A Small Modulo Lemma

This is a small lemma that seemed to be true that came up while studying Modular arithmatic with a friend. It resulted from doing this simple homework question.

Is this true? $4^{1536} - 9^{4824} mod 35 \equiv 0$

How would you reason about this problem?

Messing with this the last couple days it turns out there's an interesting pattern that begins to appear.

Given that $7 * 5 = 35$. The way that you're supposed to solve this problem, is to use the fact that $(7-1) * (5-1) = 24$ and 1536 and 4824 are multiples of 24, that means they will both result in a remainder of 1.

$$4^{1536} (mod 35) \equiv 1$$

$$9^{4825} (mod 35) \equiv 1$$

So the solution then becomes:

$$1 - 1 (mod 35) \equiv 0$$

But this is really interesting, as it turns out you can do this for any set of numbers when using modulo. Let's make a generalization, to see if this is in fact a true property that we've used here.

We've know that for some n and m, x; that $x^{nm} \% nm \equiv 0$

What we're saying is that we can raise x to the power of (n-1)*(m-1) instead, and know that if we modulo by $$$nm$$, that the result will be 1.

$$\forall n \forall m [x^{(m)(n)} \% nm \equiv 0]$$

$$\forall n \forall m [x^{(m-1)(n-1)} \% nm \equiv 1]$$

$$\forall n \forall m [x^{m+n-1} \% nm \equiv 1]$$

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