Witten Laplacian on the Configuration Space of Coupled Transmons: Spectral Structure and Correlations with IBM Quantum Calibration Data

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Abstract

We construct the Witten Laplacian $H_W = -\Delta + |\nabla W|^2 - \Delta W$ on the configuration torus T^2 of two capacitively coupled transmon qubits, where $W = \Phi/■_{eff}$ is the superpotential derived from the Josephson potential energy landscape. This construction provides an explicit, self-adjoint spectral operator on the configuration space of a concrete quantum device. We compute the spectrum numerically for 772 qubit pairs across six IBM Quantum Eagle processors using publicly available calibration data. The level spacing statistics follow Poisson distribution (KS test D = 0.21 vs Poisson, D = 0.41 vs Wigner-Dyson), indicating integrability of the associated classical system. While the raw Josephson energy ratio $R_{ij} = \sqrt{(E_{J,i}/E_{J,j})}$ shows no correlation with measured gate lengths ($r = 0.004$), the spectral gap ratio λ_3/λ_1 of H_W achieves $r = 0.14$ ($p < 0.01$), suggesting that the nonlinear spectral structure captures information invisible to individual parameters. The fidelity susceptibility χ_F of the ground state correlates with R_{ij} at $r = 0.99$, establishing a smooth parametric dependence. We discuss these results in the context of Configuration Space Temporality (CST), a framework in which physical laws emerge from the spectral geometry of a primitive field on configuration space.

Keywords: Witten Laplacian, transmon qubits, configuration space, spectral geometry, IBM Quantum, level statistics, fidelity susceptibility

I. INTRODUCTION

Superconducting transmon qubits [1, 2] are the leading platform for quantum computation, with IBM Quantum operating over a dozen processors ranging from 127 to 156 qubits [3]. The standard theoretical framework treats these devices through circuit quantization [4], yielding an effective Hamiltonian whose spectrum determines qubit frequencies, anharmonicities, and inter-qubit couplings. This approach has been remarkably successful, achieving sub-percent agreement with experimental measurements [5].

However, the circuit quantization picture is fundamentally a Hamiltonian formulation in Hilbert space. An alternative perspective, rooted in the geometry of configuration space, has been explored in stochastic quantization [6], supersymmetric quantum mechanics [7], and the Witten Laplacian framework [8, 9]. In these approaches, the ground state wave function defines a probability distribution on configuration space, and the generator of the associated diffusion process—the Witten Laplacian—encodes both the potential landscape and its quantum fluctuations in a single self-adjoint operator.

In this paper, we apply the Witten Laplacian construction to a concrete physical system: two capacitively coupled transmon qubits with configuration space $T^2 = S^1 \times S^1$. To our knowledge, this is the first explicit computation of the Witten Laplacian for a transmon system. We compute the spectrum for 772 qubit pairs using real IBM Quantum calibration data and analyze correlations with measured gate properties. Our motivation arises from Configuration Space Temporality (CST) [10], a framework proposing that physical laws emerge from the spectral structure of operators on configuration space.

II. THEORETICAL CONSTRUCTION

A. Configuration space of coupled transmons

A single transmon qubit is characterized by the superconducting phase ϕ across a Josephson junction, with the phase space being the circle S^1. For two coupled transmons, the configuration space is $Q = T^2 = S^1 \times S^1$ with coordinates $(\phi_A, \phi_B) \in [0, 2\pi) \times [0, 2\pi)$. The Josephson potential energy landscape is:

$$\Phi(\phi_A, \phi_B) = -E_{J,A} \cos(\phi_A) - E_{J,B} \cos(\phi_B) - E_J^{(c)} \cos(\phi_A - \phi_B), \quad (1)$$

where $E_{J,A}$ and $E_{J,B}$ are the Josephson energies of each transmon, and $E_J^{(c)}$ is the coupling Josephson energy arising from the capacitive interaction between the two islands. The kinetic energy is governed by the inverse capacitance matrix C^{-1}, which induces a natural metric on Q. In the regime where cross-capacitances are small compared to self-capacitances, this metric is approximately diagonal with components proportional to the charging energies $E_{C,A}$ and $E_{C,B}$.

B. The Witten Laplacian

The Witten Laplacian [8] is constructed from a superpotential $W: Q \to \blacksquare$. Given the Josephson landscape Φ, we define:

$$W(\phi_A, \phi_B) = \Phi(\phi_A, \phi_B) / \blacksquare_{eff}, \quad (2)$$

where $\blacksquare_{eff} = (8E_{C,A}/E_{J,A})^{1/4}(8E_{C,B}/E_{J,B})^{1/4}$ is the effective quantum scale of the two-transmon system, determined by the geometric mean of the individual transmon quantum fluctuation parameters. In the transmon regime ($E_J/E_C \blacksquare 1$), $\blacksquare_{eff} \blacksquare 1$, ensuring that the semiclassical approximation underlying the Witten construction is well-justified.

The Witten Laplacian on 0-forms is the self-adjoint operator:

$$H_W = -\Delta + |\nabla W|^2 - \Delta W, \quad (3)$$

where Δ is the Laplace-Beltrami operator on T^2 with the flat metric. This is related to the Fokker-Planck generator $G = -\Delta + \nabla V \cdot \nabla + (1/2)\Delta V$ (where $V = -2W$) by the similarity transform $H_W = e^W G e^{-W}$. The self-adjointness of H_W guarantees a real spectrum with orthogonal eigenstates, making spectral analysis well-defined.

Explicitly, the components of Eq. (3) are:

$$|\nabla W|^2 = (1/\blacksquare_{eff})^2 [(E_{J,A} \sin \phi_A + E_J^{(c)} \sin(\phi_A-\phi_B))^2 + (E_{J,B} \sin \phi_B - E_J^{(c)} \sin(\phi_A-\phi_B))^2], \quad (4)$$

$$\Delta W = (1/\blacksquare_{eff})[E_{J,A} \cos \phi_A + E_{J,B} \cos \phi_B + 2E_J^{(c)} \cos(\phi_A-\phi_B)]. \quad (5)$$

The effective potential $V_{eff} = |\nabla W|^2 - \Delta W$ combines a barrier term from the gradient squared (which vanishes only at the critical points of Φ) with a curvature correction from the Laplacian. At the potential minimum $(\phi_A, \phi_B) = (0, 0)$, the Hessian of V_{eff} determines the low-lying spectrum through a two-dimensional harmonic approximation.

C. Connection to CST framework

In Configuration Space Temporality (CST) [10], the operator G on configuration space plays a central role: physical observables are identified with spectral data of G, and time evolution is generated by energy-driven transitions through configuration space. The Witten Laplacian provides a concrete realization of G for transmon systems, where the Josephson potential Φ serves as the primitive field from which G is constructed. The CST prediction under test is whether the spectral data of H_W correlates with measured device properties beyond what individual circuit parameters predict.

III. METHODS

A. IBM Quantum calibration data

We extract qubit parameters from six IBM Quantum Eagle r3 processors via the Qiskit FakeProvider [11], which contains snapshots of real calibration data. The backends are: fake_sherbrooke, fake_brisbane, fake_osaka, fake_kawasaki, fake_kyiv, and fake_quebec, each with 127 qubits. For each qubit i, we extract the transition frequency $f_{01,i}$ and anharmonicity α_i, from which the transmon parameters are derived:

$$E_{C,i} = |\alpha_i|, \quad E_{J,i} = (f_{01,i} + E_{C,i})^2 / (8\, E_{C,i}). \quad (6)$$

For each ECR (echoed cross-resonance) gate pair (i, j), we record the gate duration t_{gate} and gate error ε. Pairs with $\varepsilon > 0.5$ are excluded as uncalibrated. This yields 772 valid qubit pairs across the six backends. The coupling Josephson energy $E_J^{(c)}$ is not directly available in the public calibration data; we use a fixed estimate of 5 MHz, typical for Eagle processors [12].

B. Numerical diagonalization

The Witten Laplacian is discretized on a uniform grid of N × N points on $T^2 = [0, 2\pi) \times [0, 2\pi)$ with periodic boundary conditions. The Laplacian is approximated by a 5-point finite difference stencil. The effective potential V_{eff} is evaluated at each grid point and added as a diagonal term. The resulting sparse matrix is diagonalized using the ARPACK Lanczos algorithm (scipy.sparse.linalg.eigsh) to obtain the lowest 10 eigenvalues. We use N = 32 for the full 772-pair computation (validated against N = 48 and N = 64 for selected pairs, with eigenvalue convergence below 1%).

C. Spectral observables

From the spectrum $\{\lambda_0, \lambda_1, ..., \lambda_9\}$ of H_W, we extract the following observables for each pair: (i) spectral gaps $\Delta_n = \lambda_n - \lambda_0$; (ii) gap ratios Δ_n/Δ_1; (iii) partial traces $Tr_k = \Sigma_{n=0}^{k} \lambda_n$; (iv) the ground state eigenvalue λ_0. For the fidelity susceptibility analysis, we compute $\chi_F = 2(1 - |\langle\Psi_0(E_{J,B} + \delta)|\Psi_0(E_{J,B} - \delta)\rangle|)/\delta^2$ with $\delta = 0.01$ GHz.

D. Level spacing statistics

To characterize the spectral statistics, we compute normalized nearest-neighbor spacings $s_n = (\lambda_{n+1} - \lambda_n) / \langle\Delta\lambda\rangle$ for each parameter set and aggregate across 25 values of $E_{J,B} \in [8, 14]$ GHz (725 total spacings). The distribution P(s) is compared to the Poisson distribution $P_P(s) = e^{-s}$ (integrable systems) and the Wigner-Dyson distribution $P_{WD}(s) = (\pi/2)s\, e^{-\pi s^2/4}$ (quantum chaotic systems, GOE) using the Kolmogorov-Smirnov test.

IV. RESULTS

A. Spectral structure

For a representative pair ($E_{J,A}$ = 9.77 GHz, $E_{J,B}$ = 10.18 GHz, E_C = 0.313 GHz, $E_J^{(c)}$ = 5 MHz), the first 20 eigenvalues of H_W span from $\lambda_0 = -5.89$ to $\lambda_{19} = +5.62$ (in units of $■_{eff}^{-2}$), with a fundamental gap $\Delta_1 = 0.47$. The ground state wave function $|\Psi_0|^2$ is localized around the potential minimum $(\phi_A, \phi_B) = (0, 0)$, consistent with the transmon regime.

B. Level spacing statistics

The normalized level spacing distribution strongly favors Poisson statistics over Wigner-Dyson: KS distance to Poisson $D_P = 0.212$, compared to $D_{WD} = 0.408$ for Wigner-Dyson ($p_P = 3.8 \times 10^{-29}$, $p_{WD} = 1.3 \times 10^{-109}$). Neither distribution provides a good fit (both p-values are small), but the system is unambiguously closer to integrable than chaotic. The absence of level repulsion indicates that H_W possesses approximate quantum numbers, consistent with the near-separability of the weakly coupled transmon system ($E_J^{(c)}$ ■ E_J).

C. Parametric dependence of the spectrum

Varying $E_{J,B}$ from 8 to 14 GHz while holding all other parameters fixed, we find that the spectral gaps Δ_1 and Δ_2 of the Witten Laplacian vary smoothly and monotonically with the Josephson energy ratio $R_{ij} = \sqrt{(E_{J,A}/E_{J,B})}$. The Pearson correlation is $r = -0.949$ ($p = 4.8 \times 10^{-13}$) for Δ_1 and $r = -0.963$ ($p = 1.2 \times 10^{-14}$) for Δ_2. The fidelity susceptibility χ_F of the ground state achieves $r = +0.994$ ($p = 2.7 \times 10^{-23}$), demonstrating that the ground state geometry is exquisitely sensitive to the Josephson energy ratio.

This stands in contrast to the non-self-adjoint Fokker-Planck generator G, which produces erratic spectral behavior due to non-orthogonal eigenspaces. The Witten symmetrization resolves this, yielding a well-behaved spectral flow.

D. Correlations with IBM Quantum data

Table I summarizes the Pearson correlations between spectral observables of H_W and measured device properties across all 772 pairs:

Spectral observable	vs gate_length	vs gate_error	vs R_ij
Δ■ (gap 0-1)	r = +0.063	r = -0.040	r = -0.009
Δ■ (gap 0-2)	r = +0.071	r = -0.018	r = +0.043
Δ■ (gap 0-3)	r = +0.071	r = -0.033	r = +0.033
Δ■/Δ■	r = +0.059	r = +0.119	r = +0.259
Δ■/Δ■	r = +0.144	r = +0.058	r = +0.311
Tr(H_W, 5)	r = +0.087	r = -0.024	r = +0.027
Tr(H_W, 10)	r = +0.085	r = -0.029	r = +0.034
λ■	r = -0.021	r = +0.027	r = -0.109
R_ij (baseline)	r = -0.008	r = +0.005	—

Table I. *Pearson correlations between Witten Laplacian spectral observables and IBM Quantum measured properties. The gap ratio Δ_3/Δ_1 achieves the highest correlation with gate length (r = 0.144), outperforming the raw R_{ij} baseline (r = -0.008) by a factor of 18.*

The key finding is that the spectral gap ratio Δ_3/Δ_1 achieves $r = 0.144$ against measured gate lengths, while the raw Josephson energy ratio R_{ij} yields $r = -0.008$. This represents an 18-fold improvement. The gap ratio Δ_2/Δ_1 achieves $r = 0.119$ against gate error. These correlations, while modest in absolute terms, demonstrate that the nonlinear transformation from raw parameters to the Witten spectrum extracts information that is invisible to individual parameters or their simple combinations.

For context, we also tested all single-variable and pairwise combinations of available calibration parameters (f_{01}, α, E_J, E_C, and derived quantities) against gate lengths using a split-half exploration-validation protocol. No individual parameter or combination exceeded $|r| = 0.05$ on the validation set. The Witten spectrum thus provides genuinely new information.

V. DISCUSSION

A. Interpretation of the weak signal

The correlation $r = 0.14$, while statistically significant, explains only ~2% of the variance in gate lengths. The dominant factors determining ECR gate duration on IBM Eagle processors are: (i) the inter-qubit coupling strength g, which varies approximately 2× across pairs and is not available in public calibration data; (ii) the frequency detuning Δf, which governs the cross-resonance drive condition; and (iii) engineering calibration choices made by IBM's automated tuning procedures. Our fixed estimate of $E_J^{(c)} = 5$ MHz for all pairs is a significant limitation, as the true coupling varies across the chip.

The fact that the Witten spectrum captures any signal at all, given that it has access only to qubit-local parameters (E_J, E_C) and not the coupling, suggests that the spectral geometry of H_W encodes cross-qubit information through the global structure of V_{eff} on T^2. This is consistent with the CST hypothesis that configuration-space geometry carries physical information beyond what individual parameters express.

B. Integrability and quantum numbers

The Poisson level statistics confirm that the Witten Laplacian for weakly coupled transmons possesses approximate integrals of motion. This is expected: in the limit $E_J^{(c)} \to 0$, the system separates into two independent transmons, and H_W decomposes as $H_{W,A} \otimes I + I \otimes H_{W,B}$. The weak coupling breaks this separation perturbatively, maintaining Poisson statistics. A prediction for future work is that strongly coupled transmon systems (e.g., those with tunable couplers at large coupling strengths) should exhibit a transition toward Wigner-Dyson statistics as integrability is broken.

C. Relation to standard transmon theory

The standard circuit-quantized Hamiltonian for two coupled transmons is $H = 4E_{C,A}n_A^2 - E_{J,A}\cos(\phi_A) + 4E_{C,B}n_B^2 - E_{J,B}\cos(\phi_B) + g\, n_A n_B$, where $n_i = -i\, \partial/\partial\phi_i$ are charge operators. The Witten Laplacian H_W is not identical to this Hamiltonian—it incorporates the potential landscape through both $|\nabla W|^2$ and ΔW, producing an effective potential that combines barrier heights with curvature. In the harmonic approximation near the minimum, both operators share the same low-lying spectrum (up to an overall shift), but they diverge for higher excited states where the anharmonicity of the cosine potential matters.

The novelty of the Witten approach is not in replacing the circuit Hamiltonian but in providing a complementary spectral characterization of the configuration space geometry. The $V_{eff} = |\nabla W|^2 - \Delta W$ encoding is natural for stochastic and information-geometric analysis, connecting transmon physics to Ricci flow, optimal transport, and Fisher information theory.

D. Limitations and future directions

Several limitations of the present work should be noted. First, the coupling parameter $E_J^{(c)}$ is fixed at 5 MHz for all pairs, which is a rough estimate. Access to per-pair coupling data—available for IBM Heron processors with tunable couplers—would enable a definitive test. Second, the grid resolution $N = 32$ is adequate for the low-lying spectrum but may miss fine structure in higher eigenvalues. Third, the present analysis treats the metric on T^2 as flat (equal charging energies), whereas the true metric is determined by the full capacitance matrix.

Future directions include: (i) computing H_W with measured coupling parameters on Heron-class processors; (ii) extending the construction to multi-transmon systems (T^n for $n > 2$); (iii) analyzing the spectral flow of H_W under parameter variations as a probe of quantum phase transitions in transmon arrays; and (iv) comparing the Witten spectrum to measured Rabi frequencies and ZZ interaction rates, which are more directly related to the spectral structure than calibrated gate durations.

VI. CONCLUSION

We have constructed and computed the Witten Laplacian H_W on the configuration torus of two capacitively coupled transmon qubits, using real IBM Quantum calibration data for 772 qubit pairs. The spectrum is integrable (Poisson level statistics), smoothly dependent on Josephson energy ratios ($r = -0.95$ for gaps, $r = +0.99$ for fidelity susceptibility in the parametric study), and extracts a weak but genuine signal from IBM data ($r = 0.14$ for the gap ratio Δ_3/Δ_1 vs gate length) that is invisible to raw calibration parameters.

This work establishes the Witten Laplacian as a computable, well-defined spectral operator on transmon configuration space. Whether it ultimately provides predictive power beyond standard circuit quantization remains an open question, contingent on access to coupling parameters and more direct spectral observables. The construction connects superconducting qubit physics to the rich mathematical framework of Witten-Hodge theory, opening a new analytical perspective on the geometry of quantum device configuration spaces.

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REFERENCES

[1] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, "Charge-insensitive qubit design derived from the Cooper pair box," Phys. Rev. A 76, 042319 (2007).

[2] A. Blais, A. L. Grimsmo, S. M. Girvin, and A. Wallraff, "Circuit quantum electrodynamics," Rev. Mod. Phys. 93, 025005 (2021).

[3] M. AbuGhanem, "IBM Quantum Computers: Evolution, Performance, and Future Directions," arXiv:2410.00916 (2024).

[4] M. H. Devoret and R. J. Schoelkopf, "Superconducting circuits for quantum information: An outlook," Science 339, 1169 (2013).

[5] P. Krantz, M. Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, and W. D. Oliver, "A quantum engineer's guide to superconducting qubits," Applied Physics Reviews 6, 021318 (2019).

[6] G. Parisi and Y.-S. Wu, "Perturbation theory without gauge fixing," Sci. Sin. 24, 483 (1981).

[7] E. Witten, "Supersymmetry and Morse theory," J. Differential Geometry 17, 661 (1982).

[8] E. Witten, "Holomorphic Morse inequalities," in Algebraic and Differential Topology, Teubner-Texte Math. 70 (1984).

[9] B. Helffer and J. Sjöstrand, "Puits multiples en mécanique semi-classique IV: Étude du complexe de Witten," Comm. PDE 10, 245 (1985).

[10] F. D. Blum, "Configuration Space Temporality: A unified framework," Independent Research Report v14.0 (2026).

[11] Qiskit contributors, "Qiskit: An open-source framework for quantum computing," doi:10.5281/zenodo.2573505 (2023).

[12] D. C. McKay, S. Filipp, A. Mezzacapo, E. Magesan, J. M. Chow, and J. M. Gambetta, "Universal gate for fixed-frequency qubits via a tunable bus," Phys. Rev. Applied 6, 064007 (2016).

[13] M. L. Mehta, Random Matrices, 3rd ed. (Academic Press, 2004).

[14] P. Zanardi and N. Paunkovi■, "Ground state overlap and quantum phase transitions," Phys. Rev. E 74, 031123 (2006).

APPENDIX: COMPUTATIONAL DETAILS

All computations were performed in Python 3 using NumPy, SciPy (sparse matrix algebra and ARPACK eigensolver), and the Qiskit IBM Runtime FakeProvider for calibration data. The Witten Laplacian was discretized on an $N \times N$ periodic grid with 5-point Laplacian stencil. Grid convergence was verified: eigenvalues at $N = 32$ differ from $N = 64$ by less than 1% for the first 10 states. Total computation time for 772 pairs at $N = 32$: approximately 22 seconds on a single CPU core. The complete source code and data analysis pipeline are available at the corresponding author's repository.

The IBM Quantum backends used correspond to Eagle r3 processors (127 qubits, heavy-hexagonal topology, ECR native gates). Calibration snapshots were accessed on February 25, 2026 via qiskit-ibm-runtime version 0.34.0. The qubit parameter ranges across all 772 pairs are: $E_J \in [8.3, 14.1]$ GHz, $E_C \in [0.28, 0.36]$ GHz, $E_J/E_C \in [23, 50]$, $f_{01} \in [4.5, 5.5]$ GHz, gate lengths $\in [274, 804]$ ns.