

WEEK 9 - Long Answer Question 1

Instructions

From a pack of cards, two cards are drawn at random. What is the probability of getting a King and a Spade card?

Total Cards

- Total cards in a deck: 52

Scenario 1: King first, then Spade

1. Probability of drawing a King first: There are 4 Kings in the deck.

$$P(\text{King first}) = \frac{4}{52}$$

2. Probability of drawing a Spade second: After drawing a King, there are 51 cards left. If the King drawn is not a Spade, there are still 13 Spades left. If the King drawn is the King of Spades, there will be 12 Spades left.

- If the King is not a Spade:

$$P(\text{Spade second}) = \frac{13}{51}$$

- If the King is the King of Spades:

$$P(\text{Spade second}) = \frac{12}{51}$$

3. Combined Probability for Scenario 1:

- Probability of drawing a non-Spade King and then a Spade:

$$P(\text{non Spade King}) = \frac{3}{52} \quad (3 \text{ non Spade Kings}) \quad P(\text{non Spade King}) = \frac{3}{52}$$

$$P(\text{Spade second}) = \frac{13}{51}$$

Combined:

$$P(\text{non Spade King and Spade}) = \frac{3}{52} \times \frac{13}{51} \quad P(\text{non Spade King and Spade}) = \frac{3}{52} \times \frac{13}{51}$$

- Probability of drawing the King of Spades and then another Spade:

$$P(\text{King of Spades}) = \frac{1}{52} \quad P(\text{King of Spades}) = \frac{1}{52}$$

$$P(\text{Spade second}) = \frac{12}{51} \quad P(\text{Spade second}) = \frac{12}{51}$$

Combined:

$$P(\text{King of Spades and another Spade}) = \frac{1}{52} \times \frac{12}{51} \quad P(\text{King of Spades and another Spade}) = \frac{1}{52} \times \frac{12}{51}$$

Scenario 2: Spade first, then King

1. Probability of drawing a Spade first: There are 13 Spades in the deck.

$$P(\text{Spade first}) = \frac{13}{52} \quad P(\text{Spade first}) = \frac{13}{52}$$

2. Probability of drawing a King second: After drawing a Spade, there are still 4 Kings in the remaining 51 cards.

$$P(\text{King second}) = \frac{4}{51} \quad P(\text{King second}) = \frac{4}{51}$$

Final Calculation

Now we can sum up all the probabilities:

$$P(\text{King and Spade}) = P(\text{non Spade King and Spade}) + P(\text{King of Spades and another Spade}) + P(\text{Spade and then King})$$

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Calculating each part:

1. Non-Spade King and then Spade:
 - $\frac{3}{52} \times \frac{13}{51} = \frac{39}{2652}$ $\frac{3}{52} \times \frac{13}{51} = \frac{39}{2652}$
2. King of Spades and then another Spade:
 - $\frac{1}{52} \times \frac{12}{51} = \frac{12}{2652}$ $\frac{1}{52} \times \frac{12}{51} = \frac{12}{2652}$
3. Drawing a Spade first and then any King:
 - $\frac{13}{52} \times \frac{4}{51} = \frac{52}{2652}$ $\frac{13}{52} \times \frac{4}{51} = \frac{52}{2652}$

Adding these together gives:

$$P(\text{King and Spade}) = \frac{(39+12+52)}{2652} = \frac{103}{2652}$$

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Simplifying

The final probability can be simplified:

$$P(\text{King and Spade}) = \frac{1}{26} P(\text{King and Spade}) = \frac{1}{26}$$