Week 10 Lecture Notes

1 7/15 Lecture

Torque $\vec{\tau}$

- Analogous to force but for rotation
- Accelerates rotational motion
- Has units of Nm

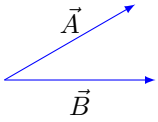
$$\sum \vec{F}_{net} = m\vec{a}$$
$$\vec{\tau} = I\vec{\alpha}$$

If there is some net force then there must be some acceleration. $\sum F_{net} \neq 0$

$$\tau = \vec{r} \times \vec{F} = |r||F| \sin(\varphi)$$

$$\vec{A} \times \vec{B} = \vec{C}$$

$\vec{A} \times \vec{B}$ = Inward
$\vec{B} \times \vec{A}$ = Outward



$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin(\theta)\hat{n}$$

Use the right hand rule to determine which direction
Anticommutative property: $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
The cross product between two parallel vectors is zero

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}_{\parallel}| = |\vec{A}_{\parallel}||\vec{B}|$$
$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}_{\perp}|$$

Comparison of Force and Torque

$$\vec{F}_{tot} = m\vec{a} = \sum \vec{F}_i$$
$$\vec{\tau} = I\vec{\alpha} = \sum \vec{r}_i \times \vec{F}_i$$

Example: Fly Fishing Event

$$-I_{cm} = \frac{1}{2}MR^2 = \frac{1}{2}(0.1)(6 \times 10^{-2}\text{ m})^2$$
$$= 1.8 \times 10^{-4}\text{ kgm}^2$$

$$I_{cm} = \frac{1}{2}mr^2$$
$$I'_{cm} = I_{cm} + mR^2$$
$$= \frac{1}{2}mr^2 + mR^2 = 7.3 \times 10^{-5}\text{ kgm}^2$$
$$I_{reel} = I_{cm} + I'_{cm} = 2.5 \times 10^{-4}\text{ kgm}^2$$

If you apply a force of 200 N tangent to the reel, what is the direction and magnitude of $\vec{\alpha}$:

$$\tau = R \times F = |R||F| \sin(90°) = I\vec{\alpha}$$
$$6 \times 10^{-2} \times (200 \text{ N}) = 12 \text{ Nm}$$
$$\alpha = \frac{RFsin(\theta)}{I} = \frac{12}{2.5 \times 10^{-4}} = 4.8 \times 10^{4} rad\ s^{-2}$$