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Review: Static Equilibrium
For a particle

- $F_{net} = 0$

For an extended object

- $F_{net} = 0$
- $\tau_{net} = 0$

$$\tau = r \times F = I\alpha$$

For linear momentum $\vec{P} = m\vec{v}$, for angular momentum, $\vec{L} = I\omega$

$$\begin{aligned} \vec{\tau} &= r \times F \\ \vec{L} &= \vec{r} \times \vec{P} \\ L = \vec{r} \times m\vec{v} &= mvrsin(\theta) \\ L_{max} &= mvr \ (v = r\omega) \\ &= (mr \times r)\omega = Iom\vec{e}ga \end{aligned}$$

Practice: Rotating Disk What is the angular momentum about the axle of a 2.0 kg, 4.0 cm diameter disk rotating at 600 rpm
Known: m, d, ω, Want: L

$$\begin{aligned} L &= I\omega \\ I &= \frac{1}{2}mR^2 \\ I &= \frac{1}{2}m(^d/_2)^2 \\ I = \frac{1}{2}m\frac{d^2}{4} = \frac{1}{8}md^2 L &= \frac{1}{8}md^2\omega \\ L &= 0.025 \text{ kg m}^2/\text{s} \end{aligned}$$

$$\vec{\tau}_{net} = I\alpha = \sum \tau_i = \sum \vec{r}_i \times \vec{F}_i$$

In the absence of external torques: $I\alpha = \dfrac{d(I\vec{\omega})}{dt} = 0$, $I\vec{\omega} = L =$ constant

Example: Krunchy on a Turntable

Krunchy of mass m rides on a disk of mass $6m$ and radius R as shown. The disk rotates bround its central axis at angular speed 1.5 rad s^{-1}

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Final will be in class, online, no work submitted, no partial credit (Do lots of MOI calculations to prepare for final)

Example: Putty on a Turntable

A small blob of putty of mass m falls from the ceiling and lands on the outer rim of a turntable of radius R and moment of inertia I_0 that is rotating freely with angular speed ω_0 about a vertical axis passing through the center of the turntable and perpendicular to the surface of the turntable

1. What is the post-collision angular speed of the turntable-putty system?
2. After several turns, the blob flies off the edge of the turntable. What is the angular speed of the turntable after the blob's departure?

   (a) $I_{d,cm} = I_0,\ \omega_d = \omega_0$

   (b) $I_{mass} = mR^2,\ \omega' =?,\ \tau_{ext} = 0$
   $L_i = L_f \ \rightarrow\ \omega_0 = (I_0 + mR^2)$

   (c) $v = R\omega'$
   $L = L_{I_0} + L_m$
   $R \times P \rightarrow L = Rmv\ L_m = RmR\omega'$
   $L_3 = I_0\omega'' + (mR^2)\omega'$
   $L_2 = L_3$
   $(I_0 + mR^2)\omega' = I_0\omega'' + (mR^2)\omega'$
   $I_0\omega' = I_0\omega''$
   $\omega' = \omega'' \neq \omega_0$

Example: Spinning Disks

Two uniform disks with masses $M_1 = 2$ kg, $M_2 = 5$ kg, and radii $R_1 = 0.10$ m, $R_2 = 0.15$ m are spinning freely about their center axis at frequencies $f_1 = 1200$ rpm, and $f_2 = 1500$ rpm. The cylinders are brought together and come to the same angular velocity via frictional contact. The moment of inertia of a uniform cylinder is given by $I = \frac{1}{2}MR^2$

A. Find the angular speed of each cylinder before they are joined
   $\omega = 2\pi f$
   $\omega_1 = 2\pi(1200 \text{ rev/min})(1 \text{ min}/60 \text{ s}) = 126 \text{ rad s}^{-1}$
   $\omega_2 = 2\pi(1500 \text{ rev/min})(1 \text{ min}/60 \text{ s}) = 157 \text{ rad s}^{-1}$

B. Find the total kinetic energy of the two cylinders before they are joined
   $M_1 = 2 \text{ kg},\ R_1 = 0.1 \text{ m},\ \omega_1 = 126 \text{ rad s}^{-1}\ M_2 = 5 \text{ kg},\ R_2 = 0.15 \text{ m},\ \omega_2 = 157 \text{ rad s}^{-1}$
   $L_i = L_f$
   $L_{i,1} + L_{i,2} = L_f$
   $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$
   $\frac{1}{2}M_1R_1^2\omega_1 + \frac{1}{2}M_2R_2^2\omega_2 = (\frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2)\omega_f$
   $\omega_f = \frac{\frac{1}{2}M_1R_1^2\omega_1 + \frac{1}{2}M_2R_2^2\omega_2}{\frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2}$
   $\frac{1}{2}M_1R_1^2\omega_1 = 1.26\ \frac{1}{2}M_2R_2^2\omega_2 = 8.83\ \frac{1}{2}M_1R_1^2 = 0.1$

C. Find the angular speed of each cylinder after they couple

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Example: Merry-Go-Round

A 25 kg child in a playground runs with an initial speed of 2.5 ms^{-1} along a path tangent ot the rim of a merry-go-round, whos radius is $R = 2$ m. The child carries a ball of mass 0.5 kg. The merry-go-round which is initially at rest, has a moment of inertia of 500 kg m^2. The child then jumps on.

(a) Find the angular velocity of the child and merry-go-round together. You may assume that the child and the ball are the same distance from the center.

(b) After a few rotations, the child throws the ball with a speed of 20 ms^{-1} at an angle 45° to the vertical. What is the new angular speed of the merry-go-round and the child together