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On the class of nilpotent Markov chains. I. The spectrum of covariance operator. (English summary)

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The authors study the central limit theorem and the structure of the corresponding covariance operator for the Markov chains generated by successive (overlapping) k -tuples, where $k \geq 1$ is fixed, $(X_{n+1}, \dots, X_{n+k})$, $n = 0, 1, 2, \dots$, are taken from a sequence $(X_n)_n$ of independent, identically distributed random variables. Such chains have finite radius of correlations so that the transition generators P are nilpotent and $(P - \Pi)^k = 0$, where $\Pi = \lim_{t \rightarrow \infty} P^t$. The main results concern the spectrum of the operator $B = I + P + P^* + \dots + P^{k-1} + (P^*)^{k-1}$ (the “covariance operator” of the chain) which is shown to be exactly $\{1, 2, \dots, k\}$; moreover, f is in the kernel of B if, and only if, $f = g - Pg$ where g satisfies $g = P^*Pg$. These results are then applied to the spectral analysis of the covariance matrices related to Marsaglia’s k -permutation test for $k = 2, 3, 4, 5$.

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