COMBINATIONS

Each of the voices or selections which can be formed by taking some or all of a given number of things is called a combination.

Considering the three letters A, B and C of English alphabet, the different arrangements or permutations by taking 2 letters at a time are AB, BA, BC, CB, CA and AC.

In permutation, the order must be considered. Thus, AB and BA are different permutations. But in combination, the selection AB and BA are alike (the same objects have been taken into consideration-irrespective of the order in which they have been taken).

The order is immaterial in forming combinations. The different combinations that can be made by taking two at a time from the letters A, B and C are AB, BC and AC.

Number of Combinations of n different things by taking r (n ≥ r) at a time

Symbolically, it is denoted by nCr or C(n,r).

$$\mathrm{nCr} = \frac{\mathrm{nPr}}{\mathrm{r!}} = \frac{n!}{(n-r)!r!}$$

Remember

$nCr = nCn = 1$; $nC_1 = nC_{n-1}$; $nC_2 = nC_{n-2}$ and in general $nC_r = nC_{n-r}$

Example

Find the number of ways in which 11 players can be selected from 15 if

(i) There is no restriction
(ii) Two of them must be included
(iii) At least two of the three leg spinners must be included
(iv) One of them who is in bad form, must be excluded

Solution:

(i) Number of ways of selecting 11 players out of 15 without any restriction = $15C_{11} = 15C_4 = \frac{15x14x13x12}{4x3x2} = 1365$

(ii) Here, two of the players must be included. So, one has to select 9 more players out of remaining 13. The number of ways in which this can be done is $13C_9 = \frac{13x12x11x10}{4x3x2} = 715$

(iii) There are three leg spinners, of which two must be included. So, the choices are (a) 2 leg spinners out of three and another 9 out of 12 (b)all the three leg spinners and another 8 from remaining 12 players. The number of ways in this case,

$3C_2 \text{ x } 12C_9 + 3C_3\text{x}12C_8 = 3X\frac{12x11x10}{3x2} + 1X\frac{12x11x10x9}{4x3x2}$

$= 660 + 495 = 1155$

(iv) As the player who is in bad form must be excluded, one has to select all the 11 players out of the remaining 14. Number of ways in which this can be done, $14C_{11} = 14C_3 = \frac{14x13x12}{3x2} = 364$

Example

There are 12 points on a plane, no three of which are collinear except 5 which are in one line. Find the number of (i) straight lines (ii) triangles that can be made by joining them.

Solution

(i) If all the 12 points were scattered in such a manner that no three of them are collinear, then one would expect $12C_2$ lines by joining them. But as 5 of them are collinear, no straight line can be obtained by joining them, except the one which is the line containing those 5 points.

Thus, instead of getting $5C_2$ lines by joining them, one would get only one line. So, number of straight lines in this case is $12C_2 - 5C_2 + 1$

$= 66\text{-}10\text{+}1 = 57$.

(ii) If all the 12 points were scattered on the plane in such a manner that no three of them are collinear, then one would expect $12C_3$ triangles by joining them. But as 5 of them are collinear, no triangle can be obtained by joining any three of them at a time. Thus, required number of triangles is $12C_3 – 5C_3$ = 220-10 = 210.

Example

Find the number of diagonals of a decagon.

Solution

A decagon is a 10 sided polygon. It has 10 vertices.

Total number of line segments that can be obtained by taking 2 at a time is = $10C_2$.

These line segments include the sides of the decagon also. Hence, the number of diagonals of a decagon = $10C_2$-10 = 35.

Note: Number of diagonals of a n sided polygon = nC_2-n

1. Find the value of $100C_{98}$
A. 4900 B. 4950 C. 5000 D. 5050

Solution:

$100C_{98} = 100C_{100-98}$ $(\because nC_r = nC_{n-r})$

:. $100C_2 = \frac{100\times99}{2\times1} = 4950$

2. A committee of 5 is to be formed out of 6 gents and 4 ladies. In how many ways this can be done, when at-least 2 ladies are included?
A. 185 B. 186 C. 126 D. 180

Solution:

We have to make a selection of:

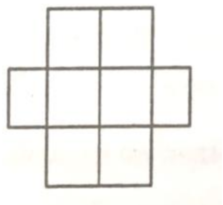
I. (2 ladies out of 4) and (3 gents out of 6)
$= 4c_2 \times 6c_3 = 6 \times 20 = 120$

II. (3 ladies out of 4) and (2 gents out of 6)
$= 4c_3 \times 6c_2 = 4 \times 15 = 60$

III. (4 ladies out of 4) and (1 gents out of 6)

$= 4c_4 \times 6c_1 = 1 \times 6 = 6$
:. Required number of ways = 120+60+6 = 186

3. Six 'X's have to be placed in the squares of the figure given below such that each row contains at-least one X. In how many different ways can this be done?



A. 22 B. 24 C. 26 D. 28

Solution:

Number of x in

	1^{st} row	2^{nd} row	3^{rd} row	No of ways
A.	1	3	2	$2c_1 \times 4c_3 \times 2c_2 = 8$
B.	1	4	1	$2c_1 \times 4c_4 \times 2c_1 = 4$
C.	2	2	2	$2c_2 \times 4c_2 \times 2c_2 = 6$
D.	2	3	1	$2c_2 \times 4c_3 \times 2c_1 = 8$

:. Required number of ways = 8+4+6+8 = 26

4. A set of 7 parallel lines is intersected by another set of 5 parallel lines. How many parallelograms are formed by this process?
A. 60 B. 280 C. 210 D. 140

Solution:

Any two parallel lines from the first set and any two from the second set will form a parallelogram.

The number of parallelograms that are formed = $7C_2$ x $5C_2$ = 21× 10 = 210.

5. There are n teams participating in a football championship. Every two teams played one match with each other. There were 171matches on the whole. What is the value of n?

A. 15 B. 17 C. 19 D. 21

Solution:

Total number of matches played = nC_2

Given, nC_2 = 171

$\Rightarrow \frac{n(n-1)}{2}$ = 171

$\Rightarrow n^2 - n - 342 = 0$

$\therefore$ (n+18) (n - 19) = 0

$\Rightarrow$ n = 19 ($\because$ n cannot be negative)

6. To fill 12 vacancies, there are 25 candidates of which 5 are from local institutes. If 3 of the vacancies are reserved for local institute candidates, while the rest are open to all, find the number of ways in which the selection can be made?

A. $20C_9$ X $5C_3$ B. $25C_{12}$ C. $5C_3$ x$20C_8$ D. $20C_9$ x$5C_2$

Solution:

A. No. of ways of making the required selection =

$$\begin{pmatrix} no.\,of\ ways\ of\ selecting \\ 3\ local\ institute \\ candidates\ out\ of\ 5) \end{pmatrix} \times \begin{pmatrix} no.\,of\ ways\ of\ selecting \\ remaining\ non-local \\ candidates\ out\ of\ 20 \end{pmatrix}$$

= $5c_3 \times 20c_9$

7. In an examination, a candidate has to pass in each of the 6 subjects. In how many ways can he fail?

A. 6 B. 6! C. 2^6 D. 63

Solution:

D. The student can fail in the 6 subjects in any of the following ways.

No. of subjects failed	Total no. of ways
6	$6C_6 = 1$
5	$6C_5=6$
4	$6C_4=15$
3	$6C_3=20$
2	$6C_2=15$
1	$6C_1=6$

Hence, the total no. of ways = 1+6+15+20+15+6 = 63 ways

8. Out of 5 men and 2 women, a committee is to be formed. In how many ways can it be formed, if at least one woman is to be included?

A. 20 B. 25 C. 30 D. 35

Solution:

The committee can be formed by choosing 1 woman and 2 men or 2 women and 1 man

1 woman and 2 men can be selected in = $2C_1 \times 5C_2 = 2 \times 10 = 20$ ways

2 women and 1 man can be selected in = $2C_2 \times 5C_1 = 1 \times 5 = 5$ ways

Total no. of ways = 20+5 = 25 ways

9. Given that 2 mC_3 : mC_2 =44:3, find the value of m.

A. 4 B. 5 C. 6 D. 7

Solution:

Given, $\frac{2mc3}{mc2} = \frac{44}{3}$

$$\frac{\frac{2m(2m-1)(2m-2)}{3 \times 2 \times 1}}{\frac{m(m-1)}{1 \times 2}}$$

= $\frac{44}{3}$

$\Longrightarrow$2 m (2m-1) (2m-2) = 44m (m-1)

4m (2m-1) (m-1) = 44m (m-1)

8m -4 = 44 $\Longrightarrow$ 8m = 48 $\Longrightarrow$ m = 6

10. Out of 30 players, 3 teams are to be formed having equal number of players. If three particular players are to be included in 3 different teams, in how many ways can the teams be formed?

A. 27! B . $\frac{3!27!}{(9!)3}$ C. $\frac{27!}{3!9!}$ D. $\frac{30!}{39!}$

Solution:

B. Let the 3 teams be A, B and C and the three particular players, be P, Q and R. We have to put the three particular players P, Q and R in different teams. This can be done in 3! Ways.

In each team 9 places remain to be filled, out of the 27 players left.
Team A can now be filled in = $27C_9$ ways.
After filling all the places in team A, team B can be filled in $18C_9$ ways.
Finally, the remaining 9 players have to be placed in team C automatically.
This can be done in 1 way.
The number of ways in which the 3 teams can be formed

$$3! \times \frac{27!}{9!\,8!} \times \frac{18!}{9!\,9!} = \frac{3!\,27!}{(9!)^3}$$

11. Ten speakers are there for a conference. Among them speaker A wants to speak before speaker B and B wants to speak before C. The number of ways in which the sequence of 10 speakers can be arranged is

A. $\frac{10!}{9}$ B. $\frac{10}{6!}$ C. $\frac{10!}{6}$ D. $\frac{10!}{6!}$

Solution:

Imagine 10 seats to be arranged in a row for 10 speakers. 3 seats can be chosen out of 10 in 10C3 ways and A, B, and C can be seated in that order in the seats. The remaining 7 speakers can be arranged in the remaining 7 seats in 7! Ways.

∴ The total no. of ways in which the 10 speakers can be listed

= 10C3 × 7! = $\frac{10!}{7!3!} \times 7! = \frac{10!}{3!} = \frac{10!}{6}$

12. In order to pass an examination, a student has to score a prescribed minimum in each of 7 subjects. Find the number of ways a student can fail.
A. 63 B. 127 C. 255 D. 124

Solution:
B. The number of ways in which a student can fail = The total number of ways in which he can fail in (i) 1 subject or (ii) 2 subjects….. (or) (vii) 7 subjects.

$= 7C_1 + 7C_2 + 7C_3 + 7C_4 + 7C_5 + 7C_6 + 7C_7 = 2^7 - 1$
($\because$ $7C_0 + 7C_1 + \ldots\ldots.. + 7C_7 = 2^7$)
= 128 -1 = 127

13. In how many ways can a mixed double tennis game be arranged out of 7 married couples, with the restriction that no couple can play in the same game?

A. 360 B. 400 C. 420 D. 240

Solution:

The game can be arranged by the following procedure.

Choose 2 men out of the 7 men available. This can be done $7C_2$ ways (i.e) 21 ways.

The wives of the two men cannot be included and only 5 women are available for choosing 2 women for the game.

This can be done in $5C_2$ ways. With the 2 men and 2 women selected 2 games can be arranged.

The total number of ways = $2 \times 7C_2 \times 5C_2 = 2 \times 21 \times 10 = 420$ ways

14. In an examination there are 1 0 questions divided into 2 sections of 5 questions in each. A candidate is required to answer 6 out of the 10 questions. He is not permitted to attempt more than 4
questions from each section. In how many different ways can be choose the questions?

A. 215 B. 200 C. 225 D. 250

Solution:

B. The following options are available for choosing questions from the two sections A and B.

Section A	Section B
4	2
3	3
2	4

The number of ways of choosing the questions

$= (5C_4 \times 5C_2) + (5C_3 \times 5C_3) + (5C_2 \times 5C_4)$

$= (5 \times 10) + (10\times 10) + (10 \times 5) = 50 + 100+ 50 = 200$