Cryptographic Administrators for Secure Group Messaging

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Full Version

Abstract

Many real-world group messaging schemes support group administrators, who exert control over a given group chat. This functionality is delegated in practice to the application level, and protocols do not generally provide formal guarantees regarding administration. Taking a cryptographic approach to group administration can prevent both implementation and protocol design pitfalls resulting in attacks on group membership.

To this end, we provide a cryptographic treatment of the group administration problem. Extending the continuous group key agreement (CGKA) paradigm used in the IETF MLS group messaging standardisation process, we introduce the administrated CGKA (A-CGKA) primitive. Our primitive natively enables a subset of group members – admins – to control the addition and removal of parties and to update their own keying material. We embed A-CGKA with a novel correctness notion which provides guarantees for group evolution and consistency, and a security model that prevents even corrupted (non-admin) members from forging messages that affect group membership. Moreover, we formalise two efficient and modular constructions of group administrators, that we prove correct and secure with respect to a semi-active adversary. In our first scheme, individual admin signatures (IAS), built on top of a CGKA, admins track signature key pairs used to attest to CGKA group state updates. Our second scheme, dynamic group signature (DGS), runs two synchronised CGKAs and provides additional performance/security trade-offs over IAS. Finally, we propose, implement, and benchmark an extension of MLS that integrates cryptographic administrators. Our constructions admit little overhead over running a CGKA and can be extended to support advanced admin functionalities.

Keywords: Group Messaging, Group Administrators, Continuous Group Key Agreement, MLS, TreeKEM, CGKA

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1 Introduction

In our current era of unprecedented digital communication, billions of people use instant messaging services daily. Building messaging protocols that provide security guarantees to users is a nontrivial task for many reasons. One of them is that protocol participants must be able to exchange messages *asynchronously* and should not be required to be online and available at all times. Besides this, they must always be ready to send or receive messages spontaneously (i.e. without additional interaction). On the other hand, sessions are long-lived, in contrast to protocols such as TLS, and the secrets are stored in potentially vulnerable mobile devices.

Messaging protocols are designed either for two-party conversations, such as the Signal [1] and OTR [2] protocols; or for group conversations, such as the MLS protocol [3]. One difficulty that is more pronounced in group messaging is *efficiency*: simple approaches such as running two-party messaging protocols between all pairs of users scale poorly (linearly in general for each party) as the group size grows. Another problematic is group *evolution* or *dynamics*: the list of group members may change at any point in time, requiring complex key agreement protocols. As a baseline for ensuring practical security guarantees, formal security proofs in a realistic adversarial model are essential, especially in a complex setting like group messaging.

1.1 Secure Group Messaging

In the modern secure messaging literature, both for two-party (for example [4, 5, 6, 7, 8]) and group ([9, 10, 11, 12, 13]) protocols, two standard security notions prevail. The first is *forward security* (FS), which protects the confidentiality of past messages in the event of a key exposure, which can be achieved using just symmetric cryptography (such as via hash chaining). The second is *post-compromise security* (PCS), which ensures that security guarantees can be restored after a key exposure in certain adversarial settings [14], typically when the adversary is passive for some period of time. To achieve PCS, a protocol must use public-key cryptography [6] and perform a continuous key exchange in which parties introduce fresh randomness for key derivation.

There exist different approaches to group messaging in practice. As remarked above, pairwise communication, dubbed the *pairwise channels* approach in the context of Signal [15], is one such approach. WhatsApp [16] uses the *sender keys* approach, where each group member encrypts messages under their own symmetric key which is shared with all group members, providing forward security.

1.1.1 Group key agreement

Most solutions in the literature, however, follow a *group key agreement (GKA)* approach where all group members run a protocol to derive a single, *common* group key.

Early GKA work considered both static and dynamic groups without key ratcheting, precluding FS and PCS guarantees [17]. The scenario in which dynamic groups can *efficiently* update their keys for FS and PCS was addressed only recently by protocols such as Asynchronous Ratchet Trees (ARTs) [18] and TreeKEM [9]. Notably, Alwen et al. [10] introduced the *continuous group key agreement* (CGKA) primitive, which captures the fundamental requirements of a group key agreement primitive for messaging, including support for asynchrony, dynamic groups and key ratcheting. CGKA, and group key agreement in general, is then used for secure

group messaging by, for example, deriving application secrets from the (C)GKA secret [13] that are used in a symmetric cryptosystem.

The CGKA approach is adopted in practice by the Messaging Layer Security (MLS) [3] work-group of the Internet Engineering Task Force (IETF). At the core of MLS, we find the *TreeKEM* protocol [9], which is a CGKA that generates and distributes fresh key material among the members of the group and achieves so-called 'fair-weather' logarithmic performance, namely in per-operation communication and time complexity, on its main operations. That is, in some executions performance can be $O(\log n)$, and in worst-case executions performance necessarily degrades to $O(n)$ [19]. By contrast, pairwise messaging incurs $O(n)$ overhead for *every* message.

1.1.2 Delivery and servers

Any messaging protocol needs to rely on a *delivery service* (DS) that distributes messages among group members. Often, a total message ordering is needed, and is achieved via centralised infrastructure (hereafter referred to as a *central server*) or state machine replication via consensus [20]. In the worst case, a malicious DS combined with an insecure protocol could enable a DS to learn all messages and keys and forge messages arbitrarily within a given conversation. Therefore, a basic requirement is that confidentiality and authenticity should never be affected by a malicious DS. In MLS, the delivery service forwards control messages to all parties for membership and key updates over time and allows for out-of-order delivery of application messages.

1.1.3 Adversarial models

Among the diverse adversarial models in the literature [17], the most general consider active adversaries who control the network (modelling the delivery service) and who can forge and inject messages in the protocol [12, 21]. CGKAs that offer strong security against active adversaries are computationally expensive and complex [12]. However, some guarantees against some active attacks can still be provided at relatively low cost on top of CGKA (e.g. via so-called parent hashing [13]). Some protocols are proven secure with respect to models that assume authentic message delivery, with [11] or without [10] adversarial message scheduling. Some aim to model so-called insider adversaries who deviate from protocol execution by crafting messages registering adversarial key material in a PKI [21].

1.2 Group Administrators

1.2.1 Motivation

There is a strong trend in practice to distinguish between at least two types of users in a group: administrators and standard users. Generally, a group administrator (or admin) has all the capabilities of a standard user plus a set of administrative rights. In practice, these capabilities are implemented at the application level via policies enforced either by the central server or users. Examples are the popular messaging apps WhatsApp, Telegram, and Signal (as of 2022).

- In WhatsApp, only the group administrators can add and remove users, create a group invite link, and govern the admin subgroup. All groups must have at least one admin; when the last admin leaves, a user is selected randomly as the new admin.

- In Telegram, the group creator can designate other admins with diverse sets of capabilities. Besides adding and removing users, admins can impose partial bans on any user's capabilities, such as sending or receiving messages, and can even restrict the content that users can send [22]. Such "fine-grained administration" is possible and practical due to the lack of end-to-end encryption. By default, Telegram relies on a central server that decrypts all messages.
- In Signal Messenger, admins can specify whether all members or only admins can add and remove users from a group (in the latter case non-admins can request to add users) and create a group invite link.

Despite that administration mechanisms are widely deployed, there is little mention of admins in the literature. Existing CGKA approaches make no formal distinction between admin and non-admin users, which results in giving admin capabilities to all users.

1.2.2 Capabilities

Let $G = \{\mathsf{ID}_1, \ldots, \mathsf{ID}_n\}$ be a group of users within a messaging or key exchange protocol execution and $G^* \subseteq G$ be a non-empty subset of group administrators. Unlike regular group members, the administrators $\mathsf{ID} \in G^*$ that we consider in this work can additionally:

- add and remove members from the group,
- approve and decline join and removal requests,
- designate other group administrators,
- give up their admin status, and
- remove the admin status of other users.

These correspond to the common administration features among the solutions presented above. In the case of Telegram, many of additional capabilities are incompatible with the schemes we introduce below (due to their lack of end-to-end encryption).

1.2.3 Security goals

There are four main security goals that cryptographic administrators aim to achieve.

1. Reduce the trust put on the delivery service, such that the service (e.g. a central server) does not control the list of admins, and potentially should not even know the identities of users.
2. Mitigate the impact of insider attacks [23, 21] on protocol execution. Insider adversaries, or compromised group members, will not be able to gain control of a group unless they are administrators[1].

[1]Note that denial-of-service attacks from malicious non-admin insiders as in [12] are not necessarily prevented. This particular issue will be discussed in further sections

3. Increase the robustness of implementations of messaging protocols, preventing pitfalls such as the *burgle into a group attack* [24], which is a vulnerability that allows an adversary to enter a group given a partial control of the central server. This particular issue affected group chats in Signal and WhatsApp.

4. Reduce concurrency issues [25] when the delivery service is not a central server [26], since only a reduced set of members are able to commit group changes. Fully decentralized protocols where admins execute consensus can be envisioned.

In a decentralized setting [26], these issues are especially apparent as admin functionality can no longer be deferred to a central server. Concurrency issues are intrinsic to these protocols, which admins can help mitigate.

Separately, there is motivation from an efficiency point of view. Existing CGKA schemes such as Tainted TreeKEM [11] present efficiency gains if groups are administrated by a reduced number of users.

1.3 Contributions

Group administration can be seen as an authentication problem; we observe that authentication issues in group messaging are not easy to solve. First, we require that the security of the authentication mechanism suits secure messaging, providing FS and PCS guarantees. Besides, the end goal is to authenticate a group of users as a whole, which is a different problem than authenticating single users. Finally, the complexity of secure messaging requires modular constructions, which is our main goal in this paper.

In this work, we cast group administration as a formal *cryptographic* problem. Our core contributions are as follows.

1. Extending the continuous group key agreement (CGKA) primitive, we introduce the *administrated* CGKA (A-CGKA) primitive. The syntax and semantics of A-CGKA provide first-class support for group administration.

2. We introduce a novel game-based correctness notion for both CGKA and A-CGKA which, unlike previous CGKA notions, emphasises the role of *group dynamics* which we argue is centrally linked to group administration.

3. Extending existing CGKA key indistinguishability security notions, we introduce a game-based security notion which additionally aims to prevent even fully corrupted non-admin users from modifying group membership.

4. We present two A-CGKA constructions, IAS and DGS, each built on top of a CGKA protocol. Both approaches offer different security and also differ in efficiency and other properties. We formalise both protocols in great detail, analyze their performance, and provide correctness and security proofs.

5. We propose an extension to MLS that provides efficient secure administration, that we also implement and benchmark locally.

6. We consider additional administration mechanisms and discuss their possible implementation.

1.4 Overview

From CGKA to A-CGKA. Inspired by newer versions of the MLS draft standard, CGKA has been increasingly formalised in the so-called *propose and commit* paradigm [12, 21, 13]. In CGKA, each user maintains a state which is input to and updated by local CGKA algorithms. Users in a given group can create proposal messages to propose to add or remove users, or to update their keying material for PCS reasons. Given proposal messages are propagated to parties, zero or more proposal messages can be combined by a party to form a *commit* message which is then *processed* by users which make the committed changes effective. Moreover, every time a commit is processed, a new group secret is derived by the processing party, and the party is said to be in a new *epoch*.

We extend CGKA to A-CGKA to support administration on the primitive level. We support additional proposal types, namely for adding and removing admins, as well as for admin key updates. In A-CGKA, only admins can make admin proposals, and moreover only admins can authorise all types of commit messages. That is, users only process group changes that have been attested to by admins.

Correctness. Our notion of A-CGKA correctness broadly enforces that users that process the same sequence of commit messages for a given group derive consistent views of both the evolution of group members and admins, and also of the shared key. Our game explicitly checks that the group membership and key can only change as a result of processing a well-formed commit message. We also enforce that honest proposals have their intended effect when embedded in commit messages upon processing.

Existing CGKA game-based correctness notions are either limited to key consistency and embedded in the security game [10, 13] or are not formally specified [11]. Moreover, existing CGKA formulations both with games and in the universal composability (UC) framework do not consider *group evolution* correctness guarantees as we do here. In these models, it is only by inspection of the constructions that one can determine whether e.g., consistent messages lead to consistent group views for processes. Given the length of our IAS construction and corresponding correctness proof, it is possible that subtle bugs concerning group evolution are hidden in existing CGKA constructions or implementations. In our constructions, we spotted inconsistencies while trying to prove our protocols correct.

Security. Our security notion captures two core guarantees. Firstly, like previous work [10, 11], we consider a key indistinguishability game where the adversary drives CGKA execution via oracles and may compromise parties. Namely, the goal of the adversary is to determine whether (common group) keys provided by the challenger on-demand correspond to actual A-CGKA keys or are uniformly sampled and independent. We prevent the adversary from winning the game trivially in so-called cleanness predicates, similar to previous work in two-party messaging [6] and in CGKA (denoted as safety predicates [10]). These predicates are protocol-dependent, i.e. can be strengthened or relaxed as needed.

Secondly, differing from standalone CGKA, we require that the adversary is unable to forge an (admin) commit message that results in a change in group structure for the processing party. We model this by allowing the adversary to inject commit messages to particular parties which process the messages (albeit without updating their state). Security is ensured insofar as the adversary does not trivially compromise an administrator, i.e., they are permitted to compromise many non-admins. As in the case of key indistinguishability, we also specify a

separate admin cleanness predicate to capture trivial attacks for this attack vector. Our security notion allows for FS and PCS guarantees with respect to the admin keying material.

Our game allows the (semi-active) adversary to adaptively corrupt participants and make challenge queries. Like much of the CGKA literature, we model and prove security for a single group. However, the proof can be adapted straightforwardly with a tightness loss of q, given the adversary creates q groups, under a natural extension of the security model to the multi-group case (assuming an incorruptible PKI). We also do not allow for randomness manipulation or active adversarial behaviour outside of the context of commit injections described above.

Authentication and PKI. Our constructions are mostly independent of the way that the underlying authentication mechanism or PKI is implemented. Notably, we only rely on a PKI-based authentication for group additions; authentication is later maintained during protocol execution. In IAS, we only require that each party's single-use public key can be retrieved (by other parties) during protocol execution. Therefore, we assume a simplified PKI model that provides this functionality. For DGS, we do not require additional PKI since we rely on the authentication guarantees of the two underlying CGKAs. We expect our approach to be compatible with other authentication mechanisms (supporting for instance out-of-band verification), in particular if A-CGKAs are embedded into larger protocols such as MLS.

Constructions. In this work, we provide two separate, modular constructions of A-CGKA. Both constructions extend an underlying CGKA to an A-CGKA.

IAS, or individual admin signatures, is our first construction. In IAS, admins in each group keep track of their own signature key pair. Admin proposals and commits which change the group or admin structure or keys are signed using the committing admin's signature key. Admins update their signature keys via admin update proposals or by crafting commit messages.

Our second construction, dynamic group signature (DGS), relies on a secondary CGKA. Instead of maintaining individual signatures, admins instead execute within this CGKA and use the common secret to derive a signature key pair for each epoch. Non-admins keep track of the signature public key over time and verify that commits are signed using it.

In Section 4, we formally specify IAS and DGS. We note that, despite the conceptual simplicity of the protocols, the requisite pseudocode is quite extensive, in part due to extensive case checking when creating or parsing messages.

Besides, in Section 4.3, we embed the MLS protocol with A-CGKA functionality more organically, by sharing the use of signature keys. We describe the main modifications needed and propose an extension of MLS that admits secure administration. Moreover, we implement and benchmark the efficiency of our MLS extension; we present our results in Section 5.

Proofs. In the appendices, we formally prove that our protocols IAS and DGS are correct and secure with respect to our A-CGKA definitions. The main theorems are the following:

Theorem 1 (Simplified)**.** *Let* CGKA *be a correct and* $(t_{\mathsf{cgka}}, q, \epsilon_{\mathsf{cgka}})$*-secure CGKA. Let* $\mathcal{S} = (\mathsf{SigGen}, \mathsf{Sig}, \mathsf{Ver})$ *be a* $(t_{\mathcal{S}}, q, \epsilon_{\mathcal{S}})$ *SUF-CMA secure signature scheme. Let* H_4 *be a* $(t_F, 1, \epsilon_F)$*-secure 4-PRF. Then, the IAS protocol (Figures 5 and 6) is correct and* $(t, q, q \cdot \epsilon_F + \epsilon_{\mathsf{cgka}} + q^2 \cdot \epsilon_{\mathcal{S}})$*-secure (Definition 4) where* $t_{\mathsf{cgka}} \approx t_{\mathcal{S}} \approx t_F \approx t$*.*

We prove IAS secure with respect to a sub-optimal admin cleanness predicate (somewhat weak forward secrecy). We argue that the protocol and proof can be very easily modified to satisfy

optimal security using forward-secure signatures with no asymptotic overhead; this is discussed in Section 4.1.4.

Theorem 2 (Simplified)**.** *Let* CGKA *(resp.* CGKA^**) be a correct and* $(t_{\mathsf{cgka}}, q, \epsilon_{\mathsf{cgka}})$*-secure (resp.* $(t_{\mathsf{cgka}*}, q, \epsilon_{\mathsf{cgka}*})$*)-secure CGKA. Let* $\mathcal{S} = (\mathsf{SigGen}, \mathsf{Sig}, \mathsf{Ver})$ *be a* $(t_{\mathcal{S}}, q, \epsilon_{\mathcal{S}})$ *SUF-CMA secure signature scheme. Let* H_4 *be a* $(t_F, 1, \epsilon_F)$*-secure 4-PRF and* H_{ro} *a random oracle queried at most* q_{ro} *times. Then, the DGS protocol (Figures 5 and 6) is correct and* $(t, q, \epsilon_{\mathsf{cgka}} + q \cdot (\epsilon_F + \epsilon_{\mathcal{S}} + \epsilon_{\mathsf{cgka}*} + 2^{-\lambda}))$*-secure (Definition 4) for security parameter* λ *in the random oracle model where* $t_{\mathsf{cgka}} \approx t_{\mathcal{S}} \approx t_F \approx t$*.*

1.5 Additional Related Work

Signal Private Groups. The Signal Private Group System [27] proposes to keeps track of group membership via a central server whilst hiding the set of group members from non-members. The advantage of this approach is that users no longer have to track the state of group membership individually as in (A)-CGKA. By relying on a trusted membership oracle, disagreement is prevented which would arise in (A)-CGKA when users apply different group updates over time. Conceptually, the same approach could be applied to also track group *administrators*.

However, the system has not been analysed in conjunction with the underlying group message protocol. Since the membership oracle is effectively independent of the communication transcript, care is required when composing the modules. For example, some users may send messages with the perspective that a given party P is in the group, whereas others may send messages with the knowledge that P has been removed from the group. By contrast, in CGKA, there is an explicit relationship between the group secret (which is then used for secure group messaging [13]) and the current group membership. It remains open to analyse Signal pairwise channels itself in addition to analysing how pairwise channels composes with the Private Group System.

We note that our protocols do not make any synchronisation assumptions beyond what is made by MLS itself, hence being compatible with out-of-order delivery of application messages. That is, our solutions are amenable to deployments where users are expected to be apply to apply CGKA commit messages in a consistent order. Signal previously managed group administration locally as in IAS/DGS, citing desynchronisation issues as a conceptual motivation to centralise group membership and build a new system [27]. In a similar vein, Signal Private Groups cannot straightforwardly be adapted to the decentralized setting, since the central server issues credentials for users. We leave designing a comparable solution compatible with (A)-CGKA as interesting future work (e.g. where admins play the role of the central server); we also discuss the possibility of administration privacy in Section 6.2.1.

Messaging. Many works in the two-party messaging literature laid the foundations for modern group messaging protocols, especially regarding FS and PCS. Initial work like OTR [2] was followed by the Signal protocol [1] (formalized in [7, 28]), and by the formalization of PCS [14]. Multiple works in two-party messaging and ratcheted key exchange have since appeared [4, 5, 6, 8, 29, 30, 31, 32].

The TreeKEM protocol in MLS, initially proposed in [9], was inspired by Asynchronous Ratchet Trees [18]. Later, variants of TreeKEM arose like Tainted TreeKEM [11], Insider-Secure

TreeKEM [21], Re-randomized TreeKEM [10], and Causal TreeKEM [33]. The security of the MLS key schedule has been studied in [34].

CGKA was introduced in [10] in reference to MLS, inspired by the two-party continuous key agreement module in [7]. CGKAs have been recently used to formally build full group messaging protocols [13]. Besides TreeKEM, other CGKA variants include [12, 35, 36] and the decentralization work in [26]. Side works related to group messaging, but not proposing CGKA constructions deal with multi-group security [37], as well as efficient key schedules for multiple groups [38]. Concurrency has also been studied [25]. Separately, [17] surveys group key exchange protocols (also beyond CGKAs).

2 Notation

A *user*, *participant*, or *party* is an entity that takes part in a protocol. Users (resp. groups) are identified by a unique, public identifier ID (resp. gid). Users keep an internal *state* γ with all information used for protocol execution. This includes keys, message records, dictionaries and parameters. If this state is leaked, we say ID suffers a *state compromise* or a *corruption*.

The definitions, games, and constructions that we introduce in our work use standard notation. Algorithms, oracle names, and cryptographic parameters are denoted in sans-serif font. To assign the output of an algorithm Alg on input x to a variable a, we write $a \leftarrow \mathsf{Alg}(x)$ if the algorithm is deterministic. If the algorithm is randomized, we write $a \leftarrow\!\!\$\, \mathsf{Alg}(x)$. Whenever we want to make the randomness explicit, we write $a \leftarrow \mathsf{Alg}(x; r)$, meaning that r is the randomness used by Alg. An algorithm can input or return blank values, represented by $\perp$. The security parameter is denoted by λ.

The security games in this paper are played between a challenger and an adversary $\mathcal{A}$, which is an idealized external entity which can interact with the protocol using several oracles. In oracles and algorithms, some special predicates are used. The predicate '**require** P' enforces that a logical condition P is satisfied; otherwise the oracle/algorithm finishes immediately and returns $\perp$. The predicate '**reward** P' is executed in games and is such that if P holds, the adversary satisfies a winning condition in a game. Namely, in games where the advantage is defined by the probability that the adversary outputs 1 (unpredictability), a game variable win is set to 1, and in indistinguishability games the game reveals the bit $b \in \{0, 1\}$ to the adversary. The keyword '**public** var' indicates that the adversary has read access to the variable var.

To store and retrieve values, we often use dictionaries: $\mathsf{A}[k] \leftarrow a$ adds the value a to the dictionary A under key k (overwriting the previous value associated to k). $b \leftarrow \mathsf{A}[k]$ retrieves $\mathsf{A}[k]$ and assigns it to variable b. A dictionary A is initialized as $\mathsf{A}[\cdot] \leftarrow a$; where all values of the dictionary are set to a (notice that a can also be $\perp$). To reduce verbosity, we occasionally use the prefix operator ++ in our games. $\mathsf{Alg}(\mathsf{++x})$, is equivalent to writing first $x \leftarrow x + 1$ and then $\mathsf{Alg}(\mathsf{x})$.

3 (Administrated) Continuous Group Key Agreement

In this section, we introduce Continuous Group Key Agreement (CGKA) and then extend it to formalize our Administrated CGKA (A-CGKA) primitive. We also introduce our correctness and security definitions for both CGKA and A-CGKA.

3.1 Continuous Group Key Agreement

The aim of the Continuous Group Key Agreement (CGKA) primitive [10] is to provide shared secrets (denoted by k) to dynamic groups of users over time. In particular, each group, labelled with a group identifier gid, is subject to additions (add), removals (rem), and user state refreshes/key updates (upd).

The definition of a CGKA is introduced below, in the so-called *propose and commit* paradigm [21, 12], in which different operation proposals in a given group (e.g. adding/removing members) are eventually collated into a commit message by a group member which is processed by users. The evolution of a CGKA in time is captured by *epochs*; a group member advances to a new epoch every time they successfully process a commit message, at which point there is a change in the group structure and/or shared secret from their perspective.

Note that the primitive is *stateful*: each user keeps their own state γ and calls each of the following algorithms locally which may update the state.

Definition 1. *A continuous group key agreement (CGKA) scheme is a tuple of algorithms* $\mathsf{CGKA} = (\mathsf{init}, \mathsf{create}, \mathsf{prop}, \mathsf{commit}, \mathsf{proc}, \mathsf{prop\text{-}info})$ *such that:*

- $\gamma \leftarrow\!\$\, \mathsf{init}(1^\lambda, \mathsf{ID})$ *takes a security parameter* 1^λ *and an identity* ID *and outputs an initial state* γ*.*
- $(\gamma', T) \leftarrow\!\$\, \mathsf{create}(\gamma, \mathsf{gid}, G)$ *takes a state* γ*, a group identifier* gid*, and a list of group members* $G = \{\mathsf{ID}_1, \ldots, \mathsf{ID}_n\}$ *and outputs a new state* γ' *and a control message* T *(welcome), where* $T = \perp$ *indicates failure.*
- $(\gamma', P) \leftarrow\!\$\, \mathsf{prop}(\gamma, \mathsf{gid}, \mathsf{ID}, \mathsf{type})$ *takes a state, a group identifier, an* ID*, and a proposal type* $\mathsf{type} \in \mathsf{types} = \{\mathsf{add}, \mathsf{rem}, \mathsf{upd}\}$*, and outputs a new state* γ' *and a proposal message* P*, where* $P = \perp$ *indicates failure.*
- $(\gamma', T) \leftarrow\!\$\, \mathsf{commit}(\gamma, \mathsf{gid}, \vec{P})$ *takes a state, a group identifier, and a vector of proposals* $\vec{P}$*, and outputs a new state* γ' *and a control message* T*, where* $T = \perp$ *indicates failure.*
- $(\gamma', \mathsf{acc}) \leftarrow \mathsf{proc}(\gamma, T)$ *takes a state and a control message* T*, and outputs a new state* γ' *and an acceptance bit* acc*, where* $\mathsf{acc} = \mathsf{false}$ *indicates failure.*
- $(\mathsf{gid}, \mathsf{type}, \mathsf{ID}, \mathsf{ID}') \leftarrow \mathsf{prop\text{-}info}(\gamma, P)$ *takes a state and a proposal* P*, and outputs the group identifier of the proposal* gid*, its type* type*, the* ID *of the user* affected *by the proposal and the proposal creator* ID'*.*

Finally, given ID*'s state* γ *and* gid*, the (possibly empty) set of group members in* gid *from* ID*'s perspective is stored as* $\gamma[\mathsf{gid}].G$*, and the group secret* k *for* gid *is* $\gamma[\mathsf{gid}].k$*.*

3.1.1 Protocol execution

For simplicity, we assume all users and groups are associated with a unique identifier ID and gid, respectively. Once every user has initialized their state using init, a group is created when a party calls create on a list of IDs. The init algorithm can also serve to authenticate and register keys on a PKI when appropriate, as in [10, 11, 12]. We expand on the use of PKI and authentication issues in Section 4.1. The create algorithm outputs a control message T that must be processed by prospective group members, including the group creator, in order to join the group gid.

In our formalism, any user can propose a member addition (add), member removal (rem) or key update (upd, only available for the caller) at any time. This is done via the prop method, which outputs a proposal message P. Proposals encode the information needed to make a change in the group structure or keying material, but the encoded changes are not immediately applied to the group. We emphasise that only the caller of prop can use argument $\mathsf{type} = \mathsf{upd}$ to propose an update to their keying material, in which case the input ID is ignored. Following [13], we define prop-info which outputs proposal attributes, rather than allowing for their direct access, to support possibly encrypted proposals (e.g., as in MLSCiphertext [3]).

Proposed changes become effective once a user commits a (possibly empty) vector of proposals $\vec{P} = (P_1, \ldots, P_m)$ using commit. The commit algorithm outputs a control message T that contains the information needed by all current and incoming group members to process the changes. Typically, such as in our schemes, the commit algorithm also updates the keying material of the caller. Control messages are processed via proc, which updates the caller's state and outputs a bit acc indicating success or failure. We note that proc does not require a group identifier as input; this models the standard behaviour of a messaging protocol where, upon reception of a message, the user needs to determine which group the message corresponds to.

3.1.2 Example

Consider 5 parties $\{\mathsf{ID}_1, \ldots, \mathsf{ID}_5\}$ executing a CGKA protocol. After they each initialize their states as $\gamma_i \leftarrow\!\$\, \mathsf{init}(1^\lambda, \mathsf{ID}_i)$, the following actions take place:

1. ID_1 calls $\mathsf{create}(\gamma_1, \mathsf{gid}, \{\mathsf{ID}_1, \mathsf{ID}_2, \mathsf{ID}_3, \mathsf{ID}_4\})$, which updates γ_1 and outputs a control message T_0. At this stage, the group is still empty, $G = \emptyset$.

2. Each ID_i (including ID_1) processes the group creation as $\mathsf{proc}(\gamma_i, T_0)$, which updates each state γ_i and outputs $\mathsf{acc} = \mathsf{true}$ to each user. At this stage, $G = \{\mathsf{ID}_1, \mathsf{ID}_2, \mathsf{ID}_3, \mathsf{ID}_4\}$, and the group members share a common secret k_1.

3. Several users propose changes in the group:

   - ID_2 proposes to add ID_5 to the group by calling $(\gamma_2', P_1) \leftarrow\!\$\, \mathsf{prop}(\gamma_2, \mathsf{gid}, \mathsf{ID}_5, \mathsf{add})$.
   - ID_3 wants to update its key material and thus calls $(\gamma_3', P_2) \leftarrow\!\$\, \mathsf{prop}(\gamma_3, \mathsf{gid}, \mathsf{ID}_3, \mathsf{upd})$.
   - ID_1 proposes to remove ID_4 from the group and calls $(\gamma_1', P_3) \leftarrow\!\$\, \mathsf{prop}(\gamma_1, \mathsf{gid}, \mathsf{ID}_4, \mathsf{rem})$.

   This is shown in Figure 1. The group remains the same.

4. ID_2 collates all proposals in a commit message by calling $(\gamma_2', T_1) \leftarrow\!\$\, \mathsf{commit}(\gamma_2, \mathsf{gid}, (P_1, P_2, P_3))$. The group remains the same, since parties have not yet processed T_1.

5. All parties process T_1 by calling $\mathsf{proc}(\gamma_i, T_1)$ and updating their states. Now, $G = \{\mathsf{ID}_1, \mathsf{ID}_2, \mathsf{ID}_3, \mathsf{ID}_5\}$ and these members share a new common secret k_2, which is not known to ID_4. In addition, ID_3 (due to the update) and ID_2 (due to the commit) have refreshed their entire keying material.

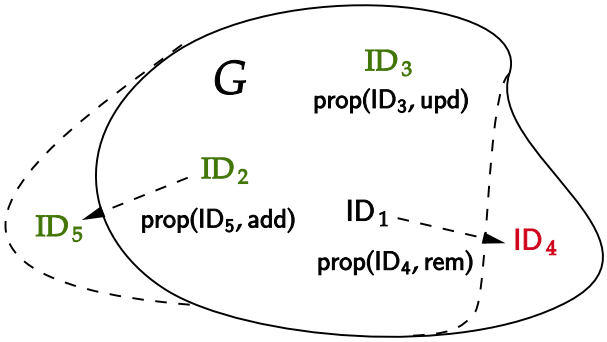

Figure 1: Diagram of a sample CGKA execution with 5 parties. Parties in green ($\mathsf{ID}_2, \mathsf{ID}_3, \mathsf{ID}_5$) will update their key material after ID_2 commits, while ID_4 will leave the group.

3.1.3 Commit semantics

We assume that proposals input to commit are processed in some deterministic, publicly-known, a priori determined order, that we call the *policy* and that we will define explicitly. It is possible to extend the syntax of A-CGKA with a dedicated policy algorithm that defines this order as in [39].

3.1.4 Alternative definitions

In previous CGKA definitions [10, 11] and in old versions of MLS, group changes are made effective immediately by processing proposals. To this end, the commit and prop algorithms are replaced by specific 'action' algorithms such as add, remove, update. The *propose and commit paradigm* used in this work was introduced in version 8 of MLS to enable many group members to refresh their keys with less latency [3]. As mentioned in [11], the older protocols can be written in the propose and commit paradigm, which is more flexible (and also better suited for group administrators).

A relevant difference between the definition in [10] (and other game-based formulations such as in [11]) and ours, is that we work in the multi-group setting, and so we consider group identifiers of the form gid. Multi-group CGKAs have not been formalized in the literature, although multi-group security has been studied [37], as well as efficient key schedules for multiple groups [38]. According to our definition, a user can be in many groups, identified by different values gid and interleave operations from each group arbitrarily. There are formulations of CGKA in the universal composability (UC) model [12, 21] which use group identifiers. In fact, composability guarantees in the UC model rely on the existence of unique, a priori established 'session identifiers' [40, 41]; these can be established in practice via a central server or a distributed protocol [42]. Note [37] considers cross-group security for messaging but does not treat CGKA formally.

Finally, we note that there are other small differences in the literature. The semantics of Tainted TreeKEM [11] enable users to speculatively execute operations; our syntax could be modified to support this. In [21, 12], additional algorithms such as getKey are provided; we treat k as a state variable instead. As mentioned, it is also possible to conceive a policy algorithm.

3.2 Administrated CGKA

An administrated continuous group key agreement (A-CGKA) is a CGKA where only a group G^* of ID's, the so-called *group administrators*, can commit (and therefore make effective) changes to the group structure, such as adding and removing users. As with the group of users G in both CGKA and A-CGKA, the group of administrators G^* is dynamic.

Definition 2. *An administrated continuous group key agreement (A-CGKA) scheme is a tuple of algorithms* $\mathsf{A\text{-}CGKA} = (\mathsf{init}, \mathsf{create}, \mathsf{prop}, \mathsf{commit}, \mathsf{proc}, \mathsf{prop\text{-}info})$ *such that:*

- *Algorithms* $\mathsf{init}, \mathsf{proc}, \mathsf{prop\text{-}info}$ *are defined as for a* CGKA *(Definition 1).*
- *In* prop *and* prop-info*,* types *is redefined as* $\mathsf{types} = \{\mathsf{add}, \mathsf{rem}, \mathsf{upd}, \mathsf{add\text{-}adm}, \mathsf{rem\text{-}adm}, \mathsf{upd\text{-}adm}\}$.
- $(\gamma', T) \leftarrow\!\$\, \mathsf{create}(\gamma, \mathsf{gid}, G, G^*)$ *additionally takes a group of admins* G^*.
- $(\gamma', T) \leftarrow\!\$\, \mathsf{commit}(\gamma, \mathsf{gid}, \vec{P}, \mathsf{com\text{-}type})$ *additionally takes a commit type* $\mathsf{com\text{-}type} \in \mathsf{com\text{-}types} = \{\mathsf{std}, \mathsf{adm}, \mathsf{both}\}$.

Given ID*'s state* γ *and* gid*,* $\gamma[\mathsf{gid}].G$ *and* $\gamma[\mathsf{gid}].k$ *are defined as in Definition 1, and* $\gamma[\mathsf{gid}].G^*$ *stores the set of admins in* gid *from* ID*'s perspective.*

The execution of an A-CGKA is analogous to that of a CGKA. Besides the introduction of the group of admins, we introduce three more proposal types add-adm, rem-adm, and upd-adm, which concern administrative changes. Namely, an admin may propose to add another admin to the group of administrators, revoke the admin capabilities from a party, or update their administrative key material. The commit type com-type specifies the scope of a commit operation; that is, whether it affects the general group (std), the administration of the group (adm), or both at the same time (both). For the latter, a simple example is when an admin is both adding a member (group modification) and refreshing its admin keys (admin modification).

We note that the create algorithm enforces the condition $\emptyset \subset G^* \subseteq G$; our correctness and security notions will ensure this holds throughout execution. Thus, the group administrators will always be a subset of the group members. We take this approach following previous CGKAs [10, 11, 12] and group messaging protocols [3], [13], where only group members can perform commits or make changes in the group. In these works, it is impossible for an external user to administrate a group, since external commits are not allowed. We elaborate on this in Section 6.

Real-world administrators. A-CGKA captures the main admin features in commercial applications such as WhatsApp and Signal as mentioned in Section 1.2.2. We remark that the fact that non-admins are not allowed to make changes is a desired consequence of our formulation of A-CGKA. If this is not desired (such as when invite links without admin approval are enabled), one can simply run a CGKA, or designate everyone to be an admin. A more fine-grained solution, at the expense of additional A-CGKA formalism, is to allow admins to send a policy change proposal, to (for instance) modify the ability of all members to form valid commit messages that add new users. Separately, we note that any user can effectively leave the group by sending a remove proposal and erasing their state, assuming all proposals are eventually committed by admins.[2]

[2]Note that the MLS specification enforces that users send commits whenever they receive new proposals before sending application messages, ensuring their timely processing.

In group messaging protocols used in practice like pairwise Signal and Sender Keys (used in WhatsApp [16]) each user is associated with their own key or keying material (rather than a common group secret as in CGKA). An IAS-like protocol can be implemented in this setting straightforwardly. For Sender Keys, where users only ratchet symmetric keys (at the cost of PCS confidentiality guarantees), admins could replace their keying material at a low cost (a signature on their new signing key) for PCS authentication guarantees. A DGS-like protocol could conceivably be implemented in which admins track common keying material via e.g. a CGKA. We leave it as useful future work to formalise group administration in this setting, noting the additional complexity of providing guarantees for users that relates the keying material or messages with the set of admins over time.

3.3 Correctness

Due to their similarity, we define the correctness of CGKA and A-CGKA together. Correctness of an (A)-CGKA scheme $\mathsf{(A)\text{-}CGKA}$ under the notion $\mathsf{CORR}_{\mathsf{(A)\text{-}CGKA}}$ is defined by game $\mathsf{CORR}^{\mathcal{A}}_{\mathsf{(A)\text{-}CGKA},\mathsf{C}_{\mathsf{corr}}}$ played by adversary $\mathcal{A}$ in Figure 13. We relegate the game and the full description to Appendix B.

Definition 3 ($\mathsf{CORR}_{\mathsf{(A)\text{-}CGKA}}$)**.** *A CGKA* CGKA *(resp. A-CGKA* A-CGKA*) is correct w.r.t. a predicate* $\mathsf{C}_{\mathsf{corr}}$ *if, for all* 1^λ *and all computationally unbounded adversaries* $\mathcal{A}$*, it holds that:*

$$\Pr[\mathsf{CORR}^{\mathcal{A}}_{\mathsf{(A)\text{-}CGKA},\mathsf{C}_{\mathsf{corr}}}(1^\lambda) = 1] = 0$$

where the probability is taken over the choice of the random coins of the challenger and adversary.

The main properties captured by the game (Figure 13) are the following:

- *View consistency:* All users who transition to the same epoch (i.e. which process the same sequence of commit messages) have the same group view (i.e., G, G^* and key k).
- *Message processing:* The group structure (G/G^*) and k can only be modified due to calls to proc.
- *Forking states:* If the group is partitioned into subgroups that process different sequences of commit messages (thus leading to different group views), the game ensures that members in each partition have consistent views.
- *Multiple groups:* The adversary may create groups via $\mathcal{O}^{\mathsf{Create}}$ on behalf of different users, and interact with different IDs in multiple groups.

Separately, we ensure that a user's state is not modified whenever a particular algorithm call fails. As observed for two-party messaging [29], we require incorrect inputs to not affect the functionality of the protocol, and in particular to not cause a denial of service.

3.4 Security

A-CGKA is a primitive that extends the functionality of a CGKA to provide secure administration mechanisms, but whose end purpose is that the members of a given group derive a

common group key. Therefore, any A-CGKA construction must satisfy at least CGKA security (i.e. key indistinguishability).

The main additional goal of A-CGKA over standard CGKA is to prevent unauthorized (standard) users from deciding on changes to a group, capturing the security of the group evolution. Note that an A-CGKA in which the adversary fully controls a standard group member is not secure with respect to key indistinguishability, but it should still be secure with respect to group evolution. We define (A)-CGKA security in Definition 4.

Definition 4 (Security of (A)-CGKA)**.** *A CGKA* CGKA *(resp. A-CGKA* A-CGKA*) is* (t, q, ϵ)*-secure w.r.t. the predicates* $\mathsf{C}_{\mathsf{cgka}}, (\mathsf{C}_{\mathsf{adm}})$ *if, for any adversary* $\mathcal{A}$ *limited to* q *oracle queries and running time* t*, the advantage of* $\mathcal{A}$ *in the* $\mathsf{KIND}_{\mathsf{(A)\text{-}CGKA}}$ *game (Figure 2) given by*

$$\left|\Pr[\mathsf{KIND}^{\mathcal{A}}_{\mathsf{(A)\text{-}CGKA},\mathsf{C}_{\mathsf{cgka}},(\mathsf{C}_{\mathsf{adm}})}(1^\lambda) = 1] - \frac{1}{2}\right|$$

is bounded by ϵ*, where the probability is taken over the choice of the challenger and adversary's random coins.*

3.4.1 Overview

At its core, the security game in Figure 2 is a key indistinguishability game that captures the security of the common group secret, extending the game in [10]. The game considers a partially active adversary who can make forgery attempts and schedule messages but cannot totally control message delivery. Namely, the adversary can inject a control message to a specific party ID, but this message is not stored in the array T that keeps track of all honestly generated messages after proc is called. The main consequence of this is that injected proposals cannot be included into commits; nevertheless, the adversary can make commits on arbitrary proposals created via $\mathcal{O}^{\mathsf{Prop}}$.

Informally, the adversary can win the game if it plays a clean game where it either 1) correctly guess the challenge bit by distinguishing between correct and uniformly sampled keys or (for A-CGKA) 2) manages to forge a message which, after being processed by a user, changes its view of (G, G^*). A detailed description follows.

3.4.2 Epochs

Messages output by successful create, commit, and prop calls are uniquely labelled by the challenger via counters prop-ctr, com-ctr. Whenever such a call is made, the corresponding messages are stored in variable T with (incremented) last argument ++com-ctr. The evolution of the group after parties process such control messages is modelled using epochs (differing from the epochs considered for correctness), which are each represented as an integer t_s (for CGKA) or a pair of integers (t_s, t_a) (for A-CGKA). The *standard epoch* t_s represents the period of time between two successive key evolutions, where a different key should be derived in each t_s. The *administrative epoch* t_a represents the time between two changes in the group administration (i.e. between two sequences of simultaneous admin updates, adds and/or removals).

Epochs advance every time a commit is processed; t_s advances if the commit type com-type $\in \{\mathsf{std}, \mathsf{both}\}$ in A-CGKA and t_a advances if com-type $\in \{\mathsf{adm}, \mathsf{both}\}$. Group members can be in different epochs, captured by the variable ep[ID] which stores the current epoch pair for a given group member ID. If a participant ID is not in the group, then ep[ID] $= -1$ (CGKA) or ep[ID] $= (-1, -1)$ (A-CGKA) holds.

$\mathsf{KIND}^{\mathcal{A}}_{\text{(A)-CGKA},\mathsf{C}_{\mathsf{cgka}},\mathsf{C}_{\mathsf{adm}}}(1^\lambda)$

$b \leftarrow\$ \{0,1\}$
$\mathsf{K}[\cdot], \mathsf{ST}[\cdot] \leftarrow \bot$
public $\mathsf{T}[\cdot], \mathsf{G}[\cdot], \mathsf{ADM}[\cdot] \leftarrow \bot$
$\mathsf{prop\text{-}ctr}, \mathsf{com\text{-}ctr}, \mathsf{exp\text{-}ctr} \leftarrow 0$
$\mathsf{ep}[\cdot], \mathsf{exp}[\cdot] \leftarrow (-1,-1)$; $\mathsf{C}[\cdot] \leftarrow -1$
$\mathsf{chall}[\cdot], \mathsf{forged} \leftarrow \mathsf{false}$
$\mathsf{ST}[\mathsf{ID}] \leftarrow\$ \mathsf{init}(1^\lambda, \mathsf{ID})\ \forall\ \mathsf{ID}$
$b' \leftarrow\$ \mathcal{A}^{\mathcal{O}}(1^\lambda)$
require $\mathsf{C}_{\mathsf{cgka}} \vee \mathsf{forged}$
return $1_{b=b'}$

$\mathcal{O}^{\mathsf{Create}}(\mathsf{ID}, G, G^*)$

$(\gamma, T) \leftarrow\$ \mathsf{create}(\mathsf{ST}[\mathsf{ID}], G, G^*)$
if $T = \bot$ **return** // failure
$\mathsf{T}[(-1,-1), \texttt{'com'}, \texttt{++com-ctr}] \leftarrow (T, \mathsf{both})$
$\mathsf{ST}[\mathsf{ID}] \leftarrow \gamma$

$\mathcal{O}^{\mathsf{Prop}}(\mathsf{ID}, \mathsf{ID}', \mathsf{type})$

$(\gamma, P) \leftarrow\$ \mathsf{prop}(\mathsf{ST}[\mathsf{ID}], \mathsf{ID}', \mathsf{type})$
$\mathsf{T}[\mathsf{ep}[\mathsf{ID}], \texttt{'prop'}, \texttt{++prop-ctr}] \leftarrow P$
$\mathsf{ST}[\mathsf{ID}] \leftarrow \gamma$

$\mathcal{O}^{\mathsf{Commit}}(\mathsf{ID}, (i_1, \ldots, i_k), \mathsf{com\text{-}type})$

$\vec{P} \leftarrow (\mathsf{T}[\mathsf{ep}[\mathsf{ID}], \texttt{'prop'}, i])_{i=(i_1,\ldots,i_k)}$
$(\gamma, T) \leftarrow\$ \mathsf{commit}(\mathsf{ST}[\mathsf{ID}], \vec{P}, \mathsf{com\text{-}type})$
if $T = \bot$ **return** // failure
$\mathsf{T}[\mathsf{ep}[\mathsf{ID}], \texttt{'com'}, \texttt{++com-ctr}] \leftarrow (T, \mathsf{com\text{-}type})$
$\mathsf{T}[\mathsf{ep}[\mathsf{ID}], \texttt{'vec'}, \mathsf{com\text{-}ctr}] \leftarrow \vec{P}$ // for cleanness
$\mathsf{ST}[\mathsf{ID}] \leftarrow \gamma$

$\mathcal{O}^{\mathsf{Challenge}}(t_s)$

require $(\mathsf{K}[t_s] \neq \bot) \wedge \neg\mathsf{chall}[t_s]$
$\mathsf{chall}[t_s] \leftarrow \mathsf{true}$
if $b = 0$ **return** $\mathsf{K}[t_s]$
if $b = 1$ **return** $r \leftarrow\$ \{0,1\}^\lambda$

$\mathcal{O}^{\mathsf{Deliver}}(\mathsf{ID}, (t_s, t_a), c)$

require $\mathsf{ep}[\mathsf{ID}] \in \{(t_s, t_a), (-1,-1)\}$
$(T, \mathsf{com\text{-}type}) \leftarrow \mathsf{T}[(t_s, t_a), \texttt{'com'}, c]$ // honest deliv.
if $\mathsf{C}[(t_s, t_a)] \in \{c, -1\}$, $\mathsf{C}[(t_s, t_a)] \leftarrow c$
else return // bad commit for epoch
$(\gamma, \mathsf{acc}) \leftarrow \mathsf{proc}(\mathsf{ST}[\mathsf{ID}], T)$
if $\neg\mathsf{acc}$ **return** // failure
if $\mathsf{ID} \notin \gamma.G$ // ID removed
  $\mathsf{ep}[\mathsf{ID}] \leftarrow (-1,-1)$
else // ID in group, update dictionaries
  $\mathsf{ep}[\mathsf{ID}] \leftarrow (t_s, t_a)$
  if $\mathsf{com\text{-}type} \in \{\mathsf{std}, \mathsf{both}\}$
    $\mathsf{K}[t_s + 1] \leftarrow \gamma.\mathsf{k}$
    $\mathsf{G}[t_s + 1] \leftarrow \gamma.G$
    $\mathsf{ep}[\mathsf{ID}] \leftarrow \mathsf{ep}[\mathsf{ID}] + (1, 0)$
  if $\mathsf{com\text{-}type} \in \{\mathsf{adm}, \mathsf{both}\}$
    $\mathsf{ADM}[t_a + 1] \leftarrow \gamma.G^*$
    $\mathsf{ep}[\mathsf{ID}] \leftarrow \mathsf{ep}[\mathsf{ID}] + (0, 1)$
$\mathsf{ST}[\mathsf{ID}] \leftarrow \gamma$

$\mathcal{O}^{\mathsf{Reveal}}(t_s)$

require $\neg\mathsf{chall}[t_s]$
$\mathsf{chall}[t_s] \leftarrow \mathsf{true}$
return $\mathsf{K}[t_s]$

$\mathcal{O}^{\mathsf{Expose}}(\mathsf{ID})$

$\mathsf{exp}[\mathsf{ID}, \texttt{++exp-ctr}] \leftarrow \mathsf{ep}[\mathsf{ID}]$
return $\mathsf{ST}[\mathsf{ID}]$

$\mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}, m, t_a)$

require $\mathsf{C}_{\mathsf{adm}} \wedge (\mathsf{ep}[\mathsf{ID}] = (\cdot, t_a)) \wedge (t_a \neq -1)$
require $(m, \cdot) \notin \mathsf{T}$ // external forgery
$(\gamma, \bot) \leftarrow \mathsf{proc}(\mathsf{ST}[\mathsf{ID}], m)$
if $(\gamma.G, \gamma.G^*) \neq (\mathsf{ST}[\mathsf{ID}].G, \mathsf{ST}[\mathsf{ID}].G^*)$
  $\mathsf{forged} \leftarrow \mathsf{true}$ // successful forgery
  return b // adversary wins
else return $\bot$

Figure 2: Key indistinguishability (KIND) security game for (single-group) (A)-CGKA, parametrized by the $\mathsf{C}_{\mathsf{cgka}}$ and $\mathsf{C}_{\mathsf{adm}}$ predicates. Highlighted code is executed only when considering an A-CGKA.

Challenges. At any point in the game, the adversary can challenge with respect to a standard epoch t_s by calling $\mathcal{O}^{\mathsf{Challenge}}$. In a challenge, the adversary is given the group key $\mathsf{K}[t_s]$ if the challenger's bit is $b = 0$, and a random string $r \leftarrow\!\!\$\, \{0,1\}^\lambda$ if $b = 1$. The adversary must try to determine the value of b by outputting a guess b' of b. A given execution is considered valid when either the *standard cleanness predicate* $\mathsf{C}_{\mathsf{cgka}}$ is true or, for A-CGKA, the adversary makes a forgery and the admin predicate is true. Cleanness ensures that no trivial attacks on the game are possible; we elaborate on this below.

Exposure mechanisms. In order to capture group key ratcheting (for FS and PCS), the adversary has two mechanisms to obtain secret group material: it can *expose* a user ID and *reveal* the group secret k. An exposure leaks the entire current state of ID stored in $\mathsf{ST}[\mathsf{ID}]$. We keep track of the specific epochs in which each ID was exposed using the $\mathsf{exp}[\cdot]$ variable. On the other hand, a reveal leaks the group key to the adversary on a specified epoch t_s – in this case, $\mathsf{chall}[t_s]$ is set to true to prevent the adversary from challenging on t_s (conversely, the reveal fails when $\mathsf{chall}[t_s] = \mathsf{true}$).

Injections. For A-CGKA, the adversary can also win the game by *injecting* a forged commit. An injection can be attempted by calling $\mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}, m, t_a)$, given that ID is in admin epoch t_a, where ID is the target group member and m is the forged message. Note we require $t_a \neq -1$ since the adversary could otherwise trivially invite a new user into a new group that it controls. The adversary wins the game if the forgery is accepted by any group member $\mathsf{ID} \in G$ and the *administrative predicate* $\mathsf{C}_{\mathsf{adm}}$ is not violated. As discussed below, $\mathsf{C}_{\mathsf{adm}}$ captures administration security by excluding trivial attacks. An example scenario where security cannot be provided is when the adversary has exposed an administrator immediately before a forgery attempt.

3.4.3 CGKA cleanness predicate

The security game in Figure 2 is parametrized by two cleanness predicates, $\mathsf{C}_{\mathsf{cgka}}$ and $\mathsf{C}_{\mathsf{adm}}$. The first predicate $\mathsf{C}_{\mathsf{cgka}}$ follows approaches like [8, 10] to parametrise the security of the common (A)-CGKA key. Namely, this predicate excludes trivial attacks on the protocol, i.e., those that break security unavoidably such as revealing the group secret and issuing a challenge in the same epoch. Further, it captures the exact security of the protocol (with respect to key indistinguishability), which in our case comprises forward security and post-compromise security after updates. If an independent CGKA is used to construct an A-CGKA, the predicate $\mathsf{C}_{\mathsf{cgka}}$ may mostly depend on the security of the CGKA.

A more fine-grained characterization of this predicate is to write it as $\mathsf{C}_{\mathsf{cgka}} = \mathsf{C}_{\mathsf{cgka\text{-}opt}} \wedge \mathsf{C}_{\mathsf{cgka\text{-}add}}$, where $\mathsf{C}_{\mathsf{cgka\text{-}opt}}$ is an *optimal*, generic cleanness predicate that excludes only unavoidable trivial attacks, and $\mathsf{C}_{\mathsf{cgka\text{-}add}}$ is an additional cleanness predicate that depends on the scheme and excludes other attacks. We define $\mathsf{C}_{\mathsf{cgka\text{-}opt}}$ in a similar way to the safe predicate in [10]. Namely, we exclude the following cases: (i) the group secret in challenge epoch t_s^* was already challenged or revealed, and (ii) a group member ID whose state was exposed in epoch $t_{\mathsf{exp}} \leq t_s^*$ did not update their keys (i.e., processed their own commit, or processed a commit in which they were involved in an add, remove, or update proposal) or was not removed before the challenge epoch t_s^*. The optimal cleanness predicate is given in Figure 3 for an adversary that makes oracle queries $q_1, \ldots, q_q$ in the game.

The predicate is the logical disjunction of three clauses: for every exposure, adversarial challenge, and party ID, we require that either 1) ID was not a group member at the challenge

$$\boxed{\mathsf{C}_{\mathsf{cgka\text{-}opt}}:}\ \forall\ (i, \mathsf{ID}, \mathsf{ctr} \in (0, \mathsf{exp\text{-}ctr}]) : q_i = \mathcal{O}^{\mathsf{Challenge}}(t_i^*),$$
$$\big(\mathsf{ID} \notin \mathsf{G}[t_i^*]\big) \vee$$
$$\big(\exists (t_i, c) : (\mathsf{tExp}(\mathsf{ID}, \mathsf{ctr}).t_s < t_i \leq t_i^*) \wedge$$
$$\mathsf{hasUpd}_{\mathsf{std}}\,(\mathsf{ID}, \mathsf{T}[(t_i, \cdot), \texttt{'com'}, c], \mathsf{T}[(t_i, \cdot), \texttt{'vec'}, c]) \wedge$$
$$(\mathsf{C}[(t_i, \cdot)] = c)\big) \vee \big(t_i^* < \mathsf{tExp}(\mathsf{ID}, \mathsf{ctr}).t_s\big).$$

Figure 3: Optimal CGKA predicate where the adversary makes oracle queries $q_1, \ldots, q_n$.

$$\boxed{\mathsf{C}_{\mathsf{adm\text{-}opt}}:}\ \forall\ (i, \mathsf{ID}, \mathsf{ID}', \mathsf{ctr} \in (0, \mathsf{exp\text{-}ctr}]) : q_i = \mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}', \cdot, t_i^*),$$
$$\big(\mathsf{ID} \notin \mathsf{ADM}[t_i^*]\big) \vee$$
$$\big(\exists (t_i, c) : (\mathsf{tExp}(\mathsf{ID}, \mathsf{ctr}).t_a < t_i \leq t_i^*) \wedge$$
$$\mathsf{hasUpd}_{\mathsf{adm}}\,(\mathsf{ID}, \mathsf{T}[(\cdot, t_i), \texttt{'com'}, c], \mathsf{T}[(\cdot, t_i), \texttt{'vec'}, c]) \wedge$$
$$(\mathsf{C}[(\cdot, t_i)] = c)\big) \vee \big(t_i^* < \mathsf{tExp}(\mathsf{ID}, \mathsf{ctr}).t_a\big).$$

Figure 4: Optimal administrative predicate where the adversary makes oracle queries $q_1, \ldots, q_n$.

time, 2) the challenged epoch is previous to the exposure (forward security), or 3) ID updated before the challenge (post-compromise security). To avoid cluttering the predicate, our game already enforces that only one challenge or reveal can be performed per epoch (which is optimal for our game).

We have used the following auxiliary functions. The function tExp is such that $\mathsf{tExp}(\mathsf{ID}, \mathsf{ctr}) = \mathsf{exp}[\mathsf{ID}, \mathsf{ctr}]$ if $\exists k : q_k = \mathcal{O}^{\mathsf{Expose}}(\mathsf{ID})$, and -1 (for CGKA) or $(-1, -1)$ (for A-CGKA) otherwise. Given $\vec{P}$, the function $\mathsf{hasUpd}_{\mathsf{std}}(\mathsf{ID}, (T, \mathsf{com\text{-}type}), \vec{P})$ (sans $\mathsf{com\text{-}type}$ for CGKA) outputs true if either: (i) ID has processed a commit of his own, where $\mathsf{com\text{-}type} \in \{\mathsf{std}, \mathsf{both}\}$, or (ii) ID is a user affected by an add, update, or removal proposal in $\vec{P}$.

3.4.4 Admin cleanness predicate

The second predicate $\mathsf{C}_{\mathsf{adm}}$ models administration security. This predicate should be more permissive in some aspects than $\mathsf{C}_{\mathsf{cgka}}$, since a forgery attempt should be permitted even if the adversary knows the state of a (standard) group member. Following the approach above, we can decompose $\mathsf{C}_{\mathsf{adm}} = \mathsf{C}_{\mathsf{adm\text{-}opt}} \wedge \mathsf{C}_{\mathsf{adm\text{-}add}}$.

$\mathsf{C}_{\mathsf{adm\text{-}opt}}$ is symmetric to $\mathsf{C}_{\mathsf{cgka\text{-}opt}}$ and excludes the following family of attacks: the adversary attempts a forgery on a member ID' at an administrative epoch t_a^* while having exposed the state of an administrator $\mathsf{ID} \in G^*$ at an administrative epoch $t_{\mathsf{exp}} \leq t_a^*$, such that ID has not updated at some point between them. The predicate is optimal, as any attack that it excludes must occur while an administrator is directly under state exposure. In the game itself, we also require that ID' is in the specified challenge epoch, i.e., $\mathsf{ep}[\mathsf{ID}'] = (\cdot, t_a^*)$. Notice that this predicate is unrelated to the common group secret and standard epochs t_s, and only relates to administration dynamics.

The optimal administrative predicate $\mathsf{C}_{\mathsf{adm\text{-}opt}}$ is captured in Figure 4. In the expression, the function $\mathsf{hasUpd}_{\mathsf{adm}}$ is defined as in $\mathsf{hasUpd}_{\mathsf{std}}$, except it is defined with respect to $\mathsf{com\text{-}type} \in \{\mathsf{adm}, \mathsf{both}\}$ (rather than $\mathsf{com\text{-}type} \in \{\mathsf{std}, \mathsf{both}\}$).

3.4.5 Limitations

Our security definition does not allow arbitrary message injections to participants. Thus, attacks on robustness are not captured by our security model. In particular, so long as non-admins are allowed to make commits, our A-CGKA schemes will only provide as much security as the underlying core CGKA: using MLS's TreeKEM, for example, a malicious non-admin can deny service by sending a malformed commit message that can be processed only by some of the users. This can be fixed at the expense of using NIZKs within TreeKEM [12, 43].

If only admins are allowed to commit, then our schemes (to be introduced) may nevertheless be safe against this attack vector for some non-strongly robust variants of TreeKEM, such as the one used in MLS [3] (up to version 13 at the time of writing). Standard users can still attain FS and PCS guarantees, and in particular PCS when their update proposals are committed. Nevertheless, efficiency may decrease significantly; in the case of TreeKEM this is due to tree blanking.

Among the broader family of group key agreement protocols, where long-lived sessions and PCS are not always considered, modelling fully active adversaries is common [17]. We also do not model authentication (we implicitly assume an incorruptible PKI) and randomness manipulation. We leave these for future work.

We also note that we can capture multi-group security rather easily. The main difference (besides increased notation complexity introduced by the gids as in Figure 13) appears in the state exposure oracle; exposing the state of a party implies a security loss in all groups that the party is a member of simultaneously. The feature is not included as our security proofs are in the single-group setting.

4 Constructions

A first attempt of A-CGKA is that group members keep a list of who the administrators are. Whenever an admin wants to make a commit, it can simply check whether the admin-changing proposals have been made by administrators, then commit them, and the other users will verify the admin condition upon processing. This approach is functional, but not secure in our model due to a lack of admin authentication. An adversary can easily forge a commit message and impersonate an admin unless this message is authenticated (for example signed). CGKA security [10, 11] does not imply such level of authentication.

One partial fix is that admins sign using a key derived from a long-term identity key (as provided in protocols such as WhatsApp [16]). Then, security cannot be recovered if the admin is compromised once, resulting in the adversary winning the A-CGKA game too. Our constructions provide FS and PCS to admin authentication mechanisms in order to circumvent this problem.

We note that authentication of group members individually is not required for administration; this distinction is made in our second protocol, DGS.

4.1 Individual Admin Signatures

In our first construction, *individual admin signatures* (IAS), we build a generic and modular administration mechanism on top of an arbitrary CGKA protocol denoted by CGKA. Each group administrator $\mathsf{ID} \in G^*$ maintains a (unique) signature key pair $(\mathsf{ssk}, \mathsf{spk})$. Each such key pair is independent from the keys used in CGKA, which is mostly used as a black-box. Group

members keep track of the list of admins G^* which is updated after every control message is processed. Proposed changes to the group and to the administration are signed using an admin's keying material.

4.1.1 Protocol

The IAS construction is presented in Figures 5 and 6, 7. The first figure describes the A-CGKA algorithms, and the second describes helper functions and auxiliary methods. We note that the algorithms defined in Figure 5 are incomplete without the helper functions; therefore, the construction spans both figures.

States. We represent the state of a participant by the symbol γ, which is in part a dictionary of states, indexed by group identifiers i.e. $\gamma[\mathsf{gid}]$. Users further maintain a common state via $\gamma.\mathsf{s0}$ encoding the underlying CGKA state, security parameter 1^λ in $\gamma.1^\lambda$ and the user's ID in $\gamma.\mathsf{ME}$. For each group gid, users keep a separate state that encodes the list of group administrators $\gamma[\mathsf{gid}].\mathsf{adminList}$ and two administration-related signature key pairs. The state also keeps the group members as $\gamma[\mathsf{gid}].G = \gamma[\mathsf{gid}].\mathsf{s0}.G$, the admins as $\gamma[\mathsf{gid}].G^* = \{\gamma[\mathsf{gid}].\mathsf{adminList}[\mathsf{ID}].\mathsf{ID}, \forall\, \mathsf{ID}\}$, and the CGKA key as $\gamma[\mathsf{gid}].\mathsf{k} = \gamma[\mathsf{gid}].\mathsf{s0}.\mathsf{k}$.

All implemented A-CGKA algorithms, including init, are stateful as if executed by the same party and, as written, *do not explicitly return the updated local state*. Instead, they modify the state during runtime. In the event of algorithm failure, the state is not modified and appropriate failure values are output.

In our functions in Figures 5 and 6, 7, we often omit the group identifier of the state to simplify presentation. We assume that γ refers to $\gamma[\mathsf{gid}]$ whenever gid is a subject of the algorithm, such as when it is a parameter of the function.

Randomness. In our construction, we make randomness used by protocol algorithms explicit, including sampled randomness $r_0 \in \{0,1\}^\lambda$ as input. Namely, for the input randomness r_0 used in any randomised method, we apply a PRF $(r_1, \ldots, r_k) \leftarrow H_k(r_0, \gamma)$ that combines the entropy of r_0 and the state γ. We do this to reduce the impact of randomness leakage and manipulation attacks [8]: without prior knowledge of γ (and assuming it has a sufficient entropy), an adversary that reads or manipulates r_0 will not be able to derive a corresponding r_i value. This is an additional feature that aims to maintain certain security properties in stronger adversarial models than the considered here, and that does not interfere with the rest of the protocol.

PKI. As mentioned in the introduction, IAS assumes a basic, incorruptible PKI functionality where all parties are authenticated with the PKI. The PKI provides a fresh signature public key spk for which only the party ID can retrieve the corresponding secret key ssk. This functionality is used only when the group of administrators expands; namely, when a group gid is created (by some party ID') or when an admin add proposal is crafted by ID'. For this purpose, we define a getSpk algorithm, which on input $(\mathsf{ID}, \mathsf{ID}', \mathsf{gid})$ outputs spk.

Parties upload such signature key pairs $(\mathsf{ssk}, \mathsf{spk})$ to the PKI upon initialization, via an abstract $\mathsf{registerKeys}(\mathsf{ID})$ method[3]. We also assume a method of the form $\mathsf{getSsk}(\mathsf{spk}, \mathsf{ID})$ that returns the ssk associated to spk when called by ID given they uploaded it.

[3]This abstraction is made to reduce notational complexity. We also assume that if all keys are depleted during protocol execution (for example due to multiple groups), a party can upload new key packages.

We assume that the underlying CGKA implements its own authentication or key retrieval mechanism, which may be a similar PKI, for instance as in [10, 11].

4.1.2 Description

Initialization. Before the creation of a group, a participant starts by calling the init method, which initializes the state γ. In turn, init calls the corresponding CGKA.init method of the underlying CGKA to initialize its state $\gamma.\mathsf{s0}$. The remaining fields are set to their corresponding values. $(\mathsf{ssk}, \mathsf{spk})$ and $(\mathsf{ssk}', \mathsf{spk}')$ are two signature key pairs for group administration. The first pair is the valid admin signature key pair using during protocol execution, while the second pair stores updated keys after a commit or a key update operation is done by the participant but before it is processed (i.e. acts as a temporary variable). After successfully processing a commit message, the second key pair replaces the first.

Group creation. The create algorithm creates the group gid from the list of members G, the admin list from G^*, and outputs a (signed) control message T for the new members in G. The adminList variable includes pairs of the form $(\mathsf{ID}, \mathsf{spk}_{\mathsf{ID}})$ for parties $\mathsf{ID} \in G^*$. The public signature keys are obtained via getSpk and each admin's private key can be retrieved from the PKI via getSsk when they process T. The group creator directly stores such key pair as $(\gamma.\mathsf{ssk}', \gamma.\mathsf{spk}')$.

Proposals. Any group member can use prop to create a proposal of a non-admin type; the algorithm calls CGKA.prop in this case. Administrative proposals are restricted to admins and crafted by makeAdminProp, which includes an administrative signature in the proposal. The signature is included to prevent an (insider) adversary from forging the sender of the proposal in an attempt to impersonate an admin. Proposal creation does not have any effect on the state other than the storage of temporary keys given type = upd-adm. In the case of an add-adm proposal to promote ID admin status, the proposer $\gamma.\mathsf{ME}$ retrieves a public signature key spk from ID using getSpk.

The prop-info method simply retrieves the main information of a proposal. As mentioned, it could be adapted to support CGKAs and A-CGKAs where proposals are encrypted[4] (under some key derived from the group key, for instance).

Commits. The commit algorithm can only be called by group administrators (except for the special case in which only key updates are proposed, when standard users can commit), and performs the following actions:

1. Clean the input vector of proposals $\vec{P}$, ensuring that they are well-formed. This is done via the propCleaner algorithm, which in turn calls the enforcePolicy method. For security reasons, we adopt the main features of the MLS policy (removing duplicates and prioritizing removals) [3] in our construction, but extensions to this policy can be implemented. In addition, we verify the legitimacy of the admin proposals via verifyPropSigs, and their validity via the predicate VALID_P. This predicate verifies that the gid matches, that added users (respectively admins) do not belong to G (resp. G^*), that removed users do

[4]Special precaution must be taken with respect to security when proposals are encrypted under the CGKA key, as the adversary gains access to multiple additional ciphertexts and this can result in a security loss.

$\mathsf{init}(1^\lambda, \mathsf{ID}; r_0)$

1 : $\gamma.\mathsf{s0} \leftarrow \mathsf{CGKA.init}(1^\lambda, \mathsf{ID}; r_0)$
2 : $\gamma.\mathsf{ME} \leftarrow \mathsf{ID}; \quad \gamma.1^\lambda \leftarrow 1^\lambda$
3 : $\gamma[\cdot].\mathsf{adminList}[\cdot] \leftarrow \bot$ // stores (ID, spk) pairs
4 : $\gamma[\cdot].\mathsf{ssk}, \gamma[\cdot].\mathsf{spk} \leftarrow \bot$ // active admin key pair
5 : $\gamma[\cdot].\mathsf{ssk}', \gamma[\cdot].\mathsf{spk}' \leftarrow \bot$ // temporary key pair
6 : registerKeys(ID) // Upload keys to PKI

$\mathsf{prop}(\mathsf{gid}, \mathsf{ID}, \mathsf{type}; r_0)$

1 : $P \leftarrow \bot$; $(r_1, r_2) \leftarrow H_2(r_0, \gamma)$
2 : **if** type = $*$-adm
// Note if type = upd-adm, keys are updated
3 : **require** $\gamma.\mathsf{ME} \in \gamma.\mathsf{adminList}$
4 : $P \leftarrow \mathsf{makeAdminProp}(\mathsf{gid}, \mathsf{type}, \mathsf{ID}; r_1, r_2)$
5 : **else** $(\gamma.\mathsf{s0}, P) \leftarrow$
$\mathsf{CGKA.prop}(\gamma.\mathsf{s0}, \mathsf{gid}, \mathsf{ID}, \mathsf{type}; r_1)$
6 : **return** P

$\mathsf{commit}(\mathsf{gid}, \vec{P}, \mathsf{com\text{-}type}; r_0)$

1 : **require** $\gamma.\mathsf{ME} \in \gamma.\mathsf{s0}.G$
2 : **require** com-type $\in \{\mathsf{adm}, \mathsf{std}, \mathsf{both}\}$
3 : $(r_1, \ldots, r_4) \leftarrow H_4(r_0, \gamma)$
4 : $(\vec{P}_0, \vec{P}_A, \mathsf{admReq}) \leftarrow \mathsf{propCleaner}(\vec{P}, \mathsf{gid})$
5 : **if** admReq $\vee$ (com-type $\in \{\mathsf{adm}, \mathsf{both}\}$)
6 : **require** $\gamma.\mathsf{ME} \in \gamma.\mathsf{adminList}$
7 : **if** com-type $\in \{\mathsf{adm}, \mathsf{both}\}$
8 : **require** $\mathsf{verifyPropSigs}(\vec{P}_A)$ // admin sigs.
9 : $C_A \leftarrow \vec{P}_A$
10 : **if** com-type $\in \{\mathsf{std}, \mathsf{both}\}$
11 : $(C_0, W_0, \mathsf{adminList}) \leftarrow$
$\mathsf{c\text{-}Std}(\mathsf{gid}, \vec{P}_0, \vec{P}_A; r_1)$
12 : **require** $C_0 \neq \bot$
13 : // generate new key pair and sign new spk
14 : $(\gamma.\mathsf{ssk}', \gamma.\mathsf{spk}') \leftarrow \mathsf{SigGen}(\gamma.1^\lambda; r_2)$
15 : $T_C \leftarrow (\mathsf{gid}, \text{'comm'}, \gamma.\mathsf{ME}, C_0, C_A, \gamma.\mathsf{spk}')$
16 : **if** $W_0 \neq \bot$ // share updated admin list
17 : $T_W \leftarrow (\mathsf{gid}, \text{'wel'}, \gamma.\mathsf{ME}, W_0, \mathsf{adminList})$
18 : **else** $T_W \leftarrow \bot$
19 : $\sigma_T \leftarrow \mathsf{Sig}(\gamma.\mathsf{ssk}, (T_C, T_W); r_4)$
20 : **else** // no group changes - no sign required
21 : $(C_0, \bot, \bot) \leftarrow \mathsf{c\text{-}Std}(\mathsf{gid}, \vec{P}_0, \bot; r_3)$
22 : $T_C \leftarrow (\mathsf{gid}, \text{'comm'}, \gamma.\mathsf{ME}, C_0, \bot, \bot, \bot)$
23 : $T_W \leftarrow \bot$; $\sigma_T \leftarrow \bot$
24 : **return** (T_C, T_W, σ_T)

$\mathsf{create}(\mathsf{gid}, G, G^*; r_0)$

1 : **require** $(\gamma.\mathsf{ME} \in G^*) \wedge (G^* \subseteq G)$
2 : $(r_1, r_2) \leftarrow H_2(r_0, \gamma)$
3 : $(\gamma.\mathsf{s0}, W_0) \leftarrow \mathsf{CGKA.create}(\gamma.\mathsf{s0}, \mathsf{gid}, G; r_1)$
4 : **if** $W_0 = \bot$ **return** $\bot$
5 : $\mathsf{adminList}[\cdot] \leftarrow \bot$ // this is not $\gamma.\mathsf{adminList}$
6 : **for** ID $\in G^*$
7 : $\mathsf{adminList}[\mathsf{ID}] \leftarrow (\mathsf{ID}, \mathsf{getSpk}(\mathsf{ID}, \gamma.\mathsf{ME}))$
8 : $\gamma.\mathsf{spk}' \leftarrow \mathsf{adminList}[\mathsf{ME}]$
9 : $\gamma.\mathsf{ssk}' \leftarrow \mathsf{getSsk}(\gamma.\mathsf{spk}', \mathsf{ME})$
10 : $T_W \leftarrow (\mathsf{gid}, \text{'wel'}, \gamma.\mathsf{ME}, W_0, \mathsf{adminList})$
11 : **return** $(T_W, \mathsf{Sig}(\gamma.\mathsf{ssk}', T_W; r_2))$

$\mathsf{proc}(T)$

1 : $(T_C, T_W, \sigma_T) \leftarrow T$
2 : $\mathsf{acc} \leftarrow \mathsf{false}$; $\mathsf{gid} \leftarrow T_C.\mathsf{gid}$
3 : **if** $T_W.\mathsf{gid} \neq \bot$ $\mathsf{gid} \leftarrow T_W.\mathsf{gid}$
4 : **if** $(\gamma.\mathsf{ME} \notin \gamma[\mathsf{gid}].\mathsf{s0}.G) \wedge (T_W \neq \bot)$
5 : $(\cdot, \mathsf{msg\text{-}type}, \cdots) \leftarrow T_W$
6 : **require** msg-type = qwel'
7 : $\mathsf{acc} \leftarrow \mathsf{p\text{-}Wel}(T_W)$ // Welcome message
8 : **else if** $(\gamma.\mathsf{ME} \in \gamma[\mathsf{gid}].\mathsf{s0}.G) \wedge (T_C \neq \bot)$
9 : $(\bot, \mathsf{msg\text{-}type}, C_0, \cdots) \leftarrow T_C$
10 : **require** msg-type = qcomm'
11 : **if** $\sigma_C = \bot$ // no sign - check no changes to G
12 : $(\gamma', \mathsf{acc}) \leftarrow \mathsf{CGKA.proc}(\gamma[\mathsf{gid}].\mathsf{s0}, C_0)$
13 : **if** $\neg\mathsf{acc} \vee (\gamma'[\mathsf{gid}].\mathsf{s0}.G \neq \gamma[\mathsf{gid}].\mathsf{s0}.G)$
14 : **return** false
15 : $\gamma[\mathsf{gid}].\mathsf{s0} \leftarrow \gamma'$; **return** true
// Signature verification
16 : **if** $\neg[(\mathsf{ID} \in \gamma[\mathsf{gid}].\mathsf{adminList}) \wedge$
$(\mathsf{Ver}(\gamma[\mathsf{gid}].\mathsf{adminList}[\mathsf{ID}].\mathsf{spk}, (T_C, T_W), \sigma_T))]$
17 : **return** false
18 : $\mathsf{acc} \leftarrow \mathsf{p\text{-}Comm}(T_C)$ // Admin commit message
19 : **return** acc

$\mathsf{prop\text{-}info}(P)$

1 : **if** P is a CGKA proposal
$(P.\mathsf{gid}, P.\mathsf{type}, P.\mathsf{ID}, P.\mathsf{ID}') \leftarrow$
$\mathsf{CGKA.prop\text{-}info}(\gamma.\mathsf{s0}, P)$
2 : **else if** P is an admin proposal
3 : $(P.\mathsf{gid}, P.\mathsf{type}, P.\mathsf{ID}, P.\mathsf{ID}', \bot, \bot) \leftarrow P$
4 : **return** $(P.\mathsf{gid}, P.\mathsf{type}, P.\mathsf{ID}, P.\mathsf{ID}')$

Figure 5: Individual admin signatures (IAS) construction of an A-CGKA, built from a CGKA, a signature scheme $\mathcal{S} = (\mathsf{SigGen}, \mathsf{Sig}, \mathsf{Ver})$, and n-PRFs $H_n : R \times \mathsf{ST} \to R^n$ for $n \leq 4$, randomness space R and state space ST. The state values representing the group, the admins, and the group key are assigned as: $\gamma[\mathsf{gid}].G = \gamma[\mathsf{gid}].\mathsf{s0}.G$, $\gamma[\mathsf{gid}].G^* = \{\gamma[\mathsf{gid}].\mathsf{adminList}[\mathsf{ID}].\mathsf{ID}, \forall\, \mathsf{ID}\}$, and $\gamma[\mathsf{gid}].\mathsf{k} = \gamma[\mathsf{gid}].\mathsf{s0}.\mathsf{k}$.

VALID$_P$

// Predicate checks validity of admin proposal
$(P.\mathsf{gid}, P.\mathsf{type}, P.\mathsf{ID}, P.\mathsf{ID}') \leftarrow \mathsf{prop\text{-}info}(P)$
$S_1 := (P.\mathsf{gid} = \mathsf{gid})$ // correct group
$S_2 := (P.\mathsf{ID} \in \gamma[\mathsf{gid}].G)$ // ID member
$S_3 := (P.\mathsf{ID}' \in \gamma[\mathsf{gid}].G^*)$ // ID$'$ admin
$C_1 := (P.\mathsf{type} = \mathsf{rem\text{-}adm})$
$S_4 := (P.\mathsf{ID} \in \gamma[\mathsf{gid}].G^*)$ // ID admin
$C_2 := (P.\mathsf{type} = \mathsf{add\text{-}adm})$
return $S_1 \wedge S_2 \wedge S_3 \wedge (\neg C_1 \vee S_4) \wedge \neg(C_2 \wedge S_4)$

makeAdminProp(gid, type, ID; r_1, r_2)

$P_0 \leftarrow \perp$
if type = add-adm
  $\mathsf{spk}_{\mathsf{pki}} \leftarrow \mathsf{getSpk}(\mathsf{ID}, \gamma.\mathsf{ME})$
  $P_0 \leftarrow (\mathsf{gid}, \mathsf{type}, \mathsf{ID}, \gamma.\mathsf{ME}, \mathsf{spk}_{\mathsf{pki}})$
else if type = rem-adm
  $P_0 \leftarrow (\mathsf{gid}, \mathsf{type}, \mathsf{ID}, \gamma.\mathsf{ME}, \perp)$
else if type = upd-adm
  if $(\gamma.\mathsf{ssk}', \gamma.\mathsf{spk}') \neq (\perp, \perp)$
    return $\perp$ // only one update per epoch
  $(\gamma.\mathsf{ssk}', \gamma.\mathsf{spk}') \leftarrow \mathsf{SigGen}(\gamma.1^\lambda; r_1)$
  $P_0 \leftarrow (\mathsf{gid}, \mathsf{type}, \gamma.\mathsf{ME}, \gamma.\mathsf{ME}, \gamma.\mathsf{spk}')$
else return $\perp$
return $(P_0, \mathsf{Sig}(\gamma.\mathsf{ssk}, P_0; r_2))$

c-Std(gid, $\vec{P}_0, \vec{P}_A; r_1$)

$(\gamma.\mathsf{s0}, C_0, W_0) \leftarrow \mathsf{CGKA.commit}(\gamma.\mathsf{s0}, \mathsf{gid}, \vec{P}_0; r_1)$
if $W_0 \neq \perp$
  $\mathsf{adminList}' \leftarrow \mathsf{updAL}(\gamma.\mathsf{adminList}, \vec{P}_A)$
return $(C_0, W_0, \mathsf{adminList}')$
else return $(C_0, \perp, \perp)$

verifyPropSigs($\vec{P}_A$)

for $(P, \sigma_P) \in \vec{P}_A$:
  $(\perp, \perp, \perp, \mathsf{ID}') \leftarrow \mathsf{prop\text{-}info}(P)$
  $\mathsf{spk}_P \leftarrow \mathsf{adminList}[\mathsf{ID}'].\mathsf{spk}$
  if $\neg(\mathsf{Ver}(\mathsf{spk}_P, P, \sigma_P) \wedge \mathsf{VALID_P})$
    return false
return true

propCleaner($\vec{P}$, gid)

$\mathsf{admReq} \leftarrow \mathsf{false}$; $\vec{P}_0, \vec{P}_A \leftarrow []$
for $P \in \vec{P}$:
  $(\mathsf{gid}, P.\mathsf{type}, P.\mathsf{ID}, P.\mathsf{ID}') \leftarrow \mathsf{prop\text{-}info}(P)$
  if $(P.\mathsf{type} = *\text{-adm}) \wedge \mathsf{VALID}_P$
    $\vec{P}_A \leftarrow [\vec{P}_A, P]$
    $\mathsf{admReq} \leftarrow \mathsf{true}$
  else // $\vec{P}_0$ is handled by CGKA
    $\vec{P}_0 \leftarrow [\vec{P}_0, P]$
    if $P.\mathsf{type} \in \{\mathsf{add}, \mathsf{rem}\}$
      $\mathsf{admReq} \leftarrow \mathsf{true}$
    // admin rem from $G \implies$ rem from G^*
    if $(P.\mathsf{type} = \mathsf{rem}) \wedge (P.\mathsf{ID} \in \gamma[\mathsf{gid}].G^*)$
      $P' \leftarrow \mathsf{makeAdminProp}(\mathsf{gid}, \mathsf{rem}, \mathsf{ID}; \perp)$
      $\vec{P}_A \leftarrow [\vec{P}_A, P']$
$(\vec{P}_0, \vec{P}_A) \leftarrow \mathsf{enforcePolicy}(\vec{P}_0, \vec{P}_A)$
return $(\vec{P}_0, \vec{P}_A, \mathsf{admReq})$

p-Wel(T_W)

$(\mathsf{gid}, \perp, \perp, W_0, \mathsf{adminList}) \leftarrow T_W$
$(\gamma[\mathsf{gid}].\mathsf{s0}, \mathsf{acc}) \leftarrow \mathsf{CGKA.proc}(\gamma.\mathsf{s0}, W_0)$
if acc $\gamma[\mathsf{gid}].\mathsf{adminList} \leftarrow \mathsf{adminList}$
if acc $\wedge$ $(\mathsf{adminList}[\mathsf{ME}] \neq \perp)$
  $\gamma.\mathsf{spk} \leftarrow \mathsf{adminList}[\mathsf{ME}].\mathsf{spk}$
  $\gamma.\mathsf{ssk} \leftarrow \mathsf{getSsk}(\mathsf{spk}, \mathsf{ME})$
return acc

updAL(adminList, $\vec{P}_A$)

for $P \in \vec{P}_A$
  $(\mathsf{gid}, \mathsf{type}, \mathsf{ID}, \perp, \mathsf{spk}, \perp) \leftarrow P$
  if $\mathsf{type} \in \{\mathsf{add\text{-}adm}, \mathsf{upd\text{-}adm}\}$
    if $(\mathsf{type} = \mathsf{add\text{-}adm}) \wedge (\mathsf{ID} = \gamma.\mathsf{ME})$
      $\gamma.\mathsf{spk} \leftarrow \mathsf{spk}$
      $\gamma.\mathsf{ssk} \leftarrow \mathsf{getSsk}(\mathsf{spk}, \mathsf{ID})$
    $\mathsf{adminList}[\mathsf{ID}] \leftarrow (\mathsf{ID}, \mathsf{spk})$
  if $\mathsf{type} = \mathsf{rem\text{-}adm}$
    $\mathsf{adminList}[\mathsf{ID}] \leftarrow \perp$
    if $(\mathsf{ID} = \gamma.\mathsf{ME})$
      $(\gamma.\mathsf{ssk}, \gamma.\mathsf{spk}) \leftarrow (\perp, \perp)$
return adminList

Figure 6: Helper functions for the IAS construction in Figure 5 (part I).

p-Comm(T_C)

1 : $(\mathsf{gid}, \perp, \mathsf{ID}, C_0, C_A, \mathsf{spk}) \leftarrow T_C$
// check signatures in proposals
2 : **if** $C_A \neq \perp$
3 : **if** $\neg$verifyPropSigs$(C_A = \vec{P}_A)$ **return** false
// apply commit
4 : **if** $C_0 \neq \perp$
5 : $(\gamma', \mathsf{acc}) \leftarrow \mathsf{CGKA.proc}(\gamma.\mathsf{s0}, C_0)$
6 : **if** acc = false **return** false
7 : **if** $\gamma.\mathsf{ME} \notin \gamma'.G$ // user removed
8 : $\gamma[\mathsf{gid}] \leftarrow \perp$ // reinitialize state (only for gid)
9 : **else** $\gamma[\mathsf{gid}].\mathsf{s0} \leftarrow \gamma'$
// set temporary updated keys
10 : **if** $(\mathsf{ID} = \gamma.\mathsf{ME}) \vee (\exists P \in C_A : P.\mathsf{ID} = \gamma.\mathsf{ME})$
11 : $(\gamma.\mathsf{ssk}, \gamma.\mathsf{spk}) \leftarrow (\gamma.\mathsf{ssk}', \gamma.\mathsf{spk}')$
12 : $\gamma.\mathsf{ssk}', \gamma.\mathsf{spk}' \leftarrow \perp$
13 : $\gamma.\mathsf{adminList} \leftarrow \mathsf{updAL}(\gamma.\mathsf{adminList}, C_A)$
14 : $\gamma.\mathsf{adminList}[\mathsf{ID}].\mathsf{spk} \leftarrow \mathsf{spk}$ // committer's key
15 : **return** true

enforcePolicy($\vec{P}_0, \vec{P}_A$)

// This method can be extended to other policies
1 : numAdmins $\leftarrow |G^*|$
2 : **for** $P \in [\vec{P}_0, \vec{P}_A]$
3 : $(\mathsf{gid}, P.\mathsf{type}, P.\mathsf{ID}, P.\mathsf{ID}') \leftarrow \mathsf{prop\text{-}info}(P)$
4 : **if** P.type = rem // If duplicates, removal prevails
5 : *delete any other* P' *s.t.* $P.\mathsf{ID} = P'.\mathsf{ID}$
6 : *except for* rem-adm *proposals*
7 : **else if** P.type = rem-adm
8 : *delete any other admin* P' *s.t.* $P.\mathsf{ID} = P'.\mathsf{ID}$
9 : **else if** P.type = rem-adm, numAdmins−−
10 : **else if** P.type = add-adm, numAdmins++
11 : **require** numAdmins ≥ 1 // Ensures $\emptyset \neq G^* \subseteq G$
12 : **return** $(\vec{P}_0, \vec{P}_A)$

Figure 7: Helper functions for the IAS construction in Figure 5 (part II).

belong to G (resp. G^*), and that the proposer is an admin. Finally, we ensure that all users removed from G are also removed from the adminList.

2. Carry out the administrative and the standard commits and produce an administrative commit message C_A (which is the clean admin proposal vector), a standard CGKA commit C_0, and an updated adminList. We split the CGKA commit in two components C_0 and W_0 as usual in the literature [10, 21, 11, 12]. If the CGKA does not allow for this, it is easy to modify the protocol without compromising security[5].

3. Generate a new (temporary) administrative signature key pair $(\gamma.\mathsf{ssk}', \gamma.\mathsf{spk}')$.

4. Produce the final control message T which includes the new spk′. The message T is again split into two components: A first component T_W (for welcome) includes all the required information for A-CGKA members, such as the administrative updates. A second component T_C (for commit) contains the updating information for group members. Both components are signed together using the committer's current $\gamma.\mathsf{ssk}$, but may also be signed separately.

Processing control messages. The proc method takes a control message T as input and updates the state accordingly. The algorithm returns an acceptance bit acc which is true if the processing succeeds, in which case the state is updated. Otherwise, the state remains the same. During an execution of proc, some checks must pass before the state is updated. For newly added

[5]This division is made for clarity, but may be used to improve efficiency too. Namely, the welcome part W_0 of a commit message does not need to be sent to group members, and C_0 can be sent only to these. In A-CGKAs such as IAS, this can also be applied if signatures are handled carefully to prevent insider attacks.

users, p-Wel verifies the message signature on the adminList, attempts to process the message via the underlying CGKA, and updates the state given this succeeds. For group members, p-Com verifies the administrator signature and the signatures in the admin proposals. The state is updated if all verification succeeds; a removed user blanks their state, and temporary keys are updated if necessary. The case in which the T is not signed is handled by proc directly by verifying that no changes to the group structure are made (only key updates).

4.1.3 Features

We first note that the IAS protocol can be built over any CGKA. Since signatures are often already present in CGKAs such as [21], the extension from CGKA to A-CGKA can be more direct (and less expensive) than presented here.

Commit and propose policies. Our construction allows standard users to perform a commit if there are no changes in the group structure or in the administration. This feature is an optional design choice that does not affect security in our model (and could be reflected in a correctness predicate), although, as previously discussed, adversarial group members may deny service if the underlying CGKA is not robust. However, we do not allow standard users to propose administrative changes (even if these could be later ignored by admins).

Security mechanisms. The security in the group administration is provided by the admin signatures; an adversary should not be able to commit changes to the group unless it compromises the state of one of the group administrations. The update mechanism provides optimal post-compromise security.

On the other hand, administrative actions are undeniable and traceable both by group members and by the message delivery service. Separately, achieving security guarantees for incoming users is not straightforward; additional protections are needed to ensure that the list of group administrators is not forged.

4.1.4 On optimal forward security

Note that, as defined, our construction does not satisfy forward security with respect to injection queries even if the underlying CGKA provides optimal forward security. Concretely, suppose that ID makes their last update in epoch 3, and then their state is exposed in epoch 5. Then ID can trivially forge commit messages for parties that are in epochs 3 and 4 since their keying material has not been updated. A broadly similar issue was present in earlier drafts of MLS with respect to confidentiality [10].

Optimal security can be easily achieved by replacing regular signatures with *forward-secure signatures* [44]. Forward-secure signatures allow signers to non-interactively update their secret keys and provide forward security given state exposure. In IAS, it suffices to use forward-secure signatures such that whenever an epoch passes and an admin has not signed a new key, they update their secret key, where new signature keys are otherwise derived as in the construction. We note that forward-secure signatures involve an overhead that may be undesirable in some cases, and also they are not used in current protocols (signatures are already used in MLS' CGKA, for instance). In Theorem 1, we characterize the exact security of IAS using standard primitives via our sub-optimal predicate. In this way, the security of both alternatives is fully characterised.

4.2 Dynamic Group Signature

In our second construction, *dynamic group signature* (DGS), the group administrators agree on a *common* signature key pair that they use for signing administrative messages on an underlying CGKA. To agree on a secret and generate a common key pair, they run a separate CGKA. As opposed to IAS, now group administrators may be opaque to group members if the concrete CGKA which is used allows for it. The reason is that they authenticate admin messages using the admin signature key that is shared among all admins. Notably, group members do not need to keep track of an administrator list; admins implicitly track this via their CGKA.

4.2.1 Protocol

The DGS protocol is introduced in Figures 8 and 9. In the algorithm, we refer to the primary (or standard) CGKA as CGKA, and to the administrative CGKA as CGKA^*. The first CGKA allows group members to agree on a common secret and group composition as in IAS, whereas the second exists only for administrative purposes (i.e., admins deriving a common signature key). Note that CGKA^* is not necessarily implemented in the same way as the primary CGKA. This feature can be exploited by a protocol designer either for performance reasons or if, for instance, stronger FS and PCS guarantees are required for the administrative CGKA.

States. Each party stores $\gamma.\mathsf{s0}$, corresponding to the primary CGKA, as well as $\gamma.\mathsf{sA}$, corresponding to CGKA^*, which are used for each group gid they consider. Besides, the state includes a field for the administrative public key $\gamma[\mathsf{gid}].\mathsf{spk}$, which is known by all group members (and can be a public group parameter, known for instance by a central server) to enable verification. The state variables are now $\gamma[\mathsf{gid}].G = \gamma.\mathsf{s0}[\mathsf{gid}].G$, $\gamma[\mathsf{gid}].G^* = \gamma.\mathsf{sA}[\mathsf{gid}].G$, and $\gamma[\mathsf{gid}].\mathsf{k} = \gamma.\mathsf{s0}[\mathsf{gid}].\mathsf{k}$.

Authentication. As opposed to IAS, our specification of DGS does not include an explicit PKI. It can simply rely on the authentication mechanism implemented by CGKA^*, if any. We note that authentication could be done while preserving anonymity, such that a member authenticates his group membership but not his identity; such a feature cannot be provided by IAS without modification.

4.2.2 Description

Initialization. The init procedure initializes the state γ and has to be run by every party before executing any other algorithm. It calls the $\mathsf{CGKA.init}$ and $\mathsf{CGKA}^*.\mathsf{init}$ algorithms to initialize $\gamma.\mathsf{s0}$ and $\gamma.\mathsf{sA}$, respectively, and sets $\gamma[\cdot].\mathsf{spk}, \gamma[\cdot].\mathsf{ssk} \leftarrow \perp$.

Group creation. The create algorithm creates a group for the two separate CGKAs by calling the corresponding $\mathsf{CGKA.create}$ and $\mathsf{CGKA}^*.\mathsf{create}$ methods. These calls output new states $\mathsf{s0}$ and sA, which overwrite the stored states, as well as control messages W_0 and W_A, which are collated into a *create* control message $T = T_{CR}$.

Proposals. The prop algorithm generates a proposal message P by using $\mathsf{CGKA.prop}$ in case the input type is standard and $\mathsf{CGKA}^*.\mathsf{prop}$ if it is administrative (i.e. of the form $*\text{-}\mathsf{adm}$). A validity check on the caller ID' and the target ID of the proposal is made using the VALID_P

$\mathsf{init}(1^\lambda, \mathsf{ID})$

$\gamma.\mathsf{s0} \leftarrow\!\$\ \mathsf{CGKA.init}(1^\lambda, \mathsf{ID})$
$\gamma.\mathsf{sA} \leftarrow\!\$\ \mathsf{CGKA}^*\mathsf{.init}(1^\lambda, \mathsf{ID})$
$\gamma.\mathsf{ME} \leftarrow \mathsf{ID}; \quad \gamma.1^\lambda \leftarrow 1^\lambda$
$\gamma[\cdot].\mathsf{spk}, \gamma[\cdot].\mathsf{ssk} \leftarrow \bot$

$\mathsf{create}(\mathsf{gid}, G, G^*; r_0)$

require $(\gamma.\mathsf{ME} \in G^*) \wedge (G^* \subseteq G)$
$(r_1, r_2) \leftarrow H_2(r_0, \gamma)$
$(W_0, \gamma.\mathsf{s0}) \leftarrow \mathsf{CGKA.create}(\mathsf{gid}, G, \gamma.\mathsf{s0}; r_1)$
$(W_A, \gamma.\mathsf{sA}) \leftarrow \mathsf{CGKA}^*\mathsf{.create}(\mathsf{gid}^*, G^*, \gamma.\mathsf{sA}; r_2)$
$T_{CR} \leftarrow (\mathsf{gid}, \text{'create'}, W_0, W_A)$
return $(T_{CR}, \bot, \bot, \bot)$

$\mathsf{prop}(\mathsf{gid}, \mathsf{ID}, \mathsf{type}; r_0)$

$(r_1, r_2) \leftarrow H_2(r_0, \gamma)$
if $\mathsf{type} = *\text{-}\mathsf{adm}$
  require $\gamma.\mathsf{ME} \in \gamma.\mathsf{sA}.G$
  $(\gamma.\mathsf{sA}, P_0) \leftarrow \mathsf{CGKA}^*\mathsf{.prop}(\gamma.\mathsf{sA}, \mathsf{gid}^*, \mathsf{ID}, \mathsf{type}; r_1)$
  $P \leftarrow (P_0, \mathsf{Sig}(\gamma.\mathsf{ssk}, P_0; r_2))$
else
  $(\gamma.\mathsf{s0}, P) \leftarrow \mathsf{CGKA.prop}(\gamma.\mathsf{s0}, \mathsf{gid}, \mathsf{ID}, \mathsf{type}; r_1)$
return P

$\mathsf{prop\text{-}info}(P)$

if P is a CGKA proposal
  $(P.\mathsf{gid}, P.\mathsf{type}, P.\mathsf{ID}, P.\mathsf{ID}') \leftarrow$
    $\mathsf{CGKA.prop\text{-}info}(\gamma.\mathsf{s0}, P)$
else if P is an admin proposal
  $(P.\mathsf{gid}, P.\mathsf{type}, P.\mathsf{ID}, P.\mathsf{ID}') \leftarrow$
    $\mathsf{CGKA}^*\mathsf{.prop\text{-}info}(\gamma.\mathsf{sA}, P)$
  $P.\mathsf{type} \leftarrow P.\mathsf{type}||\text{-}\mathsf{adm}$
return $(P.\mathsf{gid}, P.\mathsf{type}, P.\mathsf{ID}, P.\mathsf{ID}')$

$\mathsf{commit}(\mathsf{gid}, \vec{P}, \mathsf{com\text{-}type}; r_0)$

require $\gamma.\mathsf{ME} \in \gamma.G$
require $\mathsf{com\text{-}type} \in \{\mathsf{adm}, \mathsf{std}, \mathsf{both}\}$
$(r_1, r_2, r_3) \leftarrow H_3(r_0, \gamma)$
$C_0, C_A, W_0, W_A \leftarrow \bot$
$(\vec{P}_0, \vec{P}_A, \mathsf{admReq}) \leftarrow \mathsf{propCleaner}(\vec{P}, \mathsf{gid})$
if $\mathsf{admReq} \vee (\mathsf{com\text{-}type} \in \{\mathsf{adm}, \mathsf{both}\})$
  require $\gamma.\mathsf{ME} \in \gamma.G^*$
  $(\mathsf{ssk}, \mathsf{spk}) \leftarrow \mathsf{getSigKey}(\gamma.\mathsf{sA}.k)$ // old keys
  if $\mathsf{com\text{-}type} \in \{\mathsf{adm}, \mathsf{both}\}$ // update spk
    $(\mathsf{spk}, C_A, W_A) \leftarrow \mathsf{c\text{-}Adm}(\mathsf{gid}, \vec{P}_A; r_1)$
  if $\mathsf{com\text{-}type} \in \{\mathsf{std}, \mathsf{both}\}$
    $(C_0, W_0) \leftarrow \mathsf{c\text{-}Std}(\mathsf{gid}, \vec{P}_0; r_2)$
  $T_C \leftarrow (\mathsf{gid}, \text{'comm'}, C_0, C_A, W_A, \mathsf{spk})$
  $T_W \leftarrow (\mathsf{gid}, \text{'wel'}, W_0, \mathsf{spk})$
  $\sigma_T \leftarrow \mathsf{Sig}(\mathsf{ssk}, (T_C, T_W); r_3)$
else // can be done by non-admins
  $(C_0, \bot) \leftarrow \mathsf{c\text{-}Std}(\mathsf{gid}, \vec{P}_0; r_1)$
  $T_C \leftarrow (\mathsf{gid}, \text{'comm'}, C_0, \bot, \bot, \bot, \bot)$
  $T_W, \sigma_T \leftarrow \bot$
return $(\bot, T_C, T_W, \sigma_T)$

$\mathsf{proc}(T)$

$(T_{CR}, T_W, T_C, \sigma_T) \leftarrow T; \quad \mathsf{acc} \leftarrow \mathsf{false}$
$\mathsf{gid} \leftarrow$ select from $T_{CR}, T_W,$ or T_C
if $T_{CR} \neq \bot$
  if $\gamma.\mathsf{ME} \in \gamma[\mathsf{gid}].\mathsf{s0}.G$ **return** false
  $\mathsf{acc} \leftarrow \mathsf{p\text{-}Create}(T_{CR})$
else if $(\gamma.\mathsf{ME} \notin \gamma[\mathsf{gid}].G) \wedge (T_W \neq \bot)$
  $\mathsf{acc} \leftarrow \mathsf{p\text{-}Wel}(T_C, T_W, \sigma_T)$
else if $(\gamma.\mathsf{ME} \in \gamma[\mathsf{gid}].G) \wedge (T_C \neq \bot)$
  $\mathsf{acc} \leftarrow \mathsf{p\text{-}Comm}(T_C, T_W, \sigma_T)$
return acc

Figure 8: Dynamic group signature (DGS) construction of an A-CGKA, built from two (possibly different) CGKAs, a signature scheme $\mathcal{S} = (\mathsf{SigGen}, \mathsf{Sig}, \mathsf{Ver})$, and n-PRFs $H_n : R \times \mathsf{ST} \to R^n$ for $n \leq 4$, randomness space R and state space ST. The state values representing the group, the admins, and the group key are assigned as: $\gamma[\mathsf{gid}].G = \gamma[\mathsf{gid}].\mathsf{s0}.G$, $\gamma[\mathsf{gid}].G^* = \gamma[\mathsf{gid}].\mathsf{sA}.G$, and $\gamma[\mathsf{gid}].\mathsf{k} = \gamma[\mathsf{gid}].\mathsf{s0}.\mathsf{k}$.

c-Adm(gid, $\vec{P}_A$; r_1)

1 : **for** $P \in \vec{P}_A$
2 : **if** P.type = add-adm
3 : **require** P.ID $\in \gamma.G$
4 : $(C_A, W_A, \gamma.\mathsf{sA}) \leftarrow$ CGKA*.commit($\gamma.\mathsf{sA}$, gid*, $\vec{P}_A$; r_1)
5 : **if** $C_A = \perp$ **return** $\perp$
// state processed temporarily
6 : $\mathsf{st}_{\mathsf{temp}} \leftarrow$ CGKA*.proc(C_A)
7 : (ssk, spk) $\leftarrow$ getSigKey($\mathsf{st}_{\mathsf{temp}}$.sA.$k$)
8 : **return** (spk, C_A, W_A)

c-Std(gid, $\vec{P}_0$; r_2)

1 : $(C_0, W_0, \gamma.\mathsf{s0}) \leftarrow$ CGKA.commit(gid, $\gamma.\mathsf{s0}$, $\vec{P}_0$; r_2)
2 : **return** (C_0, W_0)

propCleaner($\vec{P}$, gid)

1 : admReq $\leftarrow$ false; $\vec{P}_0, \vec{P}_A \leftarrow []$
2 : **for** $P \in \vec{P}$
3 : (gid, P.type, P.ID, P.ID′) $\leftarrow$ prop-info(P)
4 : **if** P.type $\neq$ gid **continue**
5 : **if** P.type = $*$-adm $\wedge$ IAS.VALID$_P$
6 : $\vec{P}_A \leftarrow [\vec{P}_A, P]$
7 : admReq $\leftarrow$ true
8 : **else**
9 : $\vec{P}_0 \leftarrow [\vec{P}_0, P]$
10 : **if** P.type $\in$ {add, rem}
11 : admReq $\leftarrow$ true
// admin rem from $G \implies$ rem from G^*
12 : **if** (P.type = rem) $\wedge$ (P.ID $\in \gamma.G^*$)
13 : $P' \leftarrow$ CGKA*.prop($\gamma.\mathsf{sA}$, gid*, P.ID, rem)
14 : $\vec{P}_A \leftarrow [\vec{P}_A, P']$
15 : $(\vec{P}_0, \vec{P}_A) \leftarrow$ enforcePolicy($\vec{P}_0, \vec{P}_A$)
16 : **return** $(\vec{P}_0, \vec{P}_A$, admReq)

getSigKey(r)

1 : (ssk, spk) $\leftarrow$ SigGen(1^λ; $H_{\mathsf{ro}}(r)$)
// Deterministic key generation from r
// Random oracle H_{ro} required
2 : **return** (ssk, spk)

enforcePolicy($\vec{P}_0, \vec{P}_A$)

1 : // Same syntax as in IAS
2 : **return** $(\vec{P}_0, \vec{P}_A) \leftarrow$ IAS.enforcePolicy($\vec{P}_0, \vec{P}_A$)

p-Wel(T_C, T_W, σ_T)

1 : (gid, msg-type, W_0, spk) $\leftarrow T_W$
2 : **require** msg-type = 'wel'
3 : **if** $\neg$Ver(spk, $(T_C, T_W), \sigma_T$) **return** false
4 : $(\gamma', \mathsf{acc}) \leftarrow$ CGKA.proc($W_0, \gamma.\mathsf{s0}$)
5 : **if** $\neg$acc **return** false
6 : γ[gid].s0 $\leftarrow \gamma'$
7 : γ[gid].spk $\leftarrow$ spk
8 : **return** true

p-Comm(T_C, T_W, σ_T)

1 : (gid, msg-type, C_0, C_A, W_A, spk) $\leftarrow T_C$
2 : **require** msg-type = 'comm'
3 : $\gamma' \leftarrow \gamma.\mathsf{sA}$
4 : **if** $\sigma_T = \perp$ // no sig $\Rightarrow$ check no changes to G
5 : $(\gamma', \mathsf{acc}) \leftarrow$ CGKA.proc($C_0, \gamma.\mathsf{s0}$)
6 : **if** $\neg$acc $\vee$ (γ'[gid].$G \neq \gamma$[gid].G)
7 : **return** false
8 : γ[gid].s0 $\leftarrow \gamma'$
9 : **return** true
10 : **else if** $\neg$Ver(spk, $(T_C, T_W), \sigma_T$) **return** false
11 : **if** γ.ME $\in \gamma.G^*$
12 : $(\gamma', \mathsf{acc}) \leftarrow$ CGKA*.proc($C_A, \gamma.\mathsf{sA}$)
13 : **if** $\neg$acc **return** false
14 : **else if** $W_A \neq \perp$
15 : $(\gamma', \perp) \leftarrow$ CGKA*.proc($W_A, \gamma.\mathsf{sA}$)
16 : **if** $C_0 \neq \perp$
17 : $(\gamma^\dagger, \mathsf{acc}^\dagger) \leftarrow$ CGKA.proc($C_0, \gamma.\mathsf{s0}$)
18 : **if** $\neg\mathsf{acc}^\dagger$ **return** false
19 : γ[gid].s0 $\leftarrow \gamma^\dagger$
20 : **if** γ.ME $\notin \gamma^\dagger.G$ // removed user
21 : γ[gid] $\leftarrow \perp$
22 : **return** true
23 : γ[gid].sA $\leftarrow \gamma'$; γ[gid].spk $\leftarrow$ spk
24 : **return** true

p-Create(T_{CR}, σ_T)

1 : (gid, msg-type, W_0, W_A) $\leftarrow T_{CR}$
2 : **require** msg-type = 'create'
3 : $(\gamma_0, \mathsf{acc}) \leftarrow$ CGKA.proc($W_0, \gamma.\mathsf{s0}$)
4 : $(\gamma_A, \perp) \leftarrow$ CGKA*.proc($W_A, \gamma.\mathsf{sA}$)
5 : **if** γ_A[gid].$G \subseteq \gamma_0$[gid].$G \neq \emptyset$
6 : $(\gamma.\mathsf{s0}, \gamma.\mathsf{sA}) \leftarrow (\gamma_0, \gamma_A)$
7 : **else return** false
8 : **return** acc

Figure 9: Helper functions for the DGS construction in Figure 8 with respect to random oracle H_{ro}.

predicate, as in IAS. Administrative proposals are signed with $\gamma.\mathsf{spk}$; this is done to protect against insider adversaries that may re-send previously crafted administrative proposals (i.e., those that are legitimate but correspond to a previous epoch), or even create new ones if these are sent in plaintext. The prop-info algorithm is fully based on the respective CGKA's prop-info algorithms (which may not output ID' if anonymous proposals are allowed).

Commits. For a given gid, administrative changes are committed via CGKA^*.commit (which outputs a C_A) and standard group changes via CGKA.commit (outputting C_0 as usual). Note that in C_A, we update the CGKA^* secret $\mathsf{k_{adm}}$, that is used to update the admin key pair $(\mathsf{ssk}', \mathsf{spk}')$.

The new admin key spk' is included in the final A-CGKA commit message, so that group members can process it. Hence, we need that the committer processes (speculatively[6]) the commit message in order to obtain k_{adm} and then $(\mathsf{ssk}', \mathsf{spk}')$. In order to prove the authenticity of the commit (and of spk'), the committer signs the whole commit message including C_A, C_0, spk' with the old admin key $\gamma.\mathsf{ssk}$. Besides, the committer must verify all proposal signatures in advance.

As before, a commit can be split into a welcome message T_W for newly added users, and a commit message T_C for group members. These are signed jointly in our construction to simplify the security proof, but may also be signed separately. In T_C, we also include the welcome messages to CGKA^*, since they must always be addressed to current group members. Commits in both CGKAs are independent: CGKA can be updated while CGKA^* is not, and vice-versa.

Processing control messages. The proc method takes a control message T, determines the type of message (create, welcome, or commit) and the gid, and updates the state only if processing succeeds ($\mathsf{acc} = \mathsf{true}$). Newly added users verify the admin signature, process the welcome message using CGKA.proc and store the new public admin key (spk', provided in T) in $\gamma.\mathsf{spk}$.

Group members verify the administrator signature (if the commit requires administrative rights) using $\gamma.\mathsf{spk}$. Then, depending on the commit type, CGKA, CGKA^*, or both are updated via the corresponding CGKA proc algorithm. Given CGKA^* or both CGKAs are updated, the updated admin key is set as $\gamma.\mathsf{spk} \leftarrow \mathsf{spk}'$. In case T contains a create message T_{CR}, both CGKAs process the respective welcome messages contained in T_{CR} separately.

4.2.3 Features

DGS allows the use of two distinct and independent CGKA protocols that authenticate admins as a group, providing some notable features that differ from IAS.

Minimal information reveal. As opposed to IAS, the set of group administrators can be opaque to the central server and to the rest of the group (whenever the underlying CGKAs preserve the anonymity of group members with respect to external parties). We discuss this further in Section 6.2.1.

[6]Speculative execution can be avoided if the syntax of commit directly outputs the new group secret. We opted for simplifying syntax.

Incoming users. Verification issues for newly added users present in unauthenticated IAS and in other works (as mentioned in [21]) are mitigated. The administrative spk can be a public value that a server can store. Hence, an incoming member can verify the authenticity of an administrative signature by verifying spk with the server, or using a different channel other than the welcome message itself. Another possibility is out-of-band authentication, such as via safety numbers, a feature provided in some messaging services.

Limitations. A drawback of DGS is that enforcing different "levels of administration", for which IAS can be easily extended, is not straightforward. Nevertheless, one can still implement minor policies which do not have such security impact (such as muting users) at an application level (as hitherto done in practice). We also note that admins may not have a reliable view of who they are if CGKA* is susceptible to internal state forks[7]. If these attacks are relevant, one can deploy heavier protocols such as the P-Act-Rob in [12]. A third limitation is that admins cannot give up their admin status immediately; they must send a self rem-adm proposal, erase their admin state, and wait for another admin to commit. This occurs generally in CGKA when a member leaves a group; in MLS the policy enforces removals to be committed before any application message is sent.

Security mechanisms. We note the conceptual simplicity of achieving PCS and FS in the group administration keys (in the adversarial model for CGKA*) given the existence of secure CGKA schemes in the literature, since both properties are ensured by CGKA* itself. Update mechanisms are largely simplified due to a single admin key being used. Delegation and revocation of admin keys are also straightforward.

4.3 Integrating A-CGKA in MLS

Some group messaging protocols already authenticate group members using signatures. Namely, the current MLS specification relies on credentials, which are essentially public signature keys for each protocol user that are certified by a PKI; such keys authenticate messages originating from that user[8]. Therefore, it is possible to extend the CGKA used in MLS to an A-CGKA in a more efficient way than using a compiled A-CGKA construction (resembling IAS, since individual signatures are already provided in the credentials). We note that, in practice, it is feasible to support secure administration in MLS via an *MLS extension*, a feature that enables additional proposal types and new actions in the protocol [3]. Constructing such an extension is almost straightforward and we identify three main necessary changes:

- Credentials are not necessarily refreshed in MLS, meaning that admins (and users in general) whose state is compromised at some point lack forward security and post-compromise security on their authentication keys (unless they proactively update them). Our solution is to introduce an IAS-like credential update mechanism for administrators.

[7]This scenario is out of the scope of our security model where admins are fully trusted.

[8]Signatures play an important role in MLS: *"...group members can verify a message originated from a particular member of the group. This is guaranteed by a digital signature on each message from the sender's signature key. The signature keys held by group members are critical to the security of MLS against active attacks..."* [3].

- Group members need to keep track of the administrators (for an IAS-like extension). New admin proposal types, which must be signed accordingly, shall be introduced together with corresponding update policies and modifications in the commit and proc algorithms.
- Ratcheting admins' credentials should be compatible with maintaining the current guarantees for incoming users; this means that all admin keys must be registered with the PKI. An alternative solution is to use a PKI-registered key to authenticate the admin key used in the commit.

4.3.1 Modifications

We propose an extension of the main algorithms of the MLS protocol (in particular, of the CGKA-related prop, commit, proc) in Figure 10, that we also benchmark in Section 5.3. Our goal is to show how IAS can be easily integrated with relatively low overhead. We follow [13] (in particular, Figure 8 in the full version [45]), as this is the most comprehensive formalization of MLS in the literature at the time of writing; therefore, we also work in the single-group setting. We omit the send, rcv algorithms as these are used to send application messages only. We note also that their create algorithm only creates groups with one participant; hence its integration with IAS is trivial.

Protocol details. The main modifications are to (1) the admins' credentials, which are regularly updated via upd-adm proposals and admin commits; and (2) in the introduction of the three additional proposal types from A-CGKA. For brevity, we omit several parts of the protocol, such as sanity checks (for example, **require** predicates), and functions that we do not need to modify, such as the MAC. Also for simplicity, we extend the CGKA state γ to include the state variables used in IAS. Following [13], we split the processing algorithms in two: one for commit messages, and one for welcome messages, that a committer produces for each incoming user separately. Overall, the overhead with respect to bare-bone MLS is minimal; we essentially only need to support the new types of proposals and to refresh admin credentials for admin updates. Most of the protocol logic relates to updating signatures and the adminList.

5 Results

5.1 Correctness

Proposition 1. *Let* CGKA *be a correct CGKA (Definition 3) and* $\mathcal{S} = (\mathsf{SigGen}, \mathsf{Sig}, \mathsf{Ver})$ *be a correct signature scheme. Then, the IAS protocol (Figures 5 and 6) is correct with respect to Definition 3.*

Proposition 2. *Let* CGKA *and* CGKA* *be correct CGKAs, and* $\mathcal{S} = (\mathsf{SigGen}, \mathsf{Sig}, \mathsf{Ver})$ *be a correct signature scheme. Then, the DGS protocol is correct with respect to Definition 3.*

The correctness proofs for both protocols can be found in the Appendix.

5.2 Security

Observe that IAS, which uses a (regular) digital signature scheme, does not provide optimal forward security. Therefore, we prove security with respect to a sub-optimal admin cleanness

prop(ID, type; r_0)

1 : **if** type $= *$-adm
2 : **require** γ.ME $\in \gamma$.adminList
3 : $(P, \perp) \leftarrow$ IAS.makeAdminProp(type, ID; r_0)
// Redefined getSpk (used in makeAdminProp)
4 : **if** type $\in$ {add, rem, upd}
5 : $(\gamma, P) \leftarrow$ CGKA.prop(γ, ID, type; r_0)
// Added users' keys retrieved from contact list
6 : $\sigma \leftarrow$ Sig(γ.ssk, (P))
7 : **return** (P, σ)

commit($(\vec{P}_0, \vec{P}_A)$, com-type; r_0)

1 : $(r_1, r_2, r_3) \leftarrow H_3(r_0, \gamma)$
2 : **if** com-type $\in$ {adm, both}
3 : **require** γ.ME $\in \gamma$.adminList
4 : **require** IAS.verifyPropSigs($\vec{P}_A$)
5 : $C_A \leftarrow \vec{P}_A$
6 : adminList$' \leftarrow$ IAS.updAL(adminList, $\vec{P}_A$)
7 : $(\gamma.\text{ssk}', \gamma.\text{spk}') \leftarrow$ SigGen($\gamma.1^\lambda$; r_1)
8 : $\gamma \leftarrow$ updSpk(γ, ID, spk$'$)
9 : **if** com-type $\in$ {std, both}
10 : $(\gamma, C_0, W_0) \leftarrow$ CGKA.commit($\vec{P}_0$; r_2)
11 : **if** $W_0 \neq \perp$ // share updated adminList
// prepare wel. msgs as [13] in $\vec{W}$
12 : **for** $W \in \vec{W}$:
13 : $W \leftarrow W$||adminList$'$
14 : $\sigma \leftarrow\$$ Sig(γ.ssk, W) // rand.
15 : $T \leftarrow$ ('com', γ.ME, C_0, C_A, γ.spk$'$)
16 : $\sigma \leftarrow$ Sig(γ.ssk, T), r_3)
17 : **return** $((T, \sigma), \vec{W})$

proc-WM(W)

1 : **require** ID $\in W$.adminList
2 : run Proc-WM(W) in [13]
3 : γ.adminList $\leftarrow W$.adminList
// adminList[ID].spk should be checked with PKI

proc-CM(T, σ)

1 : ('com', ID, C_0, C_A, spk$'$) $\leftarrow T$
2 : **if** ID $\notin$ adminList
3 : **require** Ver(getSpk(ID), T, σ)
4 : run Proc-CM(T) in [13]
5 : **require** no membership changes to $\gamma.G$
6 : **if** ID $\in$ adminList
7 : **require** Ver(adminList[ID].spk, T, σ)
8 : **if** spk$' \neq \perp$
9 : **require** spk$'$ = getSpk(ID) // spk was registered
// Update keys and adminList as in IAS
10 : IAS.p-Comm(T)
// Should check new adminList keys with PKI

getSpk(ID)

// Get spk from ID's credential
1 : **return** Cred[ID].spk

updSpk(γ, ID, spk$'$)

// Register spk$'$ with the PKI
1 : $\gamma \leftarrow$ registerPKI(γ, ID, spk$'$)
// Update ID's credential
2 : γ.Cred[ID].spk $\leftarrow$ spk$'$

Figure 10: Construction of an MLS extension that supports group administrators, effectively turning the CGKA in MLS to an A-CGKA. Highlighted lines correspond to our main modifications in the original SGM construction in [13]. Cred[·] denotes an array that stores the credentials of all ID's. We also use the abstract function registerPKI for standard PKI functionality of registering signature keys, which can also be verified. Some technical details are omitted.

predicate $\mathsf{C}_{\mathsf{adm}}$ where $\mathsf{C}_{\mathsf{adm}} = \mathsf{C}_{\mathsf{adm\text{-}opt}} \wedge \mathsf{C}_{\mathsf{adm\text{-}add}}$ and $\mathsf{C}_{\mathsf{adm\text{-}opt}}$ is defined in Section 3.4. We define $\mathsf{C}_{\mathsf{adm\text{-}add}}$ in the appendices, together with the full security proof.

Theorem 1. *Let* CGKA *be a correct and* $(t_{\mathsf{cgka}}, q, \epsilon_{\mathsf{cgka}})$*-secure CGKA with respect to cleanness predicate* $\mathsf{C}_{\mathsf{cgka}}$*, according to Definitions 3 and 4. Let* $\mathcal{S} = (\mathsf{SigGen}, \mathsf{Sig}, \mathsf{Ver})$ *be a* $(t_{\mathcal{S}}, q, \epsilon_{\mathcal{S}})$ *SUF-CMA secure signature scheme, as in Definition 7. Let* H_4 *be a* $(t_F, 1, \epsilon_F)$*-secure 4-PRF, as in Definition 5. Then, the IAS protocol (Figures 5 and 6) is* $(t, q, q \cdot \epsilon_F + \epsilon_{\mathsf{cgka}} + q^2 \cdot \epsilon_{\mathcal{S}})$*-secure (Definition 4) with respect to predicates* $\mathsf{C}_{\mathsf{cgka}}, \mathsf{C}_{\mathsf{adm}}$*, where* $t_{\mathsf{cgka}} \approx t_{\mathcal{S}} \approx t_F \approx t$ *and* $\mathsf{C}_{\mathsf{adm}}$ *is defined in Figure 4.*

Proof idea. We first bound an adversary $\mathcal{A}$'s advantage in distinguishing between the $\mathsf{KIND}^{\mathcal{A}}_{\mathsf{A\text{-}CGKA},\mathsf{C}_{\mathsf{cgka}},\mathsf{C}_{\mathsf{adm}}}$ game and a game G_1 which replaces calls to hash functions H_i by uniformly sampling the output (modelling each H_i in IAS as a PRF). Then, we divide $\mathcal{A}$'s behaviour in G_1 into two events based on whether they successfully query the $\mathcal{O}^{\mathsf{Inject}}$ oracle (event E_1) or not (event E_2). Given E_1, we reduce security via a number of SUF-CMA adversaries. Otherwise, we reduce directly with $\mathsf{KIND}^{\mathcal{A}}_{\mathsf{CGKA},\mathsf{C}_{\mathsf{cgka}}}$ adversary, at which point the claim follows.

Theorem 2. *Let* CGKA *be a correct and* $(t_{\mathsf{cgka}}, q, \epsilon_{\mathsf{cgka}})$*-secure CGKA with respect to cleanness predicate* $\mathsf{C}_{\mathsf{cgka}}$*, according to Definitions 3 and 4. Let* CGKA^* *be a correct and* $(t_{\mathsf{cgka}*}, q, \epsilon_{\mathsf{cgka}*})$ *secure CGKA with respect to predicate* $\mathsf{C}_{\mathsf{cgka}*}$*. Let* $\mathcal{S} = (\mathsf{SigGen}, \mathsf{Sig}, \mathsf{Ver})$ *be a* $(t_{\mathcal{S}}, q, \epsilon_{\mathcal{S}})$ *SUF-CMA secure signature scheme, as in Definition 7. Let* H_4 *be a* $(t_F, 1, \epsilon_F)$*-secure 4-PRF, as in Definition 5. Let* H_{ro} *be a random oracle queried at most* q_{ro} *times. Then, the DGS protocol (Figures 8 and 9) is* $(t, q, \epsilon_{\mathsf{cgka}} + q \cdot (\epsilon_F + \epsilon_{\mathcal{S}} + \epsilon_{\mathsf{cgka}*} + 2^{-\lambda}))$*-secure (Definition 4) in the random oracle model with respect to predicates* $\mathsf{C}_{\mathsf{cgka}}, \mathsf{C}_{\mathsf{adm}}$*, where* $t_{\mathsf{cgka}} \approx t_{\mathsf{cgka}*} \approx t_{\mathcal{S}} \approx t_F \approx t$ *and* $\mathsf{C}_{\mathsf{adm}}$ *is a function of* $\mathsf{C}_{\mathsf{cgka}*}$*.*

Proof idea. We first describe $\mathsf{C}_{\mathsf{adm}}$. Intuitively, $\mathsf{C}_{\mathsf{adm}}$ ensures that the set of safe oracle queries for DGS adversary $\mathcal{A}$ given inject queries of the form $q_i = \mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}, m, t_a)$ are those that are safe for CGKA^* adversary $\mathcal{A}'$ under essentially the same queries, replacing $\mathcal{O}^{\mathsf{Inject}}(\cdot, \cdot, t_a)$ queries with queries of the form $\mathcal{O}^{\mathsf{Challenge}}(t_a)$. For example, noting the symmetry between the optimal admin and regular predicates $\mathsf{C}_{\mathsf{adm\text{-}opt}}$ and $\mathsf{C}_{\mathsf{cgka\text{-}opt}}$ respectively, that if CGKA^* is secure with respect to CGKA predicate $\mathsf{C}_{\mathsf{cgka\text{-}opt}}$, then DGS is secure with respect to admin predicate $\mathsf{C}_{\mathsf{adm\text{-}opt}}$.

To prove security, we use a similar game-hopping argument as in IAS. We first replace H_i calls using the PRF assumption. We then consider E_1 (a successful injection is made) and E_2 (otherwise) as in IAS. Given E_2, we can simulate directly via the CGKA adversary. Given E_1, we simulate differently depending on whether $\mathcal{A}$ makes a query to random oracle H_{ro} with a correct CGKA^* key or not. Here, if $\mathcal{A}$ is successful, we reduce security to the CGKA^* adversary by intercepting the relevant random oracle query and guessing the correct bit using information from $\mathcal{O}^{\mathsf{Challenge}}$. Otherwise, we can simulate via an EUF-CMA adversary as in IAS.

5.3 Benchmarking

We implemented the protocol in Section 4.3 to obtain a realistic estimate of the overhead of securely administrating a real-world messaging protocol. We modified an open-source implementation of MLS in Go[9] and compare the running times of MLS (which also performs e.g. parent hashing and non-admin proposal signing), with the running times of administrated

[9]The original source code is available at `https://github.com/cisco/go-mls`.

MLS in different scenarios. In particular, we analyze the commit and proc algorithms in Figure 10, where the latter includes proc-CM and also processing proposals when relevant (done separately in the implementation). We ran our benchmarks on a laptop with a 4-core 11th Gen Intel i5-1135G7 processor and 16 GB of RAM using Go's `testing` package[10]. Core cryptographic operations were implemented as HPKE with ciphersuite `DHKEM(P-256, HKDF-SHA256)`, `HKDF-SHA256`, `AES-128-GCM` from Go standard libraries. For each data point, we took the average of 50 iterations (using argument `-count 50`) that randomized the group members and admins performing group operations.

In Figure 11, we analyse the running time of the commit algorithm with respect to group size $|G|$ with a fixed member/admin ratio $|G|/|G^*| = 4$, and $t = |G^*|$ users updating. In Figure 12, we show running time of the proc algorithm for fixed $|G| = 64$, $|G^*| = 16$, with respect to the number of updates t in a commit. In both, we compare (1) standard commits (com-type = std, omitted in Figure 12), (2) standard commits with t update proposals, (3) standard+admin commits (com-type = both) with $t/2$ admin-update proposals, and (4) standard+admin commits with t update and $t/2$ admin-update proposals.

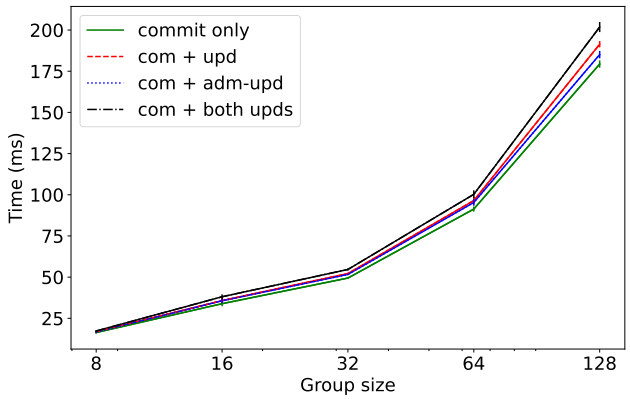

Figure 11: commit benchmarks for variable $|G|$ on constant member/admin $|G|/|G^*| = 4$ ratio and constant update ratio.

[10] `https://pkg.go.dev/testing`

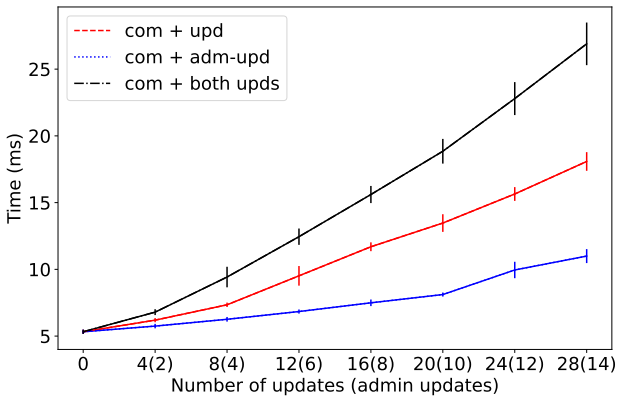

Figure 12: proc benchmarks for variable number of updates and $|G| = 64$.

6 Discussion

6.1 Efficiency

6.1.1 Protocols

The results in Section 5.3 show that the additional cost (for users) of running a securely-administrated MLS is minimal. Figure 11 shows that the commit algorithm involves less than a 10% overhead when up to $|G|/8$ members carry out admin updates simultaneously. Figure 12 shows that the processing time of admin and standard updates is very similar, and increases linearly in the number of updates.

Separately, we analyze the overhead of IAS and DGS for group members, both for number of operations and for message size. We note that this assumes that IAS and DGS are implemented modularly and not integrated with an existing CGKA as before. In IAS, admins generate a signature key pair and sign every time they carry out a commit or a proposal, and verify a small amount t of signatures (typically $t \leq |G^*|$) in admin proposals before a commit. If we denote the cost (time/length) of a message signature or verification by s, and the cost (time/length) of a signature key pair generation by k, we obtain the values[11] in Table 1.

The overhead of DGS depends heavily on the cost of CGKA^* (an optimistic estimation can be $\mathcal{O}(\log m)$ [10, 11, 12]). CGKA operations only affect administrators. Note that a DGS admin-only commit does not have to be sent to standard members (only the signed new admin key).

	Length (Adm)	Length (All)	Time (Adm)	Time (All)
IAS	$\mathcal{O}(ts + tk)$	$\mathcal{O}(ts + tk)$	$\mathcal{O}(ts + k)$	$\mathcal{O}(ts)$
DGS	$\mathcal{O}(C + s + k)$	$\mathcal{O}(s + k)$	$\mathcal{O}(C + s + k)$	$\mathcal{O}(s + k)$

Table 1: Additional cost of IAS and DGS with respect to a plain CGKA (per group) where t is the number of admin proposals and C (for DGS only) refers to the cost of running CGKA^*.

[11] We neglect the size of user identifiers IDs as we assume it small and constant with respect to the security parameter. Besides, identifiers may already be present in protocols that construct admins at the application level.

The ratio of additional messages sent, which is application-specific, is hard to estimate. Admin-only commits and admin modifications are expected to be less frequent than standard operations. The number of update proposals (although very cheap) is expected to be at most linear in $|G|$.

Forward-secure signatures (for optimally-secure IAS) can be instantiated with essentially constant amortized overhead in space and time relative to a regular signature scheme, while supporting unbounded secret key updates [46].

We conclude that IAS presents a generally affordable overhead for all users, while DGS introduces basically no cost for standard users and is more costly for administrators if $|G^*|$ is relatively large.

6.1.2 Admins in TreeKEM variants

The Tainted TreeKEM protocol provides efficiency advantages if only a subset of users carry out tree-changing operations (adds and removals) [11]. Tainted TreeKEM, however, is not formalised in the propose-and-commit-paradigm, which complicates the efficiency comparison; such an analysis remains open. When standard users are allowed to commit updates, the tree blanking issue with MLS TreeKEM is not worsened by administrators, hence efficiency should not decrease either.

6.2 Additional admin mechanisms

We consider possible extensions of A-CGKA as a primitive and corresponding construction ideas. We note that these extensions may provide stronger security guarantees, or additional functionality, at reduced cost if the number of admins is small.

6.2.1 Private administrators

In some applications, it may be desirable to hide the set of admins from users within a group. DGS could achieve some notion of administrative privacy if the underlying admin CGKA provides privacy guarantees itself. To achieve anonymity guarantees, admins could sign commits using ring signatures [47]. However, there is overhead with ring signatures over regular signatures, at a minimum to parse the anonymity set, and privacy is compromised e.g. if a single admin performs an admin update then signs after, so admins would need to batch updates for privacy. In addition, DGS provides a natural consensus mechanism for tracking the list of admins over time.

In MLS' TreeKEM protocol, proposals are constant-sized, but commits are variable, which leaks information about the contents of the commit even if it is encrypted. Thus, padding is required at a minimum. In the MLS standard, ciphertexts, i.e. MLSCiphertexts, leak the group ID, epoch and message content type (proposal or commit) in plaintext, which would need to be hidden for additional privacy. In practice, additional attack vectors like timing and traffic analysis preclude privacy also, which are considered by related work [48, 49], however it remains open to provide formal FS and PCS guarantees and in particular to adapt CGKA to defend against these attack vectors. As discussed in the introduction, the Signal Private Group System [27] hides the group membership from non-members; the mechanism could be extended for admins but adapting the technique to (A)-CGKA is also open.

6.2.2 External admins

Note that our A-CGKA constructions assumes the admins comprise a subset of all group members, i.e. $G^* \subseteq G$. It may be sometimes useful to enable *external* administration. For example, an online platform may wish to control the set of conversation participants to ensure they are subscribers but nevertheless protect their privacy. Admins who attempt to add users that group members do not trust can be detected on the protocol level, rather than the less well-defined application level.

Considering IAS and DGS, given that the underlying CGKA allows for external commits, it is straightforward to administer a group externally. Namely, the admin who is approving the change can inspect proposals and make commit messages for the corresponding parties. However, TreeKEM and its variants are not ideal for this since the committer is tasked to derive a new group secret and can thus violate privacy. One way around this issue is to essentially write an wrapper around each CGKA algorithm which declares that some CGKA group members are not actually in the group. Here, the wrapper would also force admins to delete group secrets as soon as they derive them, and would not consider admins as part of the group; the solution is however clearly vulnerable to corrupted admins.

One conceptually simple solution is to allow commits from regular users which play the role of proposals which have to be "committed" (e.g. signed) by admins. Additional machinery is however required in the malicious setting to enable admins to verify that such commits are well-formed.

6.2.3 Hierarchical admins

In messaging apps like Telegram and WhatsApp, the group creator has stronger capabilities than other admins. For instance, the group creator can never be removed by another admin. Extending this concept, one can conceive a hierarchy of administrators of several levels, e.g., of the form $G^{**} \subseteq G^* \subseteq G$, where G^{**} are super-administrators. Extending IAS, one can imagine using signatures that attest to other signatures in a chain-of-trust fashion. DGS can be extended by considering many CGKAs where CGKA $i+1$ must sign commit messages for CGKA i for each $i \geq 1$. Attribute-based admins would enable greater flexibility.

6.2.4 Muting admins

It is possible to provide some cryptographic guarantees to the process of muting conversation participants. One solution entails a DGS-like construction in which members must sign messages using a common signature key spk derived from a secondary CGKA; honest group members would then process application messages if and only if they are signed using the common signature key (i.e. only from the set of unmuted users). Then, muted members will be able to filter messages from other muted users (since they could still be informed of the state of spk over time), but they will not able to sign their own messages. Mechanisms that enable a central server to filter messages while maintaining privacy like [50] can be also be integrated into the encryption layer over A-CGKA. Nevertheless, we note that in messaging services where the identity of the group members is known, muted members can generally bypass a ban by sending individual messages to all group members using two-party messages.

6.2.5 Threshold admins

One issue with our A-CGKA constructions is that security breaks down if a single admin is compromised. To improve the robustness of the protocol, a protocol can enforce that some $k > 1$ admins must attest to a particular commit before it may be processed, which can be achieved straight-forwardly using regular signatures or more efficiently using threshold cryptography [51].

6.2.6 Decentralized admins

To allow for network decentralization, it is straightforward in theory for a given messaging group G to simply execute a state machine replication protocol [20] to order commit messages and require that users reliably broadcast [52] all proposal messages. Given that group members who are expected to execute the protocol on e.g. cellphones may not be available often, thus leading to liveness (and possibly unintended safety) violations in protocol execution, a natural solution is to entrust administrators to provide messages to users. These admins could indeed execute consensus.

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A Primitives

Definition 5 (PRF)**.** *We say that a function $f_n : \mathcal{K} \times X \to Y^n$ is a (t, q, ϵ)-secure n-pseudorandom function (n-PRF) if, for any polynomial-time adversary $\mathcal{A}$ limited to q oracle queries, the advantage of $\mathcal{A}$ in the $\mathsf{SUF\text{-}CMA}^{\mathcal{A}}_{\Pi}$ game below given by*

$$\left|\Pr[\mathsf{PRF}^{\mathcal{A}}_{f_n,1}(1^\lambda) = 1] - \Pr[\mathsf{PRF}^{\mathcal{A}}_{f_n,0}(1^\lambda) = 1]\right|$$

is bounded by ϵ, where the probability is taken over the choice of the challenger and adversary's random coins.

$\mathsf{PRF}^{\mathcal{A}}_{f_n,b}(1^\lambda)$

1 : $k \leftarrow\$ \mathcal{K}$
2 : sample fct $F : X \to Y^n$
3 : $b' \leftarrow \mathcal{A}^{\mathcal{O}}(\lambda)$
4 : **return** b'

$\mathcal{O}^{\mathsf{Eval}}(m)$

1 : **if** $b = 0$
2 : **return** $F(m)$
3 : **return** $f_n(k, m)$

Definition 6 (Digital signature)**.** *A digital signature scheme is a triple of algorithms* $(\mathsf{SigGen}, \mathsf{Sig}, \mathsf{Ver})$ *such that:*

- $(\mathsf{sk}, \mathsf{pk}) \leftarrow\$ \mathsf{SigGen}(1^\lambda)$ *creates a public-private key pair.*
- $\sigma \leftarrow\$ \mathsf{Sig}(\mathsf{sk}, m)$ *generates a signature σ from a message m and the secret key* sk*.*
- $b \leftarrow \mathsf{Ver}(\mathsf{pk}, \sigma, m)$ *outputs $b \in \{0, 1\}$, indicating acceptance or rejection, given a signature σ, a message m and a public key* pk*.*

We say that the signature scheme is correct if for any λ, $m \in \mathcal{P}$, and all choices of randomness, if $(\mathsf{sk}, \mathsf{pk}) \leftarrow\$ \mathsf{SigGen}(1^\lambda)$ *and* $\sigma \leftarrow\$ \mathsf{Sig}(\mathsf{sk}, m)$*, then* $\mathsf{Ver}(\mathsf{pk}, \sigma, m) = 1$*.*

Definition 7 (Digital signature security: SUF-CMA)**.** *A digital signature scheme Π is (t, q, ϵ)-secure against strong existential forgery under chosen message attacks (SUF-CMA) if, for any polynomial-time adversary $\mathcal{A}$ limited to q oracle queries, the advantage of $\mathcal{A}$ in the* $\mathsf{SUF\text{-}CMA}^{\mathcal{A}}_{\Pi}$ *game below given by* $\Pr[\mathsf{SUF\text{-}CMA}^{\mathcal{A}}_{\Pi}(1^\lambda) = 1]$ *is bounded by ϵ, where the probability is taken over the choice of the challenger and adversary's random coins.*

$\mathsf{SUF\text{-}CMA}^{\mathcal{A}}_{\Pi}(1^\lambda)$

1 : $(\mathsf{sk}, \mathsf{pk}) \leftarrow\$ \Pi.\mathsf{SigGen}(1^\lambda)$
2 : $Q \leftarrow \emptyset$
3 : $(m, \sigma) \leftarrow \mathcal{A}^{\mathcal{O}}(\mathsf{pk})$
4 : **require** $(m, \sigma) \notin Q$
5 : **return** $1_{\Pi.\mathsf{Ver}(\mathsf{pk}, m, \sigma)}$

$\mathcal{O}^{\mathsf{Sign}}(m)$

1 : $\sigma \leftarrow\$ \Pi.\mathsf{Sig}(\mathsf{sk}, m)$
2 : $Q \leftarrow Q \cup \{(m, \sigma)\}$
3 : **return** σ

$\mathsf{CORR}^{\mathcal{A}}_{\text{(A)-CGKA},\mathsf{C}_{\mathsf{corr}}}(1^\lambda)$

1 : **public** ep-view$[\cdot]$, ST$[\cdot]$, T$[\cdot] \leftarrow \bot$
2 : **public** first-crt$[\cdot] \leftarrow \bot$
3 : **public** prop-ctr, com-ctr $\leftarrow 0$ // msg counters
4 : **public** ep$[\cdot] \leftarrow (-1,-1)$ // user epoch tracker
5 : win $\leftarrow 0$
6 : ST[ID] $\leftarrow\$$ init$(1^\lambda,$ ID$)$ $\forall$ ID
7 : $\mathcal{A}^{\mathcal{O}}(1^\lambda)$
8 : **require** $\mathsf{C}_{\mathsf{corr}}$ // optional predicate
9 : **return** win // 1 if $\mathcal{A}$ is rewarded

$\mathcal{O}^{\mathsf{Create}}$(ID, gid, G, G^*)

1 : $(\gamma, T) \leftarrow\$$ create(ST[ID], gid, G, G^*)
2 : **if** $T = \bot$ **return**
3 : **reward** $\neg(\emptyset \neq G^* \subseteq G)$
4 : CheckSameGroupState(ST[ID], γ, gid)
5 : T[gid, $(-1,-1)$, 'com', ++com-ctr] $\leftarrow T$
6 : ST[ID] $\leftarrow \gamma$

$\mathcal{O}^{\mathsf{Deliver}}$(ID, gid, (t,c), c')

1 : **require** ep[gid, ID] $\in \{(t,c),(-1,-1),\bot\}$
2 : $T \leftarrow$ T[gid, (t,c), 'com', c'] // honest delivery
3 : $(\gamma,$ acc$) \leftarrow$ proc(ST[ID], T)
4 : **if** $\neg$acc **return** // failure
5 : **reward** $\neg(\emptyset \neq \gamma$[gid].$G^* \subseteq \gamma$[gid].$G)$
6 : **if** $(t,c) = (-1,-1)$ // create msg
7 : UniqueCreatePerGID$(\gamma$, gid, $c')$
8 : **if** ID $\notin \gamma$[gid].G // ID removed
9 : ep[gid, ID] $\leftarrow \bot$
10 : **reward** γ[gid].k $\neq \bot$ // key deleted
11 : **else** // ID in group
12 : UpdateView$(\gamma$, gid, $t, c')$
13 : ep[gid, ID] $\leftarrow (t+1, c')$
14 : ST[ID] $\leftarrow \gamma$

$\mathcal{O}^{\mathsf{Prop}}$(ID$'$, gid, ID, type)

1 : **require** type $\in$ types
2 : $(\gamma, P) \leftarrow\$$ prop(ST[ID$'$], gid, ID, type)
3 : **if** $P = \bot$ **return** // failure
4 : (gid*, type*, ID*, ID$'^*$) $\leftarrow$ prop-info(γ, P)
5 : **reward** (gid*, type*, ID*, ID$'^*$)
$\neq$ (gid, type, ID, ID$'$)
6 : CheckSameGroupState(ST[ID$'$], γ, gid)
7 : T[gid, ep[gid, ID$'$], 'prop', ++prop-ctr] $\leftarrow P$
8 : ST[ID$'$] $\leftarrow \gamma$ // upd. ST of proposer ID$'$

$\mathcal{O}^{\mathsf{Commit}}$(ID, gid, $I = (i_1, \ldots, i_k)$, com-type)

1 : **require** com-type $\in$ com-types
2 : **require** ep[gid, ID] $\notin \{(-1,-1),(\bot,\bot)\}$
3 : $\vec{P} \leftarrow (T$[gid, ep[gid, ID], 'prop', $i])_{i \in I}$
4 : $(\gamma, T) \leftarrow\$$ commit(ST[ID], gid, $\vec{P}$, com-type)
5 : **if** $T = \bot$ **return** // failure
6 : **reward** ID $\notin$ ST[ID][gid].G // no external comm.
7 : CheckSameGroupState(ST[ID], γ, gid)
8 : T[gid, ep[gid, ID], 'com', ++com-ctr] $\leftarrow T$
9 : T[gid, ep[gid, ID], 'vec', com-ctr] $\leftarrow \vec{P}$
10 : ST[ID] $\leftarrow \gamma$

UpdateView$(\gamma$, gid, $t, c')$

1 : $v \leftarrow$ ep-view[gid, $t+1, c']$
2 : **if** $v = \bot$
3 : ep-view[gid, $t+1, c'] \leftarrow \gamma$
4 : **else** CheckSameGroupState$(v, \gamma$, gid$)$

CheckSameGroupState$(\gamma_1, \gamma_2$, gid$)$

1 : **reward** γ_1[gid].k $\neq \gamma_2$[gid].k
2 : **reward** γ_1[gid].$G \neq \gamma_2$[gid].G
3 : **reward** γ_1[gid].$G^* \neq \gamma_2$[gid].G^*

UniqueCreatePerGID$(\gamma$, gid, $c')$

1 : **if** first-crt[gid] $\neq \bot$
2 : **require** $c' =$ first-crt[gid]
3 : **else** first-crt[gid] $\leftarrow c'$

Figure 13: Correctness game for (A)-CGKA with respect to predicate $\mathsf{C}_{\mathsf{corr}}$. Highlighted code is executed only when considering an A-CGKA. Note that when **reward** P is true for predicate P, the variable win is set to 1.

B Correctness Game

In Figure 13, we provide the full correctness game corresponding to Definition 3. We also provide a description below.

Overview The game starts by setting up several public dictionaries. The main two are $\mathsf{ST}[\cdot]$, which is a dictionary indexed by ID which keeps the states of each of the users ID throughout the game; and $\mathsf{T}[\cdot]$, which keeps all control messages and proposals generated by the A-CGKA algorithms. A message T stored in T is indexed by the corresponding gid, epoch, type of message (`'prop'`, `'com'` or `'vec'`, standing for proposal, commit, or proposal vector respectively), and a message counter prop-ctr or com-ctr. After initialization, we let the adversary $\mathcal{A}$ interact with the oracles with respect to multiple groups. The variable win is set to 1 if one of the **reward** clauses is true, which leads to $\mathcal{A}$ winning given the (optional) predicate $\mathsf{C}_{\mathsf{corr}}$ is also true when $\mathcal{A}$ finishes executing.

Epochs. Control messages output by successful create and commit calls are labelled uniquely by the challenger. For correctness, an *epoch* is a pair (t, c), where t is an integer relative to a particular group and party which increments upon each successful proc call while in the group, and c is the value of the global variable com-ctr when the corresponding control message was output. For a given group, each party's epoch value is stored in $\mathsf{ep}[\cdot]$ and initialised to $(-1, -1)$, and is set to $\perp$ when they leave a group. To this end, we model correctness in the presence of an adversary who maintains arbitrary network partitioning, so long as they provide a consistent view of messages to parties in each such 'partition'.

Group consistency. We enforce that, for each group member, that each group is only (possibly) updated upon a successful call to proc (via CheckSameGroupState). For A-CGKA, we ensure that, for a group gid, $\emptyset \neq G^* \subseteq G$ must hold at all times. In proc, we enforce that all users who transition to the same epoch have the same view of the group and set of admins when relevant (via UpdateView). The dictionary ep-view stores the state of the first party who transitions to a given epoch (t, c) for a gid. We also require that, even if there are multiple calls to create, only one of them is processed by group members (i.e. states do not fork from the initial epoch). For this endeavour, the variable first-crt tracks the commit number of the first succesfully processed create message for a given gid. This check is made in UniqueCreatePerGID.

Key partnering. Note that new epoch keys are derived upon successful proc calls, and that a new key k (for group members) is always derived in this case. Correctness ensures that all users who transition to the same epoch derive the same key k. Moreover, whenever a user derives a key $k \neq \perp$, they must be a group member (line 10 of $\mathcal{O}^{\mathsf{Deliver}}$).

Liveness. We enforce that some algorithms, such as prop, always succeed on 'valid' input, since without such a check, a (A)-CGKA with algorithms that always fail is considered correct. Since the precise semantics of a (A)-CGKA vary between applications, additional checks are delegated to a correctness predicate $\mathsf{C}_{\mathsf{corr}}$ which, in part, parameterises the correctness game. Without extra checks, the predicate should be set to $\mathsf{C}_{\mathsf{corr}} = \mathsf{true}$.

$$\boxed{\mathsf{C}_{\mathsf{adm}}:}\ \forall\ (i, \mathsf{ID}, \mathsf{ID}', \mathsf{ctr} \in (0, \mathsf{exp\text{-}ctr}]) : q_i = \mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}', \cdot, t_i^*),$$
$$(\mathsf{ID} \notin \mathsf{ADM}[t_i^*]) \vee$$
$$\big(\exists (t_i, c) : (\mathsf{tExp}(\mathsf{ID}, \mathsf{ctr}).t_a < t_i \leq t_i^*) \wedge$$
$$\mathsf{hasUpd}_{\mathsf{adm}}(\mathsf{ID}, \mathsf{T}[(\cdot, t_i), \texttt{'com'}, c], \mathsf{T}[(\cdot, t_i), \texttt{'vec'}, c]) \wedge$$
$$(\mathsf{C}[(\cdot, t_i)] = c)\big).$$

Figure 14: Sub-optimal administrative predicate where the adversary makes oracle queries $q_1, \ldots, q_n$.

C Security Proofs

C.1 Proof of Theorem 1 (IAS security, Section 5.2)

Note that IAS uses a (regular) signature scheme, which results in sub-optimal security,. since an admin may not update their signature key at the beginning of every admin epoch. We start with the definition of the predicate $\mathsf{C}_{\mathsf{adm}}$ predicate we use to prove security for IAS. After proving security, we discuss how to (easily) extend the proofs to the optimal setting by use of forward-secure signatures. $\mathsf{C}_{\mathsf{adm}}$ is presented in Figure 14. Note that it differs from the optimal predicate (Figure 4) only by the lack of $(t_i^* < \mathsf{tExp}(\mathsf{ID}, \mathsf{ctr}).t_a)$ condition, so it holds that $\mathsf{C}_{\mathsf{adm}} \wedge \mathsf{C}_{\mathsf{adm\text{-}opt}} = \mathsf{C}_{\mathsf{adm}}$. That is, the forward security guarantees are weaker, since e.g. if a party updates their admin key in admin epoch 3 then if they are exposed in epoch 5 then the adversary can make a trivial forgery in the construction (and thus it is considered a trivial attack by $\mathsf{C}_{\mathsf{adm}}$. Towards proving security, we prove the following lemma.

Lemma 1. *Let $\mathcal{A}$ be a $\mathsf{KIND}^{\mathcal{A}}_{\mathsf{A\text{-}CGKA},\mathsf{C}_{\mathsf{cgka}},\mathsf{C}_{\mathsf{adm}}}$ adversary playing with respect to IAS. Consider any query $\mathcal{A}$ makes of the form $\mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}, m, t_a)$ which results in a response $v \neq \perp$. Then $\mathcal{A}$ can parse $m = (T_C, T_W, \sigma_T)$ and efficiently derive pk such that $\mathsf{Ver}(\mathsf{pk}, \sigma_T, (T_C, T_W))) = 1$.*

Proof. Consider $\mathcal{A}$'s query $\mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}, m, t_a)$ that outputs $v \neq \perp$. Given $v \neq \perp$ and by definition of $\mathcal{O}^{\mathsf{Inject}}$, a call $(\gamma, \cdot) \leftarrow \mathsf{proc}(\mathsf{ST}[\mathsf{ID}], m)$ was previously made by the challenger such that $(\gamma.G, \gamma.G^*) \neq (\mathsf{ST}[\mathsf{ID}].G, \mathsf{ST}[\mathsf{ID}].G^*)$. Note that $\mathcal{O}^{\mathsf{Inject}}$ disallows $t_a \neq -1$, which is the case if and only if $\mathsf{ID} \notin G$, and that unsigned control messages cannot change the group structure (line 11 of proc). Thus, we only need to consider control messages that are input to p-Comm in proc and result in output $\mathsf{acc} = \mathsf{true}$, i.e. when $\sigma_T \neq \perp$. To reach p-Comm, the proc call must be such that $\mathsf{Ver}(\gamma[\mathsf{gid}].\mathsf{adminList}[\mathsf{ID}].\mathsf{spk}, (T_C, T_W), \sigma_T) = 1$. Since $\gamma[\mathsf{gid}].\mathsf{adminList}[\mathsf{ID}].\mathsf{spk}$ must have been previously sent in a message by construction of IAS, it follows that $\mathsf{pk} = \gamma[\mathsf{gid}].\mathsf{adminList}[\mathsf{ID}].\mathsf{spk}$ is efficiently computable. □

Now we are ready to prove the theorem which we restate below:

Theorem 1 (IAS security)**.** *Let* CGKA *be a correct and $(t_{\mathsf{cgka}}, q, \epsilon_{\mathsf{cgka}})$-secure CGKA with respect to cleanness predicate $\mathsf{C}_{\mathsf{cgka}}$, according to Definitions 3 and 4. Let $\mathcal{S} = (\mathsf{SigGen}, \mathsf{Sig}, \mathsf{Ver})$ be a $(t_{\mathcal{S}}, q, \epsilon_{\mathcal{S}})$ SUF-CMA secure signature scheme, as in Definition 7. Let H_4 be a $(t_F, 1, \epsilon_F)$-secure 4-PRF, as in Definition 5. Then, the IAS protocol (Figures 5 and 6) is $(t, q, q \cdot \epsilon_F + \epsilon_{\mathsf{cgka}} + q^2 \cdot \epsilon_{\mathcal{S}})$-secure (Definition 4) with respect to predicates $\mathsf{C}_{\mathsf{cgka}}, \mathsf{C}_{\mathsf{adm}}$, where $t_{\mathsf{cgka}} \approx t_{\mathcal{S}} \approx t_F \approx t$ and $\mathsf{C}_{\mathsf{adm}}$ is defined in Figure 14.*

Proof. Let G_0 be the $\mathsf{KIND}^{\mathcal{A}}_{\mathsf{IAS},\mathsf{C}_{\mathsf{cgka}},\mathsf{C}_{\mathsf{adm}}}$ game. Let G_1 be as in G_0, except that all calls of the form $(r_1, \ldots, r_i) \leftarrow H_i(r_0, \gamma)$ are replaced with calls of the form $(r_1, \ldots, r_i) \leftarrow\!\!\$\ R^i$ where R is

the space of random coins used by each (A)-CGKA algorithm. Note that in IAS we always have $i \leq 4$.

Let $G_{0,0} = G_0$. Let $G_{0,j}$ be G_0 except that the first j calls of the form $H_i(r_0, \gamma)$ that adversary $\mathcal{A}$ makes are replaced as above, and the rest remain unchanged. Note that since every oracle query that $\mathcal{A}$ makes results in at most one call to a function of the form $H_i(\cdot, \cdot)$, $G_{0,k} = G_1$ for some $k \leq q$.

Consider $G_{0,j-1}$ and $G_{0,j}$ where $j \geq 1$; suppose these games are played by adversary $\mathcal{A}$. Let $\mathcal{A}'$ be a 4-PRF adversary as in Definition 5. $\mathcal{A}'$ simulates directly except when $\mathcal{A}$ makes their oracle query which leads to the j-th call to a function of the form H_i. Upon this call, $\mathcal{A}$ simulates this call by calling $\mathcal{O}^{\mathsf{Eval}}(\gamma)$ for the input γ, and truncates the response $(r_1, \ldots, r_4)$ to $(r_1, \ldots, r_i)$ when necessary before continuing execution (i.e. its simulation). Clearly $\mathcal{A}'$ perfectly simulates $G_{0,j-1}$ when $\mathcal{A}'$'s challenger's bit is 1. When $\mathcal{A}'$'s challenger's bit is 0, note that the output of $\mathcal{A}'$'s $\mathcal{O}^{\mathsf{Eval}}$ call, namely $(r_1, \ldots, r_4)$, is of the form $F(\gamma)$ for a uniformly random function $F : \mathsf{ST} \to R^4$, where ST is the A-CGKA state space. Since F is randomly chosen, this output is distributed identically to $(r_1, \ldots, r_4)$ where each r_i is uniformly sampled from R. It follows that $\mathcal{A}'$ perfectly simulates $G_{0,j}$ when its challenge bit is 0. By combining the sequence of game hops, it follows that

$$\left|\Pr[G_0^{\mathcal{A}}(1^\lambda) = 1] - \frac{1}{2}\right| \leq \left|\Pr[G_1^{\mathcal{A}}(1^\lambda) = 1] - \frac{1}{2}\right| + q \cdot \epsilon_F,$$

where ϵ_F is the advantage of PRF adversary $\mathcal{A}'$.

In the following, we will simulate for a G_1 adversary via either a SUF-CMA or CGKA adversary. Without hopping from G_0 to G_1, the simulation would not have been identical when e.g. using the SUF-CMA $\mathcal{O}^{\mathsf{Sign}}$ oracle. Since we have transitioned to G_1, which uses randomness normally, there are no issues regarding simulation and randomness.

Let $\mathcal{A}$ be a G_1 adversary. Let E_1 be the event that $\mathcal{A}$ makes a query to $\mathcal{O}^{\mathsf{Inject}}$ such that $\mathcal{O}^{\mathsf{Inject}}$ outputs value $v \neq \perp$ (i.e. the challenge bit). Let E_2 be the event that this does not occur; clearly $\Pr[E_1] + \Pr[E_2] = 1$. We consider each event separately. Without loss of generality, we restrict our simulations given E_1 to the case where $\mathsf{C}_{\mathsf{adm}}$ is true, which is trivial for the adversary to determine, and where given E_2 to the case where $\mathsf{C}_{\mathsf{cgka}}$ is true. In the latter case, it suffices to observe that $\mathsf{C}_{\mathsf{cgka}}$ is efficiently computable by even a CGKA adversary such that they can abort during unclean executions.

$\underline{E_1}$: By construction of IAS and Lemma 1, note that, given that $\mathcal{O}^{\mathsf{Inject}}$ outputs $v \neq \perp$, that a signature forgery has occurred where the signature keying material is sampled due to an oracle call. Given E_1, we need to determine which input values ID and t_a are used by $\mathcal{A}$ on the first $\mathcal{O}^{\mathsf{Inject}}$ call which outputs $v \neq \perp$. By construction of IAS, this can happen as a result of a query to $\mathcal{O}^{\mathsf{Create}}$, $\mathcal{O}^{\mathsf{Prop}}$ or $\mathcal{O}^{\mathsf{Commit}}$. However, $\mathcal{A}$ may make at most q injection attempts with respect to this key pair, each of which may be a winning one. Thus, we have to guess both 1) the $\mathcal{O}^{\mathsf{Inject}}$ query which first outputs $v \neq \perp$ and 2) the query q_i which generates the signature key pair corresponding to this injection.

Let $E_{1,i,j}$ be the event that $\mathcal{A}$ makes query q_i which leads to the generation of signature key pair $(\mathsf{ssk}, \mathsf{spk})$ such that query q_j is the first query to $\mathcal{O}^{\mathsf{Inject}}$ resulting in output $v \neq \perp$. Note that IAS is such that each oracle query q_i results in at most one new signature key pair being

sampled by the challenger. By the union bound, we have:

$$\Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge E_1] \leq \sum_{i,j \in \{1,\ldots,q\}} \Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge E_{1,i,j}].$$

Suppose $E_{1,i,j}$ holds. Let $\mathcal{A}'$ be an SUF-CMA adversary that simulates for G_1 adversary $\mathcal{A}$. We aim to show that $\Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge E_{1,i,j}] \leq \epsilon_S$.

$\mathcal{A}'$ simulates as follows. $\mathcal{A}'$ simulates variable initialisation as in G_1 except that it lazily samples init calls as needed (ensuring it remains polynomially-bounded). $\mathcal{A}'$ simulates the first $i-1$ of $\mathcal{A}$'s oracle queries locally, i.e. simulates all relevant behaviour resulting in corresponding state and game variables being set and updated. This includes queries of the form getSsk and getSpk which $\mathcal{A}'$ simulates via signature scheme $\mathcal{S}$.

Consider $\mathcal{A}'$'s i-th oracle query q_i. Let $(\mathsf{ssk}^*, \mathsf{spk}^*)$ denote the signature key pair sampled by the SUF-CMA challenger and recall SUF-CMA adversary $\mathcal{A}'$ has access to oracle $\mathcal{O}^{\mathsf{Sign}}$. $\mathcal{A}'$ simulates as follows:

- If q_i is to $\mathcal{O}^{\mathsf{Create}}$, then $(\mathsf{ssk}^*, \mathsf{spk}^*)$ plays the role of $(\gamma.\mathsf{ssk}', \gamma.\mathsf{spk}')$. Namely, $\mathcal{A}'$ sets the output of $\mathsf{getSpk}(\mathsf{ID}, \gamma.\mathsf{ME})$ to spk^* and that of $\mathsf{getSsk}(\gamma.\mathsf{spk}', \mathsf{ME})$ to ssk^* after embedding spk^* in $\mathsf{adminList}[\mathsf{ME}]$ at line 7 of create. $\mathcal{A}'$ also calls $\mathcal{O}^{\mathsf{Sign}}(T_W)$ which outputs σ, which $\mathcal{A}'$ sets to σ_W. $\mathcal{A}'$ otherwise simulates and returns the result to $\mathcal{A}$.
- If q_i is to $\mathcal{O}^{\mathsf{Prop}}$ with input $\mathsf{type} = \mathsf{upd\text{-}adm}$, then $(\mathsf{ssk}^*, \mathsf{spk}^*)$ plays the role of $(\gamma.\mathsf{ssk}', \gamma.\mathsf{spk}')$ (line 10 of $\mathsf{makeAdminProp}$). Similarly to above, $\mathcal{A}'$ embeds spk^* in P_0 (line 11 of $\mathsf{makeAdminProp}$), and then simulates the rest of the oracle call.
- Otherwise, q_i is to $\mathcal{O}^{\mathsf{Commit}}$. Note we assume $E_{1,i,j}$ holds. Thus, the branch at line 5 in commit must be executed. As before, $(\mathsf{ssk}^*, \mathsf{spk}^*)$ plays the role of $(\gamma.\mathsf{ssk}', \gamma.\mathsf{spk}')$; $\mathcal{A}'$ simulates the remainder of the call.

All other oracle queries, except to $\mathcal{O}^{\mathsf{Inject}}$, are simulated locally by $\mathcal{A}'$ except when signatures with respect to spk^* are required, in which case SUF-CMA oracle $\mathcal{O}^{\mathsf{Sign}}$ is used, or when spk^* is to be embedded in a message. Note that at most q $\mathcal{O}^{\mathsf{Sign}}$ queries are made by $\mathcal{A}'$ since each oracle query $\mathcal{A}$ makes produces at most one signature (which may or may not require $\mathcal{O}^{\mathsf{Sign}}$ to simulate).

Note by construction of IAS that each signature key pair is sampled by a single party and is never revealed/embedded in another message, and that each key pair is uniformly and independently sampled. Thus, the simulation is valid, ignoring $\mathcal{O}^{\mathsf{Expose}}$ queries, since the challenge key pair is independent of all other keying material and challenge secret key ssk^* is never revealed. Similarly, since $\mathsf{C_{adm}}$ is true (since we assume E_1), $\mathcal{A}$ will never query $\mathcal{O}^{\mathsf{Expose}}$ with respect to the challenge key pair, a query which would otherwise not be able to be simulated.

For $\mathcal{A}$'s query $q_{j'} = \mathcal{O}^{\mathsf{Inject}}(\mathsf{ID})$ such that $j' \neq j$, $\mathcal{A}'$ simulates by returning $\perp$ to $\mathcal{A}$. When $\mathcal{A}$ makes query $q_j = \mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}, m, t_a)$, $\mathcal{A}'$ inspects $m = (T_C, T_W, \sigma_T)$ and returns the message/signature pair $((T_C, T_W), \sigma_T)$ to $\mathcal{A}$ which, by Lemma 1, exists. These two steps are valid by definition of $E_{1,i,j}$ and that $j' < j$ in the first step since we only simulate up to query j.

It thus follows that the simulation is perfect. Noting that $\mathcal{A}$ wins at most as often as $\mathcal{A}'$ (since e.g., $\mathcal{A}$ may come up with a forgery (m, σ) nevertheless rejected by proc), we have:

$$\Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge E_{1,i,j}] \leq \epsilon_{sig}.$$

$\underline{E_2}$: Note first that given E_2, we can deduce that every query to $\mathcal{O}^{\mathsf{Inject}}$ will have output $\perp$. Let $\mathcal{A}'$ be a $\mathsf{KIND}^{\mathcal{A}'}_{\mathsf{CGKA},\mathsf{C}_{\mathsf{cgka}}}$ adversary. $\mathcal{A}'$ simulates for $\mathsf{KIND}^{\mathcal{A}}_{\mathsf{A\text{-}CGKA},\mathsf{C}_{\mathsf{cgka}},\mathsf{C}_{\mathsf{adm}}}$ adversary $\mathcal{A}$ as follows. When $\mathcal{A}$ makes an oracle query, $\mathcal{A}'$ executes all code in IAS except for that which makes use of CGKA algorithms, which are processed via $\mathcal{A}'$'s oracles; $\mathcal{A}'$ then returns each response to $\mathcal{A}$. One case we must deal with is when proc is called at line 12; the call may succeed but the caller may ignore the state update, undoing the changes made; this happens given the group state changes as a result of the call. Note that $\mathcal{A}'$ can determine whether this case occurs or not using the fact that commit messages are honestly delivered by construction of the KIND game and by correctness which ensures that honestly-generated and delivered commit messages are accepted. In particular, $\mathcal{A}'$ can use the policy to determine whether or not the call would update the group state, and only call its $\mathcal{O}^{\mathsf{Deliver}}$ when the group state is not changed. Finally, $\mathcal{A}'$ outputs the same bit as $\mathcal{A}$.

To see that the simulation is perfect, first note that CGKA algorithms are used as a black box in IAS. Moreover, except in the case dealt with above, CGKA state variables $\mathsf{s0}$ for each ID are not used except as input to CGKA algorithms, after which they are immediately overwritten, exactly as done by the CGKA KIND challenger given correctness and in particular the fact that failing algorithm calls do not update the state. Thus, it suffices to simulate CGKA code using $\mathcal{A}'$'s oracles. Thus:

$$\Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge E_2] = \Pr[\mathsf{KIND}^{\mathcal{A}'}_{\mathsf{CGKA},\mathsf{C}_{\mathsf{cgka}}}(1^\lambda) = 1] = \epsilon_{\mathsf{cgka}}$$

We then have:

$$\begin{aligned}
&\left|\Pr[G_1^{\mathcal{A}}(1^\lambda) = 1] - \frac{1}{2}\right| \\
&= \left|\Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge (\vee_{i,j} E_{1,i,j} \vee E_2)] - \frac{1}{2}\right| \\
&\leq \left|\sum_{i,j} \Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge E_{1,i,j}] + \Pr[G_1^{\mathcal{A}}(1^\lambda) \wedge E_2] - \frac{1}{2}\right| \\
&\leq \left|\sum_{i,j} \Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge E_{1,i,j}]\right| \\
&\qquad + \left|\Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge E_2] - \frac{1}{2}\right| \\
&\leq q^2 \cdot \epsilon_S + \epsilon_{\mathsf{cgka}}
\end{aligned}$$

where the second and third lines follow from the union bound and triangle inequality, respectively. The result follows by combining this with the game hop earlier. □

Optimal forward security. We can achieve optimal forward security, and thus optimal admin security (i.e. security with respect to $\mathsf{C}_{\mathsf{adm\text{-}opt}}$) by replacing signatures with forward-secure signatures [44, 46]. Forward-secure signatures divide signature key pairs into epochs based on how many secret key update calls are made by the secret key holder. Relative to regular signatures, the security game for forward-secure signatures provides two additional oracles to 1) expose a secret key at a given epoch and 2) transition the secret key to a new epoch. Recall that the construction change we suggested to IAS comprised of parties calling the key update function whenever they transition to a new epoch such that do not 'update' their keys

in the CGKA sense. The logic of the security reduction is very similar to that presented with regular signatures above. The main difference is that forward-secure signature calls are replaced by oracle calls, including possibly key exposure calls, which, conditioned on the optimal admin predicate being true, will not result in a trivial attack in the forward-secure signature game.

C.2 Proof of Theorem 2 (DGS security, Section 5.2)

Predicate $\mathsf{C}_{\mathsf{adm}}$. $\mathsf{C}_{\mathsf{adm}}$ is tailored to DGS and is a function of the underlying CGKA^* predicate $\mathsf{C}_{\mathsf{cgka}*}$. Intuitively, $\mathsf{C}_{\mathsf{adm}}$ ensures that the set of safe oracle queries for DGS adversary $\mathcal{A}$ given inject queries of the form $q_i = \mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}, m, t_a)$ are those that are safe for CGKA^* adversary $\mathcal{A}'$ (i.e., the predicate $\mathsf{C}_{\mathsf{cgka}^*}$) under roughly the same queries, replacing $\mathcal{O}^{\mathsf{Inject}}(\cdot, \cdot, t_a)$ queries with queries of the form $\mathcal{O}^{\mathsf{Challenge}}(t_a)$. For example, noting the symmetry between predicates $\mathsf{C}_{\mathsf{adm\text{-}opt}}$ and $\mathsf{C}_{\mathsf{cgka\text{-}opt}}$, if CGKA^* is secure with respect to CGKA predicate $\mathsf{C}_{\mathsf{cgka\text{-}opt}}$, then DGS is secure with respect to admin predicate $\mathsf{C}_{\mathsf{adm\text{-}opt}}$.

We define $\mathsf{C}_{\mathsf{adm}}$ more formally. Let $Q = (q_i)_I$ be the ordered sequence of oracle queries made by the DGS adversary $\mathcal{A}$. To define $\mathsf{C}_{\mathsf{adm}}$, we construct an ordered sequence of queries Q^* that are made by the CGKA^* adversary $\mathcal{A}'$ in the security proof below by replacing, inserting and/or deleting queries in-order. Let $\ell \in [1, q_{inj}]$ where q_{inj} is the number of $\mathcal{O}^{\mathsf{Inject}}$ queries made by $\mathcal{A}$. To this end, consider each $q_i \in Q$ and, for each ℓ, define q_i^* to be either a single query or a sequence of queries in Q^* as follows:

- $q_i = \mathcal{O}^{\mathsf{Create}}(\mathsf{ID}, G, G^*)$: Set $q_i^* = \mathcal{O}^{\mathsf{Create}}(\mathsf{ID}, G^*)$.
- $q_i = \mathcal{O}^{\mathsf{Prop}}(\mathsf{ID}, \mathsf{ID}', \mathsf{type})$: Set $q_i^* = \perp$ if $\mathsf{type} \neq *\text{-}\mathsf{adm}$ and $q_i^* = \mathcal{O}^{\mathsf{Prop}}(\mathsf{ID}, \mathsf{ID}', \mathsf{type}^*)$ otherwise where $\mathsf{type} = \mathsf{type}^*\text{-}\mathsf{adm}$.
- $q_i = \mathcal{O}^{\mathsf{Commit}}(\mathsf{ID}, (i_1, \ldots, i_k), \mathsf{com\text{-}type})$: If the condition at line 6 of commit is false, $\mathsf{com\text{-}type} = \mathsf{std}$ or ID is not currently an admin, set $q_i^* = \perp$. Otherwise, let $\{ID_1, \ldots, ID_j\}$ be the (possibly empty) set of parties for which CGKA^* rem proposals are introduced by propCleaner (line 13). Let $\vec{P}_A$ be the value input to CGKA^*.commit at line 4 of c-Adm (or $\vec{P}_A = \perp$ if the line is not reached), and $(i_1, \ldots, i_k)$ the corresponding proposal indices in the CGKA^* KIND game. Then, set q_i^* to the sequence $(\mathcal{O}^{\mathsf{Prop}}(\mathsf{ID}, \mathsf{ID}_1, \mathsf{rem}), \ldots, \mathcal{O}^{\mathsf{Prop}}(\mathsf{ID}, \mathsf{ID}_j, \mathsf{rem}), \mathcal{O}^{\mathsf{Commit}}(\mathsf{ID}, (i_1, \ldots, i_k))$.
- $q_i = \mathcal{O}^{\mathsf{Challenge}}(t_s)$: Set $q_i^* = \perp$.
- $q_i = \mathcal{O}^{\mathsf{Deliver}}(\mathsf{ID}, (t_s, t_a), c)$: Let $(T, \mathsf{com\text{-}type}) = \mathsf{T}[(t_s, t_a), \texttt{'com'}, c]$ for DGS KIND game variable T. If proc is called by the game, $\mathsf{ID} \in G$ holds, $T_c \neq \perp$ holds and either $\mathsf{ID} \in G^*$ holds or W_A (contained in T) is $\neq \perp$, set $q_i^* = \mathcal{O}^{\mathsf{Deliver}}(\mathsf{ID}, t_a, c^*)$, where c^* is the number of times CGKA^*.commit was called after c queries to $\mathcal{O}^{\mathsf{Commit}}$. Otherwise, set $q_i^* = \perp$.
- $q_i = \mathcal{O}^{\mathsf{Reveal}}(t_s)$: Set $q_i^* = \perp$.
- $q_i = \mathcal{O}^{\mathsf{Expose}}(\mathsf{ID})$: Set $q_i^* = q_i$.
- $q_i = \mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}, m, t_a)$: Set $q_i^* = \perp$ if q_i^* is the j-th query to $\mathcal{O}^{\mathsf{Inject}}$ where $j \neq \ell$ and $\mathcal{O}^{\mathsf{Challenge}}(t_a)$ otherwise.

For each Q^*, define $|Q^*|+1$ sequences of the form Q_i^* such that a query $q^* = \mathcal{O}^{\mathsf{Challenge}}(t_s)$ is inserted between queries q_i and q_{i+1} for $i \in [0, q]$ (where $q_0 = q_{|Q^*|} = \perp$).[12] Then, $\mathsf{C}_{\mathsf{adm}}$ is defined to be true for sequence Q and value t_s whenever $\mathsf{C}_{\mathsf{cgka}^*}$ is true for all sequences Q_i^*.

We restate the theorem to be proved:

Theorem 2 (DGS security)**.** *Let* CGKA *be a correct and* $(t_{\mathsf{cgka}}, q, \epsilon_{\mathsf{cgka}})$*-secure CGKA with respect to* $\mathsf{C}_{\mathsf{cgka}}$*, according to Definitions 3 and 4. Let* CGKA^* *be a correct and* $(t_{\mathsf{cgka}*}, q, \epsilon_{\mathsf{cgka}*})$ *secure CGKA with respect to* $\mathsf{C}_{\mathsf{cgka}*}$*. Let* $\mathcal{S}$ *be a* $(t_{\mathcal{S}}, q, \epsilon_{\mathcal{S}})$ *SUF-CMA secure signature scheme, as in Definition 7. Let* H_4 *be a* $(t_F, 1, \epsilon_F)$*-secure 4-PRF, as in Definition 5. Let* H_{ro} *be a random oracle queried at most* q_{ro} *times. Then, the DGS protocol (Figures 8 and 9) is* $(t, q, \epsilon_{\mathsf{cgka}} + q \cdot (\epsilon_F + \epsilon_{\mathcal{S}} + \epsilon_{\mathsf{cgka}*} + 2^{-\lambda}))$*-secure (Definition 4) in the random oracle model with respect to predicates* $\mathsf{C}_{\mathsf{cgka}}, \mathsf{C}_{\mathsf{adm}}$*, where* $t_{\mathsf{cgka}} \approx t_{\mathsf{cgka}*} \approx t_{\mathcal{S}} \approx t_F \approx t$ *and* $\mathsf{C}_{\mathsf{adm}}$ *is a function of* $\mathsf{C}_{\mathsf{cgka}*}$*.*

Proof. Following the proof of Theorem 1, let G_0 be the $\mathsf{KIND}^{\mathcal{A}}_{\mathsf{A\text{-}CGKA},\mathsf{C}_{\mathsf{cgka}},\mathsf{C}_{\mathsf{adm}}}$ game, and G_1 be as in G_0 except that all calls of the form $(r_1, \ldots, r_i) \leftarrow H_i(r_0, \gamma)$ are replaced with calls of the form $(r_1, \ldots, r_i) \leftarrow\!\$\, R^i$. We transition between G_0 and G_1 in the exact same fashion as in Theorem 1. That is, we define hybrids $G_{0,j}$ where $G_{0,0} = G_0$, $G_{0,j} = G_1$ when $j \geq q$ and $G_{0,j}$ differs from G_0 for appropriate $0 < j < q$ by replacing the first j calls to functions of the form H_i with uniformly sampled values by the challenger. As before, we have $\left|\Pr[G^{\mathcal{A}}_{0,j-1}(1^\lambda) = 1] - \Pr[G^{\mathcal{A}}_{0,j}(1^\lambda) = 1]\right| \leq \epsilon_F$ for each $j \geq 1$, where ϵ_F is the advantage of PRF adversary $\mathcal{A}'$, which implies also that

$$\left|\Pr[G^{\mathcal{A}}_0(1^\lambda) = 1] - \frac{1}{2}\right| \leq \left|\Pr[G^{\mathcal{A}}_1(1^\lambda) = 1] - \frac{1}{2}\right| + q \cdot \epsilon_F.$$

Let E_1 be the event that $\mathcal{A}$ queries $\mathcal{O}^{\mathsf{Inject}}$ such that the oracle does not output $\perp$ (i.e. it outputs the challenge bit). Let E_2 be the event that this does not occur; clearly $\Pr[E_1] + \Pr[E_2] = 1$. To prove security, we will reduce to the security of the primary CGKA, i.e. CGKA, given E_2, and to CGKA^* and signature security given E_1.

We first consider the simpler E_2 case where no successful injection is made (and thus $\mathcal{O}^{\mathsf{Inject}}$ calls can be easily simulated); let $\mathcal{A}'$ be a KIND adversary w.r.t. CGKA simulating for G_1 adversary $\mathcal{A}$ given E_2. $\mathcal{A}'$ simulates as follows. For each oracle query, $\mathcal{A}'$ simulates relevant CGKA^* calls locally unless stated otherwise. In particular, $\mathcal{A}'$ calls $\mathsf{init}(1^\lambda, \mathsf{ID})$ for ID only as needed (i.e. lazily). Then:

- $\mathcal{O}^{\mathsf{Create}}(\mathsf{ID}, G, G^*)$: $\mathcal{A}'$ calls $\mathcal{O}^{\mathsf{Create}}(\mathsf{ID}, G)$ if needed and otherwise locally simulates.
- $\mathcal{O}^{\mathsf{Prop}}(\mathsf{ID}, \mathsf{ID}', \mathsf{type})$: $\mathcal{A}'$ simulates locally if type is of the form $*\text{-}\mathsf{adm}$ and simulates via $\mathcal{O}^{\mathsf{Prop}}(\mathsf{ID}, \mathsf{ID}', \mathsf{type})$ otherwise.
- $\mathcal{O}^{\mathsf{Commit}}(\mathsf{ID}, (i_1, \ldots, i_k), \mathsf{com\text{-}type})$: Since $\mathsf{CGKA.commit}$ is called after $\mathsf{CGKA}^*.\mathsf{commit}$, $\mathcal{A}'$ can simulate CGKA^* calls locally and call $\mathcal{O}^{\mathsf{Commit}}(\mathsf{ID}, J = (j_1, ..., j_{k'}))$, where J corresponds to the set of relevant CGKA proposal indices derived from $(i_1, \ldots, i_k)$, to simulate $\mathsf{CGKA.commit}$ calls.
- $\mathcal{O}^{\mathsf{Deliver}}(\mathsf{ID}, (t_s, t_a), c)$: $\mathcal{A}'$ simulates the relevant $\mathsf{CGKA.proc}$ call (there is at most one such call made by construction of DGS proc) via $\mathcal{O}^{\mathsf{Deliver}}(\mathsf{ID}, t_s, c')$ where c' is the index of the relevant CGKA control message. $\mathcal{A}'$ simulates locally otherwise.

[12] These extra sequences of queries are required to handle random oracle queries in the security proof.

- $\mathcal{O}^{\mathsf{Reveal}}(t_s)$ and $\mathcal{O}^{\mathsf{Challenge}}(t_s)$: $\mathcal{A}'$ simulates directly via their respective oracles.
- $\mathcal{O}^{\mathsf{Expose}}(\mathsf{ID})$: $\mathcal{A}'$ calls $\mathcal{O}^{\mathsf{Expose}}(\mathsf{ID})$ and simulates the rest of the call locally.
- $\mathcal{O}^{\mathsf{Inject}}$: By definition of E_2, $\mathcal{O}^{\mathsf{Inject}}$ always returns $\perp$, and since $\mathcal{O}^{\mathsf{Inject}}$ does not modify the state, $\mathcal{A}'$ simply outputs $\perp$ upon each $\mathcal{O}^{\mathsf{Inject}}$ call.

By construction, DGS inherits the normal safety predicate $\mathsf{C}_{\mathsf{cgka}}$ from CGKA, and so the simulation is perfect and it follows thus that:

$$\Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge E_2] \leq \epsilon_{\mathsf{cgka}}.$$

Now, consider adversary $\mathcal{A}$ playing G_1 given E_1 occurs, i.e. $\mathcal{A}$ makes a successful $\mathcal{O}^{\mathsf{Inject}}$ call. Note that there are at most q different CGKA^* epochs during a given execution and consequently at most q different CGKA^* signature key pairs computed by correct parties (since each signature key pair is derived from a given CGKA^* epoch secret).

Given E_1, let F_1 be the event that G_1 adversary $\mathcal{A}$ calls random oracle H_{ro} with input r that corresponds to any of the $k \leq q$ signature key pairs created by the G_1 challenger. That is, r is such that $(\mathsf{ssk}, \mathsf{spk}) \leftarrow \mathsf{SigGen}(1^\lambda, H_{\mathsf{ro}}(r))$ is called by the challenger at some point. Let F_2 be the event no such query is made. We consider the case with F_1 (by simulating via the CGKA^* KIND game) and F_2 (via the SUF-CMA game) separately.

Consider F_1. Let F_i' be the event that the first successful injection is the i-th query to $\mathcal{O}^{\mathsf{Inject}}$; clearly at most q such events occur. Let $\mathcal{A}_i$ be a CGKA^* adversary who simulates for G_1 adversary $\mathcal{A}$ as follows given $F_i' \wedge E_1$. Broadly, $\mathcal{A}_i$ simulates each of $\mathcal{A}$'s queries by calling their respective oracles and simulating locally for the remainder of the query. We consider internal $\mathsf{getSigKey}$ and H_{ro} queries as well as $\mathcal{O}^{\mathsf{Inject}}$ queries in the next paragraph. The CGKA^* oracle queries required by $\mathcal{A}_i$ to simulate properly are described in the description of $\mathsf{C}_{\mathsf{adm}}$ above. Note that to simulate without having to 'rewind' its oracles that $\mathcal{A}_i$ should also make earlier CGKA calls than written in the DGS code in the event that $\mathsf{CGKA.proc}$ queries are unsuccessful but $\mathsf{CGKA}^*.\mathsf{proc}$ queries succeed.

$\mathcal{A}_i$ simulates $\mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}, m, t_a)$ as follows. For the first $i-1$ queries to $\mathcal{O}^{\mathsf{Inject}}$, $\mathcal{A}_i$ returns $\perp$, since $\mathcal{A}$'s first successful injection is their i-th query to $\mathcal{O}^{\mathsf{Inject}}$. When $\mathcal{A}$ calls makes their i-th query $\mathcal{O}^{\mathsf{Inject}}(\mathsf{ID}, m, t_a)$, $\mathcal{A}_i$ first queries $\mathcal{O}^{\mathsf{Challenge}}(t_a)$ and receives k as a response. $\mathcal{A}_i$ then executes $\mathsf{getSigKey}(k)$ locally which outputs $(\mathsf{ssk}, \mathsf{spk})$. Finally, $\mathcal{A}_i$ outputs bit 0 if and only if the signature public key inside of m is exactly spk.

$\mathcal{A}_i$ simulates H_{ro} as follows. When $\mathcal{A}_i$ calls $\mathsf{getSigKey}$ while simulating commit calls, H_{ro} is queried with input r; $\mathcal{A}_i$ lazily samples in this case. If $\mathcal{A}$ has previously called H_{ro} with input r, then if $\mathcal{A}$ has made a state exposure that would allow them to trivially derive the CGKA^* key r, then $\mathcal{A}_i$ does nothing else. Otherwise, $\mathcal{A}_i$ makes a $\mathcal{O}^{\mathsf{Challenge}}$ query for the corresponding epoch, which outputs k'; $\mathcal{A}_i$ finishes simulating and returns bit 0 if and only if $k' = k$. When $\mathcal{A}$ queries H_{ro} with input k, $\mathcal{A}_i$ similarly lazily samples except in the case that k was previously queried by $\mathcal{A}_i$; in this case, $\mathcal{A}_i$ makes the same $\mathcal{O}^{\mathsf{Challenge}}$ query and returns the corresponding bit. Note by definition of F_1 that $\mathcal{A}$ must make such a query to H_{ro}.

Note that there is a problem with simulation due to the speculative $\mathsf{CGKA}^*.\mathsf{proc}$ calls made by the commit algorithm which cannot be guessed around without incurring an exponential loss in tightness in q. To get around this, one could re-define the commit algorithm to output the new key; this should not weaken security and can be trivially implemented by existing CKGAs; the safety predicate above should also be modified to account for this. One could also

strengthen the security model by allowing the $\mathcal{O}^{\mathsf{Deliver}}$ oracle to consider both past and present states for parties, which is however less standard and not necessarily realistic.

Notwithstanding this, the simulation is perfect, and when $b = 0$ $\mathcal{A}_i$ wins iff $\mathcal{A}$ wins, since $\mathcal{A}_i$ outputs 0 only if they derive the correct key or signature public key in the simulation, and when $b = 0$ the challenge oracle outputs the correct key. The $b = 1$ case is similar except in the case that $b = 1$ and the r sampled by the game is the same as the real key (which happens with probability $\frac{1}{2^\lambda}$); it follows that

$$\Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge E_1 \wedge F_1] \leq q \cdot \epsilon_{\mathsf{cgka}*} + \frac{q}{2^\lambda}.$$

We consider F_2. Let F_i' be the event, for $1 \leq i \leq q$, that a successful injection is made which uses the i-th CGKA^* key pair sampled by oracle queries during the game's execution. Note that such a key pair must exist and that a signature forgery, by algorithm construction, is a necessary but not sufficient condition to make a $\mathcal{O}^{\mathsf{Inject}}$ query with a non-bottom response. Consider $\mathcal{A}_i$ who simulates as follows. $\mathcal{A}_i$ simulates all queries locally except that $\mathcal{A}_i$ embeds his challenge key in the i-th such CGKA^* key pair in relevant control messages and uses the $\mathcal{O}^{\mathsf{Sign}}$ oracle to produce signatures as necessary. Note that the safety predicate is such that, conditioned on F_i', that an $\mathcal{O}^{\mathsf{Expose}}$ query that leaks the corresponding signature key is disallowed. In addition, $\mathcal{A}$ does not make any H_{ro} query that would lead to a trivial exposure by definition of F_2. It follows that the simulation is perfect and that:

$$\Pr[G_1^{\mathcal{A}}(1^\lambda) = 1 \wedge E_1 \wedge F_2] \leq q \cdot \epsilon_{\mathcal{S}}$$

The proof is completed by combining the sequence of game hops considered hitherto. □

D Correctness Proofs

D.1 Proof of Proposition 1 (IAS correctness)

We want to prove that no adversary $\mathcal{A}$ can win $\mathsf{CORR}_{\mathsf{A\text{-}CGKA},\mathsf{C}_{\mathsf{corr}}}$ (where we set $\mathsf{C}_{\mathsf{corr}} = \mathsf{true}$) played with respect to IAS (Figures 5 and 6) given that CGKA is correct. We analyze the different game oracles separately and sketch parts derived by direct inspection or based on CGKA correctness.

For $\mathcal{O}^{\mathsf{Prop}}$, $\mathcal{A}$ can win the game in $\mathcal{O}^{\mathsf{Prop}}$ if either prop-info incorrectly interprets the proposal or if the prop call changes the view of the group. In the first case, correctness follows from the correctness of CGKA.prop-info if the proposal is standard, and by inspection of IAS otherwise. In the second case, the group view is never changed by prop (as makeAdminProp only updates $\gamma.\mathsf{ssk}', \gamma.\mathsf{spk}'$ in case of an admin proposal, and CGKA.prop is correct in the case of a standard proposal).

$\mathcal{O}^{\mathsf{Create}}$ and $\mathcal{O}^{\mathsf{Commit}}$ can be proven correct by inspection in a similar way.

$\underline{\mathcal{O}^{\mathsf{Deliver}}}$: We examine each **reward** clause. Note that in line 2 of $\mathcal{O}^{\mathsf{Deliver}}$, T is either a commit message created by create or by commit.

We start with the clause ($\emptyset \neq \gamma[\mathsf{gid}].G^* \subseteq \gamma[\mathsf{gid}].G$). If T is a create message, then line 1 of create and the fact that variable adminList is populated ensures that this condition is fulfilled for T. Upon processing, and after a correct PKI retrieval, p-Wel overwrites γ.adminList and γ.s0 (via CGKA.proc), so the condition holds.

If T is a (standard) commit made by a non-admin user – that is, one without a signature – then there are no changes to the group as checked explicitly by the proc algorithm from line 11. Otherwise, if T an admin commit, then it is processed by p-Comm. We can distinguish some further cases depending on the proposals contained in T:

- If T contains proposals of types upd, upd-adm, add only, then the condition is trivially met.
- If T additionally contains rem proposals, for every removed ID a corresponding rem-adm proposal is generated by an honest admin in propCleaner, hence $G^* \subseteq G$.
- If T additionally contains add-adm proposals, the VALID_P predicate checks that the added admins are already group members (via S_2), hence $G^* \subseteq G$.
- If T additionally contains rem-adm proposals, the final check in enforcePolicy ensures that $G \neq \emptyset$.
- Any other combination of several contradicting proposals affecting the same ID is handled by enforcePolicy, which prioritizes removals (while preserving admin removals for the same user) which performs a final check on the size of G^*.

We conclude that, for any possible combination of proposals, the condition is always met provided that acc = true with respect to T.

The next case is the **reward** $\gamma[\mathsf{gid}].\mathsf{k} \neq \bot$ condition given proc outputs γ such that $\mathsf{ID} \notin \gamma[\mathsf{gid}].G$, which is straightforward by inspection.

The last check by UpdateView, rewards the adversary if two users processing the same commit message (on epoch (t, c)) differ in their group view. We show correctness by induction. Suppose ID_1 and ID_2 process the same commit message T. If they are in epoch $(-1, -1)$ and process a create message, correctness is easily seen. For the inductive step, we assume that their group views were equal in (t, c), and we want to show that they remain equal after moving to epoch $(t + 1, c')$. The commit is handled by p-Comm, and the only parts that can change for different users are the **if** condition in line 10 and updAL. The behaviour of both sections of code varies only on the modification of γ's signature keys, but not on the group structure. Hence, and assuming CGKA correctness, we conclude that ID_1 and ID_2 end up having consistent views.

The edge case in which a user is just added to the group is handled by p-Wel, and follows from CGKA correctness and the fact that γ.adminList $\leftarrow$ adminList is executed where adminList is directly provided in T.

D.2 Proof of Proposition 2 (DGS correctness)

We prove that no adversary $\mathcal{A}$ can win $\mathsf{CORR}_{\mathsf{A\text{-}CGKA},\mathsf{C}_{\mathsf{corr}}}$ (where we set $\mathsf{C}_{\mathsf{corr}} = \mathsf{true}$) played with respect to DGS (Figures 8 and 9) given that CGKA, CGKA* are correct. We omit some details which are analogous to IAS' correctness proof (note that IAS and DGS are designed such that they share sections of their code).

For $\mathcal{O}^{\mathsf{Prop}}$, the correctness of prop-info follows from the correctness of CGKA.prop-info, CGKA*.prop-info by assumption. Also, the adversary cannot win after the group membership check as prop only modifies the state by calling CGKA.prop and CGKA*.prop; which are correct by assumption.

The group membership check must always pass in $\mathcal{O}^{\mathsf{Create}}$ and $\mathcal{O}^{\mathsf{Commit}}$ for identical reasons.

$\underline{\mathcal{O}^{\mathsf{Deliver}}:}$ We examine each **reward** clause as done previously with IAS. As before, note that in line 2 of $\mathcal{O}^{\mathsf{Deliver}}$, T is a commit message created either by create or by commit.

The clause $(\emptyset \neq \gamma[\mathsf{gid}].G^* \subseteq \gamma[\mathsf{gid}].G)$ is met upon generation of any create message T (produced by $\mathcal{O}^{\mathsf{Create}}$) by construction (line 1 of create). When any message is processed by p-Create, the condition is enforced again.

If T is a (standard) commit made by a non-admin user – that is, one without a signature – then there are no changes to the group in the commit as checked explicitly by the auxiliary c-Std upon commit. The message must only contain a T_C which is processed by $p\text{-}Comm$, which again enforces this condition.

If T an admin commit, then it is processed by p-Comm or by p-Wel. In both cases, commit messages are processed by the underlying $\mathsf{CGKA}, \mathsf{CGKA}^*$ methods. Therefore, correctness depends on the commit algorithm. The case distinction follows the exact same logic as in IAS, since the algorithms propCleaner and enforcePolicy, and the predicate VALID_P enforce the same conditions.

The next case is the **reward** $\gamma[\mathsf{gid}].\mathsf{k} \neq \bot$ for a removed (or non-member) ID, which is enforced in DGS by p-Comm and by the correctness of the CGKAs.

For the last check by UpdateView, one can proceed by induction as in IAS. The main difference is that the admin update is not done manually as in IAS (i.e. modifying adminList), but rather by the underlying proc algorithms of CGKA^*, which yields the result easily. We omit the details.