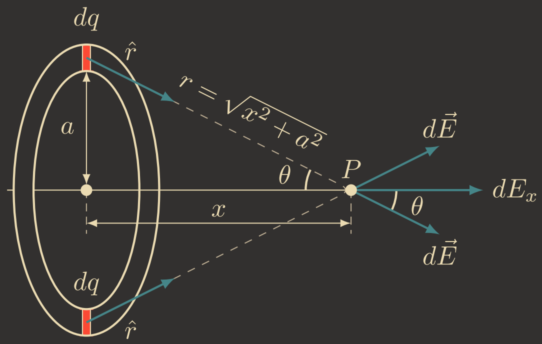
Example: Evaluating The Field - A Charged Ring

A ring of radius a has a charge Q distributed evenly over the ring. Find an expression for the electric field at any point on the axis of the ring.

Take the x-axis to coincide with the ring axis with the center of the ring at $x = 0$. The y-components of the field contributions from pairs of charge elements on opposite sides of the ring cancel so the net field points in the $+x$ direction (for $x > 0$), and only the x-components of the unit vectors are needed, namely, $\hat{r}_x = r\cos(\theta) = \frac{x}{r}$



Each charge element dq contributes the same amount to the field, $dE_x = \dfrac{k_e dq}{r^2}\hat{r}_x = \dfrac{k_e x dq}{r^3}$

Writing $r = \sqrt{x^2 + a^2}$ as $(x^2 + a^2)^{1/2}$, the integral becomes

$$E = \int_{\text{ring}} dE_x = \int_{\text{ring}} \frac{k_e x \, dq}{(x^2 + a^2)^{3/2}} = \frac{k_e x}{(x^2 + a^2)^{3/2}} \int_{\text{ring}} dq$$

Because the field point P is fixed, its coordinate x is a constant in the integration. The remaining integral is the sum of all the charge elements on the ring, the total charge Q.

$$E = \frac{k_e Q x}{(x^2 + a^2)^{3/2}}$$