Proof of Commutative & Distributive Properties

Let $\vec{u} = \langle x_1, y_1 \rangle$ and $\vec{v} = \langle x_2, y_2 \rangle$

Commutative Property

Apply the commutative property for $\mathbb{R}$

$$\vec{u} + \vec{v} = \langle x_1 + x_2, y_1 + y_2 \rangle = \langle x_2 + x_1, y_2 + y_1 \rangle = \vec{v} + \vec{u}$$

Distributive Property

Apply the distributive property for $\mathbb{R}$

$$\begin{aligned} r(\vec{u} + \vec{v}) &= r \cdot \langle x_1 + x_2, y_1 + y_2 \rangle \\ &= \langle r(x_1 + x_2), r(y_1 + y_2) \rangle \\ &= \langle rx_1 + rx_2, ry_1 + ry_2 \rangle \\ &= \langle rx_1, ry_1 \rangle + \langle rx_2, ry_2 \rangle \\ &= r\vec{u} + r\vec{v} \end{aligned}$$