Definition

The vector projection of $\vec{v}$ onto $\vec{u}$ is denoted $\text{proj}_{\vec{u}}\vec{v}$. It has the same initial point as $\vec{u}$ and $\vec{v}$, and the same direction as $\vec{u}$, and represents the component of $\vec{v}$ acting in the direction of $\vec{u}$. If θ is the angle between $\vec{u}$ and $\vec{v}$, then the length of $\text{proj}_{\vec{u}}\vec{v}$ is $|\text{proj}_{\vec{u}}\vec{v}| = |\vec{v}|\cos(\theta)$.

Expressing $\cos(\theta)$ in terms of the dot product, this becomes

$$\begin{aligned} |\text{proj}_{\vec{u}}\vec{v}| &= |\vec{v}|\cos(\theta) \\ &= |\vec{v}|\left(\frac{|\vec{u}\cdot\vec{v}|}{|\vec{u}||\vec{v}|}\right) \\ &= \frac{|\vec{u}\cdot\vec{v}|}{|\vec{u}|} \end{aligned}$$

This is multiplied by a unit vector in the direction of $\vec{u}$ to give $\text{proj}_{\vec{u}}\vec{v}$

$$\text{proj}_{\vec{u}}\vec{v} = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|}\left(\frac{1}{|\vec{u}|}\vec{u}\right) = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|^2}\vec{u} \tag{6}$$

The magnitude of this vector is known as the scalar projection of $\vec{v}$ onto $\vec{u}$ and is denoted by

$$|\text{proj}_{\vec{u}}\vec{v}| = \text{comp}_{\vec{u}}\vec{v} = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|} \tag{7}$$