Randomised Benchmarking of Universal Qutrit Gates

Explicit Randomised Benchmarking qutrit schemes are limited to Clifford gates. We introduce a **scheme to characterise a qutrit T gate**.

$$X: \quad \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$T: \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{2\pi i/9} & 0 \\ 0 & 0 & (e^{2\pi i/9})^2 \end{bmatrix}$$

$$D: \quad \begin{bmatrix} e^{2\pi i/9} & 0 & 0 \\ 0 & e^{2\pi i/9} & 0 \\ 0 & 0 & e^{2\pi i/9} \end{bmatrix}$$

Our scheme is a **feasible** extension to qutrits of the Dihedral Benchmarking scheme [Carignan-Dugas *et. al.*]. Our scheme is the synthesis of the **Fourier method** [Merkel *et. al.*] applied to non-Clifford gates.

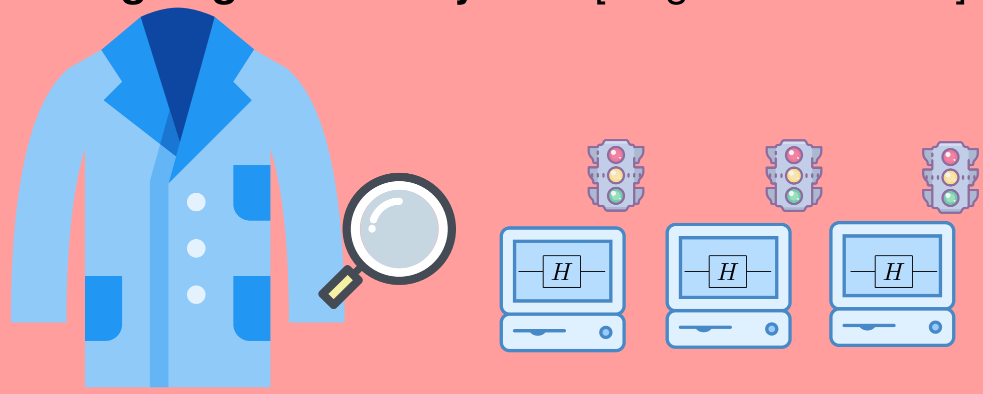
Our scheme is important for experimental groups with qutrit implementations [Morvan *et. al.*], theorists working on Randomised Benchmarking methods, and in general theorists interested in the application of Representation Theory in Quantum Information.

David Amaro-Alcalá[1,2], Barry C. Sanders[1], Hubert de Guise[1,2].

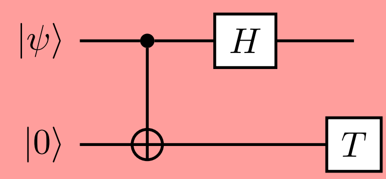
1 University of Calgary 2 Lakehead University

Background

Randomised Benchmarking (RB) estimates quantum gate quality by comparing the behaviour of ideal and noisy gates, via the **average gate fidelity** F [Magesan *et. al.*].



RB is used to characterize Clifford gates, T gates require an extension of the RB scheme for their characterization.



A **qutrit** is a three-level quantum system that offers advantages over qubits and is widely available in different quantum information implementations [Wang *et. al.*].

Results

RB assumes gates approximately implement transformations of a group; we introduce the HyperDihedral group to characterise a T gate. The HyperDihedral group is generated by X, T, and D.

The name HyperDihedral is given because it is a generalisation of the Dihedral group.

Dihedral	HyperDihedral
$C_2 \ltimes C_8$	$C_3 \ltimes C_9 \times C_9$

We obtained the expression for the average gate fidelity for the HyperDihedral group; it has **two parameters**, accessible by using two different **initial states**. Our expression

$$F = \tfrac{3}{4}(1 + 2r_0 + 6r_+) + \tfrac{1}{4}.$$

is valid for state imperfections and gate-dependent errors.

Our method is feasible because it requires the same resources in already established RB schemes [Wang *et. al.*].

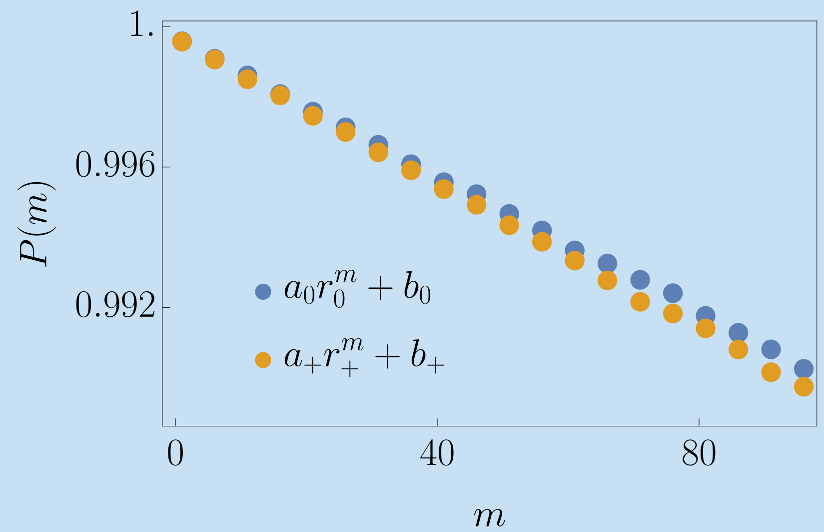


Figure 1: Survival probability P *vs* circuit depth m curve illustrating the relationship between the parameters that can be extracted from it to estimate the average gate fidelity shown in the second column. Density matrices $|0\rangle\langle 0|$ and $|+\rangle\langle +|$, denote the initial state required to obtain parameters r_0 and r_+, respectively.

References 1. Carignan-Dugas *et al*, *Phys. Rev. A.*, 2015. 2. Merkel *et al*, *Quantum*, 2021. 3. Morvan *et al*, *Phys. Rev. Lett.*, 2021. 4. Magesan *et al*, *Phys. Rev. A.*, 2012. 5. Wang *et al*, *Front. Phys.*, 2020.

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