Boelkins

Mathematics 204-04
Lab 3, Due: Friday, 3.5.21, by 5 pm
Matrix Transformations and Computer Animation

Names:

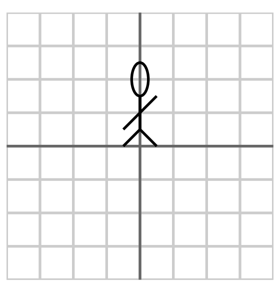
Instructions: Complete the following exercises in groups of 2 students; submit a single report for your group. It is ideal if you print this document and write directly on the handout; be sure to include complete explanations of the work you have done and justification for your conclusions. If you don't have easy access to a printer, simply write your work on separate paper and submit that instead.

When finished, scan your work to a single PDF and email that PDF to me at boelkins.grading@gmail.com. Use the subject line "Lab 3 - Name1, Name2" and name your PDF file similarly. Be sure that both lab partners are included on the email.

This lab will be marked on a scale 30 points. The points for each question are noted in parentheses following the question number. There will be 6 labs over the course of the semester, due about every other Friday. You will have the opportunity to (individually) revise and resubmit up to 3 of your labs and earn up to 2/3 of the points that were deducted. More details on revisions will be posted on Blackboard around the time this lab is returned.

Introduction. We've learned a lot so far this semester thinking about abstract ideas such as the span of a set of vectors, linear independence of a set of vectors, and how to determine matrix transformations. In this lab, we'll see how computer animators use matrix transformations and other ideas we've been studying to tell stories. In fact, if you have seen *Ratatouille* or *Incredibles 2* or any Pixar animated film, you've already spent a couple of hours of your life watching some elegant applications of linear algebra.

Now, let's meet Wooody, the loveable star of the animated kids' film *Toy Tale* due out in 2021.



The computer animators who made *Toy Tale* assembled the film frame by frame. To make us think that Wooody is moving, they make changes in Wooody's position from one frame to the next. This is where linear algebra is employed. (Actually, the characters in animated films live in a three-dimensional world, and each part of their bodies may be moving in different ways, so the situation is more complicated than is presented here. But the fundamental ideas that we will explore in two dimensions are what drive the animations you see on the big screen.)

Go to

http://gvsu.edu/s/0Jb

where you will see a figure that can make Wooody move. On the left, you see a picture of Wooody, while on the right you will see a picture of him after a particular matrix transformation has been applied.

Here is the function we will consider that changes "old Wooody" (at left in the figure) to "new Wooody" (at right in the figure); it is not quite as simple as others that we have seen in class:

$$\begin{aligned} x_{\text{new}} &= a x_{\text{old}} + b y_{\text{old}} + c \\ y_{\text{new}} &= d x_{\text{old}} + e y_{\text{old}} + f \end{aligned}$$

The six sliders along the top of the diagram represent the quantities $\begin{matrix} a & b & c \\ d & e & f \end{matrix}$. Experiment with each of the sliders a bit to see how the different values affect the resulting image of Wooody.

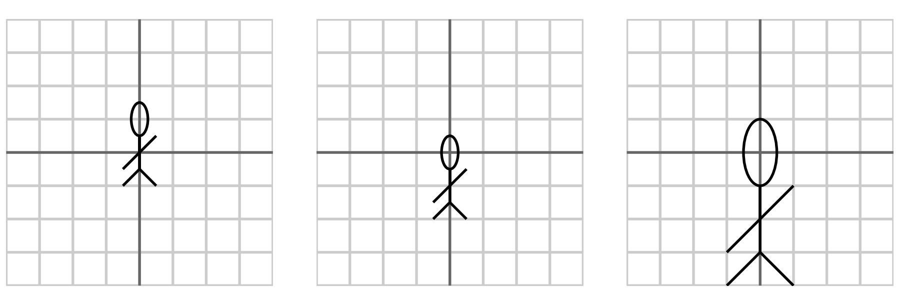
1. (3) The function that changes "old Wooody" (at left) to "new Wooody" (at right) can be represented by a matrix transformation[1] $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ given by $T(\vec{x}) = A\vec{x}$.

   Compute the matrix-vector product below and explain how this represents the equations above that relate x_{new} and y_{new} to x_{old} and y_{old}.

   $$A\vec{x} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \\ 1 \end{bmatrix} =$$

   What is the role of the "1" in the third component of the vector $\vec{x} = \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \\ 1 \end{bmatrix}$?

2. (6) Press the **Reset** button. At one point in the story, Wooody suddenly remembers a long-lost love and the camera moves in for a close up. To do so, the following three images are generated.

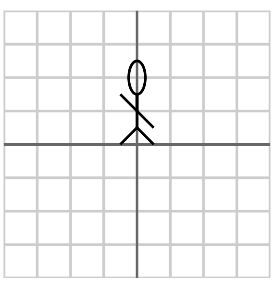


[1] It turns out that in order to be able to *translate* Wooody around the plane, we have to introduce more than four parameters, thus requiring a matrix larger than 2×2. The $\begin{bmatrix} a & b \\ d & e \end{bmatrix}$ portion of the matrix works identically to how the matrices in Section 2.5 of our text behave when generating matrix transformations.

Explain in writing how each figure is related to the previous one (the leftmost to the original, the middle one to the leftmost, and the rightmost one to the middle), and for each state the matrix of the transformation that accomplishes the change. Call these matrices A_1, A_2, and A_3.

Then, determine the single transformation (that is, the matrix of the transformation) that takes the original image of Wooody into the final close up. Clearly show your thinking and explain how you determined this single matrix, say B, that accomplishes this single transformation.

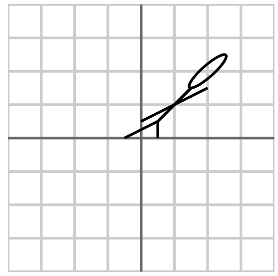
3. (3) Press the **Reset** button. Note that in the original frame, Wooody is waving with his left hand. In the next scene of the movie, Wooody comes on screen and waves with his right hand, as shown below.



Explain in words the effects of the geometric transformation that has been applied to produce the image below from the original Wooody, and state the matrix, A, of the transformation.

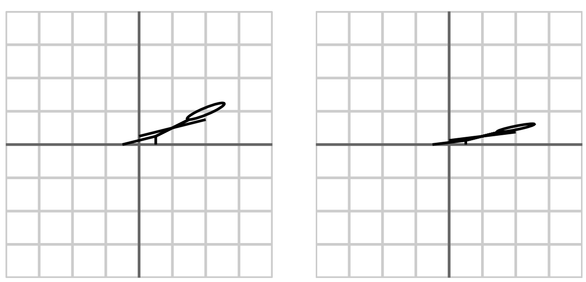
What happens if this same transformation is applied to the updated figure shown above? What image will result? Why?

4. (6) Press the **Reset** button. Wooody decides to go out for a walk. In the morning sun, he casts a shadow that looks like this:



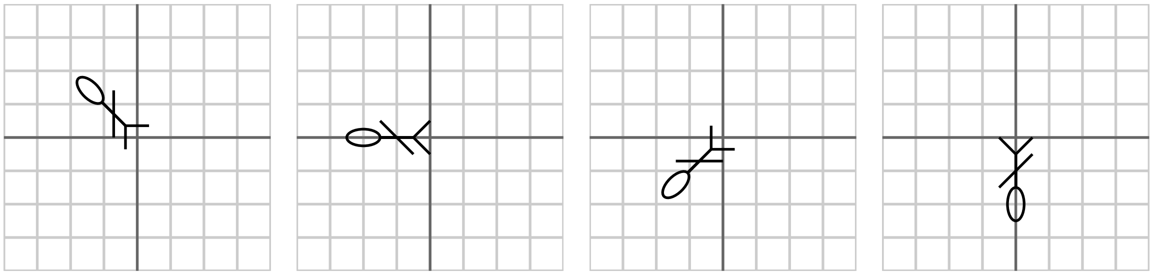
Find the matrix of the transformation that creates the shadow. State the matrix C_1 and clearly discuss your reasoning by focusing on where two vectors have to be transformed: the vector $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ that points at the center of Wooody's head in the original frame, and the vector $\begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$ that points at Wooody's left foot (on our right) in the original frame.

As the sun comes up, his shadow get shorter, and the following images are created.



Explain what transformations achieve the 2nd and 3rd images here and how you found them. For each, be sure to state the matrix of each transformation, calling them C_2 and C_3.

5. (6) Press the **Reset** button. Next, Wooody begins his morning calisthenics and performs a cartwheel. In particular, we want to generate the following sequence of images (away from his initial location):



What is the matrix, M_1, of the transformation that produces the first image in the sequence above? From that image, what matrix, M_2, produces the second image? what matrix, M_3, produces the third? and what matrix M_4 produces the fourth? (**Note**: if you press the "Compose" button, it will update "Original Wooody" to the most recent image produced.) As you answer the questions here, be sure to include the *exact* entries of the matrix, discussion of how you found them, and discussion of how the matrix is being applied to produce the images.

What single matrix M produces the fourth and final image from the original? Why?

6. (6) Write your own scene for the story and describe how to illustrate it. It must include at least four different images of Wooody (preferably screenshots or alternatively images you draw by hand), as well as appropriate commentary, the matrices that generate them, and corresponding explanation.