

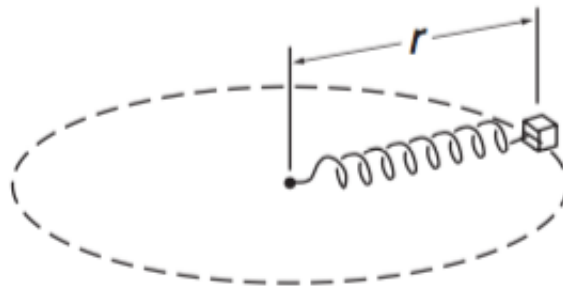
# Central Force Motion

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## **A Sample Question Question**

A  $2\text{ kg}$  mass on a frictionless table is attached to one end of a massless spring. The other end of the spring is held by a frictionless pivot. The spring produces a force of magnitude  $3r$  newtons on the mass, where  $r$  is the distance in meters from the pivot to the mass. The mass moves in a circle and has a total energy of  $12\text{ J}$ .



**a) Find the radius of the orbit and the velocity of the mass.**

For motion in a circular orbit for this mass that's connected to a spring.

$$E = \frac{1}{2}kr^2 + \frac{1}{2}mv^2$$

Using sum of forces, since the only force acting on the mass is the spring force, we have  $kr = \frac{mv^2}{r}$ .

multiply by r

$$mv^2 = kr^2$$

substitute into E for  $mv^2$

$$E = \frac{1}{2}kr^2 + \frac{1}{2}kr^2 = kr^2$$

$$E = 12J = kr^2$$

We're told in the problem that the force is of magnitude  $3r$ , meaking  $k$  is 3.

$$12J = 3r^2$$

$$r = 2m$$

For the velocity, we can simply use the relation from the sum of the forces, (sometimes called the equilibrium relation).

$$kr = \frac{mv^2}{r}$$

$$v_0 = \frac{kr^2}{m}$$

$$k = 3N/m, r = 2m, m = 2kg$$

$$v_0 = \sqrt{\frac{6m}{s}}$$

**b) The mass is struck by a sudden sharp blow, giving it instantaneous velocity of  $1m/s$  radially outward. Show the state of the system before and**

**after the blow on a sketch of the energy diagram.**

Before the blow, we have a total energy such as the following:

$$E = \frac{1}{2}kr^2 + \frac{1}{2}kr^2$$

But taking into account, this new impulse, it's extending the block radially outward.

So we can simply add 1 meter per second in kinetic energy.

$$E = \frac{1}{2}kr^2 + \frac{1}{2}kr^2$$

$$\delta E = E + \frac{1}{2}mv_f^2$$

$$\frac{1}{2}kr_i^2 + \frac{1}{2}mv_\theta^2 + \frac{1}{2}m = \frac{1}{2}k_f^2$$

And afterwards it oscillates about this new, augmented equilibrium.