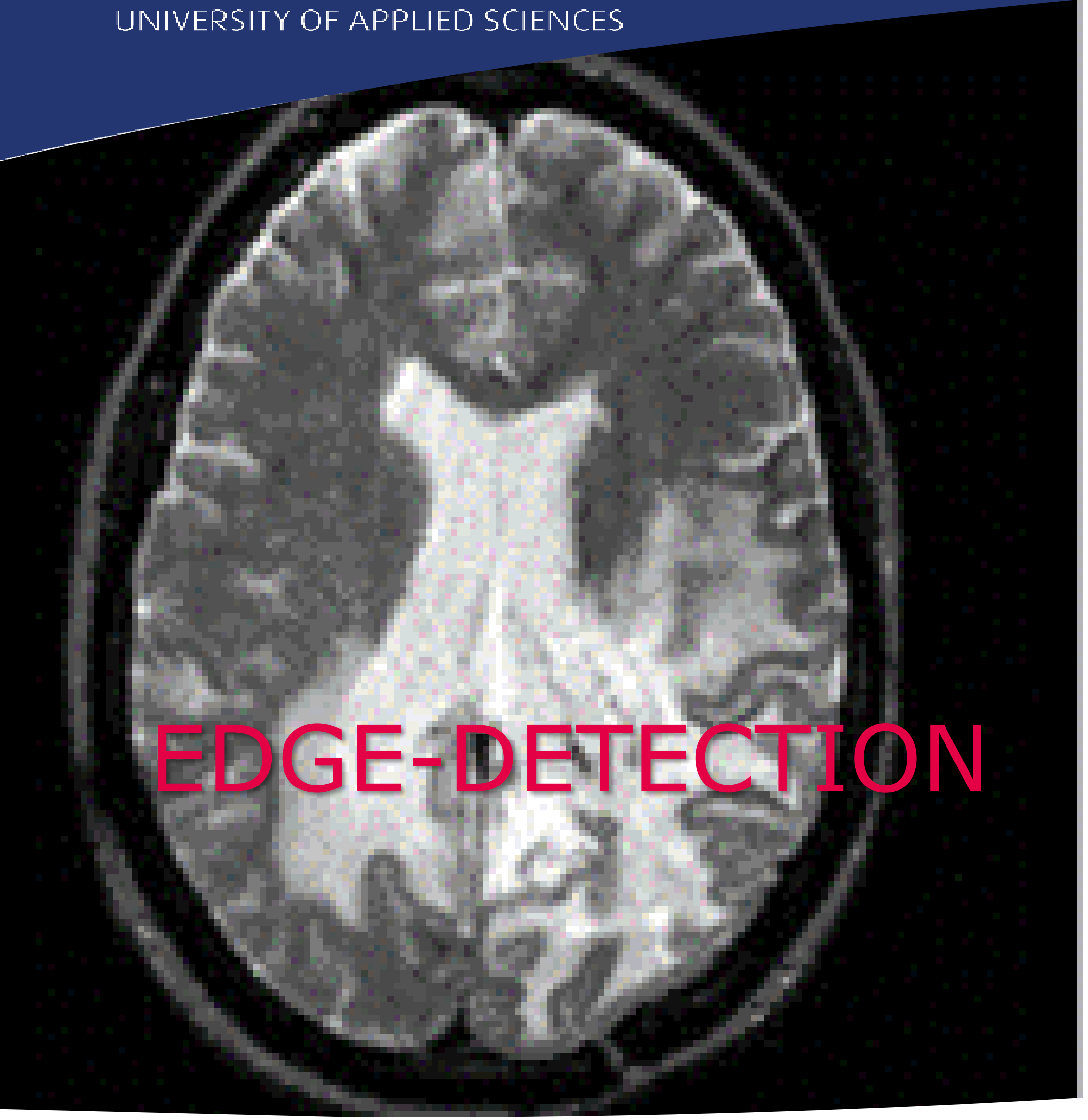


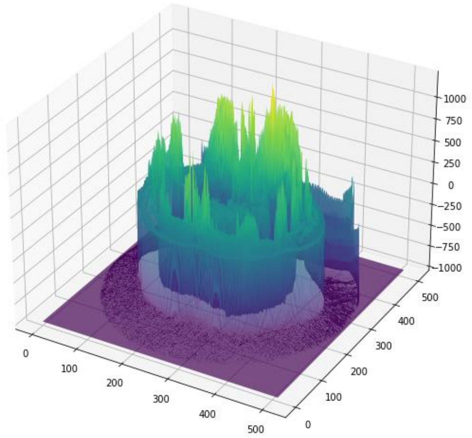
# EDGE-DETECTION

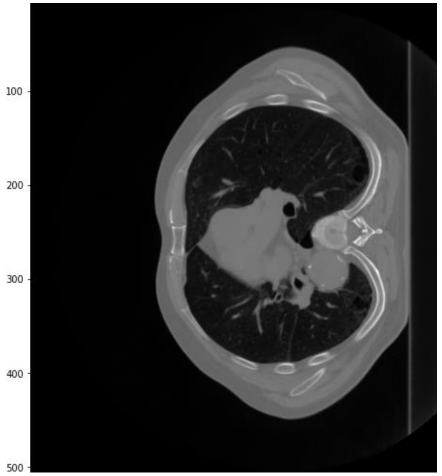
# Edge Detection

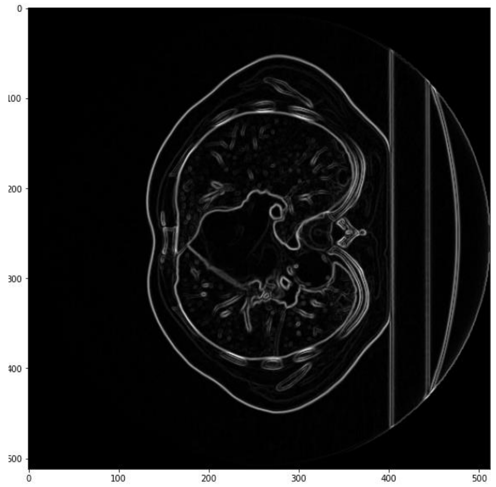
How can we detect changes?

- Edges: **Boundary** (steep-changes) between two regions with distinct gray-level properties.

- Most important visual information.



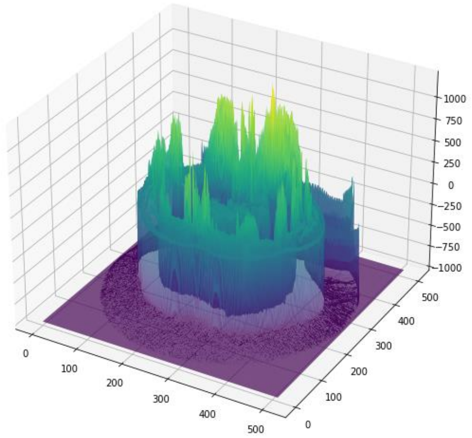



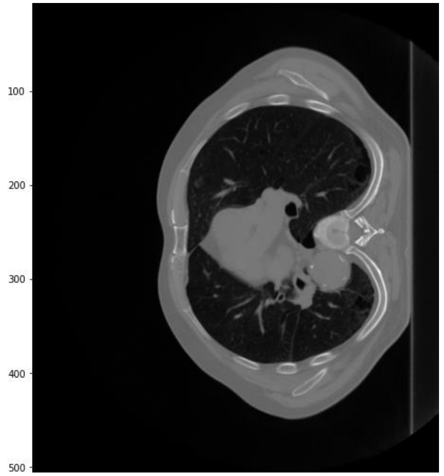



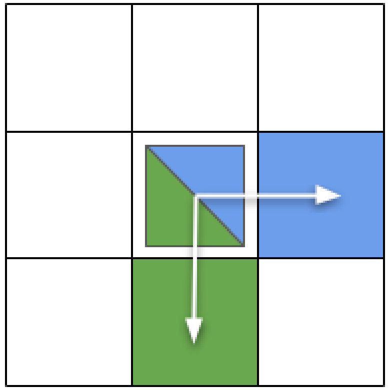
# Edge Detection – Derivatives!

- Steepness is defined by spatial derivatives: $\frac{\partial I}{\partial x}$ and $\frac{\partial I}{\partial y}$ (and $\frac{\partial I}{\partial z}$ in 3D)

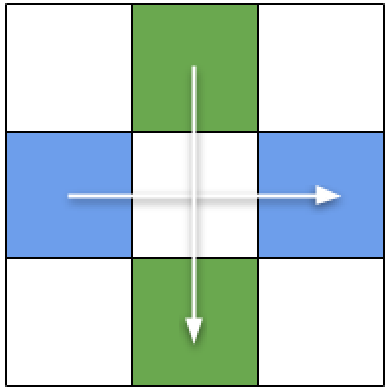
- Image edges (gradients) have direction and magnitude



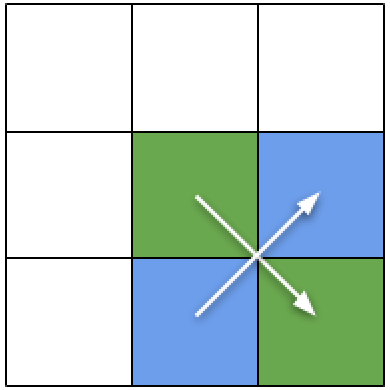



Forward difference



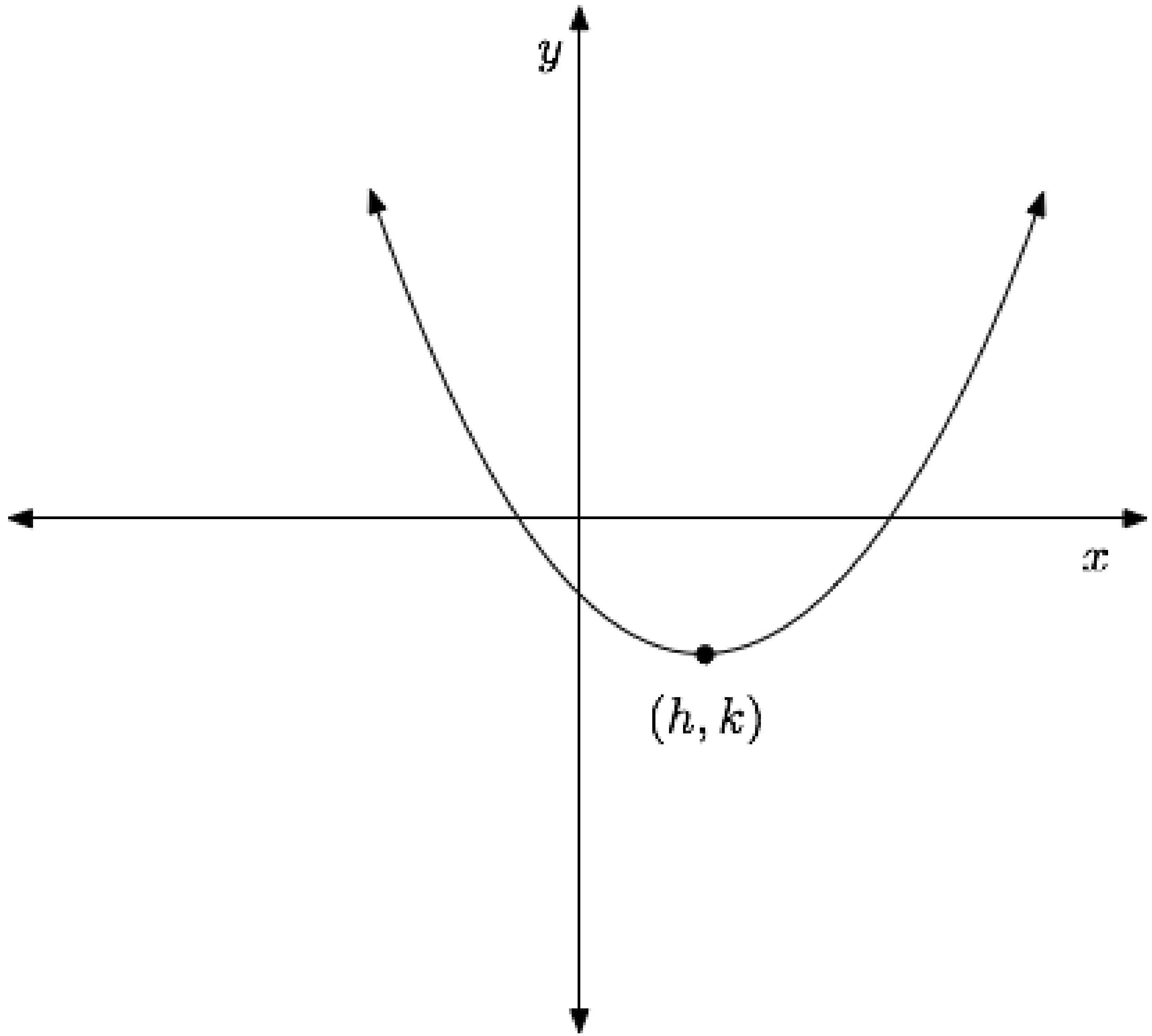
Central difference



Diagonal difference

# Derivatives

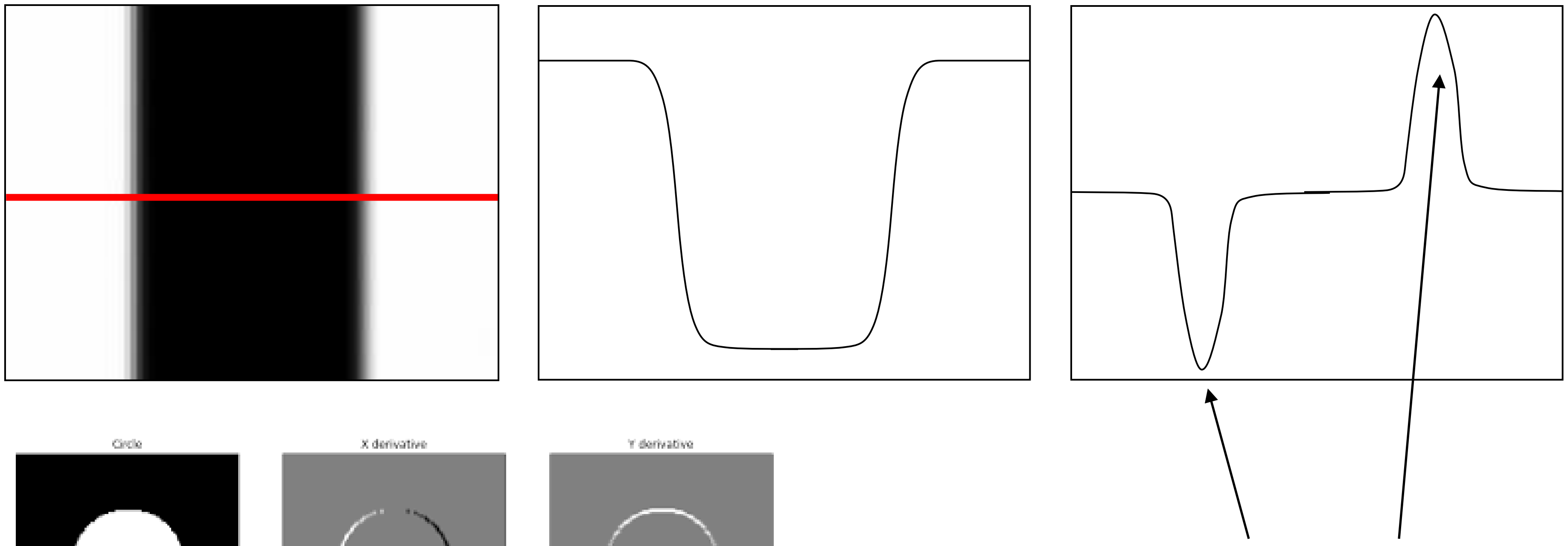
Reminder



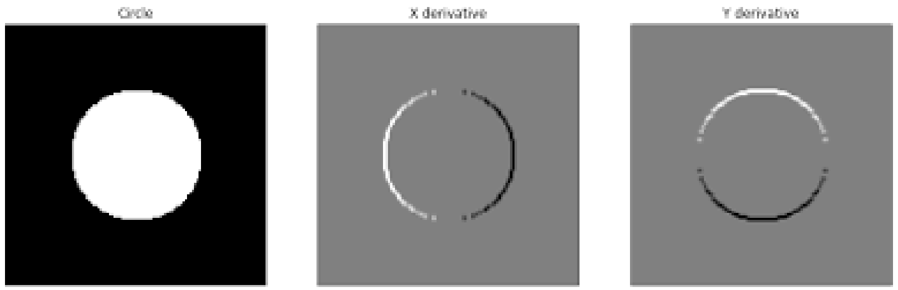
# Derivatives and edges

An edge is a place of ***rapid change*** in the image intensity function.

**image** **intensity function (along horizontal scanline)** **first derivative**



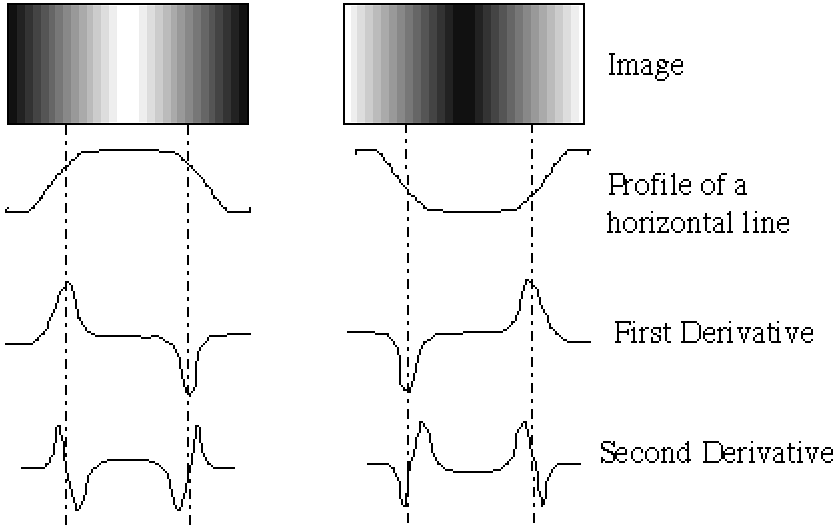
**edges correspond to extrema of derivative**

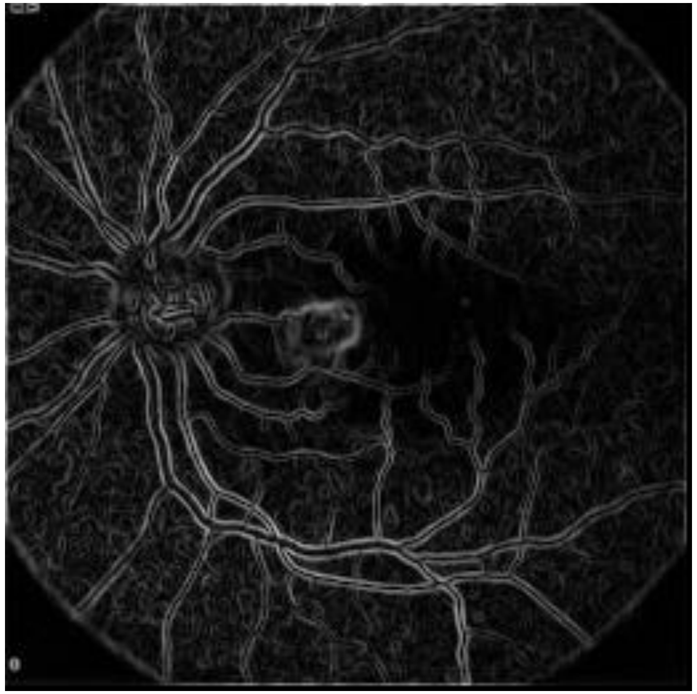



Source: L. Lazebnik

# Edge Detection

## Using derivatives



# Derivatives with convolution

Where would the function “go” if we give it a little “nudge”?

For 2D function, $f(x, y)$, the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon, y) - f(x,y)}{\epsilon}$$

For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

# Edge Detection as Derivatives

Where would the function "go" if we give it a little "nudge"?

- For continuous functions: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

- Approximate partial derivatives for both x and y:

$$\frac{\partial}{\partial x} f(i,j) \cong f(i,j) - f(i-1,j)$$

$$\frac{\partial}{\partial y} f(i,j) \cong f(i,j) - f(i,j-1)$$

- Approximate derivatives by convolution with $[1 \quad -1]$

How would you implement this as a filter?

These kernels perform well on very clean images. They don’t work as well if there’s noise on the images (kernels are too small!)

# Edge Detection Convolution Kernels

## Calculating Gradients

- Forward Finite Difference: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

| | |
|---|---|
| -1 | 1 |

| |
|---|
| 1 |
| -1 |

- Backward Finite Difference: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x) - f(x-\Delta x)}{\Delta x}$

| | |
|---|---|
| -1 | 1 |

| |
|---|
| 1 |
| -1 |

- Central Finite Difference: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$

| | | |
|---|---|---|
| -1 | 0 | 1 |

| |
|---|
| 1 |
| 0 |
| -1 |

# Derivatives Approximation with Kernel Operations

- Roberts Operator
- Prewitt Operator
- Sobel Operator

| | | |
|---|---|---|
| +1 | 0 | -1 |
| +1 | 0 | -1 |
| +1 | 0 | -1 |

Gx

| | | |
|---|---|---|
| -1 | -1 | -1 |
| 0 | 0 | 0 |
| +1 | +1 | +1 |

Gy

Prewitt Operator

| | | |
|---|---|---|
| -1 | 0 | +1 |
| -2 | 0 | +2 |
| -1 | 0 | +1 |

Gx

| | | |
|---|---|---|
| +1 | +2 | +1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

Gy

Sobel Operator

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# Gradient Operators

- 2D first derivatives - $\frac{\partial}{\partial x} f(i,j) \cong f(i,j) - f(i-1,j)$
- $\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$
- Gradient Magnitude: $\nabla f = \sqrt{G_x^2 + G_y^2} \approx |G_x| + |G_y|$
- Gradient Direction: $\alpha(x,y) = \tan^{-1}\left(\frac{G_y}{G_x}\right)$

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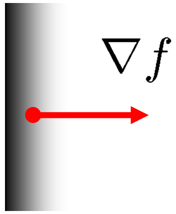
# Image Gradients

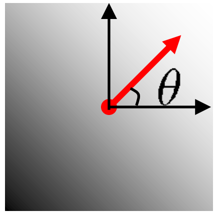
The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The gradient points in the direction of most rapid change in intensity



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$

$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



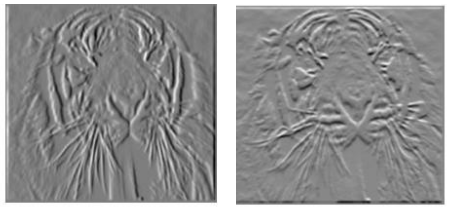

$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The **gradient direction** (orientation of edge normal) is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

The **edge strength** is given by the gradient magnitude

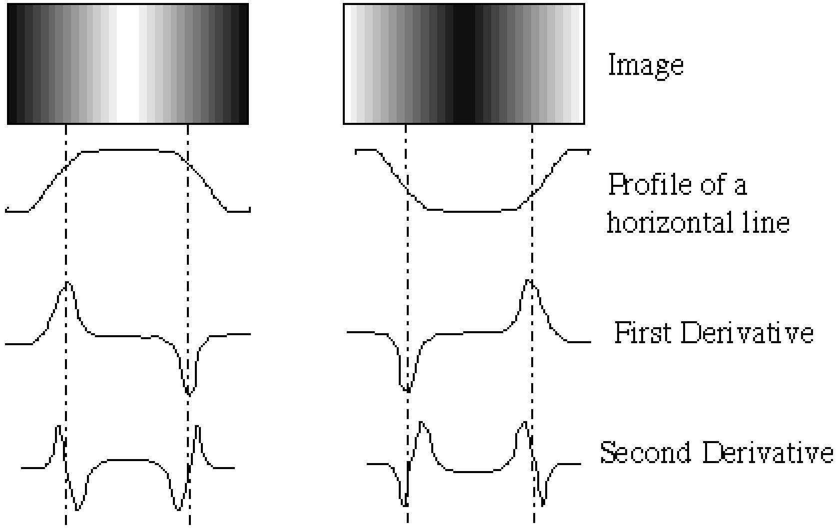
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$



Slide credit Steve Seitz

# Edge Detection

## Using derivatives for a discrete function




Gradient vector:

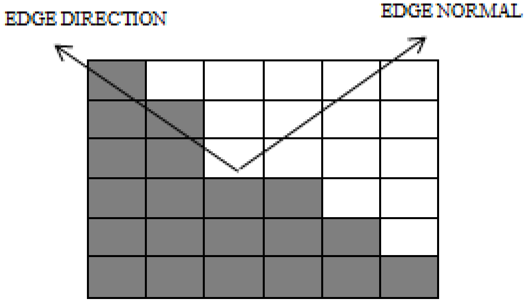
$$\nabla I = \left[\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right]^T$$

Magnitude:

$$|\nabla I| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

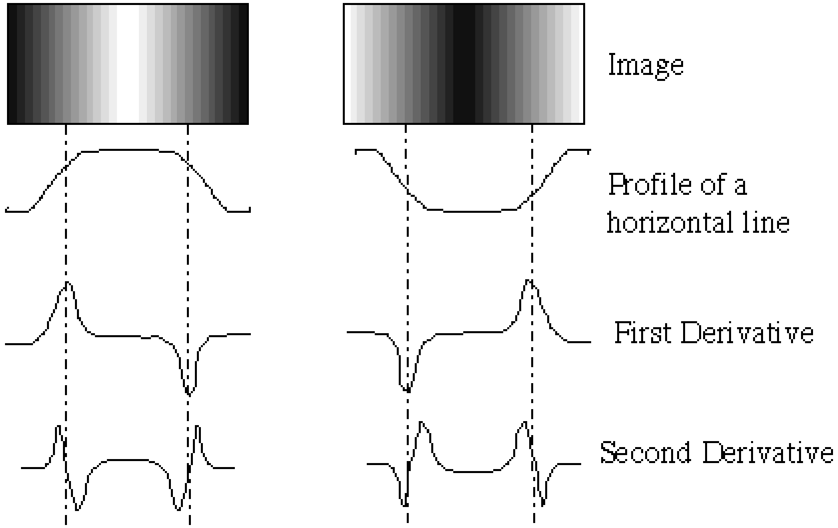
Orientation:

$$\theta = \operatorname{atan}^2\left(\frac{\partial I}{\partial y}, \frac{\partial I}{\partial x}\right)$$



# Edge Detection

## Using derivatives for a discrete function through filters




| | | |
|---|---|---|
| +1 | 0 | -1 |
| +1 | 0 | -1 |
| +1 | 0 | -1 |

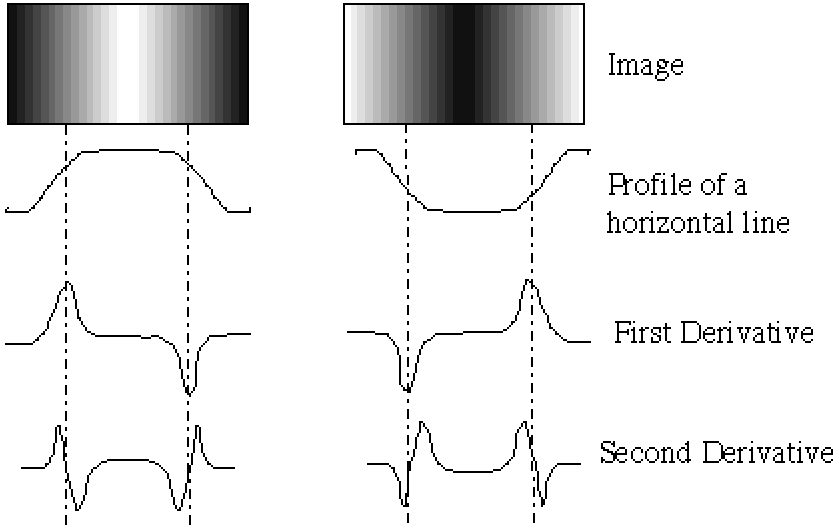
Gx

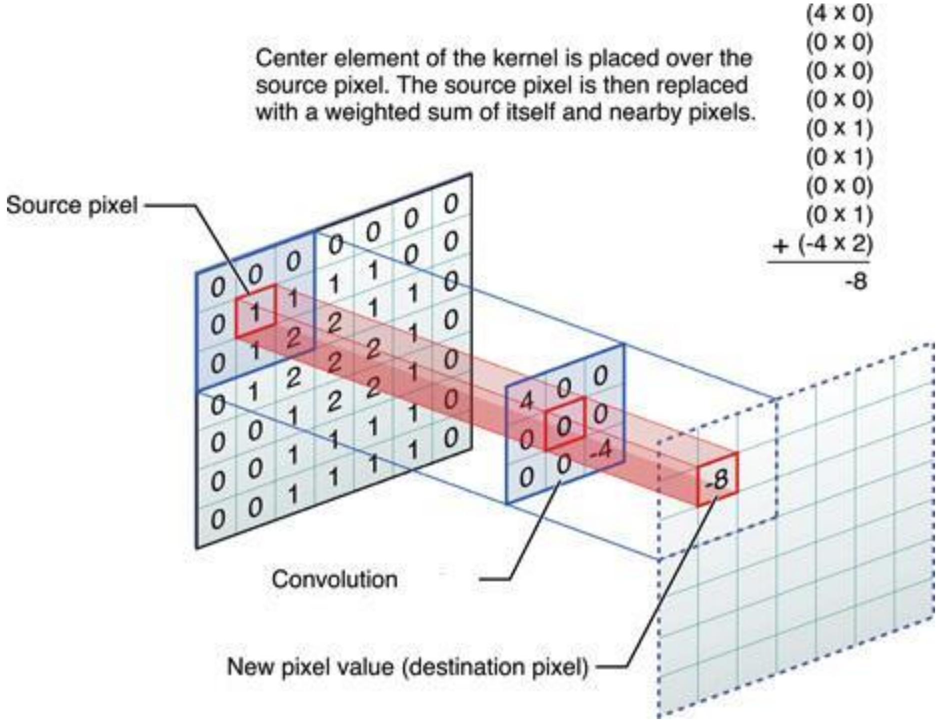
| | | |
|---|---|---|
| -1 | -1 | -1 |
| 0 | 0 | 0 |
| +1 | +1 | +1 |

Gy

# Edge Detection

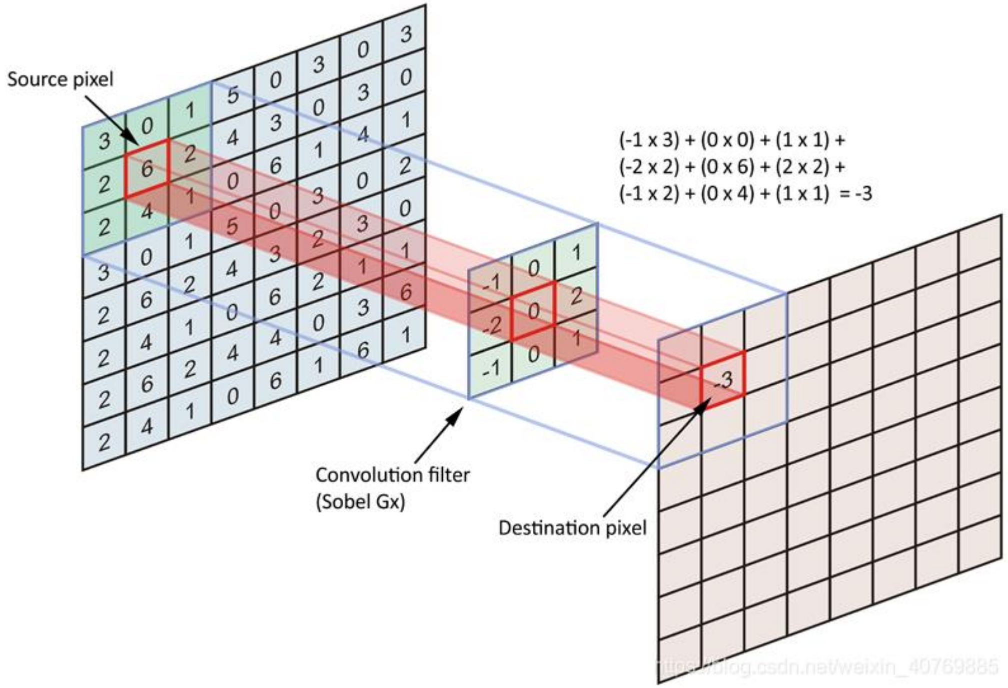
## Using derivatives for a discrete function through filters convolutions



# Sobel Operator

- Approximate gradient calculation (the image derivatives)
- The final edge is received by L2-Distance: $\sqrt{G_x^2 + G_y^2}$




| | | |
|---|---|---|
| -1 | 0 | +1 |
| -2 | 0 | +2 |
| -1 | 0 | +1 |

Gx

| | | |
|---|---|---|
| +1 | +2 | +1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

Gy

# Edge Detection Convolution Kernels

Prewitt and Sobel compute derivatives in one direction while smoothing in the other direction

## Calculating Gradients

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

| | |
|---|---|
| -1 | 1 |

| |
|---|
| 1 |
| -1 |

**Forward Finite Difference**

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x) - f(x-\Delta x)}{\Delta x}$$

| | |
|---|---|
| -1 | 1 |

| |
|---|
| 1 |
| -1 |

**Backward Finite Difference**

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

| | | |
|---|---|---|
| -1 | 0 | 1 |

| |
|---|
| 1 |
| 0 |
| -1 |

**Central Finite Difference**

x

| | | |
|---|---|---|
| -1 | 0 | 1 |
| -1 | 0 | 1 |
| -1 | 0 | 1 |

y

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| -1 | -1 | -1 |

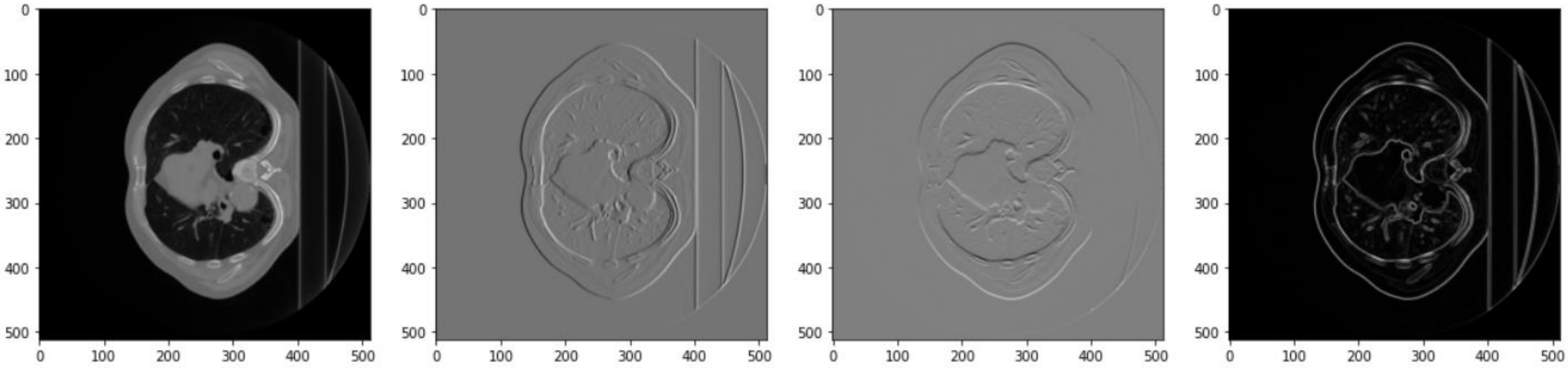
**Prewitt Operator**

| | | |
|---|---|---|
| -1 | 0 | 1 |
| -2 | 0 | 2 |
| -1 | 0 | 1 |

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

**Sobel Operator**

# Sobel Operator



# CANNY EDGE DETECTION

When noise is on the way

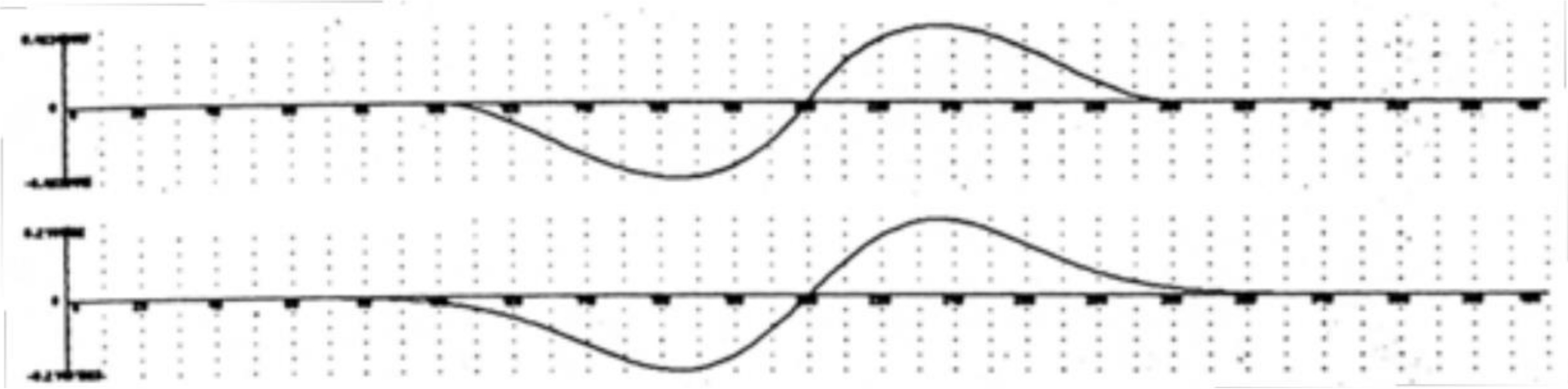
# Canny Edge Detector

## Classic Algorithm for Finding Edges in Images

- We want a kernel to simultaneously maximize SNR and localization for a step edge under white *Gaussian* noise
- Solution by numerical optimization is very close to derivative of Gaussian kernel

Numerically Optimized

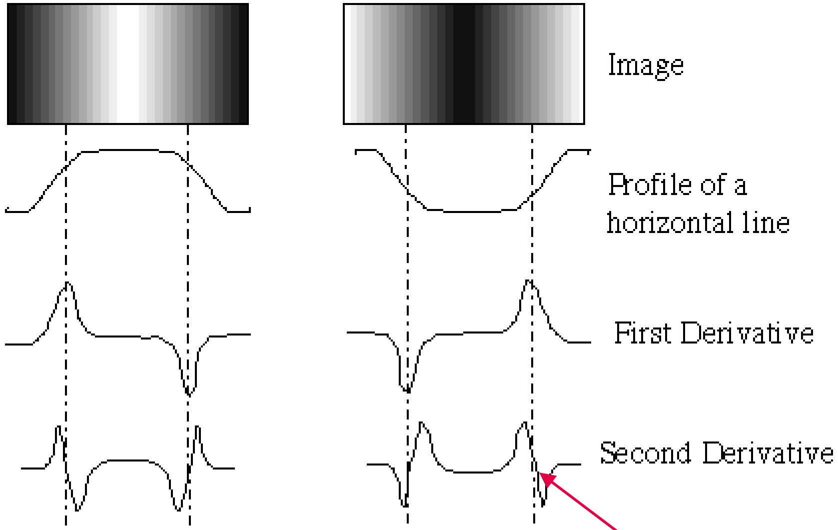
Derivative of Gaussian



Canny, IEEE TPAMI 1986

# Edge Detection - Laplacian

## Second Derivative




Detect zero-crossing

- Laplacians: sum of second derivatives:

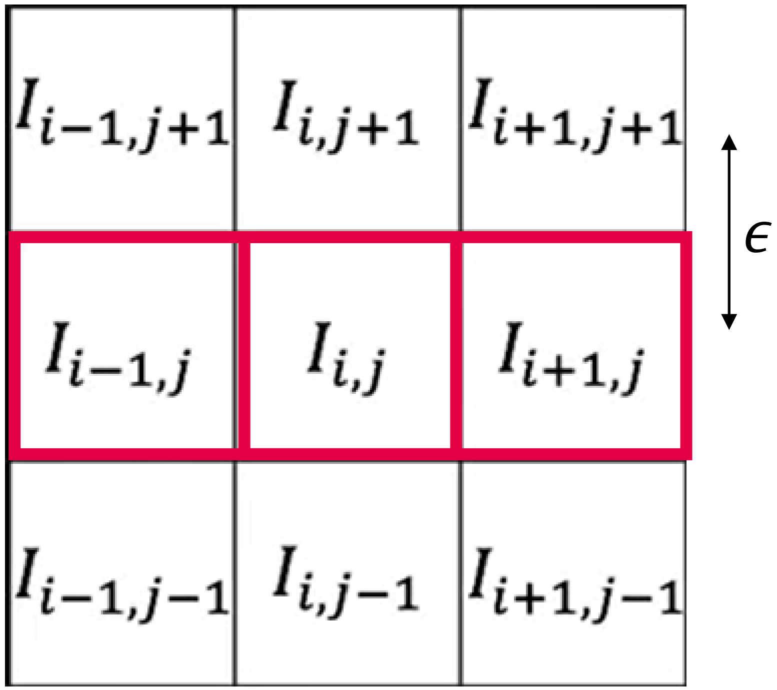
$$\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

- Zero-crossings mark edge location
- Do not provide directions of edges

# Edge Detection - Laplacian

## Second Derivative – the difference of the difference

- $\frac{\partial^2 I}{\partial x^2} \approx \frac{1}{\epsilon^2}\left(I_{i-1,j} - 2I_{i,j} + I_{i+1,j}\right)$
- $\frac{\partial^2 I}{\partial y^2} \approx \frac{1}{\epsilon^2}\left(I_{i,j-1} - 2I_{i,j} + I_{i,j+1}\right)$
- $\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$

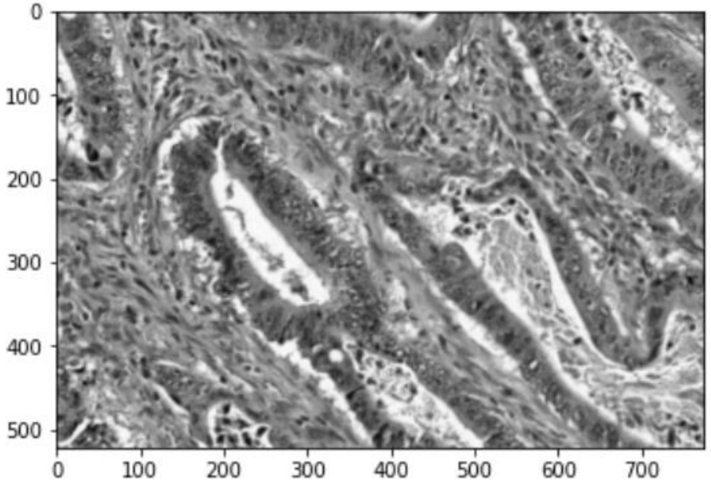


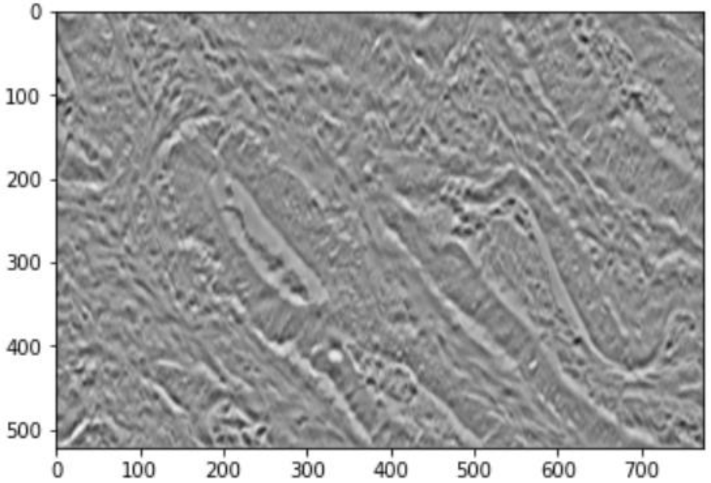

- $\nabla^2 = \frac{1}{\epsilon^2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ or (better) $\nabla^2 = \frac{1}{6\epsilon^2}\begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix}$

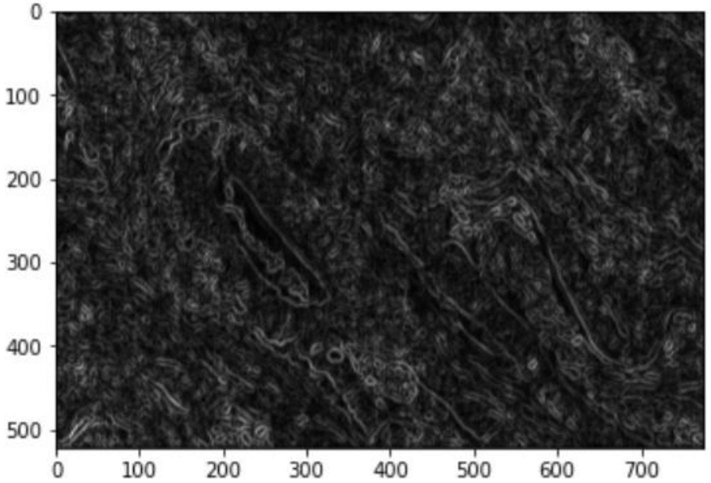
# Edge Detection - Laplacian

## Laplacian of a gaussian

$$\begin{pmatrix} 0 & 0 & 3 & 2 & 2 & 2 & 3 & 0 & 0 \\ 0 & 2 & 3 & 5 & 5 & 5 & 3 & 2 & 0 \\ 3 & 3 & 5 & 3 & 0 & 3 & 5 & 3 & 3 \\ 2 & 5 & 3 & -12 & -23 & -12 & 3 & 5 & 2 \\ 2 & 5 & 0 & -23 & -40 & -23 & 0 & 5 & 2 \\ 2 & 5 & 3 & -12 & -23 & -12 & 3 & 5 & 2 \end{pmatrix}$$

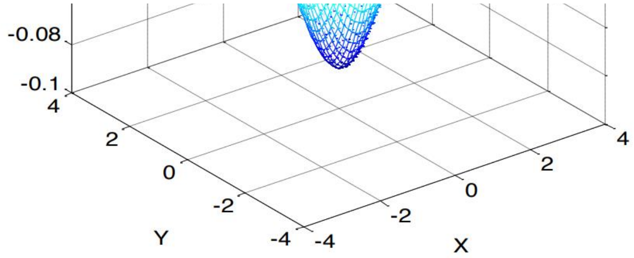


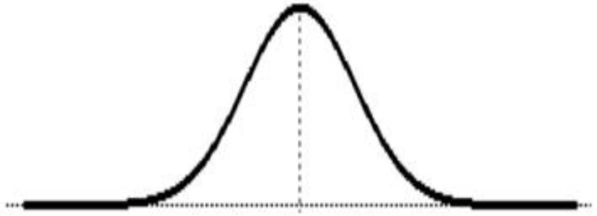
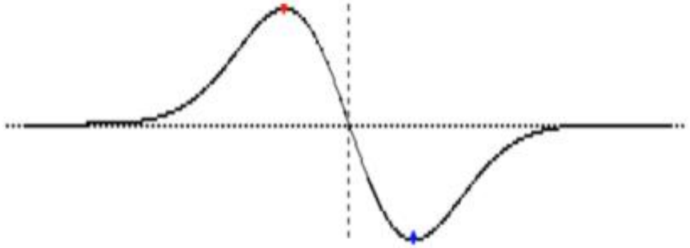
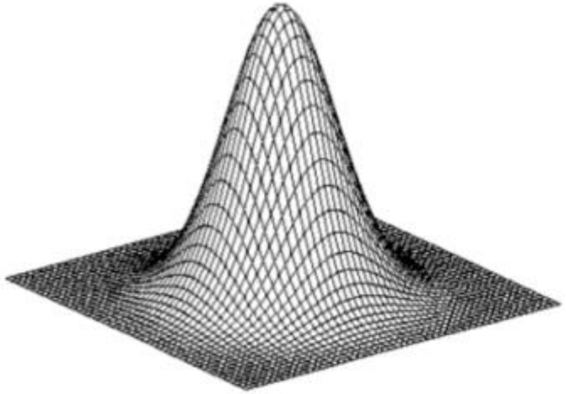
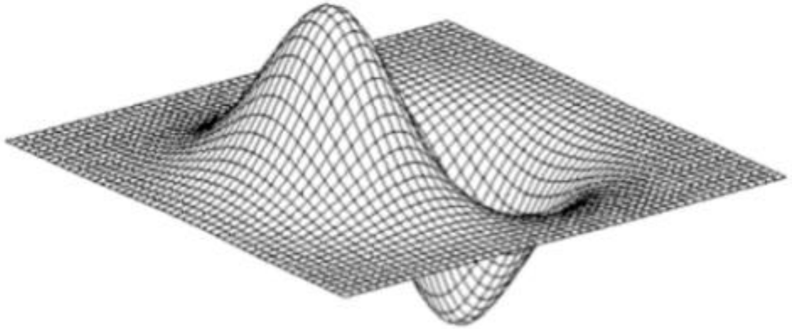




$\sqrt{2}$

$$Log(x,y) = -\frac{1}{\pi\sigma^4}\left(1 - \frac{x^2+y^2}{2\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$



# (Partial) Derivative of Gaussian Kernel

| | $G(\cdot)$ | $\dfrac{\partial G(\cdot)}{\partial x}$ |
|---|---|---|
| **1D** |  |  |
| **2D** |  |  |
| **3D** | $\dfrac{1}{\left(\sigma\sqrt{2\pi}\right)^3} e^{-\frac{\left(x^2+y^2+z^2\right)}{2\sigma^2}}$ | $-\dfrac{x}{\sigma^2\left(\sigma\sqrt{2\pi}\right)^3} e^{-\frac{\left(x^2+y^2+z^2\right)}{2\sigma^2}}$ |

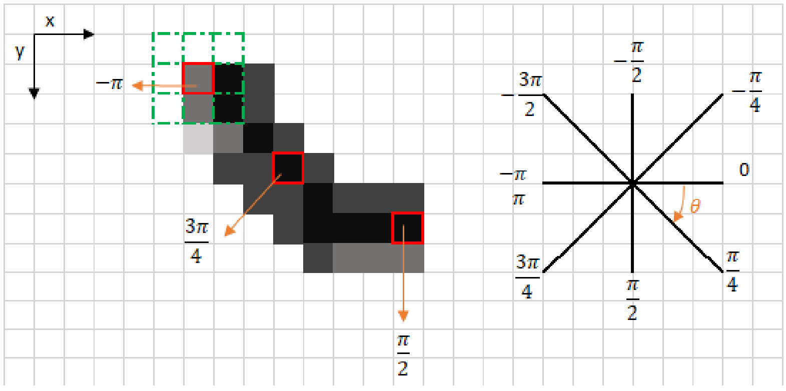
A Computational Approach to Edge Detection

John Canny

# Canny Edge Detection

Commonly used edge-detection filter

- Noise smoothing with a gaussian filter → $n_\sigma * I$
- Edge gradients (e.g., with Sobel operator) → $\nabla n_\sigma * I$
- Find Gradient Magnitude for each pixel → $\|\nabla n_\sigma * I\|$
- Find Gradient Orientation for each pixel → $\hat{n} = \frac{\nabla n_\sigma * I}{\|\nabla n_\sigma * I\|}$




- Compute the Laplacian along the gradient direction $\hat{n}$ at each pixel → $\frac{\partial^2 n_\sigma * I}{\partial \hat{n}^2}$

# Canny Edge Detection

## In other words

- Convolving with a derivative of Gaussian is the same as convolving with a Gaussian and then taking a derivative
  (…however, taking the *exact* derivative of an image is not well defined)
- It is (more or less) equivalent to blurring first, to reduce noise, then taking an exact derivative.

# Canny Edge Detection

Final steps:

- Hysteresis thresholding:
    - First, global threshold with an upper threshold (on $|\nabla I|$)
    - Then, region grow starting from those regions using a lower threshold
- Non-maximum suppression
    - Eliminate edges that are not local maxima (along the direction of the gradient)

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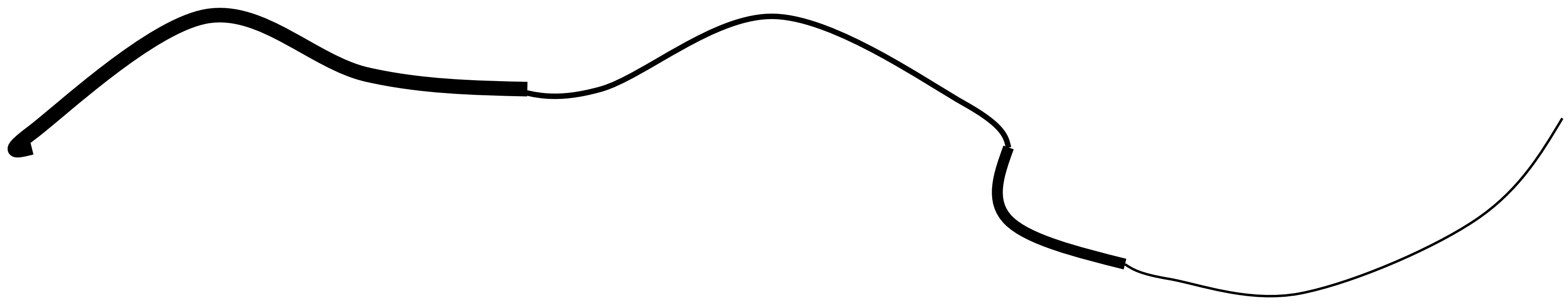
# Hysteresis thresholding

- Threshold at low/high levels to get weak/strong edge pixels
- Do connected components, starting from strong edge pixels

Credit: James Hays

# Hysteresis thresholding

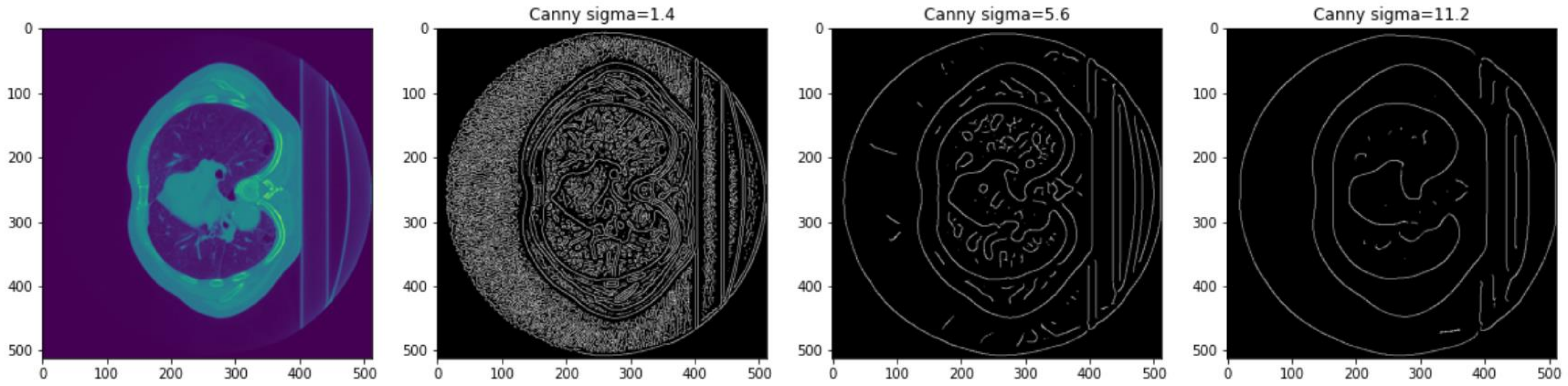
- Use a high threshold to start edge curves, and a low threshold to continue them.



Source: Steve Seitz

# Canny Edge Detection

The filtering's sigma value affects the results



# Take-Aways

- There are many different segmentation methods
  - Thresholding
  - Region Growing
  - ML-Based
  - And even more is coming up soon:
    - Level-sets,
    - Fast-Marching,
    - Graph-based
- Edge Detection
  - Can produce disconnected boundaries
  - Over/Under Segmentation
  - Harder in 3D
  - Canny smooths before applying gradients
- No one algorithm is the “best”.
  - Must match to the problem and data