An Intuitive Derivation of Escape Velocity

Ananth Rao, David Awad

So there is this notion of an **escape velocity** that exists in physics where what we're interested in what is the *minimum* initial velocity that a particle can have for it to escape the gravitational pull of some massive object, like a planet.

So let's formalize this a bit, what does it *mean* for something to be under the gravitational pull of an object. Let's imagine a smaller particle of mass m and a massive body of mass M and radius R. The gravitational potential on an object is defined as the following.

$$\frac{GMm}{R}$$

And this makes sense, because it's based on the gravity of both objects, and as the distance between the masses increases the force weakens as a result.

So imagine the particle sitting on the crust of this hypothetical planet, it's a distance R away from the center of mass of the body. So the force is as large as it's going to get since we're on the planet itself.

As R, the distance between the small mass and the planet, tends to infinity, the force eventually becomes zero. But energy is conserved in this motion, as the potential decreases, our kinetic energy is increasing to compensate.

$$E_i = -\frac{GMm}{R} + \frac{1}{2}mv_0^2$$

But we're interested in the velocity when the potential energy doesn't exist. Or when the distance is infinity.

So it is that point that must be the initial velocity. Making the total energy zero.

$$E_i = -\frac{GMm}{R} + \frac{1}{2}mv_0^2$$

$$0 = -\frac{GMm}{R} + \frac{1}{2}mv_0^2$$

$$\frac{GMm}{R} = \frac{1}{2}mv_0^2$$

$$\frac{1}{2}mv_0^2 = \frac{GMm}{R}$$

$$v_0^2 = \frac{2GM}{R}$$

Solving for v_0 and we'll have our escape velocity.

$$v_e = \sqrt{\frac{2GM}{R}}$$