Chapter 1

Eigenvectors and eigenvalues

We will study the eigenvalue-eigenvector condition

$$A\mathbf{v} = \lambda\mathbf{v}.$$

Finding eigenvalues

Remember that the eigenvalue-eigenvector condition is $A\mathbf{v} = \lambda\mathbf{v}$, which we have rewritten as

$$(A - \lambda I)\mathbf{v} = 0.$$

If there is a nonzero vector $\mathbf{v}$ that satisfies this equation, then

$$\det(A - \lambda I) = 0,$$

which we have called the *characteristic equation*.

1. Suppose we have the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

(a) Form the matrix $A - \lambda I$ and find the characteristic polynomial $\det(A - \lambda I)$.

(b) Find the eigenvalues λ by solving the characteristic equation $\det(A - \lambda I) = 0$.

(c) For each eigenvalue λ, find a basis for the corresponding eigenspace E_λ by solving the equation $(A - \lambda I)\mathbf{v} = 0$.

2. In this exercise, we will find the eigenvalues of two more matrices.

(a) Find the characteristic polynomial of the matrix $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ and use it to find the eigenvalues of B. For each eigenvalue λ, find a basis for the corresponding eigenspace E_λ.

(b) Find the characteristic polynomial of the matrix $C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. What does this tell you about the eigenvalues of C?