

PHY 2048 Formulas

Dot and Cross Product

$$\vec{A} \cdot \vec{B} = AB \cos(\varphi)$$

$$\vec{A} \times \vec{B} = AB \sin(\varphi)$$

Translational Kinematics

$$x = x_0 + vt$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2a(\Delta x)$$

Projectile Motion

$$\text{Time of flight: } T = \frac{2v_0 \sin(\theta_0)}{g}$$

Trajectory:

$$y = x \tan(\theta) - \left[\frac{g}{2(v_0 \cos(\theta))^2} \right] x^2$$

$$\text{Range: } \frac{v_0^2 \sin(2\theta_0)}{g}$$

Uniform Circular Motion

$$a_c = \frac{v_T^2}{r} \quad T = \frac{2\pi}{V}$$

$$\omega = \frac{2\pi}{T} \quad r = \frac{v^2}{g}$$

Orbital Motion

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \frac{2\pi r}{T}$$

Terminal Velocity

$$v_T = \sqrt{\frac{2mg}{\rho C A}}$$

Newton's Laws

$$1. v = \text{constant when } F_{net} = 0$$

$$2. F_{net} = ma = \frac{dp}{dt} = \frac{d}{dt}(mv)$$

$$3. \vec{F}_{AB} = -\vec{F}_{BA}$$

Common Forces

$$N = mg \cos(\theta)$$

$$F_{sp} = -k\Delta x$$

$$f_s \leq \mu_s N$$

$$f_k = \mu_k N$$

$$F_c = ma_c = m \frac{v^2}{r} = mr\omega^2$$

Conservative Force

$$\frac{dF_x}{dy} = \frac{dF_y}{dx}$$

Work

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$$

$$W = \vec{F} \cdot \vec{d} = Fd \cos(\theta)$$

$$W_g = -mg\Delta y$$

$$W_{sp} = \frac{1}{2}k(x_f^2 - x_0^2)$$

$$W_f = \mu_k N d = \mu_k mgd$$

(F_{cons} , closed path)

$$W = \oint \vec{F} \cdot d\vec{r} = 0$$

Energy

$$K_t = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$K_r = \frac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_{sp} = \frac{1}{2}k(x_f^2 - x_0^2)$$

$$\Delta K_{AB} = -\Delta U_{AB}$$

Work-Energy Theorem

$$W_{net} = \Delta K = K_f - K_0$$

$$-W_{net} = \Delta U = U_f - U_0$$

Conservation of Energy

$$K_A + U_A = K_B + U_B \quad (\text{if } W_{nc} = 0)$$

$$W_{nc} = \Delta(K + U) = \Delta E$$

Momentum & Impulse

$$\vec{p} = m\vec{v}$$

$$\vec{J} = \Delta p = m\Delta v$$

$$\vec{J} = \vec{F}_{avg} \Delta t \rightarrow \vec{F}_{avg} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\vec{J} = \int_{t_0}^{t_f} \vec{F}(t) dt$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Rolling Motion, No Slipping

$$a_{cm} = \frac{mg \sin(\theta)}{1 + (I_{cm}/r^2)}$$

Conservation of Momentum

If:

$$1. \left[\frac{dm}{dt} \right]_{sys} = 0, \text{ and}$$

$$2. \vec{F}_{ext} = 0, \text{ then:}$$

$$\frac{d}{dt} \sum_N (\vec{p}_0 + \vec{p}_f) = 0$$

$$\sum_{j=1} \vec{p}_j = \text{constant}$$

$$\vec{v}_{cm,f} = \vec{v}_{cm,0}$$

Center of Mass

System of particles:

$$\vec{r}_{cm} = \frac{1}{M} \sum_{j=1}^N m_j \vec{r}_j$$

$$v_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{1}{M} \sum_{j=1}^N m_j \frac{d\vec{r}_j}{dt}$$

$$M\vec{v}_{cm} = \sum_{j=1}^N m_j \vec{v}_j$$

Continuous object:

$$\vec{r}_{cm} = \frac{1}{M} \int \vec{r} dm$$

Rotational Variables

$$\theta = \frac{s}{r}$$

$$\omega = \frac{v_t}{r} = \frac{d\theta}{dt}$$

$$\alpha = \frac{a_t}{r} = \frac{d\omega}{dt}$$

Rotational Kinematics

$$\theta_f = \theta_0 + \omega t$$

$$\omega_f = \omega_0 + \alpha t$$

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$$

Moment of Inertia

$$I = \sum_i m_i r_i^2$$

$$I = \int r^2 dm$$

$$I_{tot} = \sum_i I_i$$

Parallel Axis Theorem

$$I_{pa} = I_{cm} + md^2$$

Torque

$$\tau = \vec{r} \times \vec{F} = rF \sin(\theta) = I\alpha$$

$$\tau_{net} = \sum_i \tau_i = I\alpha$$

$$W = \int \sum \vec{\tau} \cdot d\vec{\theta}$$

$$W_{AB} = \int_{\theta_a}^{\theta_B} \left(\sum_i \tau_i \right) d\theta$$

Angular Momentum

$$\vec{l} = \vec{r} \times \vec{p} \rightarrow l = rp \sin(\theta)$$

$$L = I\omega$$

$$\frac{d\vec{L}}{dt} = \sum_i \frac{d\vec{l}_i}{dt} = \sum_i \tau_i$$

Conservation of \vec{L}

$$\text{If: } \sum \vec{\tau} = 0$$

$$\text{Then: } \frac{d\vec{L}}{dt} = 0 \rightarrow I_f \omega_f = I_0 \omega_0,$$