CLASSIFICATION OF NUMBERS

INTRODUCTION:

An arithmetic value expressed as a word or a figure is a number.

Numbers are quantities used to count. Numbers are used for calculations. These numbers are widely classified as

- **N**: Natural Numbers: 1,2,3,4…

- **W**: Whole Numbers: 0,1,2,3,4….

- **I or Z**: Integers: ….-2,-1,0,1,2,3,…..

- **Q**: Rational Numbers: Numbers of the form p/q, q not zero , and p,q€Z. These are terminating, recurring decimals.

  Example: 4 , 2/5, 1/3, 22/7, …….

- **R**: Real Numbers: All rational numbers along with **Irrational** numbers form the Real Number system.

Irrational numbers are non-terminating, non-recurring decimals.

Example: π, $\sqrt{2}$

It should be observed here that,

N is a subset of W, W is a subset of Z, Z is a subset of Q, Q is a subset of R.

Hence **Real numbers** contains all numbers in existence**. (Not imaginary numbers, Mind it!)**

ODD, EVEN Numbers:

All Integers are classified as Odd and Even Integers.

All Integers divisible by 2 are called Even Integers, *Example: ….-4, -2, 0, 2, 4,……*

Otherwise, they are odd integers.

Remember :

- Odd number x Odd number= an Odd number
- Even X Even = Even
- Even X Odd = Even

<u>PRIME</u> <u>and</u> **<u>COMPOSITE Numbers</u>**

All Natural numbers greater than 1 are called Prime numbers if they are divisible by 1 and itself.

Example: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, ….

All those which are not prime are called composite numbers.

- 1 is neither Prime nor Composite
- 2 is the only even prime
- All prime numbers greater than 3 will be of the form 6n+1 or 6n-1.

- ***<u>To check if a number is prime:</u>***

Working rule to check if X is a prime number:

➢ *Find a number S which is greater than √X.*
➢ *List out all primes less than or equal to S.*
➢ *Check if at least one of these primes divides X.*
➢ *If not, then conclude that X is a prime number*
➢ *OR if a number is of the form 6n+1 or 6n-1, then it is a prime number*

Check it out with an example!

<u>Example 1</u>: **Is 211 a prime?**

<u>Solution:</u>

- 15 is greater than √211
- Prime numbers less than 15 are 2, 3, 5, 7, 11 and 13

- None of these divides 211 completely
- Hence 211 is a prime number

Alternate method: 211 is a prime number as it is of the form 35 x6 +1

Relatively prime or Coprime numbers:

A pair of Natural numbers whose common factor is 1.

Two consecutive numbers are always co-prime (Two prime numbers are co-primes)

Example: (2,3), (2,5), (8,15), (20, 39)

Twin primes:

Primes which are differ by 2 are twin primes.

Examples: (3, 5), (5, 7) ……

- **Multiplication by short cut method:**

  - **Multiplication of a number by 99, 999, 9999 etc.**

Rule: Place as many zeros to the right of the multiplicand as is the number of nines and from the number so formed, subtract the multiplicand to get the answer.

- **Example 2: Multiply 567958 by 99999.**

Solution:

Using the above rule, we have:
567958 x 99999 = (56795800000 – 567958) = 56795232042

  - **Multiplication of a number by 5^n.**

Rule: Put n zeros to the right of the multiplicand and divide the number so formed by 2^n.

- **Example 3: Multiply 975436 by 625.**

Solution:

We may write, $625 = 5^4$

$975436 \times 625 = 975436 \times 5^4 = 9754360000 \div 16 = 609647500$

- **Multiplication by distributive law**

We use the laws:

I. $a \times b + a \times c = a \times (b + c)$

II. $a \times b - a \times c = a \times (b - c)$

- **Example 4: Evaluate 978 × 184 + 978 × 816**

Solution:

$978 \times 184 + 978 \times 816$

$= 978 \times (184 + 816)$

$= 978 \times 1000 = 978000$

- **Example 5: Evaluate 857 × 318 – 857 × 218**

Solution:

$857 \times 318 - 857 \times 218$

$= 857 \times (318 - 218)$

$= 857 \times 100 = 85700$

<u>Divisibility test:</u>

Let us use the following rules to check the divisibility of a number by 2, 3, 4 and so on.

- ***<u>Divisibility by 2:</u>*** Any even number is divisible by 2. I.e any number whose unit digits are 0, 2, 4, 6 and 8

  <u>Example 1:</u>

  The number 16970, 217352, 67904, 81376 and 34918 end in 0, 2, 4, 6 and 8 respectively. So they are all divisible by 2.

- ***<u>Divisibility by 3:</u>*** A number is divisible by 3, if the sum of digits are divisible by 3.

  <u>Example 2:</u>Is the number 254679 divisible by 3?

  <u>Solution:</u>

  2+5+4+6+7+9 = 33, which is divisible by 3.
  Hence 254679 is divisible by 3.

  <u>Example 3</u>**: Show that 6903258 is divisible by 3.**

  <u>Solution:</u>

  Sum of is digits is 33, which is divisible by 3.
  So, the given number is divisible by 3.

  <u>Example 4</u>**: Show that 734689 is not divisible by 3.**

  <u>Solution:</u>

  Sum of its digits is 37, which is not divisible by 3.
  So, the given number is not divisible by 3.

- ***Divisibility by 4:*** A number is divisible by 4, if the last two digits are divisible by 4.

  Example 5:

  2154626 is not divisible by 4 as 26 is not divisible by 4 and 545680 is divisible by 4 as 80 is divisible by 4.

  Example 6**: Show that 738924 is divisible by 4.**

  Solution:

  Clearly the number formed by the last two digits of the given number is 24, which is divisible by 4.
  Hence, the given number is divisible by 4.

  Example 7**: Show that 8844634 is not divisible by 4.**

  Solution:

  Clearly, the number formed by the last two digits of the given number is 34, which is not divisible by 4.
  So, the given number is not divisible by 4.

- ***Divisibility by 5(or 10):*** A number whose digit is 0 or 5 (or 0) is divisible by 5(or 10)

  Example 8:
  The number 63180 and 75635 end in 0 and 5 respectively

- ***Divisibility by 6:*** A number is divisible by 6 if the number is divisible by both 2 and 3.

  Example 9:
  254679 though divisible by 3 is not divisible by 2, hence is not divisible by 6.

- ***Divisibility by 8:*** A number is divisible by 8, if the last three digits are divisible by 8.

  ***Example 10*: Show that 769573512 is divisible by 8.**

  Solution:

  Clearly, the number formed by the last three digits of the given number is 512, which is divisible by 8
  So, the given number is divisible by 8.

  Example 11: **Show that 4862412 is not divisible by 8.**

  Solution:

  Clearly, the number formed by the last three digits of the given number is 412, which is not divisible by 8.
  So, the given number is not divisible by 8.

- ***Divisibility by 9:*** A number is divisible by 9, if the sum of digits are divisible by 9

  Example 12: **Show that 7864506 is divisible by 9**.

  Solution:

  Sum of its digits is 36, which is divisible by 9.
  So, the given number is divisible by 9.

  Example13: Show that 65879413 is not divisible by 9.

  Solution:

  Sum of its digits is 43, which is not divisible by 9.
  So, the given number is not divisible by 9.

- ***<u>Divisibility by 11:</u>***A number is divisible by 11, if the difference between the sum of the digits in odd places and the sum of the digits in even places is 0 or a multiple of 11.

  <u>Example 14:</u> Will 11 divide 4763088 completely?

  <u>Solution:</u>

  Sum of digits in odd places = 4+6+0+8 = 18

  Sum of digits in even places = 7+3+8 = 18 and their difference 18-18=0, which is divisible by 11.

  <u>Example 15:</u> **Show that 684387 is divisible by 11.**

  <u>Solution:</u>

  Sum of digits at odd places = (7+3+8) = 18
  Sum of digits at even places = (8+4+6) =18
  Difference of sums = (18-18) = 0
  Hence, the given number is divisible by 11.

  Example 16: **Show that 4832718 is divisible by 11.**

  <u>Solution:</u>

  Sum of digits at odd places = (8+7+3+4) = 22.
  Sum of digits at even places = (1+2+8) =11
  Difference of sums = (22-11) = 11, which is divisible by 11.

  Example 17: **Show that 915436708 is not divisible by 11.**

  <u>Solution</u>:

  Sum of digits at odd places = (8+7+3+5+9) = 32
  Sum of digits at even places = (0+6+4+1) = 11
  Difference of sums = (32-11) = 21, which is not divisible by 11.
  Hence, the given number is not divisible by 11.

  Beyond 11 any number can be written as a combination of numbers which or co-prime to each other.

For example

- 12 can be written as a linear combination of co-primes 4 & 3.
- 28 can be written as a linear combination of co-primes 7 and 4
- 72 can be written as a linear combination of co-primes 9 and 8 and so on…

Hence to check the divisibility of any number by numbers greater than 11, write it as a combination of numbers which are coprime to each other and check for the divisibility of that numbers by these co-primes. Hence

- ***<u>Divisibility by 12 :</u>*** Check if the number is divisible by both 4 and 3.

- ***<u>Divisibility by 14:</u>*** Check if the number is divisible by both 2 and 7

- ***<u>Divisibility by 15:</u>*** Check if the number is divisible by both 5 and 3

- ***<u>Divisibility by 16:</u>*** Check if the last 4 digits of the number is divisible by 16

- ***<u>Divisibility by 18:</u>*** Check if the number is divisible by both 9 and 2 and so on.

- ***<u>Divisibility by 88:</u>*** Check if thenumber is divisible by both 8 and 11.

***<u>So we conclude that</u>*:**

If a number is divisible by p as well as by q, where p and q are co-primes, then the given number is divisible by pq. If p and q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.

<u>Example 18</u>: **Show that 569172 is divisible by both 2 and 4, but not divisible by 8. Why so?**

<u>Solution:</u>

Given number ends in 2. So, it is divisible by 2

Number formed by last 2 digits of the given number is 72, which is divisible by 4. So, the given number is divisible by 4.
But, the number formed by the last 3 digits of the given number is 172, which is not divisible by 8.
So, the given number is not divisible by 8.
Clearly, 2 and 4 are not co-primes. So the given number need not be divisible by their product, namely 8.

<u>Example 19</u>**: Show that 2784936 is divisible by 88.**

<u>Solution:</u>

The number formed by the last 3 digits of the given number is 936, which is divisible by 8.
So, the given number is divisible by 8.
In the given number, we have:
(Sum of digits at odd places) – (Sum of digits at even places)
= [(6+9+8+2) – (3+4+7)] = 11, which is divisible by 11.
So, the given number is divisible by 11.
Thus, the given number is divisible by both 8 and 11, where 8 and 11 are co-primes.
Hence, the given number is divisible by 88.

<u>Example 20:</u> **34278xyz is exactly divisible by 125. Find all the possible values z can take?**

<u>Solution:</u>

If a number is divisible by 125, it has to be divisible by 5.
Hence the last digit will be 0 or 5.

To find the number of zeros at the end of a product

Can you guess what helps to get a zero at the end of any product?
Or in simple words which two numbers when multiplied give a zero in the product?
Yes! It is 2 and 5 when multiplied gives the product which ends with a zero.
Let us look into an example to understand it clearly.

- ***Example 1***:

How many zeros will this product end with:

- i) 25 x 10 will end with *one zero*
- ii) 25 x 2 will end with *one zero*

  In both the cases, one 5 in 25 will combine with the 2 and hence one zero will be in the end

- iii) 25 x 4 will end with 2 zeros

  Reason: there are *two 5's in 25* which will combine with the *two2's in 4* and hence the product will end with two zeros

Hence it is clearly understood here that the 5's and 2's in any product will give the zeros at the end of the product.

- ***Example 2***: How many zeros are there in the end of

- i) 15 x 25 x 32 x 39 x 12

  Solution: 15 = 5 x 3

  25 = 5^2

  32 = 2^5
  39 = 13 x 3(no 2 and 5's involved, hence ignored)

  Hence by counting the 2's = 7

  and 5's =3

the 3fives will combine with 3 twos and give 3 zeros at the end.

OR the minimum of 7, 3 is 3 and hence the product will have 3 zeros at the end.

- ii) 125 x 36x 112 x 52 x 18 x 40 x 75
  Solution:

$125 = 5^3$

$36 = 3^2 \times 2^2$

$112 = 2^4 \times 7$

$52 = 2^2 \times 13$

$18 = 3^2 \times 2$

$40 = 2^3 \times 5$

$75 = 5^2 \times 3$

No. of 2's = 12
No. of 5's = 6 and the minimum of (12,6) = 6
Hence this product has 6 zeros at the end.

Now let us compile these and list a working rule for finding the number of zeros at the end of any product is tabulated as:

1. Factorise the given numbers
2. Count the number of 2's and 5's in the factors of each number
3. The minimum of 2's and 5's will be the number of zeros at the end

Now let us see how to find the number of zeros at the end of a factorial notation.

Let us understand here that

n! is the product of all natural numbers till n
ie, n!= n . (n-1) . (n-2) …. 3. 2. 1

<u>Example 3</u>:

Find the number of zeros at the end of 5!
Solution:
5! = 5 x 4 x 3 x 2 x 1
Since number of zeros are decided by the 5's and 2's present in the product,
And in any factorial notation 2's will be more in number, hence ignored and only 5's are counted.
Now number of 5's is 1 and hence,
Conclude the number of zeros is 1.

- ***Example 4***:

Find the number of zeros at the end of 155!
As we understood from the previous example that only 5's are counted and 2's ignored, let us check the number of 5's in 155!
For which divide 155 by 5 repeatedly as long as division is permitted.

5I 155
  5 I31
    5 I6
      1

Hence the number of zeros are 31 + 6 + 1 = 38(these are the quotients and reminders are ignored.

So, note here that in any factorial notation if we are to count the number of zeros then count only the 5's

Same concept will be used now for a different type of question.

Can u guess how many 5's will be there in a factorial notation?
Yes, you are perfectly right.

The number of highest powers of 5 in a factorial is obtained by dividing the number by 5 repeatedly

Let us check out on this example

- ***Example 5***:

Find the highest power of 5 in 100!
Solution:

Here the question format is different, but when you look into it, you will understand that the meaning of the question in example 3 and this is the same.

Just count the fives involved by dividing 100 by 5 repeatedly, till the number becomes lesser than 5.

5 I 100
5I 20
4

This way helps us to conclude that there are 20 + 4 = 24 5's in 100!

Working rule to find the highest power of any prime number p in n!:
1. Divide n by p repeatedly till the quotient is less than p
2. Add the quotients(ignore the remainders)
3. That will be the highest power of p in n!

How to find the highest power of a composite number 'q' in n!?
Prime factorize q, and find the number of these prime factors in n!, i.e., if $q = p_1 \times p_2$*, then find the highest power of these* p_1 *&* p_2 *in n!, whichever is minimum will give the number of zeros at the end.*

Let us understand this with an example:

Example 6:

Find the highest power of 15 in 120!
Solution:
As per the previous method explained, the factorisation of 15 = 5 x 3
Now to find the number of 5's and 3's in 150!, it is enough if we find the number of 5's alone as 3's will be more when compared to 5's in 150!
Hence dividing 150 by 5 we get

5 I 150
5 I30
5 I6
1

implies there are 37 5's in 150! and this will be the minimum when compared to the number of 3's.
So,
150! has 37 multiples of 15,
Or the highest power of 15 in 150! Is 37

Example 7:

Find the number of 9's in 180.

Solution:

Prime factorisation of 9 = 3 x 3

Now, on dividing 180 by 3, we get

3I 180
3I60
3 I20
3 I6
2

60 + 20 + 6 + 2 = 88 is the highest power of 3 ie 3^{88}

And hence,

$$3^{88} = (3^2)^{44}$$

$$= (9)^{44}$$

Hence there are 44 9's in 180!

Example 8:

Find the maximum power of 6 which can divide 120! without leaving a remainder.

Solution:

Prime factorisation of 6 is 2 and 3

Among the prime factors 2 and 3, highest power of 3 in 120! Will be less than the highest power of 2 in 120!.

Hence the highest power of 6 in 120! Is equal to highest power of 3 in 120!

Maximum power of 6 in 120! Is

$= [120/3]+[120/3^2]+[120/3^3]+[120/3^4]$

= 40+13+4+1

= 58

Hence maximum power of 6 which can divide 120! Without leaving a remainder is 58.

Example 9:

Find the maximum power of 40 in 120!
Solution:
The prime factors of 40 are 2^3x5
Maximum power of 2^3 in 120! Is
=[maximum power of 2 in 120!]/3
Maximum power of 2 in 120!
$=[120/2]+[120/2^2]+[120/2^3]+[120/2^4]+[120/2^5]+[120/2^6]$
= 60+30+15+7+3+1
= 116
Hence maximum power of 2^3 in 120! =[116/3]
=38
Maximum power of 5 in 120!
$=[120/5]+[120/5^2]$
= 24+4
= 28
Since the maximum power of 5 is less than the maximum power of 2^3, the maximum power of 40 in 120! is 28.

Note: If p is a prime number, highest power of p^a present in a factorial n is given by [Highest power of p in n!]/a

- ***Example 10***:

Find the number of zeroes at the end of 1000!
Solution:
Since number of zeros are decided by the 5's and 2's present in the product,

And in any factorial notation 2's will be more in number, hence ignored and only 5's are counted.
Hence the number of 5's in 1000! Is
$=[1000/5]+[1000/5^2]+[1000/5^3]+[1000/5^4]$
= 200 + 40 + 8 + 1
= 249
The number of zeroes at the end of 1000! is 249.

- ***Example 11***:

Find the number of zeroes at the end of the product 125^3x1024^2x199^3x47^{23}
Solution:
In any product the number of trailing zeroes depends on the pair of 2's and 5's are present in the product.
Ignore the terms which are not even and not divisible by 5.
$125=5^3$ and $1024=2^{10}$

Hence the number of 5's in 125^3 is 9
Number of 2's in 1024^2 is 20
Here the number of 5's is lesser than the number of 2's.
Hence the number of zeroes at the end is 9.

- ***Example 12***:

Find the number of zeroes in the product $127^{24}x475^{34}x555^{228}x777^9$
Solution:
In any product the number of trailing zeroes depends on the pair of 2's and 5's are present in the product.
In the given expression since there is no even number and hence there is no zero at the end of the product.

TO FIND THE UNIT DIGIT OF (ADCD…Z)^{N}

Many competitive examinations will have questions on finding the unit digits or also can be a part of a problem. It may help us to conclude to a solution just by checking on the unit digit. Let us look into the concept of finding the unit digit.

The unit digit of any number raised to the power of another (or same) number can be calculated by using cyclicity of numbers.

For example:

The unit digit of $2^1 = 2$,

$2^2 = 4$,

$2^3 = 8$,

$2^4 = 6$

$2^5 = 2$,

$2^6 = 4$,

$2^7 = 8$,

$2^8 = 6$ and so on.

It is understood here that the powers of 2's unit digit is cyclic.

Let us check the cyclicity for powers of 7 now.

$7^1 = 7$

$7^2 = 9$

$7^3 = 3$

$7^4 = 1$ and beyond this the unit digit of powers of 7 will be cyclic and are 7, 9 and 3 and 1 respectively.

Also it is to observe here that all even numbers to the power of 4 and its multiples are 6 and odd numbers to the power of 4 and its multiples are 1.

The cyclicity holds true of any number from 0 to 9, which is tabulated as below:

Digit ending with/powers	1	2	3	4	5	6	7	8	9
0*	0	0	0	0	0	0	0	0	0

1*	1	1	1	1	1	1	1	1	1
2	2	4	8	6	2	4	8	6	2
3	3	9	7	1	3	9	7	1	3
4	4	6	4	6	4	6	4	6	4
5*	5	5	5	5	5	5	5	5	5
6*	6	6	6	6	6	6	6	6	6
7	7	9	3	1	7	9	3	1	7
8	8	4	2	6	8	4	2	6	8
9	9	1	9	1	9	1	9	1	9

Using this cyclicity of powers of numbers, let us give a working rule for finding the unit digit of a number to any power.

To find the unit digit of $(aed…z)^n$, follow the following steps:

- If the unit digit z=0, 1, 5, or 6, then the unit digit of $(aed…z)^n$ is 0, 1, 5, 6 respectively. (Refer *)
- Otherwise divide n by 4.The possible remainders are 0, 1, 2 or 3.
- If the remainder is 0, then the unit digit is
    - Z^0 = 1(if z is odd)
    - = 6 (if z is even)(Refer the table above)
- If the remainder is 1, z is the unit digit
- If the remainder is 2, z^2's unit digit is the unit digit
- If the remainder is 3, z^3's unit digit is the unit digit.

Let us look into some examples to get familiarised with the concept of finding the unit digits. Let us look into few examples to understand this logic.

- ***Example 1***: Find the unit digit of 23456^{323}

  Solution:

  Any number ending with 6 to the power of any number is always 6.

  Hence unit digit of 23456^{323} is 6.

- ***Example 2:*** Find the unit digit of the number 17^{2003}

  Solution:

  Remainder when 2003 divided by 4 is 3.

Hence the unit digit is 7^3's unit digit, which is 3.

Answer is 3.

- ***Example 3*** **:** Find the unit digit of 1! + 2! + 3! + 4! + 5! + 6! + …..+ 100!

  Solution:

  The unit digit of 5! , 6! till 100! is all zero and hence the unit digits of

  1! + 2! + 3! + 4! = 1 + 2+ 6 + 24 = 33, whose unit digit is 3.

  Answer is 3

- ***Example 4***: Find the units digit of $(137^{13})^{47}$

  Solution:

  $(137^{13})^{47} = 137^{611}$[$(a^m)^n = a^{mn}$]

  611/4 leaves remainder 3,

  Hence the unit digit will be 7^3's unit digit is 3.

- ***Example 5***: The number 6 is multiplied like 6 x 6 x 6 x 6 x . . . . 100 times and then added by 6, find the last digit of the final value.

  Solution:

  6 x 6 x 6 x 6 x . . . . 100 times and then added by 6

  $= 6^{100} + 6$

  = 6 + 6, whose unit digit is 2.

  Hence answer is 2.

- ***Example 6*** **:** Find the unit digit of the number 1948^{1947}

  Solution:

  Remainder when 1947 divided by 4 is 3.

  Hence the unit digit is 8^3's unit digit, which is 2.

  Answer is 2.

- ***Example 7*** : Find the unit digit of the number $2946^{106} + 512^{117}$

Solution:

Any number ending with 6 to the power of any number is always 6.

Hence unit digit of 2946^{106} is 6.

Remainder when 117 divided by 4 is 1.

Hence the unit digit is 2^1, which is 2.

Answer is 6+ 2 = 8

- ***Example 8*** **:** Find the unit digit of the number 975^{169} - 516^{197}

Solution:

Any number ending with 5 to the power of any number is always 5.

Similarly, 6 to the power of any number is 6.

Hence the unit digit is 5 – 6

(Since 6 is greater than 5, follow subtraction method)

Answer is 9

- ***Example 9*** **:** Find the unit digit of the number 1968^{1994} + 1969^{1947}- 554^{4556}

Solution:

Remainder when 1994 divided by 4 is 2.

Hence the unit digit is 8^{2}'s unit digit, which is 4.

Any number ending with 9 to the power of odd number is always 9.

Hence unit digit of 1969^{1947} is 9.

Any number ending with 4 to the power of even number is always 6.

Hence unit digit of 554^{4556} is 6

Answer is 4 + 9 - 6 = 7

- ***Example 10*** **:** Find the unit digit of 256^{256}x444^{444} x555^{555}x777^{777}

Solution:

Any number ending with 5 to the power any number is always 5

In the given product we have an even number.

The product of an even number and 5 is always ends with zero.

Hence the unit digit is 0.

Note:

In any product, if one of the terms with unit digit 5 and another term with unit digit even, then unit digit of the resultant is always zero.

- ***Example 11*** **:** Find the unit digit of 199^{199}x947^{7377}x323^{342}x1075^{457}

  Solution:

  Any number ending with 5 to the power any number is always 5

  In the given product there is no even number.

  The product of an odd number with 5 always ends with 5

  Hence the unit digit is 5

  Note:

  In any product, if one of the terms with unit digit 5 and all the other terms are odd, then unit digit of the resultant is always 5.

- ***Example 12*** **:** Find the unit digit of 999^{999}x87^{87}x355^{103}x133^{199}

  Solution:

  Any number ending with 5 to the power any number is always 5

  In the given product there is no even number.

  The product of an odd number with 5 always ends with 5

  Hence the unit digit is 5

- ***Example 13*** **:** Find the unit digit of $999^{999}+87^{87}+355^{103}+133^{199}$

  Solution:

  Any number ending with 9 to the power of odd number is always 9.

  Hence the unit digit of 999^{999} is 9

  Remainder when 87 divided by 4 is 3.

  Hence the unit digit is 7^{3}'s unit digit, which is 3.

  Any number ending with 5 to the power any number is always 5.

  Hence the unit digit of 355^{103} is 5

Remainder when 199 divided by 4 is 3.

Hence the unit digit is 3^3's unit digit, which is 7.

Sum of the unit digits is 9+3+5+7 = 24

Answer is 4.

- ***Example 14*** **:** Find the unit digit of $1^3+2^3+3^3+...+99^3$

  Solution:

  Sum of first n cube natural numbers = $[n(n+1)/2]^2$

  Hence $1^3+2^3+3^3+...+99^3 = [99(100)/2]^2$

  Answer is 0

- ***Example 15*** **:** Find the unit digit of $11^1+12^2+13^3+14^4+15^5+16^6$

  Solution:

  The unit digit of 1 to the power any number is 1

  The unit digit of 2 to the power 2 is 4

  The unit digit of 3 to the power 3 is 7

  The unit digit of 4 to the power even number is 6

  The unit digit of 5 to the power any number is 5

  The unit digit of 6 to the power any number is 6

  Hence the unit digit in the sum is 1+4+7+6+5+6, which is 9

  Answer is 9

- ***Example 16*** **:** Find the unit digit of $171^{11}+122^{23}-133^{34}+144^{45}-155^{56}+166^{67}$

  Solution:

  The unit digit of 1 to the power any number is 1

  Remainder when 23 divided by 4 is 3

  Hence the unit digit is 2^3's unit digit, which is 8

  Remainder when 34 divided by 4 is 2

  Hence the unit digit is 3^2's unit digit, which is 9

  The unit digit of 4 to the power odd number is 4

The unit digit of 5 to the power any number is 5

The unit digit of 6 to the power any number is 6

Hence the unit digit in the expression is 1+8-9+4-5+6, which is 5

Answer is 5

- ***<u>Example 17</u>*** **:** Find the unit digit of 1971x2055x556x244

  <u>Solution:</u>

  Since, in the product we have a 5 and an even number, the unit digit is always 0

  Answer is 0

- ***<u>Example 18 :</u>*** Find the unit digit of 111! +112! + 113! +114! + 115! + 116! + …..+ 200!

  <u>Solution:</u>

  The unit digit of 5! , 6! and so on is all zero and hence the unit digit of

  111! + 112! + 113! + 114! +...+200! is 0.

  Answer is 0

- ***<u>Example 19</u>***: Find the units digit of $(15443^{19})^{17}$

  <u>Solution:</u>

  $(15443^{19})^{17} = 15443^{323}$ [$(a^m)^n = a^{mn}$]

  323 when divided by 4 leaves remainder 3,

  Hence the unit digit will be 3^3's unit digit, which is 7.

  Answer is 7.

- ***<u>Example 20</u>***: Find the units digit of $(154^{17})^{17}+(127^{13})^{13}-(108^{11})^{11}$

  <u>Solution:</u>

  $(154^{17})^{17} = 154^{289}$ [$(a^m)^n = a^{mn}$]

  The unit digit of 4 to the power odd is 4

  $(127^{13})^{13} = 127^{169}$

  169 when divided by 4 leaves remainder 1

  Hence the unit digit will be 7.

  $(108^{11})^{11} = 108^{121}$

  121 when divided by 4 leaves remainder 1

Hence the unit digit of 108^{121} is 8

The unit digit of the result is 4+7-8, which is 3

Answer is 3.