Parámetros del problema:

$$C_d(0) = 0$$

$$C_s(0) \;=\; 0.5\ \frac{kg}{m^3}$$

$$V_d \;= 550\ hm^3$$

$$V_s = 920\ hm^3$$

$$f_0 = \;440\ \frac{m^3}{s}$$

Sistema de Ecuaciones Diferenciales Ordinarias EDOs iniciales:

$$\frac{dC_s}{dt} = -\frac{C_s}{V_s} f_0$$

$$\frac{dC_d}{dt} = \frac{C_s}{V_s} f_0 \;-\; \frac{C_d}{V_d} f_0$$

'' SOLUCIÓN ANALÍTICA CON MÉTODO DE ELIMINACIÓN ''

———

Cambiamos notación de EDOs iniciales y dejamos constantes juntas

$$(1)\ C_s' = -\frac{f_0}{V_s} C_s$$

$$(2)\ C_d' = \frac{f_0}{V_s} C_s - \frac{f_0}{V_d} C_d$$

Despejamos C_s de (2):

$$(3)\ C_s = \frac{V_s C_d'}{f_0} + \frac{V_s C_d}{V_d}$$

Derivamos C_s de (3) para obtener C_s':

$$(4)\ C_s' = \frac{V_s C_d''}{f_0} + \frac{V_s C_d'}{V_d}$$

Reemplazamos (3) y (4) en (1) para tener todo en términos de C_d:

$$\frac{V_s}{f_0} C_d'' + \frac{V_s}{V_d} C_d' = -C_d' - \frac{f_0}{V_d} C_d$$

Reescribimos para obtener forma general:

$$\frac{V_s}{f_0} C_d'' + \left(\frac{V_s + V_d}{V_d}\right) C_d' + \frac{f_0}{V_d} C_d = \;0$$

Reemplazamos con nuestros parámetros:

$$\frac{92000}{440}C_d'' + \left(\frac{92000 + 55000}{55000}\right)C_d' + \frac{440}{55000}C_d = 0$$

Solucionamos ecuación homogénea con coeficientes constantes:

$$C_d = e^{mt}$$

$$e^{mt}\left(\frac{92000}{440}m^2 + \frac{92000 + 55000}{55000}m + \frac{440}{55000}\right) = 0$$

$$m_1 \approx -0.0047$$

$$m_2 \approx -0.0080$$

Tenemos caso de m reales y diferentes, entonces C_d tiene la forma:

$$C_d(t) = Ae^{m_1 t} + Be^{m_2 t}$$

Reemplazamos con nuestros valores:

$$(5)\ C_d(t) = Ae^{-0.0047t} + Be^{-0.0080t}$$

Obtenemos derivada de (5):

$$(6)\ C_d'(t) = -0.0047Ae^{-0.0047t} - 0.0080Be^{-0.0080t}$$

Reemplazando (5) y (6) en (3) para obtener valor de C_s:

$$(7)\ C_s(t) = \frac{92000}{440}\left(-0.0047Ae^{-0.0047t} - 0.0080Be^{-0.0080t}\right) + \frac{92000}{55000}\left(Ae^{-0.0047t} + Be^{-0.0080t}\right)$$

Usamos condiciones iniciales para obtener valores de constantes A y B:

$$C_d(0) = 0:$$

$$0 = A + B \implies A = -B$$

$$C_s(0) = 0.5:$$

$$0.5 = \left(\frac{92000 \times -0.0047}{440}\right)A + \left(\frac{92000 \times -0.0080}{440}\right)B + \frac{92000}{55000}A + \frac{92000}{55000}B$$

$$0.5 = -0.9827A - 1.6727B + 1.6727A + 1.6727B$$

Sabemos que A = -B, entonces:

$$0.5 = 0.9827B - 1.6727B - 1.6727B + 1.6727B$$

$$0.5 = -0.69B$$

$$B \approx -0.7246$$

$$A \approx 0.7246$$

Reemplazamos valores de A y B en (5) para obtener C_d(t):

$$C_d(t) = Ae^{-0.0047t} + Be^{-0.0080t}$$

$$C_d(t) = 0.7246e^{-0.0047t} - 0.7246e^{-0.0080t}$$

Reemplazamos valores de A y B en (7) para obtener C_s(t):

$$C_s(t) = \frac{92000}{440}\left(-0.0047Ae^{-0.0047t} \; - \; 0.0080Be^{-0.0080t}\right) + \; \frac{92000}{55000}\left(Ae^{-0.0047t} + Be^{-0.0080t}\right)$$

$$C_s(t) \; = \; 209.0909\left(-0.0047(0.7246)e^{-0.0047t} \; - \; 0.0080(-0.7246)e^{-0.0080t} \right) + \; 1.6727\left(0.7246e^{-0.0047t} - 0.7246e^{-0.0080t}\right)$$

$$C_s(t) = \; - 0.7120e^{-0.0047t} + 1.2121e^{-0.0080t} + 1.2120e^{-0.0047t} - 1.2121e^{-0.0080t}$$

$$C_s(t) = 0.5e^{-0.0047t}$$