

Caspar

A Python implementation of the $SU(n)$ factorization scheme of [arXiv:1708.00735](https://arxiv.org/abs/1708.00735).

If you run into any trouble, or find a bug, feel free to drop me a line at odimatte@uwaterloo.ca or create an issue in the repo.

Dependencies

You will need the numpy package. Caspar was developed using Python 3.5.2 but also was tested and runs smoothly in 2.7.10.

Installation

In the main directory of the folder, type `python setup.py install`.

Usage (basic)

There are two important files: `factorization_script.py`, and `user_matrix.py`. Enter the $SU(n)$ matrix you wish to factorize in the variable `SUn_mat` in `user_matrix.py`. Then, factorize it by running

```
python factorization_script.py
```

The output of this script will be a series of lines in the following format, for example:

```
4,5    [-2.8209, 2.5309, 2.3985]
3,4    [-1.7534, 1.4869, -1.753]
...
```

This is a sequence of $SU(2)$ transformations. The first two integers indicate the modes on which the transformation acts. The set of three floats are the parameters of the transformation (see parametrization below). The original matrix `SUn_mat` is obtained by embedding each $SU(2)$ transformation into the indicated modes of an $SU(n)$ transformation, and multiplying them together from top to bottom of the list (with each transformation added to the product on the right, e.g. $U = U_{45} U_{34} \dots$).

Important usage notes

At the time of writing...

- For sparser matrices, such as the m -qubit Paulis, Caspar has seen good success up to 6 qubits ($n = 2^6 = 64$, and this is just as high as I tested).
- For denser Haar-random unitaries, Caspar works well for up to about $n = 10$ before it begins to suffer from issues due to numerical precision. You can use the function `sun_reconstruction` to compare the original matrix to the one reconstructed by the parameters that Caspar outputs (see below).

Usage (detailed)

An arbitrary element of $SU(n)$ can be fully expressed using at most $n^2 - 1$ parameters. We put forth a factorization scheme that decomposes elements of $SU(n)$ as a sequence of $SU(2)$ transformations. $SU(2)$ transformations require in general 3 parameters, $[a, b, g]$, written in matrix form as $[[e^{i(a+g)/2} \cos(b/2), -e^{i(a-g)/2} \sin(b/2)], [e^{-i(a-g)/2} \sin(b/2), e^{i(a+g)/2} \cos(b/2)]]$.

There are two main functions: `sun_factorization` and `sun_reconstruction`, each contained in the appropriately named files.

The function `sun_factorization` takes an $SU(n)$ matrix (as a numpy matrix) and decomposes it into a sequence of $n(n-1)/2$ such $SU(2)$ transformations. The full set of $n^2 - 1$ parameters is returned as a list of tuples of the form $(i, i+1, [a_k, b_k, g_k])$ where i and $i+1$ indicate the modes on which the transformation acts (our factorization uses transformations only on adjacent modes).

The following code snippet can be used to factorize the $SU(3)$ matrix below.

```
import numpy as np
from caspar import sun_factorization

n = 3

SUn_mat = np.matrix([[0., 0., 1.],
                     [np.exp(2 * 1j * np.pi / 3), 0., 0.],
                     [0., np.exp(-2 * 1j * np.pi / 3), 0.]])

# Perform the decomposition
parameters = sun_factorization(SUn_mat)

# The output produced is
#
# Factorization parameters:
# 2,3 [2.0943951023931953, 0.0, 2.0943951023931953]
# 1,2 [0.0, 3.1415926535897931, 0.0]
# 2,3 [0.0, 3.1415926535897931, 0.0]
```

It is also possible to reconstruct an $SU(n)$ transformation based on a list of parameters for $SU(2)$ transformations given in the form $(i, i+1, [a_k, b_k, g_k])$. The matrix is computed by multiplication on the right. At the moment only adjacent mode transformations are supported.

```
from caspar import sun_reconstruction

parameters = [("1,2", [1.23, 2.34, 0.999]),
              ("4,5", [0.3228328, 0.23324, -0.2228])]

new_SUn_mat = sun_reconstruction(6, parameters)
```