PHY 2048 Formulas

Dot and Cross Product

$\vec{A} \cdot \vec{B} = AB\cos(\varphi)$

$\vec{A} \times \vec{B} = AB\sin(\varphi)$

Translational Kinematics

$x = x_0 + vt$

$v = v_0 + at$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a(\Delta x)$

Projectile Motion

Time of flight: $T = \frac{2v_0 \sin(\theta_0)}{g}$

Trajectory:

$y = x\tan(\theta) - \left[\frac{g}{2(v_0\cos(\theta))^2}\right]x^2$

Range: $\frac{v_0^2 \sin(2\theta_0)}{g}$

Uniform Circular Motion

$a_c = \frac{v_T^2}{r}$ $\quad T = \frac{2\pi}{V}$

$\omega = \frac{2\pi}{T}$ $\quad r = \frac{v^2}{g}$

Orbital Motion

$v = \sqrt{\frac{GM}{r}}$

$v = \frac{2\pi r}{T}$

Terminal Velocity

$v_T = \sqrt{\frac{2mg}{\rho CA}}$

Newton's Laws

1. $v =$ constant when $F_{net} = 0$
2. $F_{net} = ma = \frac{dp}{dt} = \frac{d}{dt}(mv)$
3. $\vec{F}_{AB} = -\vec{F}_{BA}$

Common Forces

$N = mg\cos(\theta)$

$F_{sp} = -k\Delta x$

$f_s \leq \mu_s N$

$f_k = \mu_k N$

$F_c = ma_c = m\frac{v^2}{r} = mr\omega^2$

Conservative Force

$\frac{dF_x}{dy} = \frac{dF_y}{dx}$

Work

$W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$

$W = \vec{F} \cdot \vec{d} = Fd\cos(\theta)$

$W_g = -mg\Delta y$

$W_{sp} = \frac{1}{2}k(x_f^2 - x_0^2)$

$W_f = \mu_k Nd = \mu_k mgd$

Energy

$K_t = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_{sp} = \frac{1}{2}k(x_f^2 - x_0^2)$

$\Delta K_{AB} = -\Delta U_{AB}$

$W = \oint \vec{F} \cdot d\vec{r} = 0 \quad \text{(closed path)}$

Work-Energy Theorem

$W_{net} = \Delta K = K_f - K_0$

$-W_{net} = \Delta U = U_f - U_0$

Conservation of Energy

With conservative forces:

$K_A + U_A = K_B + U_B$

$W_{nc} = \Delta(K + U) = \Delta E$

Momentum & Impulse

$\vec{p} = m\vec{v}$

$\vec{J} = \Delta p = m\Delta v$

$\vec{J} = \vec{F}_{avg}\Delta t \rightarrow \vec{F}_{avg} = \frac{\Delta \vec{p}}{\Delta t}$

$\vec{J} = \int_{t_0}^{t_f} \vec{F}(t)dt$

$\vec{F} = \frac{d\vec{p}}{dt}$

Rolling Motion, No Slipping

$a_{cm} = \frac{mg\sin(\theta)}{1 + (I_{cm}/r^2)}$

Conservation of Momentum

If:

1. $\left[\frac{dm}{dt}\right]_{sys} = 0$, and
2. $\vec{F}_{ext} = 0$, then:

$\frac{d}{dt}(\vec{p}_0 + \vec{p}_f) = 0$

$\sum_{j=1}^{N} \vec{p}_j = \text{constant}$

$\vec{v}_{cm,f} = \vec{v}_{cm,0}$

Center of Mass

System of particles:

$\vec{r}_{cm} = \frac{1}{M}\sum_{j=1}^{N} m_j \vec{r}_j$

$v_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{1}{M}\sum_{j=1}^{N} m_j \frac{d\vec{r}_j}{dt}$

$M\vec{v}_{cm} = \sum_{j=1}^{N} m_j \vec{v}_j$

Continuous object:

$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}dm$

Rotational Variables

$\theta = \frac{s}{r}$

$\omega = \frac{v_t}{r} = \frac{d\theta}{dt}$

$\alpha = \frac{a_t}{r} = \frac{d\omega}{dt}$

Rotational Kinematics

$\theta_f = \theta_0 + \omega t$

$\omega_f = \omega_0 + \alpha t$

$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$

Moment of Inertia

$I = \sum_i m_i r_i^2$

$I = \int r^2 dm$

$I_{tot} = \sum_i I_i$

Parallel Axis Theorem

$I_{pa} = I_{cm} + md^2$

Torque

$\tau = \vec{r} \times \vec{F} = rF\sin(\theta) = I\alpha$

$\tau_{net} = \sum_i \tau_i = I\alpha$

$W = \int \sum \vec{\tau} \cdot d\vec{\theta}$

$W_{AB} = \int_{\theta_a}^{\theta_B} \left(\sum_i \tau_i\right) d\theta$

Angular Momentum

$\vec{l} = \vec{r} \times \vec{p} \rightarrow l = rp\sin(\theta)$

$L = I\omega$

$\frac{d\vec{L}}{dt} = \sum_i \frac{d\vec{l}_i}{dt} = \sum_i \tau_i$

Conservation of $\vec{L}$

If: $\sum \vec{\tau} = 0$

Then: $\frac{d\vec{L}}{dt} = 0 \rightarrow I_f\omega_f = I_0\omega_0,$