Permutations setting arrangement problems

1. In how many ways 4 boys and 5 girls can set in a table ?

   Ans:

   Here they are not mention any condition so total 4 boys and 5 girls total 9 members can set in linear table in 9! Ways so answer is 9!

2. In how many ways 4 boys and 5 girls can set in a row table so that all boys are set together ?

   Ans:

   Here condition is all boys are set together so teat all 4 boys as a 1 unit

   And remaining 5 girls and this one unit total 6 units can arranged in 6! Ways

   And with in the boys unit 4 boys can arranged in 4!

   So total number of ways this can be done as

   6!X4!

3. In how many ways 4 boys and 5 girls can set in a row table so that all girls are set together ?

   Ans:

   It is same as previous question but condition is all girls are set together so assume all 5 girls as 1 unit

So 4 boys 4 units and all girls as 1 unit total 5 units can arranged in 5! And with in girls unit all 5 girls can arranged in 5 !

So answer is 5!X5!

4. In how many ways can 5 members can set in a row table so that two particulars persons ,Mr. A and Mr. B are never together ?

   Ans:

   We can solve this question by

   total ways – A and B are together

   total number of ways 5 member can arranged in 5!

   Number of ways that Mr. A and Mr. B together is teat this A and B as 1 unit and remaining 3 and this 1 unit 4 unit can arranged in 4! And with I the A and B unit again two members can arranged in 2! So total number of ways is

   =5! – 4!x2!

   =120-24x2

   =120-48=72

5. Find the number of ways in which 5 students and 5 teacher be seated in a row so that no two students set together

   Ans:

Here condition is no two students set together. In this case first arranged remaining 5 teacher in 5 places ,after that arrange 5 students in between the teacher or after or before the teacher

SO

--T--T--T--T--T---

5 teacher can arranged in any of the 5 places in 5! Ways

After that arrange student in between the teachers or after or before the teacher

So total possible places for student is 6 . but we have only 5 students .

In this case first select any 5 places from 6 places ,and arrange in that can be done in

6p5

So answer is 5! X 6p5

Here it is 6p5 because it is arrangement problem .

6. Find the number of ways in which 4 boys and 6 girls set in a row table. so that no two boys set together

   Ans:

   It is same as previous question first arrange ,6 girls in 6 places that can be done in 6! Ways

   --G---G---G---G---G---G---

After that arrange 4 boys in between or before or after . so now we have 7 empty places

In that we have to select 4 places and after that arrangement

That is done in 7p4

So answer for this question is

6! X7p4

7. There are 5 English books ,3 math books ,4 science books ,in how many ways all this books can arrange in a row so that same subject books are always together ?

   Ans:

   In this case all subject books always together so first make all English books as 1 unit and all math books as 1 unit and all science books as 1 unit

   2 units can arrange in a row in 3! Ways

   After that English books unit 5 books can arrange in 5! Ways. And it is same for math and science subjects so answer is

   3! X 5! X 3! X 4!

Circular setting Arrangement problems :

If 'n' objects are arranged in a circular way and if the clockwise and anti-clockwise arrangement is different, then the formula is **(n-1)!** ways.

When there is no difference between clockwise and anticlockwise arrangements. In those cases, the total possible arrangements are half of the original ways of arrangements, i.e **(n-1)!/2**

1. In how many ways can 6 boys form a ring ?

   Ans:

   Here it is given that 6 boys form a ring means circular arrangement tat can be done in

   6-1! Ways

   That is 5! ways because n objects can arrange in a circle in n-1! Ways

2. In how many ways can 6 boys be arranged at a round table so that 2 particular boys may be together ?

   Ans;

   Here condition is 2 particular boys always together means assume that two boys as 1 unit .and remaining 4 boys and this 1 unit total 5 units arrange in round table in

   4! Ways

   After that with in the boys unit that two boys can arrange in 2! Ways

   So answer is 4! X 2!

3. In how many ways 5 boys and 4 girls set in a circular table . so that all girls are always set together ?

   Ans:

   Here the condition is all girls are always set together .so assume all 4 girls asm1 unit .and remaining 5 boys and this 1 unit 6 unit can arrange in 5! ways

   After that with in the girls unit 4girls can arrange in 4! Ways

   So answer is

   5!x4!

4. In how many ways 5 boys and 4 girls can set in a round table .so that no two girls are set together ?

   Ans:

   Here the condition is no two girls set together so we have to arrange them .in between or before or after

   First arrange 5 boys in a circle in 4! Ways

   After that we have 5 gaps in between the boys .now arrange 4 girls in that 5 gaps in 5p4 ways

   So answer is 4! X 5p4

5. In how many ways can five boys and three girls sit in a row such that all boys sit together?

   Ans:

   Treat all boys as one unit. Now there are four students and they can be arranged in 4! ways. Again five boys can be arranged among themselves in 5! ways.

   Required number of arrangements = 4! * 5! = 24 * 120 = 2880.

6. In how many ways can 6 beads be strung into a necklace?

   Ans:

   Here it necklace that means circular arrangement so usually answer is 5! But that is not correct

   Because in circular arrangements if no different between clock and anti clock wise in that case divide it by 2

   Same way answer is 5!/2

7. Find the number of ways in which 10 different flowers can be strung to form a flower garland ?

   Ans:

   This is also like previous question so answer is

   (10-1)!/2

Here when we have to note that if the material can hang in that case n object can arrange in   n-1!/2 ways

8. The number of ways in which six boys and six girls can be seated in a row for a photograph so that no two girls sit together is -.

   Ans:

   We can initially arrange the six boys in 6! ways.

   Having done this, now three are seven places and six girls to be arranged. This can be done in $^{7}P_{6}$ ways.

   Hence required number of ways = 6! * $^{7}P_{6}$

9. There are 20 students in class .if each person send greeting cards to every other person once find the total number of greeting cords circulated ?

Ans:

First person send 19 cards same ways 2nd person send 19 cards like every 20 members can send 19 cards each .so answer is 20X19 i.e 380

In another way it is 20p2