Mathematics 204

Lab 3: Dynamical systems

Due: Monday, November 21 at 10pm

Please work on this lab activity in a group of 2 or 3 students. When you are ready, scan your work into a single PDF and submit through Blackboard with only one submission per group. Just make sure everyone's name is included below.

Names:

This lab has a total of 40 points with point totals given in bold next to the questions.

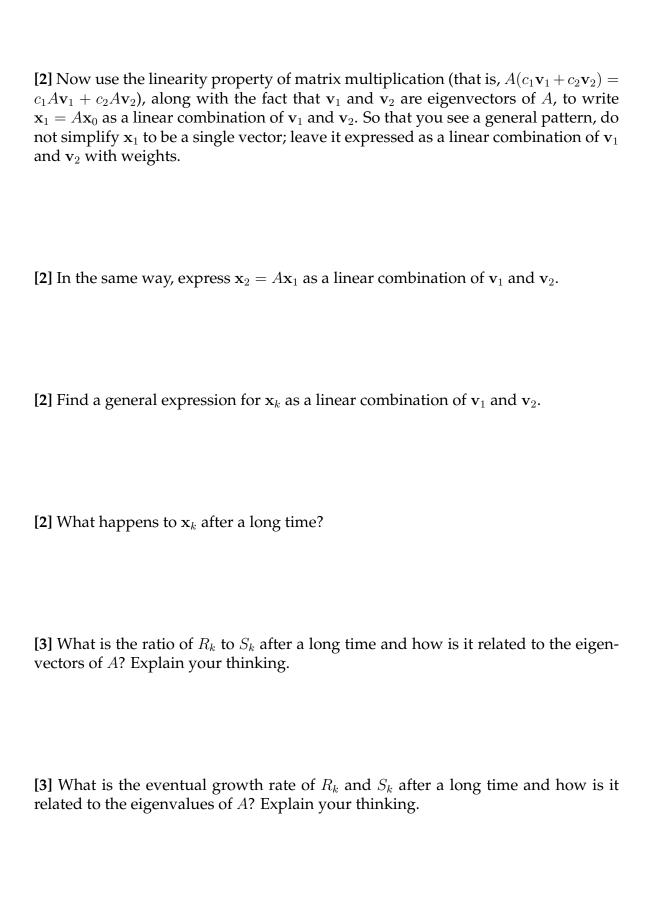
1. We'll begin by considering two species R and S whose populations k years after we begin recording them are R_k and S_k . Suppose that the population change from year to year according to the rule

$$R_{k+1} = 0.9R_k + 0.8S_k$$
$$S_{k+1} = 0.2R_k + 0.9S_k.$$

[2] We will use the state vector $\mathbf{x}_k = \begin{bmatrix} R_k \\ S_k \end{bmatrix}$ to record the populations. Find the matrix A such that

$$\mathbf{x}_{k+1} = A\mathbf{x}_k.$$

- [2] Verify that $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ are eigenvectors of your matrix A and find their respective eigenvalues.
- [2] Suppose the initial populations are given by the state vector $\mathbf{x}_0 = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$. Write \mathbf{x}_0 as a linear combination of \mathbf{v}_1 and \mathbf{v}_2 .



- 2. Now imagine that a city has a bike-sharing program where bicycles are rented and returned from two locations *P* and *Q*. Your research indicates that
 - 70% of the bicycles rented from location *P* are returned to *P* that evening with the remaining 30% returned to *Q*.
 - 40% of the bicycles rented from location Q are returned to Q that evening with the remaining 60% returned to P.

(Fun fact: a former Grand Valley math major used to work for the City of Grand Rapids and was responsible for studying and maintaining the data from their bikesharing program.)

[2] If there are initially 100 bicycles at P and none at Q, how are they distributed the next day?

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[2] We'll record the number of bicycles k days after we start recording data with the state vector $\mathbf{x}_k = \begin{bmatrix} P_k \\ Q_k \end{bmatrix}$ where P_k and Q_k are the number of bicycles at the two locations. Find the matrix A such that $\mathbf{x}_{k+1} = A\mathbf{x}_k$.

| [4] Find a linearly independent set of eigenvectors \mathbf{v}_1 and \mathbf{v}_2 of A and their associated eigenvalues. |
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| [2] Suppose that there are initially 800 bicycles at P and 700 at Q . Write the initial state vector \mathbf{x}_0 as a linear combination of \mathbf{v}_1 and \mathbf{v}_2 . |
| [2] Express \mathbf{x}_1 , the state vector for the next day, as a linear combination of \mathbf{v}_1 and \mathbf{v}_2 . |
| [2] Express \mathbf{x}_k as a linear combination of \mathbf{v}_1 and \mathbf{v}_2 . |
| [2] What happens to x_k after a very long time? |
| [2] After a very long time, how many bicycles are at P and how many at Q ? |