Lecture 8: Non-parametric Comparison of Location

GENOME 560

Doug Fowler, GS (dfowler@uw.edu)

Review

- What do we mean by nonparametric?

- What is a desirable location statistic for ordinal data?

- What are NP equivalents of a one-sample t-test?

Goals

- Comparing the medians of two samples using the Wilcoxon Rank Sum test

- Comparing the medians of many mutually independent samples using the Kruskal-Wallis test

Wilcoxon Rank Sum Test

- Used to test whether two samples are likely to be drawn from the same distribution or different ones

Wilcoxon Rank Sum Test

- Used to test whether two samples are likely to be drawn from the same distribution or different ones
- Identical to Mann-Whitney U test

Wilcoxon Rank Sum Test

- Used to test whether two samples are likely to be drawn from the same distribution or different ones
- Identical to Mann-Whitney U test

$$X_1 \ldots X_{n_x}, \quad Y_1 \ldots Y_{n_y}$$

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

Wilcoxon Rank Sum Test

- Used to test whether two samples are likely to be drawn from the same distribution or different ones
- Identical to Mann-Whitney U test

$X_1 ... X_{n_x}, \quad Y_1 ... Y_{n_y}$

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

- Pool $N = n_x + n_y$ observations

Wilcoxon Rank Sum Test

- Used to test whether two samples are likely to be drawn from the same distribution or different ones
- Identical to Mann-Whitney U test

$$X_1 ... X_{n_x}, \quad Y_1 ... Y_{n_y}$$

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

- Pool $N = n_x + n_y$ observations
- Arrange into an ordered array, preserving labels

Wilcoxon Rank Sum Test

- Used to test whether two samples are likely to be drawn from the same distribution or different ones
- Identical to Mann-Whitney U test

$$X_1 \ldots X_{n_x}, \quad Y_1 \ldots Y_{n_y}$$

$$H_0 : M_x = M_y$$

$$H_1 : M_x \neq M_y$$

- Pool $N = n_x + n_y$ observations
- Arrange into an ordered array, preserving labels
- Assign ranks to each element of the array from 1…N

Wilcoxon Rank Sum Test

- Used to test whether two samples are likely to be drawn from the same distribution or different ones
- Identical to Mann-Whitney U test

$$X_1 ... X_{n_x}, \quad Y_1 ... Y_{n_y}$$

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

- Pool N = n_x + n_y observations
- Arrange into an ordered array, preserving labels
- Assign ranks to each element of the array from 1…N
- The test statistic T_x is the sum of the ranks of *X*

$$T_x = \sum R_x$$

Wilcoxon Rank Sum Test

- Used to test whether two samples are likely to be drawn from the same distribution or different ones
- Identical to Mann-Whitney U test

$$X_1 ... X_{n_x}, \quad Y_1 ... Y_{n_y}$$

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

- Pool N = n_x + n_y observations
- Arrange into an ordered array, preserving labels
- Assign ranks to each element of the array from 1…N
- The test statistic T_x is the sum of the ranks of *X*

$$T_x = \sum R_x$$

- Reject H_0 if T_x is very large or very small compared to possible values of T_x for n = N

Wilcoxon Rank Sum Test - Example

- Let’s say we have measured a transcript level in 6 pre-operative patients (*X*) and 3 post-operative patients (*Y*). Does the surgery change transcript levels?

Wilcoxon Rank Sum Test - Example

- Let's say we have measured a transcript level in 6 pre-operative patients (*X*) and 3 post-operative patients (*Y*). Does the surgery change transcript levels?

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

Wilcoxon Rank Sum Test - Example

- Let's say we have measured a transcript level in 6 pre-operative patients (*X*) and 3 post-operative patients (*Y*). Does the surgery change transcript levels?

$$X : (2.5, 3.0, 6.2, 9.1, 14.3, 14.7)$$
$$Y : (14.1, 15.6, 16.7)$$

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

Wilcoxon Rank Sum Test - Example

- Let's say we have measured a transcript level in 6 pre-operative patients (*X*) and 3 post-operative patients (*Y*). Does the surgery change transcript levels?

$$X : (2.5, 3.0, 6.2, 9.1, 14.3, 14.7)$$
$$Y : (14.1, 15.6, 16.7)$$

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

Observation	Sample	Rank
2.5	X	
3	X	
6.2	X	
9.1	X	
14.3	X	
14.7	X	
14.1	Y	
15.6	Y	
16.7	Y	

Wilcoxon Rank Sum Test - Example

- Let's say we have measured a transcript level in 6 pre-operative patients (*X*) and 3 post-operative patients (*Y*). Does the surgery change transcript levels?

$$X : (2.5, 3.0, 6.2, 9.1, 14.3, 14.7)$$
$$Y : (14.1, 15.6, 16.7)$$

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

Observation	Sample	Rank
2.5	X	1
3	X	2
6.2	X	3
9.1	X	4
14.1	Y	5
14.3	X	6
14.7	X	7
15.6	Y	8
16.7	Y	9

Wilcoxon Rank Sum Test - Example

- Let's say we have measured a transcript level in 6 pre-operative patients (*X*) and 3 post-operative patients (*Y*). Does the surgery change transcript levels?

$$X : (2.5, 3.0, 6.2, 9.1, 14.3, 14.7)$$
$$Y : (14.1, 15.6, 16.7)$$

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

Observation	Sample	Rank
2.5	X	1
3	X	2
6.2	X	3
9.1	X	4
14.1	Y	5
14.3	X	6
14.7	X	7
15.6	Y	8
16.7	Y	9

$$T_x = (1 + 2 + 3 + 4 + 6 + 7) = 23$$

Distribution of T_x When N is Small

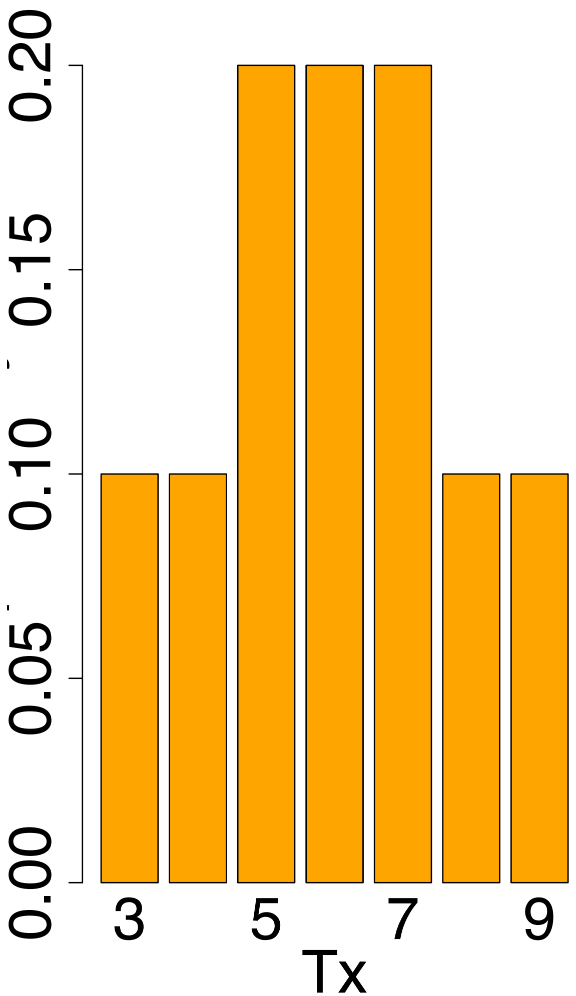
- Consider a case where $n_x = 2$ and $n_y = 3$
- We know ranks must be 1, 2, 3, 4, 5
- Again, the issue is how to assign these ranks amongst the samples *X* and *Y*

Distribution of T_x When N is Small

- Consider a case where $n_x = 2$ and $n_y = 3$
- We know ranks must be 1, 2, 3, 4, 5
- Again, the issue is how to assign these ranks amongst the samples X and Y
- There are $\binom{5}{2} = 10$ ways of assigning five ranks to two samples
- Each way is equally likely under the null hypothesis so each has a probability of 10%

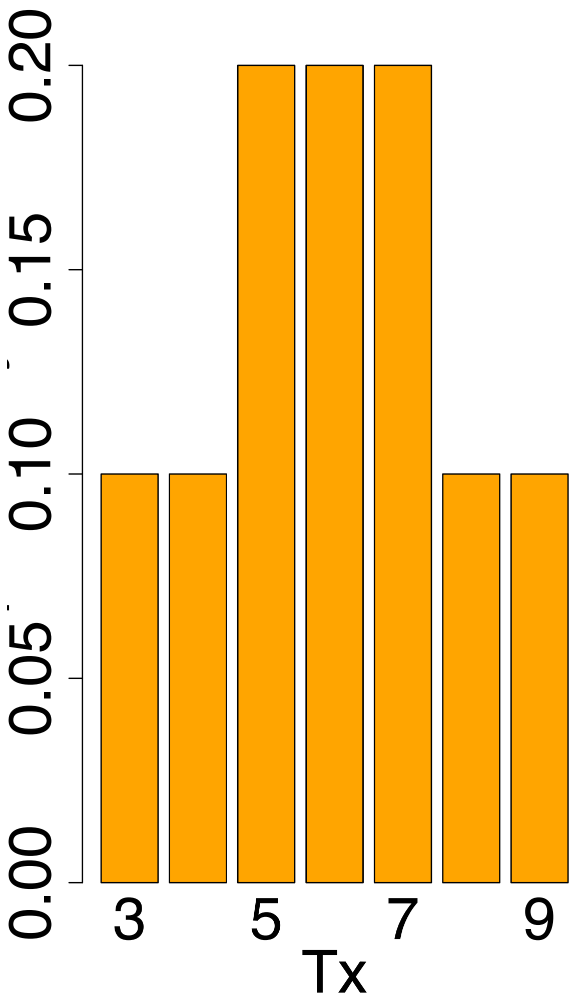
Distribution of T_x When N is Small

X Ranks	Y Ranks	Value of T_x	Probability
1, 2	3, 4, 5	3	0.10
1, 3	2, 4, 5	4	0.10
1, 4	2, 3, 5	5	0.10
2, 3	1, 4, 5	5	0.10
2, 4	1, 3, 5	6	0.10
1, 5	2, 3, 4	6	0.10
2, 5	1, 3, 4	7	0.10
3, 4	1, 2, 5	7	0.10
3, 5	1, 2, 4	8	0.10
4, 5	1, 2, 3	9	0.10



Distribution of T_x When N is Small

X Ranks	Y Ranks	Value of T_x	Probability
1, 2	3, 4, 5	3	0.10
1, 3	2, 4, 5	4	0.10
1, 4	2, 3, 5	5	0.10
2, 3	1, 4, 5	5	0.10
2, 4	1, 3, 5	6	0.10
1, 5	2, 3, 4	6	0.10
2, 5	1, 3, 4	7	0.10
3, 4	1, 2, 5	7	0.10
3, 5	1, 2, 4	8	0.10
4, 5	1, 2, 3	9	0.10



$$P(T_x \leq 4) = 0.2$$

Distribution of T_x When N is Large

Wilcoxon Rank Sum Test - Example

- Let's say we have measured a transcript level in 6 pre-operative patients (*X*) and 3 post-operative patients (*Y*). Does the surgery change transcript levels?

$$X : (2.5, 3.0, 6.2, 9.1, 14.3, 14.7)$$
$$Y : (14.1, 15.6, 16.7)$$

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

Observation	Sample	Rank
2.5	X	1
3	X	2
6.2	X	3
9.1	X	4
14.1	Y	5
14.3	X	6
14.7	X	7
15.6	Y	8
16.7	Y	9

$$T_x = (1 + 2 + 3 + 4 + 6 + 7) = 23$$
$$p = 0.095$$

Wilcoxon Rank Sum Test - Example

- Let's say we have measured a transcript level in 6 pre-operative patients (*X*) and 3 post-operative patients (*Y*). Does the surgery change transcript levels?

$$X : (2.5, 3.0, 6.2, 9.1, 14.3, 14.7)$$
$$Y : (14.1, 15.6, 16.7)$$

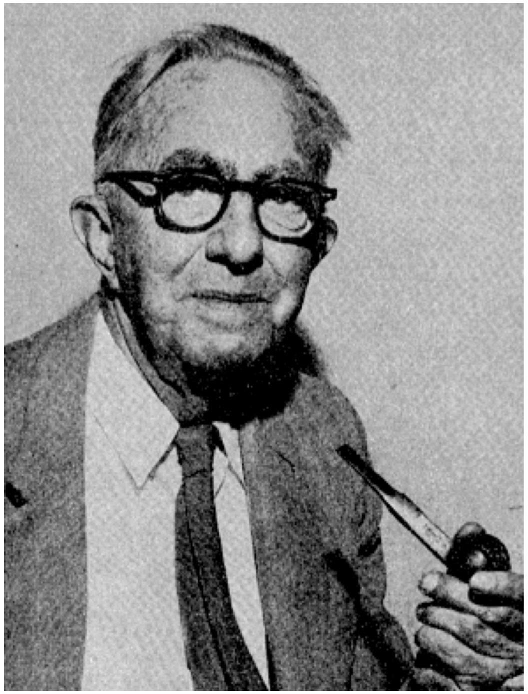
$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

Observation	Sample	Rank
2.5	X	1
3	X	2
6.2	X	3
9.1	X	4
14.1	Y	5
14.3	X	6
14.7	X	7
15.6	Y	8
16.7	Y	9

$$T_x = (1 + 2 + 3 + 4 + 6 + 7) = 23$$
$$p = 0.095$$

Accept H_0

Frank Wilcoxon



Wilcoxon lived from 1892 to 1965. He was a polymath, working as an oilman and a tree surgeon before training as a physical chemist, working in plant research and then in process control in industry. In a single paper in 1945 he published both tests that bear his name.

Goals

- Comparing the medians of two samples using the Wilcoxon Rank Sum test

- Comparing the medians of many mutually independent samples using the Kruskal-Wallis test

Kruskal-Wallis Test

- Generalization of the Wilcoxon rank sum test to 3 or more independent random samples
- Used to test whether the medians of the samples are equal
- Nonparametric version of the one-way ANOVA

Kruskal-Wallis Test

- Generalization of the Wilcoxon rank sum test to 3 or more independent random samples
- Used to test whether the medians of the samples are equal
- Nonparametric version of the one-way ANOVA

$$H_0 = M_1 = M_2 ... = M_k$$
$$H_1 = all\ medians\ not\ equal$$

Kruskal-Wallis Test

- Generalization of the Wilcoxon rank sum test to 3 or more independent random samples
- Used to test whether the medians of the samples are equal
- Nonparametric version of the one-way ANOVA
- Pool all observations

$$H_0 = M_1 = M_2 ... = M_k$$
$$H_1 = all\ medians\ not\ equal$$

Kruskal-Wallis Test

- Generalization of the Wilcoxon rank sum test to 3 or more independent random samples
- Used to test whether the medians of the samples are equal
- Nonparametric version of the one-way ANOVA

$$H_0 = M_1 = M_2 ... = M_k$$
$$H_1 = all\ medians\ not\ equal$$

- Pool all observations
- Rank the pooled samples

Kruskal-Wallis Test

- Generalization of the Wilcoxon rank sum test to 3 or more independent random samples
- Used to test whether the medians of the samples are equal
- Nonparametric version of the one-way ANOVA

$$H_0 = M_1 = M_2 ... = M_k$$
$$H_1 = all\ medians\ not\ equal$$

- Pool all observations
- Rank the pooled samples
- Sum the ranks for each sample to get individual sample rank sums $R_1, R_2, ..., R_k$

Kruskal-Wallis Test

- Under the null hypothesis, what should be true about the relationship between any two rank sums R_i, R_j?

$$R_1, R_2, ..., R_k$$

Kruskal-Wallis Test

- The sum of all the sample rank sums is

$$R_1 + R_2 + ... + R_k = \frac{N(N+1)}{2}$$

Kruskal-Wallis Test Outcome

- Given the way the test statistic/hypotheses are constructed, what does a rejection of H_0 mean?

Nonparametric Location Tests

- Can be used to perform one or two sample tests with fewer assumptions about the distribution from which the sample(s) are drawn
- Usage of sign and rank (rather than interval, as with parametric tests) enable this and confer other benefits
  - More robust (immune to outliers)
  - Can be used on ordinal data
- NP tests still have assumptions, and still must be used with care (e.g. zeroes for sign test, ties, similarity of distributions for rank-sum test)

AND THERE IS NO FREE LUNCH

- Let's say we have measured a transcript level in 6 pre-operative patients (X) and 3 post-operative patients (Y). Does the surgery change transcript levels?

$$X : (2.5, 3.0, 6.2, 9.1, 14.3, 14.7)$$
$$Y : (14.1, 15.6, 16.7)$$

$$H_0 : M_x = M_y$$
$$H_1 : M_x \neq M_y$$

Observation	Sample	Rank
2.5	X	1
3	X	2
6.2	X	3
9.1	X	4
14.1	Y	5
14.3	X	6
14.7	X	7
15.6	Y	8
16.7	Y	9

$$T_x = (1 + 2 + 3 + 4 + 6 + 7) = 23$$

$$p = 0.095$$

AND THERE IS NO FREE LUNCH

- Let's say we have measured a transcript level in 6 pre-operative patients (X) and 3 post-operative patients (Y). Does the surgery change transcript levels?

$$X : (2.5, 3.0, 6.2, 9.1, 14.3, 14.7) \qquad H_0 : M_x = M_y$$
$$Y : (14.1, 15.6, 16.7) \qquad H_1 : M_x \neq M_y$$

- If we assume normality and identicality of variance, then a two sample t-test gives:

$$p = 0.02$$

AND THERE IS NO FREE LUNCH

- Generally speaking, nonparametric tests trade fewer assumptions for less power

AND THERE IS NO FREE LUNCH

- Generally speaking, nonparametric tests trade fewer assumptions for less power
- Different nonparametric tests perform better or worse in this regard (efficiency)

AND THERE IS NO FREE LUNCH

- Generally speaking, nonparametric tests trade fewer assumptions for less power
- Different nonparametric tests perform better or worse in this regard (efficiency)
- All will do better than their parametric counterparts when assumptions are violated

AND THERE IS NO FREE LUNCH

- Generally speaking, nonparametric tests trade fewer assumptions for less power
- Different nonparametric tests perform better or worse in this regard (efficiency)
- All will do better than their parametric counterparts when assumptions are violated
- The Mann-Whitney-Wilcoxon test is particularly good, giving up little power even for normally distributed data

R Goals

- Executing nonparametric tests in R
- Playing around with different distribution shapes and test assumptions
- Examining effect size vs. test outcome

Reading/Resources

- http://www.statsoft.com/Textbook/Nonparametric-Statistics/button/2
- http://sci2s.ugr.es/keel/pdf/algorithm/articulo/wilcoxon1945.pdf
- http://www.mayo.edu/mayo-edu-docs/center-for-translational-science-activities-documents/berd-5-6.pdf
- Nonparametric statistics: an introduction, Jean Gibbons (available online through UW libraries at http://goo.gl/NERixX)