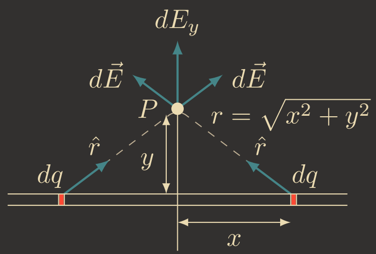
Example: Line Charge - A Power Line's Field

A long, straight electric power line coincides with the x-axis and carries a uniform line charge density λ (unit $=$ C m^{-1}). Find the electric field on the y-axis using the approximation that the wire is infinitely long.
The field point P is placed along the y-axis and the wire is divided into infinitesimal charge elements dq. The x-components of the fields $d\vec{E}$ contributed by elements dq on opposite sides of the y-axis cancel, so only the y-components of each unit vector are needed. $\hat{r}_y = \sin(\theta) = {}^{y}/_{r}$ where $r = \sqrt{x^2 + y^2}$



The wire has charge density λ C m^{-1}, so if a charge element has length dx, its charge is $dq = \lambda dx$

$$dE_y = \frac{k_e dq}{r^2}\hat{r} = \frac{k_e \lambda dx}{r^2}\frac{y}{r} = \frac{k\lambda y}{(x^2+y^2)^{3/2}}dx$$

Where $r = \sqrt{x^2 + y^2}$. Since the x-components cancel, the y-components can be integrated to get the net field

$$E = E_y = \int_{-\infty}^{\infty} \frac{k_e \lambda y}{(x^2+y^2)^{3/2}}dx = k_e \lambda y \int_{-\infty}^{\infty} \frac{1}{(x^2+y^2)^{3/2}}dx$$
$$= k_e \lambda y \left[\frac{x}{y^2\sqrt{x^2+y^2}}\right]_{-\infty}^{\infty} = k_e \lambda y \left[\frac{1}{y^2} - \left(\frac{1}{y^2}\right)\right] = \frac{2k_e\lambda}{y}$$

The result is the field's magnitude in the radial direction away from the wire for $+\lambda$ and toward the wire for $-\lambda$