Gamma-Frequency Resonance in Networks of Neurons

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Motivation - Why Gamma Oscillations?

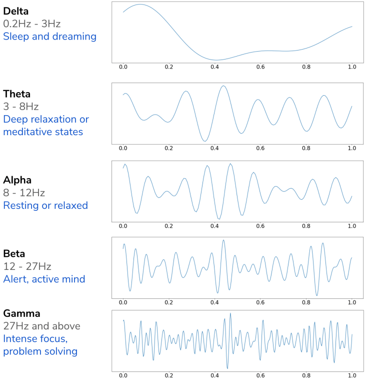


Figure: Brain waves in typical EEG [1]

- Linked to attention, memory and perception.
- Disrupted in disorders like epilepsy and schizophrenia.
- Yet, their origins in neural circuits are still unclear
- Inhibitory networks are known to support gamma rhythms — but how exactly do they do so?

Project Aims & Scope

Aim:
Understand how gamma-band oscillations emerge and remain stable in inhibitory Izhikevich networks.

Scope:

- Simulate spiking inhibitory networks with 2^{nd}-order synaptic dynamics (biologically inspired).
- Compare sparse vs. all-to-all connectivity for synchrony and robustness.
- Vary key parameters: synaptic strength (g_{ij}), time constant (τ_s), heterogeneity, and noise.
- Apply mean-field reduction to link spiking activity to population-level dynamics.

Biological Background

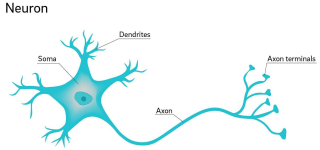


Figure: Neuron structure [2].

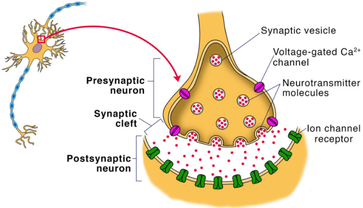


Figure: Synaptic transmission [3].

- **Neurons:** Basic units of the brain that transmit signals.
- **Action Potential:** A brief electrical spike enabling communication.
- **Oscillations:** Rhythmic activity across neurons (e.g., gamma: 30–90 Hz).
- **Synapse:** The junction where neurons communicate.
- **Excitatory:** Increases the chance of neuron firing.
- **Inhibitory:** Decreases the chance of neuron firing.

Modelling Approach – Neuron Dynamics

Neuron Model: Variation of the Izhikevich model employed by Tikidji-Hamburyan et al. [4].

$$\begin{cases} \frac{dv_j}{dt} = 0.04v_j^2 + 5v_j + 140 - u_j + I_j + J_j(t) - (v_j - E_{\text{syn}}) \sum_{i=1}^{N} g_{ij} s_i(t) \\ \frac{du_j}{dt} = a(bv_j - u_j) \end{cases} \tag{1}$$

Reset Conditions:

$$\text{If } v_j \geq 30\,\text{mV}, \quad \begin{cases} v_j \to v_{\text{reset}} \\ u_j \to u_j + u_{\text{jump}} \\ p_i \to p_i + p_0 \end{cases} \tag{2}$$

Modelling Approach – Synaptic Coupling

Coupling: Second-order synaptic coupling is used to describe $s_i(t)$:

$$\left(1 + \tau_s \frac{d}{dt}\right)^2 s_i = p_0 \sum_{t_i^q < t} \delta(t - t_i^q) \tag{3}$$

This can be written as two first order differential equations:

$$\begin{cases} \frac{ds_i}{dt} = -\frac{s_i}{\tau_s} + \frac{p_i}{\tau_s} \\ \frac{dp_i}{dt} = -\frac{p_i}{\tau_s} + \frac{p_0}{\tau_s} \sum_{t_i^q < t} \delta(t - t_i^q) \end{cases} \tag{4}$$

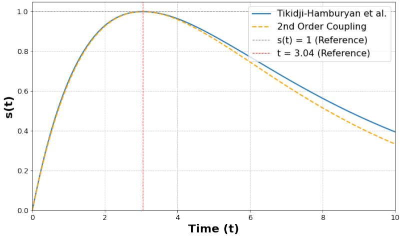
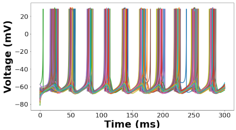


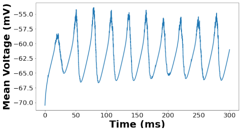
Figure: Comparison of the second-order synaptic model with model used in [4].

Simulation Results – Sparse Network

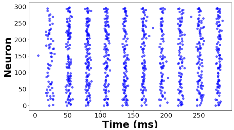
Synchronous Regime



Membrane potential traces

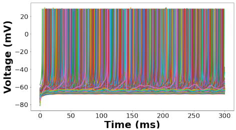


Mean membrane potential

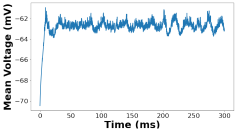


Spike raster plot

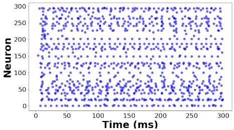
Asynchronous Regime



Membrane potential traces



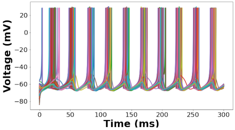
Mean membrane potential



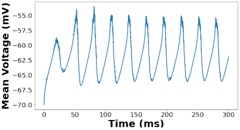
Spike raster plot

Simulation Results - All-to-All

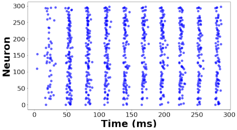
Synchronous Regime



Membrane potential traces

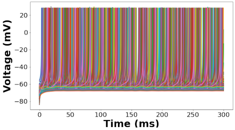


Mean membrane potential

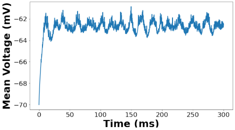


Spike raster plot

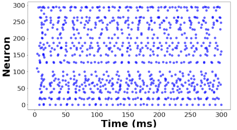
Asynchronous Regime



Membrane potential traces

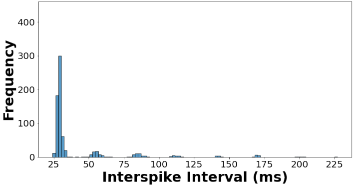


Mean membrane potential

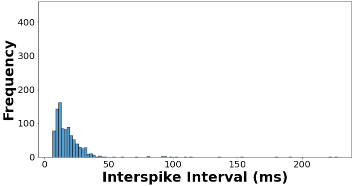


Spike raster plot

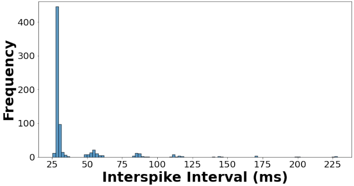
Interspike Intervals



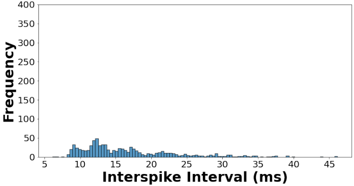
Sparse connectivity in synchronous regime



Sparse connectivity in asynchronous regime



All-to-all connectivity in synchronous regime



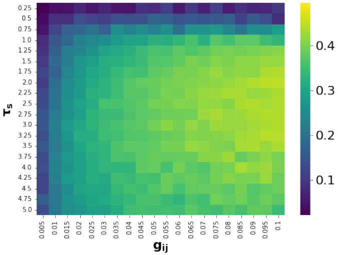
All-to-all connectivity in asynchronous regime

Quantitative Comparison of Network Dynamics

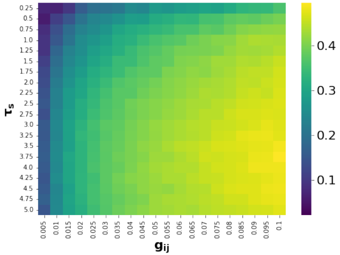
Network Type	χ^2	Reliability	Firing Rate (Hz)	Network Freq. (Hz)
Sparse Synchronous	0.2667	0.1103	10.45	34.14
Sparse Asynchronous	0.0117	0.0075	11.23	–
All-to-all Synchronous	0.2700	0.1056	10.05	35.20
All-to-all Asynchronous	0.0121	0.0068	9.83	–

Comparison of synchrony, reliability, firing rate, and network frequency across network types.

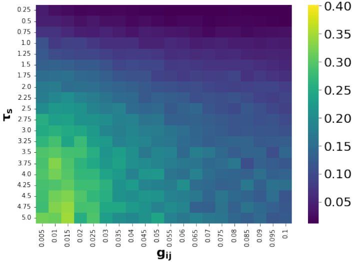
Parameter Exploration - Synchrony



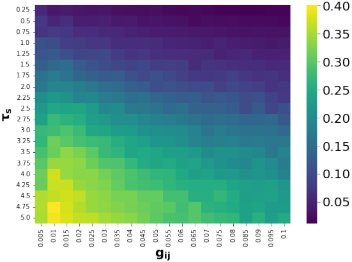
χ^2 in sparse network



χ^2 in all-to-all network

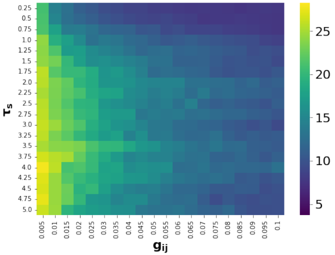


Reliability in sparse network

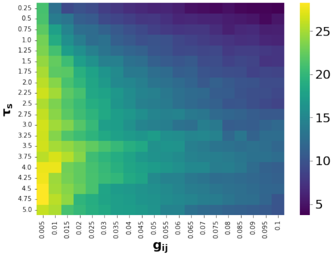


Reliability in all-to-all network

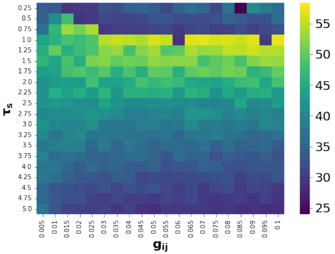
Parameter Exploration - Firing Rate and Network Freq.



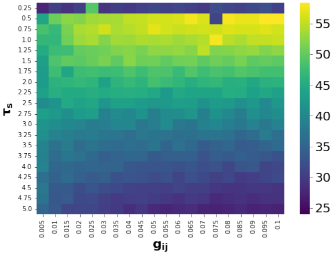
Firing rate in sparse network



Firing rate in all-to-all network



Network frequency in sparse network



Network frequency in all-to-all network

Mean-Field Reduction Equations

$$r'(t) = \frac{\Delta_\eta}{\pi} + 0.08\, rv + (5 - gs)\, r \tag{5}$$

$$\langle v(t)\rangle' = 0.04\, v^2 + 5v + 140 - u + I + \bar{\eta} - gs(v - E_{\text{syn}}) - \frac{\pi^2}{0.04}\, r^2 \tag{6}$$

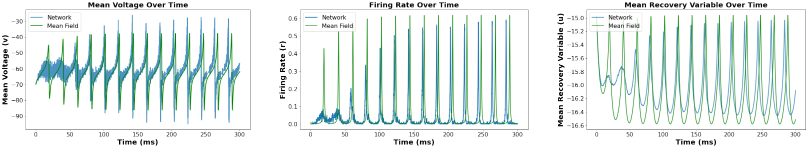
$$\langle u(t)\rangle' = a(bv - u) + u_{\text{jump}}\, r \tag{7}$$

$$\langle s(t)\rangle' = -\frac{s}{\tau_s} + \frac{p}{\tau_s} \tag{8}$$

$$\langle p(t)\rangle' = -\frac{p}{\tau_s} + p_0\, r \tag{9}$$

- Captures the macroscopic dynamics of a spiking network using population-level variables.
- Incorporates adaptation, synaptic coupling, and firing rate evolution within a low-dimensional system.

Comparison of Network and Mean-Field Dynamics



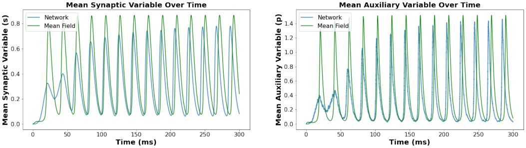
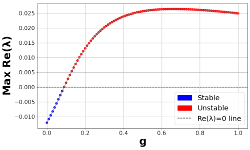
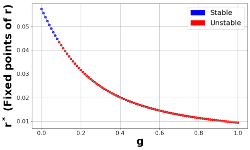


Figure: Comparison of the temporal behaviour of the network of Izhikevich neurons (blue) and the corresponding mean-field model (green).

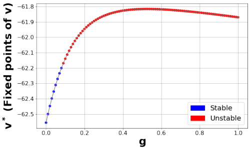
Bifurcation and Phase Plane Analysis - Synaptic Strength



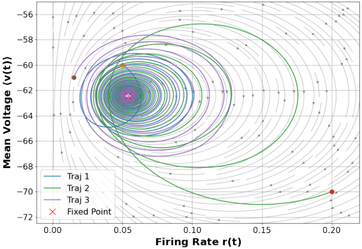
Max real part of Jacobian eigenvalues vs. g.



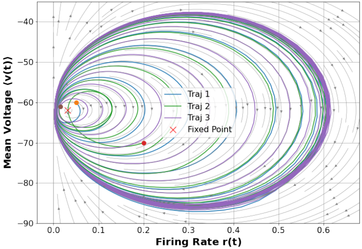
Fixed points of firing rate r^* vs. g.



Fixed points of voltage v^* vs. g.

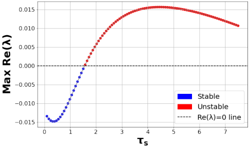


Stable spiral for $g < 0.08959$.

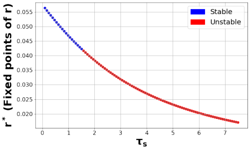


Unstable spiral for $g > 0.08959$.

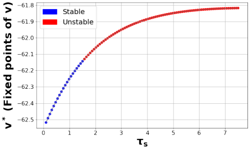
Bifurcation and Oscillation Analysis



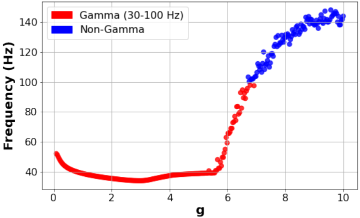
Max real part of Jacobian eigenvalues vs. τ_s.



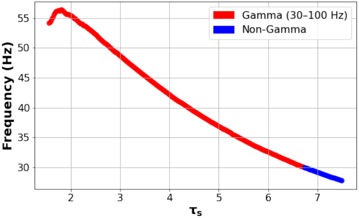
Fixed points of firing rate r^* vs. τ_s.



Fixed points of voltage v^* vs. τ_s.



Oscillation frequency vs. g.



Oscillation frequency vs. τ_s.

Conclusion

- Developed and analysed a mean-field model for Izhikevich-based spiking neural networks.
- Strong agreement with full network simulations across dynamical regimes.
- Bifurcation analysis revealed critical parameters for stability and oscillations.
- τ_s and g crucial in shaping gamma-band oscillations.
- Sparse and all-to-all networks show same quantitative dynamics under fixed g, and qualitatively similar behaviour as τ_s, g vary.
- Reduced models efficiently capture complex network dynamics.

Limitations and Future Work

Limitations

- Mean-field model assumes infinite network size and idealised input distributions.
- Valid agreement between sparse and all-to-all networks only holds for $g \leq 0.1$, $\tau_s \leq 5$ms.
- Simplifying assumptions (e.g., infinite threshold/reset) reduce biological realism.
- Quantitative mismatch in oscillation amplitude and frequency between model and full simulation.

Future Work

- Extend the model to include an excitatory population.
- Investigate structured connectivity.
- Conduct two-parameter bifurcation analysis.
- Systematically map the breakdown of mean-field accuracy across parameter space.

References

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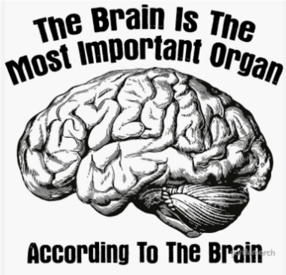
Thank you for listening!

I'd be happy to take any questions.



Synchrony and Firing Rate Measures

Synchrony – χ^2 Measure:

- Quantifies voltage synchrony across neurons.

$$\chi^2 = \frac{\sigma_V^2}{\frac{1}{N}\sum_{i=1}^{N}\sigma_{V_i}^2} \tag{10}$$

- 0: asynchronous, 1: perfect synchrony.

Firing Rate:

- Mean spikes per neuron per second after initial 100ms:

$$\mathrm{FR} = \frac{\text{Total spikes after 100ms}}{N(T-100)} \times 1000 \tag{11}$$

Next: Spike-time reliability measure using convolved spike trains.

Spike-Time Reliability – R Measure

Idea: Measures consistency in spike timing across trials.

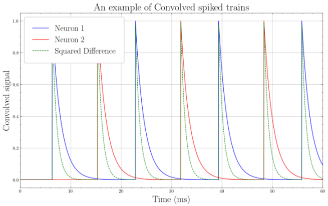
How it works:

- Convolve each spike train with an exponential kernel.
- Sum across neurons: $X(t)$.
- Compute:
  $R = \int X^2(t)\,dt - \left(\int X(t)\,dt\right)^2$

Interpretation:

- **Sharp peaks** in $X(t) \to$ high R (reliable spikes).
- **Smoother** $X(t) \to$ low R (jittered or desync'd).

Normalised: $R/R_{\text{max}} \in [0, 1]$



Example of convolved spike trains. Sharp overlap between neurons indicates reliable timing (high R).