Week 10 Lecture Notes

1 7/15 Lecture

Torque $\vec{\tau}$

- Analogous to force but for rotation
- Accelerates rotational motion
- Has units of Nm

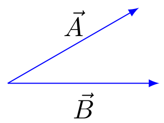
$$\sum \vec{F}_{net} = m\vec{a}$$
$$\vec{\tau} = I\vec{\alpha}$$

If there is some net force then there must be some acceleration. $\sum F_{net} \neq 0$

$$\tau = \vec{r} \times \vec{F} = |r||F|\sin(\varphi)$$

$$\vec{A} \times \vec{B} = \vec{C}$$

$\vec{A} \times \vec{B} =$ Inward
$\vec{B} \times \vec{A} =$ Outward



$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin(\theta)\hat{n}$$

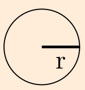
Use the right hand rule to determine which direction
Anticommutative property: $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
The cross product between two parallel vectors is zero

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}_{\parallel}| = |\vec{A}_{\parallel}||\vec{B}|$$
$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}_{\perp}|$$

Comparison of Force and Torque

$$\vec{F}_{tot} = m\vec{a} = \sum \vec{F}_i$$
$$\vec{\tau} = I\vec{\alpha} = \sum \vec{r}_i \times \vec{F}_i$$

Example: Fly Fishing Event



$$-I_{cm} = \frac{1}{2}MR^2 = \frac{1}{2}(0.1)(6 \times 10^{-2}\ \text{m})^2$$
$$= 1.8 \times 10^{-4}\ \text{kgm}^2$$

$$I_{cm} = \frac{1}{2}mr^2$$
$$I'_{cm} = I_{cm} + mR^2$$
$$= \frac{1}{2}mr^2 + mR^2 = 7.3 \times 10^{-5}\ \text{kgm}^2$$
$$I_{reel} = I_{cm} + I'_{cm} = 2.5 \times 10^{-4}\ \text{kgm}^2$$

If you apply a force of 200 N tangent to the reel, what is the direction and magnitude of $\vec{\alpha}$:

$$\tau = R \times F = |R||F|\sin(90°) = I\vec{\alpha}$$
$$6 \times 10^{-2} \times (200\text{ N}) = 12\text{ Nm}$$
$$\alpha = \frac{RFsin(\theta)}{I} = \frac{12}{2.5 \times 10^{-4}} = 4.8 \times 10^{4} rad\ s^{-2}$$

2 7/16 Lecture

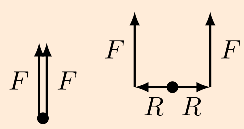
$$\begin{aligned} &\theta \\ \vec{\omega} &= \frac{d\theta}{dt}\hat{n} \\ \vec{\alpha} &= \frac{d\vec{\omega}}{dt} \\ K_{rot} &= \frac{1}{2}I\omega^2 \\ \tau_{tot} &= I\vec{\alpha} = \sum \vec{r}_i \times \vec{F}_i \end{aligned}$$

$$\begin{aligned} &\vec{r} \\ \vec{v} &= \frac{d\vec{r}}{dt} \\ \vec{a} &= \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \\ K_{lin} &= \frac{1}{2}mv^2 \\ \vec{F}_{tot} &= m\vec{a} = \sum \vec{F}_i \end{aligned}$$

Example: Space Telescope

A telescope in outer space is observing some stars. Two air thrusters are each a distance R on either side of the center of mass of the telescope. When the thrusters are activated, each produces a force F in the same direction.

A. Draw a free body diagram of the telescope, indicating forces acting directly on the mass as a point. Then draw a diagram indicating the placement of the forces relative to the center of mass

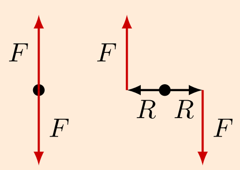


B. What is the net force acting on the telescope? What is the net torque around the center of mass?

$$\sum F_{net} = F\hat{z} + F\hat{z}$$
$$\sum F_{net} = 2F\hat{z}$$

$$\sum \tau_{net} = RF\sin(90°)\hat{y} + RF\sin(-90°)\hat{y}$$
$$\sum \tau_{net} = RF\sin(90°)\hat{y} - RF\sin(90°)\hat{y} = 0$$

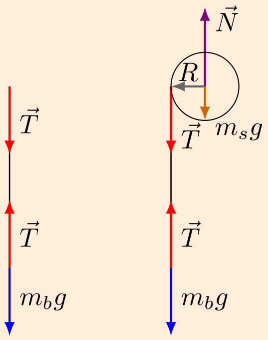
C. Would the telescope have a linear acceleration? Angular acceleration?



$$\sum F_{net} = F\hat{z} - F\hat{z}$$
$$\sum F_{net} = 0$$

$$\sum \tau_{net} = RF\sin(90°)\hat{y} + RF\sin(90°)\hat{y}$$
$$\sum \tau_{net} = 2RF\hat{y}$$

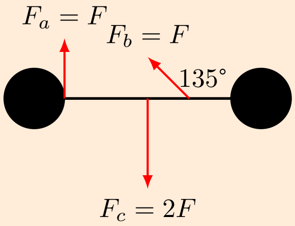
Example: Acceleration of a Bucket



$$
\begin{aligned}
F_{net} &= ma \\
T - m_b g &= m_b a \\
a &= \frac{T - m_b g}{m_b}
\end{aligned}
$$

$$\tau = r \times F$$

Example: Dumbell



(a) Find the moment of inertia of the dumbell

$$
\begin{aligned}
I &= I_{b-left} + I_r + I_{b-right} \\
&= (I_b + M_b(R + {}^{L}\!/_{2})^2) + \frac{1}{12} M_r L^2 + (I_b + M_b(R + {}^{L}\!/_{2})^2) \\
&= 2\left(\frac{2}{5} M_b R^2 + M_b(R + {}^{L}\!/_{2})^2\right) + \frac{1}{12} M_r L^2
\end{aligned}
$$

(b) Consider the applied forces as shown. Find the angular acceleration

$$
\begin{aligned}
\tau_z &= \tau_a + \tau_b + \tau_c \\
&= \left(-\frac{1}{2} FL \sin(90°)\right) + \left(\frac{1}{4} FL \sin(45°)\right) + 0 \\
&= \left(\frac{\sqrt{2}}{8} - \frac{1}{2}\right) FL \\
\tau_z &= I a_z \\
A_z &= \frac{\tau_z}{I} = \frac{\left(\frac{\sqrt{2}}{8} - \frac{1}{2}\right) FL}{2\left(\frac{2}{5} M_b R^2 + M_b(R + {}^{L}\!/_{2})^2\right) + \frac{1}{12} M_r L^2}
\end{aligned}
$$