Randomised Benchmarking of universal qutrit gates

Explicit Randomised Benchmarking qutrit schemes are limited to Clifford gates. We introduce a **scheme to characterise a qutrit T gate**.

$$X:\quad \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$T:\quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{2\pi i/9} & 0 \\ 0 & 0 & (e^{2\pi i/9})^2 \end{bmatrix}$$

$$D:\quad \begin{bmatrix} e^{2\pi i/9} & 0 & 0 \\ 0 & e^{2\pi i/9} & 0 \\ 0 & 0 & e^{2\pi i/9} \end{bmatrix}$$

Our scheme is a **feasible** extension to qutrits of the Dihedral Benchmarking scheme doi.org/brjj. Our scheme is the synthesis of the **Fourier method** doi.org/jrwr applied to non-Clifford gates.

Our scheme is important for experimental groups with qutrit implementations doi.org/gj8dt4, theorists working on Randomised Benchmarking methods, and in general theorists interested in the application of Representation Theory in Quantum Information.

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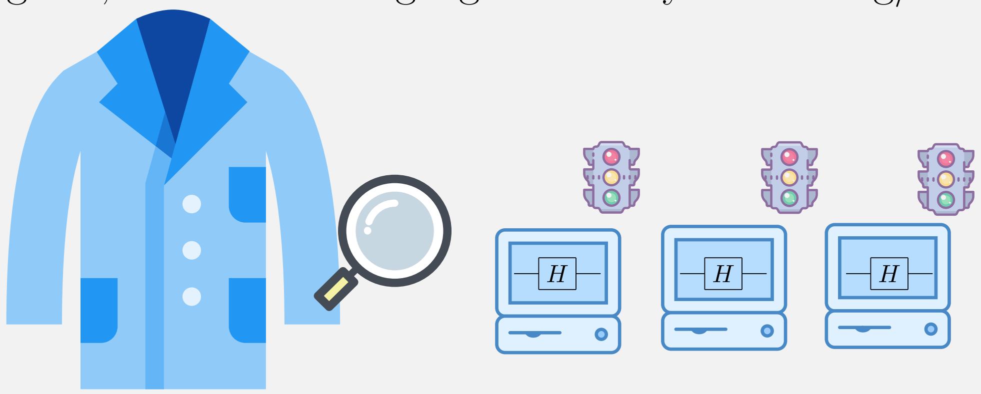
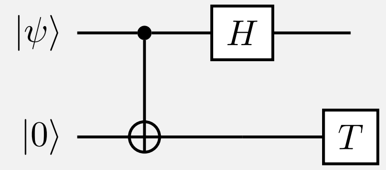
Background

Randomized Benchmarking estimates quantum gate quality by comparing the behaviour of ideal and noisy gates, via the average gate fidelity F doi.org/tfz.



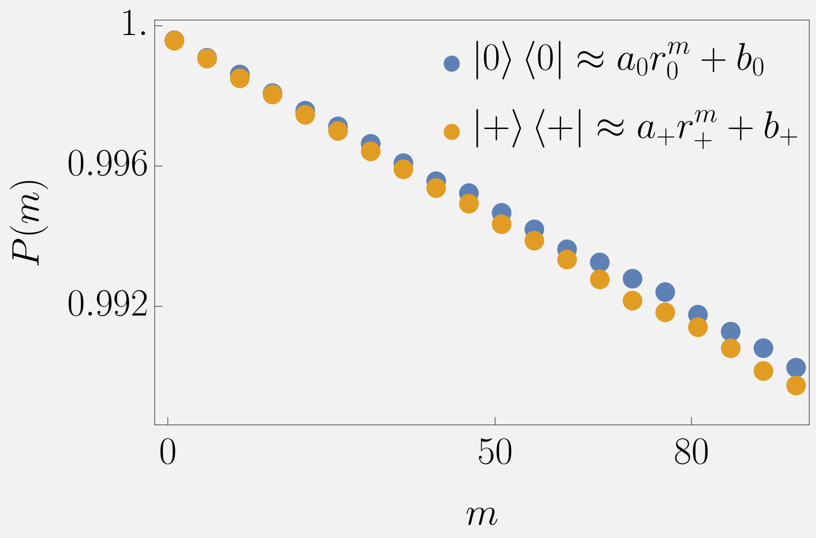
RB is used to characterize Clifford gates, T gates require an extension of the RB scheme for their characterization.



A **qutrit** is a three-level quantum system that offers advantages over qubits and is widely available in different quantum information implementations doi.org/ghptsj.

Results

RB assumes gates correspond to a physical group; we introduce the HyperDihedral group to characterise a T gate. The HyperDihedral group is generated by X, T, and D.



The name HyperDihedral is given because it is a generalisation of the Dihedral group.

Dihedral	HyperDihedral
$C_2 \ltimes C_8$	$C_3 \ltimes C_9 \times C_9$

We obtained the expression for the average gate fidelity for the HyperDihedral group; it has two parameters, accessible by using two different initial states. Our expression

$$F = \tfrac{3}{4}(1 + 2r_0 + 6r_0) + \tfrac{1}{4}.$$

is valid for state imperfections and gate-dependent errors.