Applications, computing, and the teaching of linear algebra

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Linear algebra at Grand Valley State

- GVSU is a regional comprehensive university with about 22,000 students in west Michigan
- We have a two-course linear algebra sequence:
  - Calculus is not a prerequisite
  - Required of all math majors
  - Strong enrollment from computer science, statistics, and some social sciences

Linear algebra at Grand Valley State

- First course: Systems of equations, span, linear independence, bases, eigenstuff
- Second course: Orthogonality, least squares, symmetric matrices, principal component analysis, singular value decompositions

Course objectives

- Develop students' abilities to reason mathematically and communicate their thinking with clarity and precision
- Increase students' awareness of how linear algebra impacts our society
- Improve students' computational proficiency and their capacity to deploy linear algebraic thinking in realistic situations

Computing environment

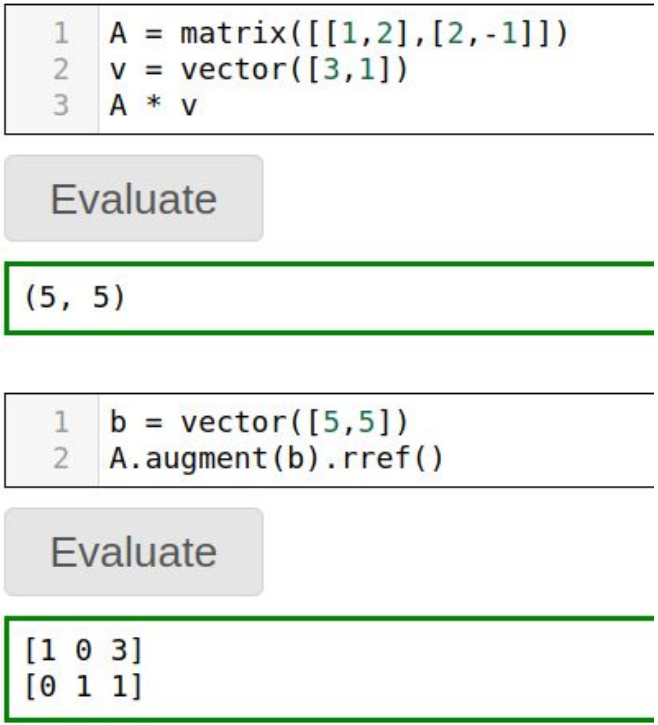
- Easy to access, ~~local installation~~
- Syntax should mirror mathematical notation and vocabulary
- Capabilities sufficient for our two-course sequence
- Notebook environment
  - Interweave text and computation
  - Sharing and collaboration
- ~~Graphing calculator~~

Sage cells

- Sage cell server: https://sagecell.sagemath.org/
- Page of Sage cells: https://gvsu.edu/s/0Ng
- Pre-populated cells

But:

- Work lost on page reload
- No ability to share, collaborate, or add text

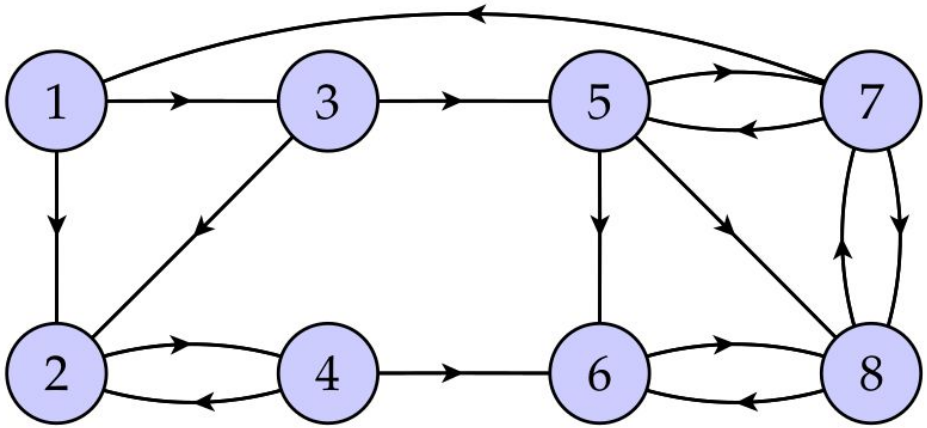


Course 1 labs

- Systems of equations
- Introduction to Markov chains
- Computer animation (geometry of 2x2 transformations)
- JPEG image compression (change of bases)
- Discrete dynamical systems
- Google PageRank

Google PageRank lab

- PageRank vector x satisfies Gx = x and is found using a Markov chain
- Application of Perron-Frobenius
- Pre-populated Sage cells to facilitate computation




```
def markov(A, x0, n):
    for i in range(n):
        x0 = A*x0
        print(N(x0, digits=4))

def modified_google_matrix(G):
    alpha = 0.85
    n = G.nrows()
    constant = 1/n*matrix(n, n, [1]*(n*n))
    return alpha*G + (1-alpha)*constant

```

Course 2 labs

- K-means clustering (introduction of geometry)
- Applications of the dot product
- Least squares and regression
- Principal component analysis
- Singular value decompositions and Supreme Court data
- Recommender systems

Next steps in computing: Google Colab

- Notebook environment
- Easy to share and for students to collaborate
- Standard tool in industry
- `Numpy` is the main linear algebraic tool
- Custom Python module allows students to use Sage syntax

Principal component analysis

- Students import the iris dataset (150 entries)
- Perform principal component analysis “by hand”

Remeber that `A` is the demeaned data matrix. Form the covariance matrix C and display it below.

```
[4] C = 1/150 * A * A.T
    C
```

```
    [[ 0.68112222 -0.03900667  1.26519111  0.51345778]
     [-0.03900667  0.18675067 -0.319568   -0.11719467]
     [ 1.26519111 -0.319568    3.09242489  1.28774489]
     [ 0.51345778 -0.11719467  1.28774489  0.57853156]]
```

Find the eigenvalues of C.

```
[5] C.eigenvalues()
```

```
    array([4.19667516, 0.24062861, 0.07800042, 0.02352514])
```

What fraction of the total variance is represented by the first principal component? (Click here and enter your response)

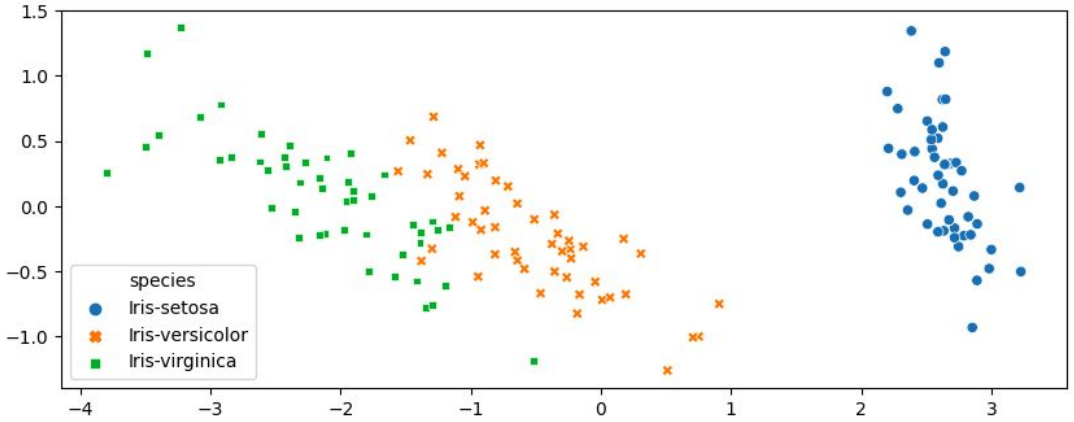
```
[ ]
```

What fraction of the total variance is represented by the first two principal components? Use the cell below to enter your response and to explain your thinking.

Double-click (or enter) to edit

Principal component analysis

- Plot the projected data
- Reason and write about their findings
- Submit by sharing their notebook electronically



Student response

When [image compression] was first pulled out in class, something clicked for me. I by no means immediately understood what was going on, but I was kind of in awe. It was so cool to me, that something that I was learning was being used on my phone every day, and that I now could understand, even if only a bit, how that function worked. It made the class seem like there was a use, and it really intrigued me. I was even more invested when the Google example came up. I have Googled many things in my life, but not once had I stopped and tried to think about how it worked.

Thank you!

Learn more: *Understanding Linear Algebra*, http://gvsu.edu/s/0ck

Slides and labs: http://gvsu.edu/s/2nQ