A Generic Random Number Generator Test Suite

Mario Rütti[1], Matthias Troyer[2], and Wesley P. Petersen[1]

[1] Seminar for Applied Mathematics, ETH Zürich, CH-8092 Zürich
maruetti@comp-phys.org
[2] Institute for Theoretical Physics, ETH Zürich, CH–8093 Zürich

Abstract. The heart of every Monte Carlo simulation is a source of high quality random numbers and the generator has to be picked carefully. Since the “Ferrenberg affair” [3] it is known to a broad community that statistical tests alone do not suffice to determine the quality of a generator, but also application-based tests are needed. With the inclusion of an extensible random number library and the definition of a generic interface into the revised C++ standard [1] it will be important to have access to an extensive C++ random number test suite. Most currently available test suites are limited to a subset of tests are written in Fortran or C and cannot easily be used with the C++ random number generator library.

In this paper we will present a generic random number test suite written in C++. The suite is based on the Boost reference implementation [2] of the forthcoming C++ standard random number generator library. The Boost implementation so far contains most modern random number generators. Employing generic programming techniques the test suite is flexible, easily extensible and can be used with any random number generator library, including those written in C and Fortran. Test results are produced in an XML format, which through the use of XSLT transformations allows extraction of summaries or detailed reports, and conversion to HTML, PDF, PostScript or any other format.

At this time, the test suite contains a wide range of different tests, including the standard tests described by Knuth [5], Vattulainen’s physical tests [14], parts of Marsaglia’s Diehard [6] test suite, and a number of number of newer test such as Gonnet’s repetition test [4]

1 Introduction

Testing random number generators is a really important part in the selection process for an adequate random number generator. But testing random number generators is also a very serious problem and not so easy to manage. In this paper we will present a generic framework (random number generator test suite, RNGTS) to manage, perform and analyze random number generator tests. The framework is written in C++ and makes an extensive use of the generic programming technique. This technique allows the usage of different data types and supports all different random number generators which fulfill the specification in the upcoming C++ standard for random number generators. An other important point is the flexibility of the framework.

Our attempt offers a flexible and simple extensible solution for further tests and other generators. Using XML to format the output opens a wide field of standardized transformations to other formats.

1.1 Random Number Generator Tests

Using a random number generator is one thing, the knowledge about its performance another. Testing the performance means first thinking about the expected properties of the random numbers, for example its application specific requirements. In a second step the most suitable tests have to be chosen. This selection is not easy and often people make "overall" tests of the generator. The problem is that we probably know what the test does, but one rarely knows which one the sensitive parameters in the application are. So, it is preferable to run more tests than fewer.

The best known source is Knuth's book [5], but there are many other tests available, some included in test benches, other as single procedures. The best known test bench is probably the DIEHARD test suite from Marsaglia [7] of which a new version has been released [8]. An other test bench, more specialized on physical tests, is available from Vattulainen [13]. Furthermore there is the SPRNG library [9] for parallel generators. In addition to the numerated test benches there are a lot of stand alone procedures like "Maurers Universal Test" [10] or the "Repeating Time Test" [4]. Different tests are a dime a dozen, so we collected the most popular ones in table 3. All tests in the table having a "Class Name" are available in the RNGTS framework.

1.2 Generic Programming

Generic programming in C++ became popular with the introduction of the Standard Template Library (STL) into the C++ Standard. The STL is a collection of often used Containers and Algorithms. The principle of generic programming is simple, instead of fixing the view onto particular data structures, the main interest are lying on the algorithms. This requires the introduction of a special data type handling mechanism. In C++ this technique is called template programming.

One simple example is the maximum operator, the ordinary maximum function. In an ordinary way, the maximum operator may be defined as follows.

```
const int max(const int& a, const int& b)
{ return (a > b) ? a : b; }
```

This method works well with integers, but we get into trouble if there are two doubles to compare. One solution is the replacement of all int's with double's. But the next problem occurs if the values are of type byte. The solution for this rapidly multiplying dilemma is, as mentioned above, templates. The specific data type is replaced by a generic placeholder which must be defined at compile time. Now, the maximum operator can be defined as followed.

```
template <typename T>
const T max(const T& a, const T& b)
{ return (a > b) ? a : b; }
```

The precondition for making this method work is that the comparison operator “>” must be defined.

The diversity of the STL gives an idea how powerful the usage of the template concept is. But this is only the tip of the iceberg, one can do much more. There is a technique called “Template Metaprogramming”, based on templates. This syntax allows compile time programming, e. g. nifty things like compile time if’s or decisions about used data types. A number of different applications of this template technique can be found in the BOOST library [2] as “Generic Programming” and “Template Metaprogramming”.

1.3 The Boost library

This section gives a short description of the BOOST project and presents the “BOOST Random Number Library”.

The following quotation from the BOOST home page describes the project in a few words.

> The Boost web site provides free peer-reviewed portable C++ source libraries. The emphasis is on libraries which work well with the C++ Standard Library. The libraries are intended to be widely useful, and are in regular use by thousands of programmers across a broad spectrum of applications.
> A further goal is to establish ”existing practice” and provide reference implementations so that Boost libraries are suitable for eventual standardization. Ten Boost libraries will be included in the C++ Standards Committee’s upcoming C++ Standard Library Technical Report as a step toward becoming part of a future C++ Standard.

The project contains already a huge collection of libraries and one of them is the `Random` library. The `Random` library is one of the libraries included in the upcoming C++ Standard. Working with this random library means working with the future of C++.

The `Random` library assembles a specific random number generator from to parts. One part is an “engine” (the raw random number generator), the other part is a “distribution”. This two parts together are represented by a so called `variate_generator`. For both parts there are a number of predefined types. Some examples are shown in table 1 and 2.

To manufacture a desired generator, one has only to specify the desired components and put them together. The code snippet below shows an example.

Boost Random Number Engine	Example
`linear_congruential`	GGL
`additive_combine`	L'Ecuyer 1988
`lagged_fibonacci`	R1279
`mersenne_twister`	MT19937
`shuffle_output`	Bays-Durham shuffle
...	

Table 1. Some Boost random library generators

Boost Random Number Distribution	Example
`uniform_01`	$x \in \mathbb{R}, x \in [0,1)$
`uniform_int`	$x \in \mathbb{N}$
`uniform_real`	$x \in \mathbb{R}, \quad x \in [\min, \max)$
`uniform_on_sphere`	$x \in \mathbb{S}^n$
`normal_distribution`	$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$
`exponential_distribution`	$P(x) = \lambda e^{-\lambda x}$
...	

Table 2. Some Boost random library distributions

```
#include <boost/random.hpp>          // the required libraries

boost::mt19937 rng;                  // the engine is a Mersenne Twister
boost::uniform_int<> six(1,6)        // the distribution maps to integer 1..6

// glue the components together
boost::variate_generator<boost::mt19937, boost::uniform_int<> >
    die(rng, six);

int x = die();                       // use the generator...
```

2 Random Number Generator Test Suite Framework

2.1 Statistics

After running a test one has to check if the obtained result is the expected one or something different. To classify the differences there are a variety of different statistical methods. There are three different statistical methods most frequently used. These are *Chi-Square* (χ^2), *Kolmogorov-Smirnov* and *Gaussian* test. To give an introduction, there is a short description below. As the final output of each statistic, one or more probabilities in the $[0, 1]$ range are expected. These values can provide a confidence level.

Chi-Square (χ^2) The χ^2-Test is perhaps the best known statistical test. It is based on a comparison between the empirical data and a theoretical distribution. The empirical data are the results of the random process.

$$\chi^2 = \sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i} = \frac{1}{n} \sum_{i=1}^{k} \left(\frac{n_i^2}{p_i} \right) - n \tag{1}$$

Kolmogorov-Smirnov The χ^2 test is applied when observations can fall into a finite number of categories. But normally one will consider random quantities which may assume an infinite number of values. In this test, the random number generators distribution function $F_n(x)$ is compared to the theoretical cumulative distribution function$F(x)$.

$$K_n^+ = \sqrt{n} \max_{-\infty < x < \infty} (F_n(x) - F(x)) = \sqrt{n} \max_{1 < i < n} \left(\frac{i}{n} - F(X_i) \right) \tag{2}$$

$$K_n^- = \sqrt{n} \max_{-\infty < x < \infty} (F(x) - F_n(x)) = \sqrt{n} \max_{1 < i < n} \left(F(X_i) - \frac{i-1}{n} \right) \tag{3}$$

Gaussian The Gaussian test is a little different from the χ^2 or the Kolmogorov-Smirnov test. In these two tests the expected distribution function is compared with the measured distribution function and based on the difference some indicators are calculated. In the Gaussian test a physical view is used. If a measurement is done, it is known that, even if the best tools are used, the result depends on a number of ruleless and uncontrolled parameters. These measurement errors are random and a combination of different single errors. This statistic calculates the probability for the deviation from the mean value. The used formula is

$$\mathrm{p(x)} = \frac{1}{2} - \frac{1}{2}\mathrm{erf}\left(\frac{1}{\sqrt{2}} x \right) \tag{4}$$

where erf denotes the "error function" $\mathrm{erf(z)} = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$.

The RNGTS-Framework implements these three statistics as base classes for the tests. They are named as `chisquare_test`, `ks_test` and `gaussian_test` and implement the needed statistical methods. Using one of these classes is not mandatory, but recommended. A benefit is the possibility of using other helpers like `iterate_test`.

2.2 Tests

Well known sources for random number generator tests are the book of Knuth [5] and Marsaglias DIEHARD test suite [7] and [8]. But there are many more tests, perhaps not so nicely packaged as in the works mentioned above, but still useful. Each of these test checks a specific property of the tested generator, e. g. the uniformity of binned values or the uniformity of 'continuous' values. The most popular tests are shown in table 3. The same table contains also the name of the class which implements the test, if it is already implemented.

To make a test work with the RNGTS framework, one only has to implement a specific interface, or in other words, there are some methods in a test class which are needed by the framework. The implementation may be done from scratch or if one of the often used statistics is needed, by deriving from a given base class.

Implementing a test: The following listing shows the base of each test, containing all required methods. This methods have to be implemented if the test is not derived from one of the provided test base classes.

```
#include "xml_helper.h"      // XML output functions

class the_new_test
{
public:
  the_new_test(...);

  template< class RNG >
  void run(RNG& rng);

  std::string test_name() const;

  template < class InputIterator >
  void analyze(xml_stream& out,
               InputIterator cl_begin,
               InputIterator cl_end) const;

  void print_parameters(xml_stream& out) const;
}
```

constructor The constructor has to take all parameters needed for further calculations, e.g. the number of random numbers to test or the number of test runs.

run The run method is the core of every test, this method has to run the test sequence and calculate the appropriate statistic. The generator to use is passed as `rng` and fulfills the Boost random number generator specification.

test_name This method has to return the name of the test which is printed in the output.

analyze The analyze method checks the test results against a given set of confidence levels, which are passed by iterators. The check also includes the decision if a test passed or failed at the confidence level. The output is written onto an XML stream. Several methods for producing valid XML output are provided. The output has to fulfill the XML Schema.

print_parameters This method has to print all relevant parameters to the given XML stream.

Test	Class Name
Equidistribution Test (Frequency Test)	ks_uniformity_test
	chisqr_uniformity_test
Gap Test	gap_test
Ising Model Test	ising_model_test
n-block test	n_block_test
Serial Test	serial_test
Poker Test (Partition Test)	poker_test
Coupon collector's Test	coupon_collector_test
Permutation Test	permutation_test
Run Test	runs_test
Maximum of t Test	max_of_t_test
Collision Test (Hash Test)	collision_test
Serial correlation Test	serial_correlation_test
Birthday-Spacing's Test	birthday_spacing_test
Overlapping Permutations Test	overlapping_permutations_test
Ranks of 31×31 and 32×32 matrices Test	bin_rank_chisqr_test
Ranks of 6×8 Matrices Test	bin_rank_ks_test
Monkey Tests on 20-bit Words	monkey_20bit_test
Monkey Tests OPSO,OQSO,DNA	monkey_[OPSO|OQSO|DNA]_test
Count the 1's in a Stream of Bytes	count_ones_stream_test
Count the 1's in Specific Bytes	count_ones_bytes_test
Parking Lot Test	parking_lot_test
Minimum Distance Test	minimum_distance_test
Random Spheres Test	random_sphere_test
The Sqeeze Test	squeeze_test
Overlapping Sums Test	overlapping_sums_test
The Craps Test	craps_test
Sum of distributions (for parallel streams)	
FFT	
Blocking Test	
2-d Random Walk	random_walk_test
Random Walkers on a line (S_n Test)	
2D Intersection Test	
2D Height Correlation Test	height_corr2d_test
Repeating Time Test	repetition_test
Gorilla Test	gorilla_test
GCD Test	gcd_test
Maurers Universal Test	maurers_universal_test

Table 3. Available tests in the RNGTS framework. A detailed description or/and the original sources may be found in [12].

2.3 Other Utilities

To perform a widespread analysis of a random number generator, the most important using is a variate of different tests. Occasionally, people want to test particular behavior of the generator, or they have to repeat a test multiple times or for different seeding strategies, etc. A short overview of the possibilities for included utilities are itemized below.

count_fails_test Repeats a test several times and count the number of failings per confidence level

iterate_test Repeats a test several times and calculates a Kolmogorov-Smirnov statistic from the results

parallel_rng_imitator Combines multiple random number generators to one generator

rng_bit_extract Interpret a random number as bit field and combine a sequence of successive bits to a new number

rng_bit_test Interpret a random number as bit field and generate sub-fields by shifting a specific mask over the field

rng_file Reads random numbers from a binary file

rng_wrapper Wrap external generators (e.g. Fortran, C) to use in the framework

2.4 Results

To rate a tested random number generator, one needs an overview over all test results with different seeds at different confidence levels. Additional interesting information to ensure the reproducibility of performed tests should also be saved.
This variety of different information are stored in an XML structure. An XML structure allows transformations of the raw results to different target representations like HTML or LaTeX. These two transformations are implemented using xslt (XML Stylesheet Language Translation). The ability of newest Internet browsers to format and display XML files makes the generation of a report really simple. If a report in LaTeX style is required, an xslt processor (like xsltproc) has to be used.

2.5 An Example

To give an impression of the usage of the RNGTS framework, a short example is presented. As we can see here, the usage is intuitive and straight forward.

```
#include <boost/random.hpp>                // import the boost random library
#define PRINT_STATUS                       // print status information at run time
#include <rngts/rng_test_suite.h>          // import the rngts framework
#include <rngts/chisqr_uniformity_test.h> // import all required tests

int main()
{
```

```
  rng_test_suite<> rngts;                 // create test-suite

  // specify confidence levels
  rngts.add_confidence_level(0.05);       // add 5% confidence level
  rngts.add_confidence_level(0.95);       // add 95% confidence level

  rngts.add_seed(331);                    // add seed 331
  rngts.add_seed(667790);                 // add seed 667790

  // register random number generators to test
  // using Mersenne-Twister and RanLux from boost
  rngts.register_rng<boost::mt19937>("mt-19937");
  rngts.register_rng<boost::ranlux64_3_01>("rl-64");

  // create and initialize a chisquare test object
  chisqr_uniformity_test chi(100000, 256);
  // register the chisquare test
  rngts.register_test<chisqr_uniformity_test>(chi);

  std::ofstream file_out("results.xml");  // file to write output
  try
  {
    rngts.run_test(file_out, true);       // run all tests
  } catch (std::exception& e)
  {
    std::cout << "failure : " << e.what();
  }
  file_out.close();                       // close output file
}
```

If the example above is compiled and executed, the file `results.xml` is written. A part of this file is printed below. The extract shows the results of a χ^2 test of the *Mersenne Twister 19937* generator. The test was performed with a seed of 331 and no "warmup" runs were executed. These are the configuration data of the generator section. In the test section, the needed test parameters are contained in the `PARAMETERS` block, and the `ANALYZE` block contains the test results. The results are included in the test name tag or in generalized result tags. Information whether a test passed or failed is represented by tags named `PASSED` or `FAILED` which also contain the appropriate confidence level.

```
<?xml version="1.0" ?><?xml-stylesheet href="xml2html.xsl" type="text/xsl"?>
<RNG_TEST_SUITE_RESULT  date="2004-04-26">
   <RNG  name="mt-19937"  warmup="0">
      <SEED  seed="331">
         <TEST  name="Chi-Square-Uniformity-Test">
            <PARAMETERS>
               <PARAMETER  name="Number of Numbers"  value="100000"/>
               <PARAMETER  name="Number of Classes"  value="256"/>
            </PARAMETERS>
            <ANALYZE>
               <CHI_SQUARE  chi2="242.33"  probability="0.706"  dof="255">
                  <PASSED  confidenceLevel="0.05"/>
                  <PASSED  confidenceLevel="0.95"/>
               </CHI_SQUARE>
            </ANALYZE>
         </TEST>
      </SEED>
      .....
   </RNG>
</RNG_TEST_SUITE_RESULT>
```

3 Summary

The presented RNGTS framework is a powerful but manageable tool to test the performance of C++ standard compatible random number generators. It provides a variety of random number generator tests an some other helpfully utilities. The ease of transforming XML to other formats allows postprocessing results to different representation like HTML or LaTeX.

But, one has to keep in mind, the final decision about using a generator or not can not be done by any program, because the usability strongly depends on the application. But, this framework should be a help to simply the decision.

4 Sources

The RNGTS framework is available at `http://www.comp-phys.org/rngts`. At this point one can also find a detailed description of the framework, some examples and related material.

5 Acknowledgements

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