Mathematics 204-04
Lab 1, Due: Friday, 2.5, by 5 pm

Names:

Instructions: Complete the following exercises in groups of 2 students; submit a single report for your group. It is ideal if you print this document and write directly on the handout; be sure to include complete explanations of the work you have done and justification for your conclusions. If you don't have easy access to a printer, simply write your work on separate paper and submit that instead.

When finished, scan your work to a single PDF and email that PDF to me at boelkins.grading@gmail.com. Use the subject line "Lab 1 - Name1, Name2" and name your PDF file similarly. Be sure that both lab partners are included on the email.

There is a page of Sage cells available at http://gvsu.edu/s/0Ng.

This lab will be marked on a scale 30 points. The points for each question are noted in parentheses following the question number. There will be 6 labs over the course of the semester, due about every other Friday. You will have the opportunity to (individually) revise and resubmit up to 3 of your labs and earn up to 2/3 of the points that were deducted. More details on revisions will be posted later on Blackboard.

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1. (5) Consider the system of linear equations:

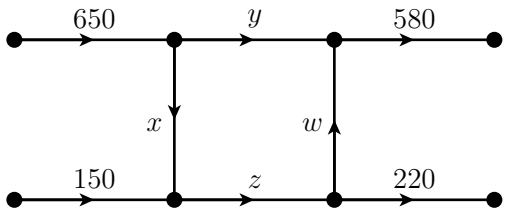
$$\begin{aligned} -x_1 + 2x_2 + 5x_3 + x_4 + x_5 &= -3 \\ 2x_1 + x_2 + 3x_4 - 7x_5 &= 16 \\ 4x_1 - 5x_2 - 14x_3 + x_4 - 13x_5 &= 26 \end{aligned}$$

   Represent this system as an augmented matrix, use Sage to find the RREF of the matrix, and fully describe the solution space of the system. If there are infinitely many solutions, describe them algebraically using parametric form.

2. (7) Shown below is the traffic pattern in one part of the downtown area of a large city. The numbers and variables give the number of cars per hour traveling along each road segment.

   Any car that drives into an intersection must also leave the intersection. This means that the number of cars entering an intersection in an hour is equal to the number of cars leaving the intersection.

   Focus on the four "nodes" of the square, and think about equations that will have to hold based on the given information.



   (a) Explain why you would expect infinitely many solutions for this particular traffic pattern.

   (b) Write a system of equations for the quantities x, y, z, and w; represent the system with an augmented matrix; find RREF; and and describe the set of solutions accordingly.

   (c) What is the smallest value of x that makes sense in the physical context of the problem, and what does this number represent?

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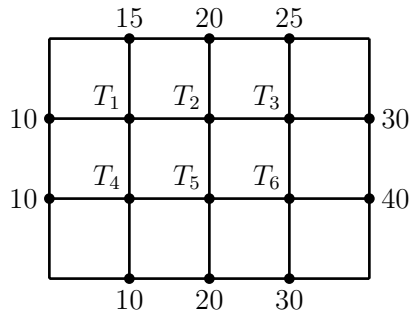
3. (8) A typical problem in thermodynamics is to determine the temperature distribution at different points in a thin plate if you know the temperature around the boundary. For example, the thin plate might represent a cross section of a metal beam with negligible heat flow in the direction perpendicular to the plate.

   For this example, let $T_1, \ldots, T_6$ be the respective temperatures at the six pictured nodes inside the beam in the figure below. The temperature at a node is approximately the average of the four nearest nodes; in this exercise, we will assume that this average tells us the exact temperature at a given node. For instance,

$$T_1 = \frac{10 + 15 + T_2 + T_4}{4} \quad \text{or} \quad 4T_1 - T_2 - T_4 = 25.$$



   In more sophisticated applications, the approximation becomes better the closer the points are together or as we add more and more into the grid.

   Write a system of equations for the quantities $T_1, \ldots, T_6$; represent the system with an augmented matrix; find RREF; and and describe the set of solutions accordingly. Write at least one sentence to explain the meaning of your result(s).

Helpful Sage hint: If you have a matrix `B` containing rational entries (that is, fractions), you can obtain a decimal approximation using `B.numerical_approx(digits=4)`. You may, of course, change "4" to any other appropriate value.

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4. (10) This exercise is about balancing chemical reactions.

   Chemists denote a molecule of water as H_2O, which means it is composed of two atoms of hydrogen (H) and one atom of oxygen (O). The process by which hydrogen is burned is described by the chemical reaction

   $$x\,H_2 + y\,O_2 \to z\,H_2O$$

   This means that x molecules of hydrogen H_2 combine with y molecules of oxygen O_2 to produce z water molecules. The number of hydrogen atoms is the same before and after the reaction; the same is true of the oxygen atoms.

   Note that there are $2x$ hydrogen atoms before the reaction, and $2z$ hydrogen atoms after the reaction, which tells us that $2x = 2z$.

   (a) Find and state a second linear equation in x, y, and z by equating the number of oxygen atoms before and after the reaction.

   (b) Why should you expect infinitely many solutions when balancing a chemical reaction?

   (c) Describe the solutions of the linear system determined by the two equations we've considered above. Show the augmented matrix you use to represent the system and its RREF.

   (d) In the physical setting of a chemical reaction, x, y, and z should be positive integers. Find the solution where x, y, and z are the smallest possible positive integers.

   (e) Consider the reaction where potassium permanganate and manganese sulfate combine with water to produce manganese dioxide, potassium sulfate, and sulfuric acid:

   $$x_1\,KMnO_4 + x_2\,MnSO_4 + x_3\,H_2O \to x_4\,MnO_2 + x_5\,K_2SO_4 + x_6\,H_2SO_4.$$

   As in the questions about the previous chemical reaction, find the appropriate positive integer values for $x_1, x_2, \ldots, x_6$ to balance this different chemical reaction. Show your work and thinking fully in the spirit of other questions in this lab.

   There is more room to work on the following blank page

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(Room for additional work on #4e)