ReLU Recurrent Neural Networks

Daniel and David

October 27, 2017

Outline

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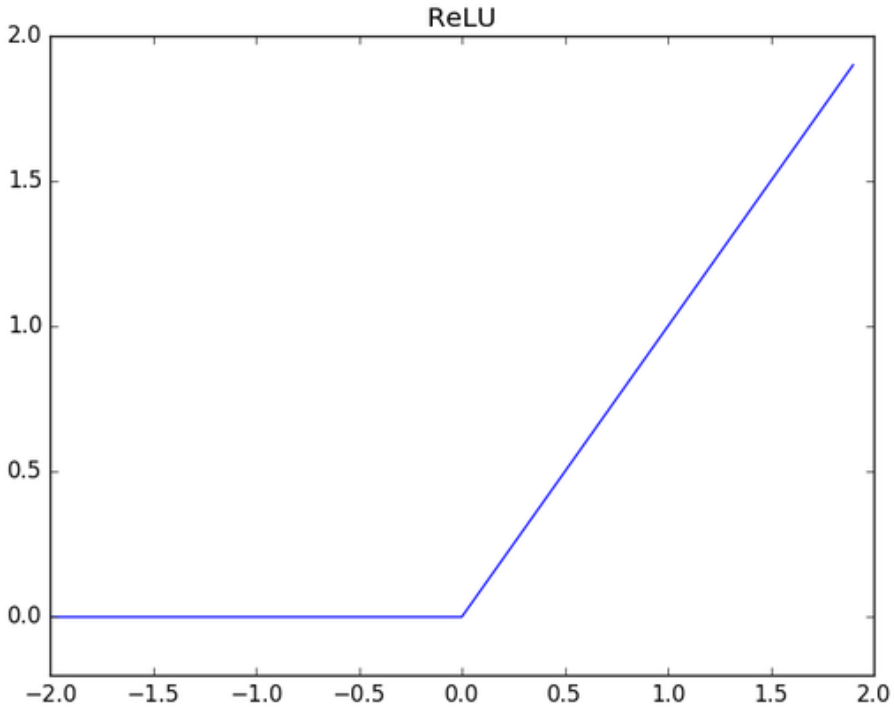
ReLU

$$f(x) = \max(x, 0)$$



Network Equations

Continuous

$$\tau \dot{x} = -x + W_{rec} f(x) + b_{rec} + W_{in} u + Noise$$

Network Equations

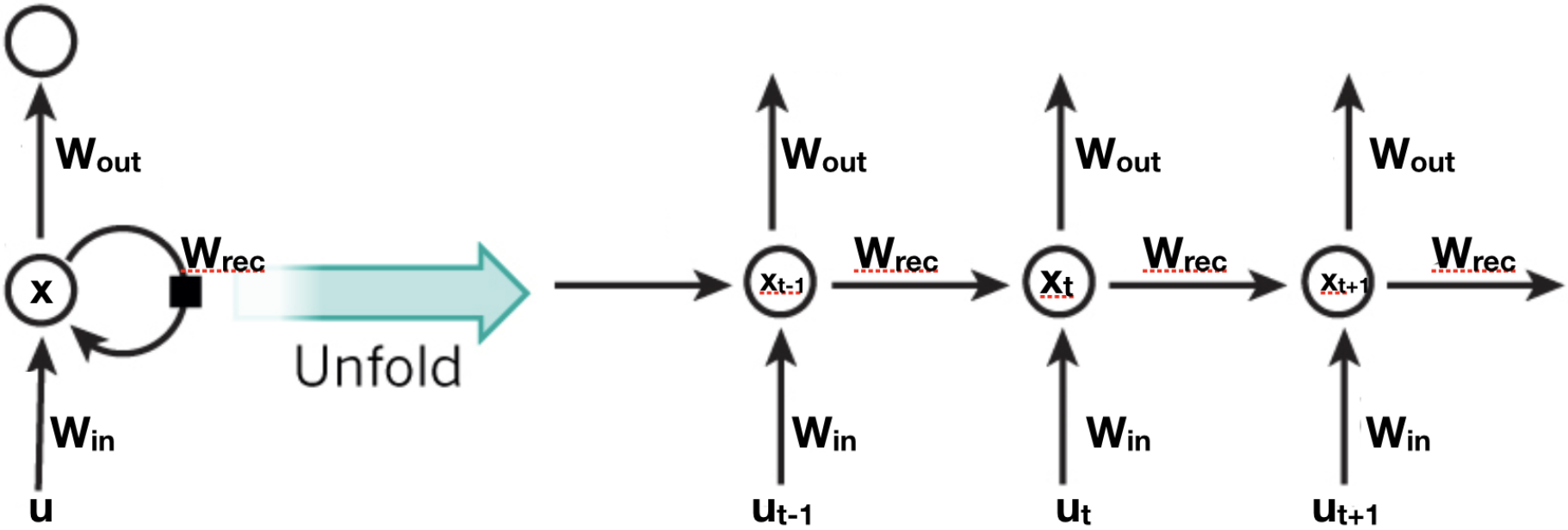
Continuous

$$\tau \dot{x} = -x + W_{rec} f(x) + b_{rec} + W_{in} u + Noise$$

Discrete

$$x_{t+1} = \left(1 - \frac{dt}{\tau}\right) x + \left(\frac{dt}{\tau}\right) (W_{rec} f(x_t) + b_{rec} + W_{in} u_t) + Noise$$

Network Visualization



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Hahnloser, 1997

Simplified model:

$$\tau \dot{x} = -x + W_{rec} f(x) + J$$

Hahnloser, 1997

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Note

$$\begin{pmatrix} w & w & w \\ w & w & w \\ w & w & w \end{pmatrix} \begin{pmatrix} x \\ 0 \\ x \end{pmatrix} = \begin{pmatrix} w & 0 & w \\ w & 0 & w \\ w & 0 & w \end{pmatrix} \begin{pmatrix} x \\ x \\ x \end{pmatrix}$$

Hahnloser, 1997

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So, define W^+ such that

$$W^+ x = W_{rec} \max(x, 0) = W_{rec} f(x)$$

Hahnloser, 1997

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So, define W^+ such that

$$W^+ x = W_{rec} \max(x, 0) = W_{rec} f(x)$$

The region where W^+ is the recurrent matrix is called a *partition*.

Hahnloser, 1997

At a fixed point

$$\begin{aligned} 0 &= -x + W^+ x + J \\ x - W^+ x &= J \\ x &= (1 - W^+)^{-1} J \end{aligned}$$

Hahnloser, 1997

At a fixed point

$$\begin{aligned} 0 &= -x + W^+ x + J \\ x - W^+ x &= J \\ x &= (1 - W^+)^{-1} J \end{aligned}$$

Subtelty: x must be in the partition defined by W^+

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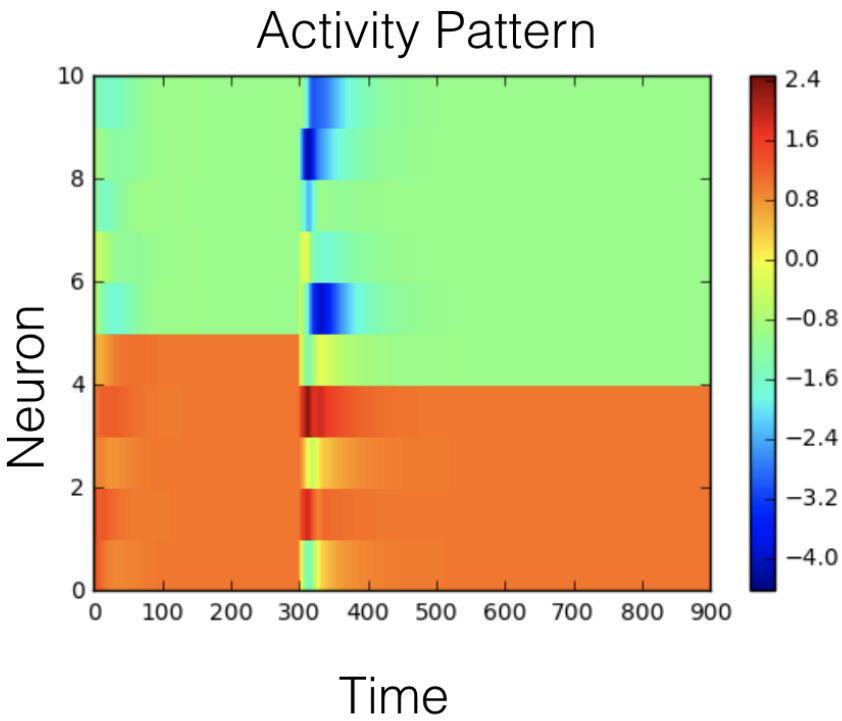
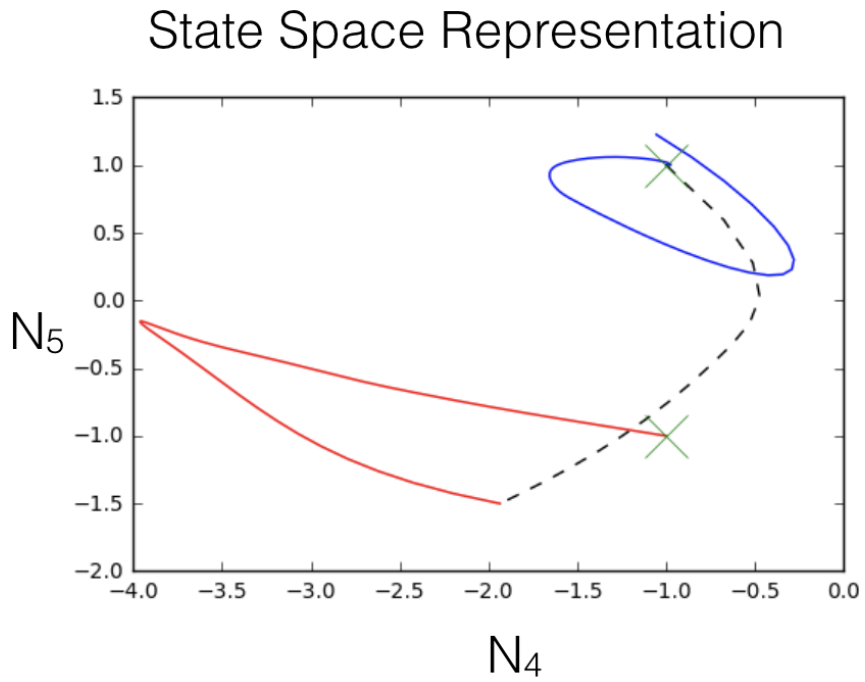
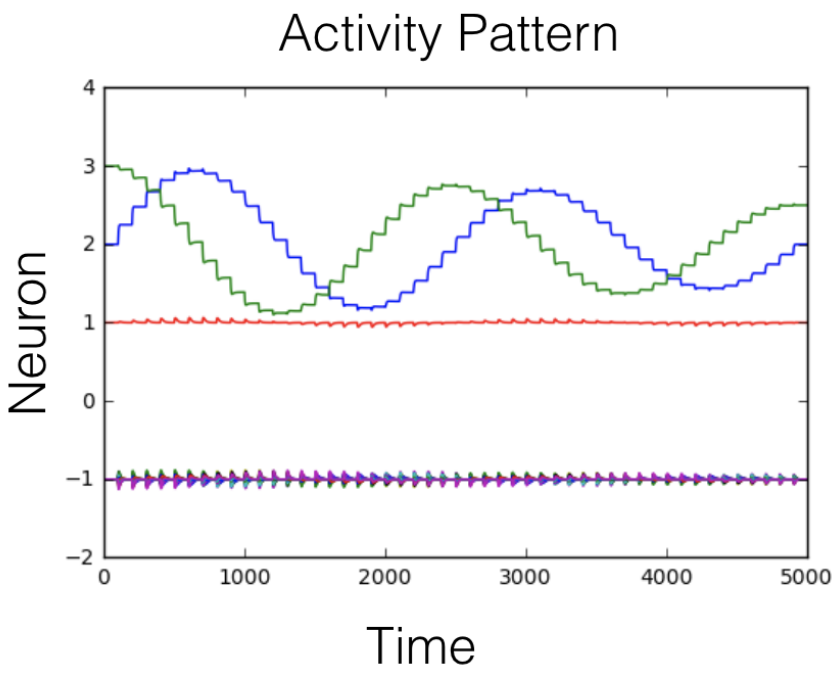
Hand Built Networks

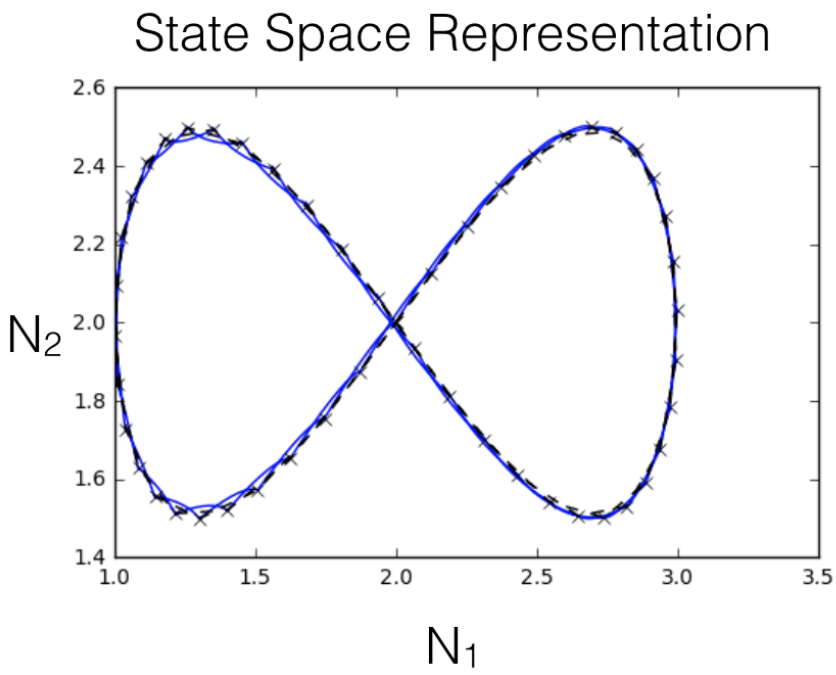
$$x = (1 - W^{+})^{-1} J$$



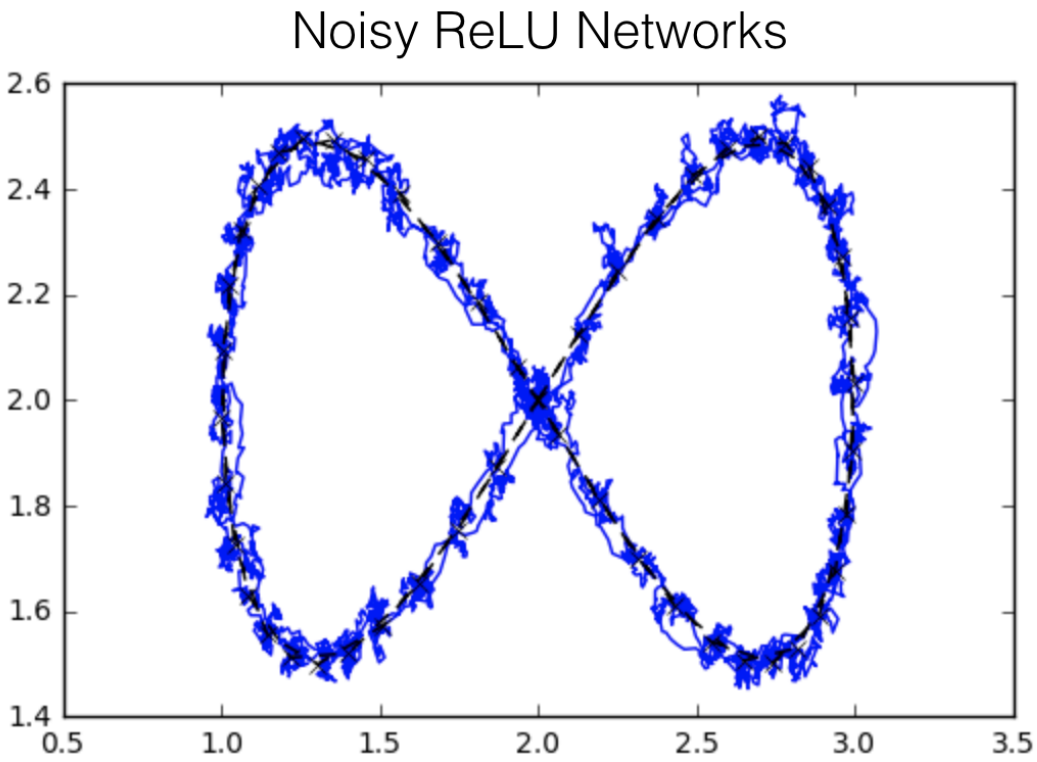


Hand Built Networks

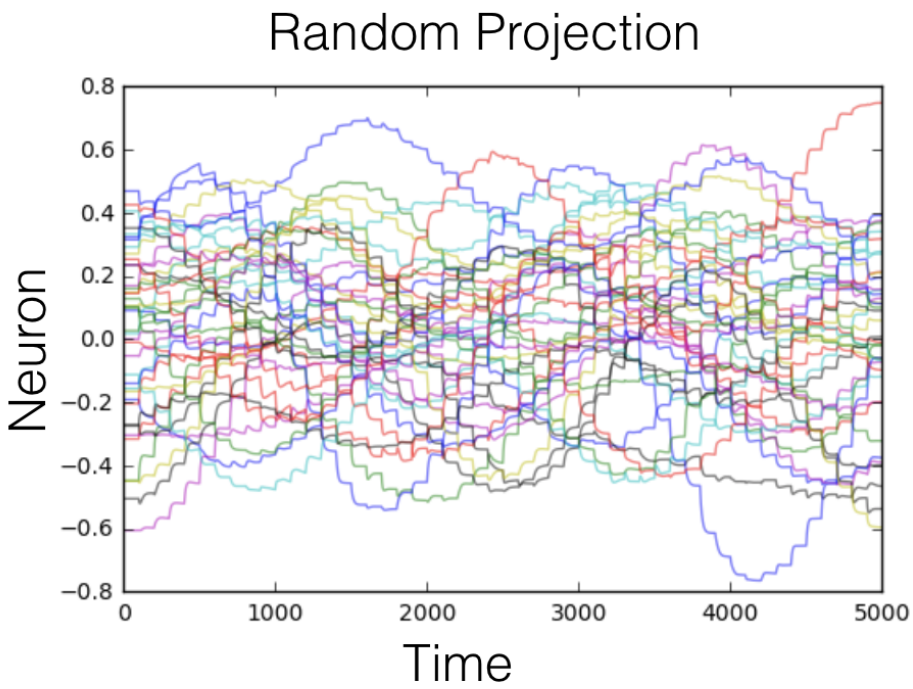


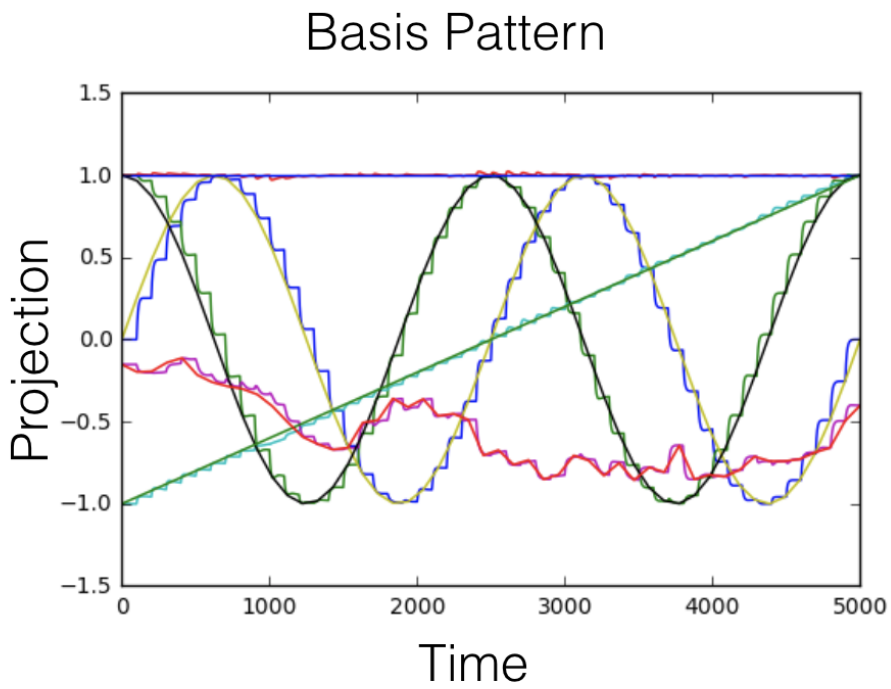


Hand Built Networks

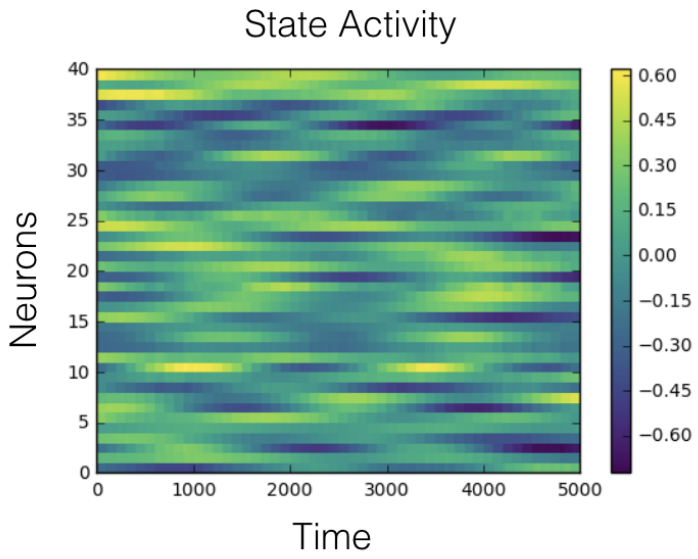


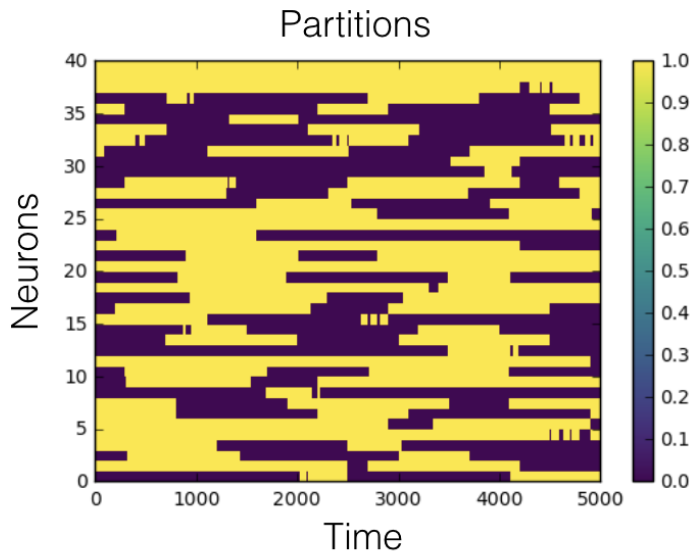
Hand Built Networks

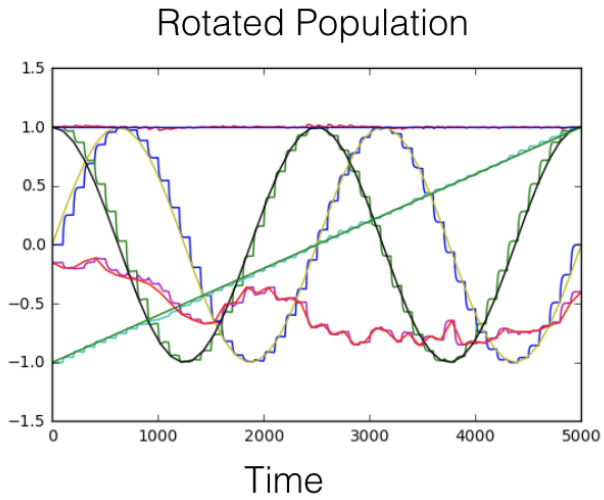


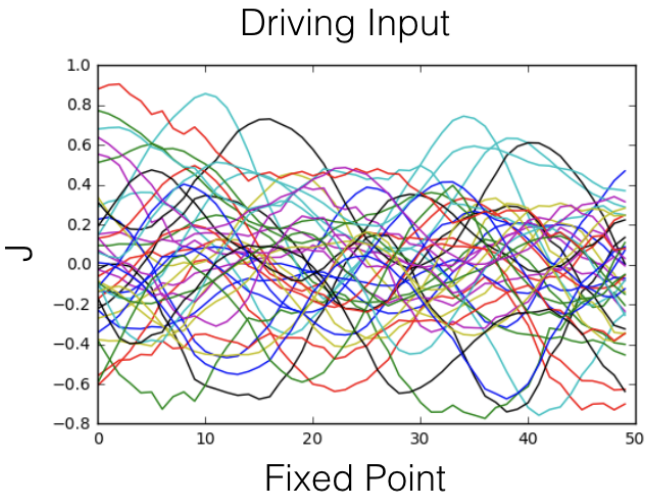


Hand Built Networks









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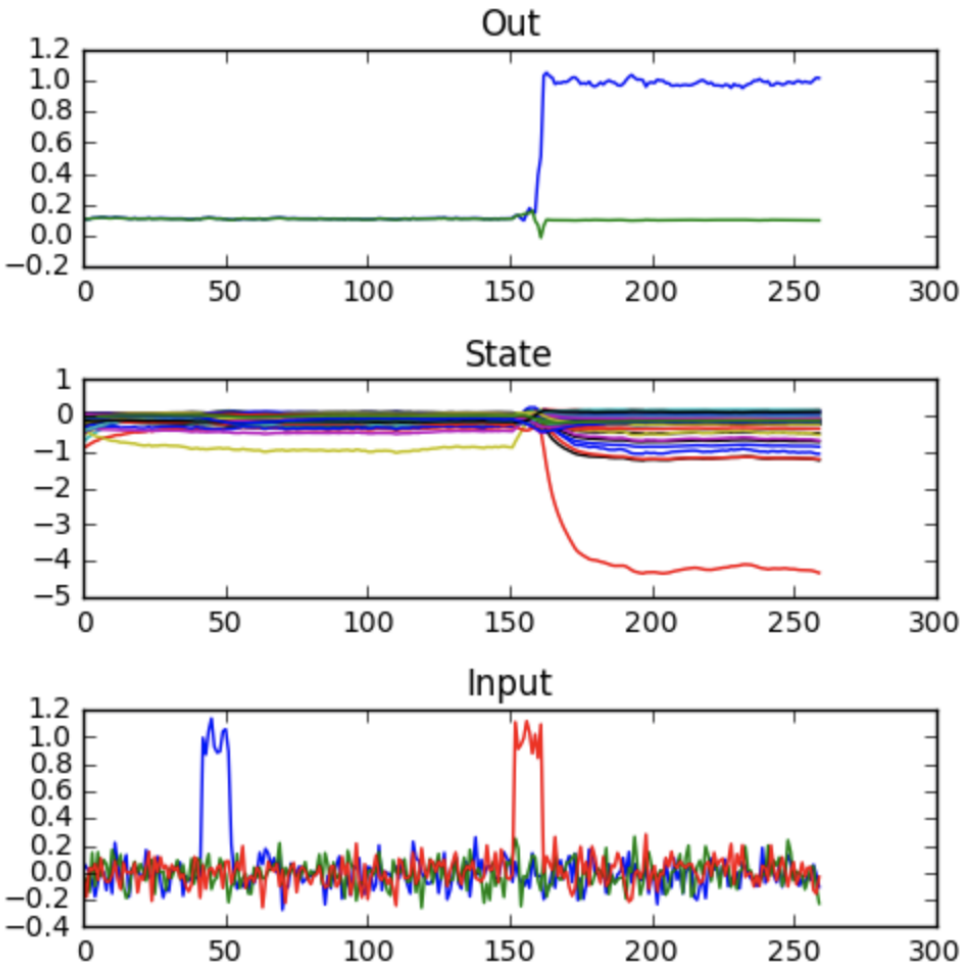
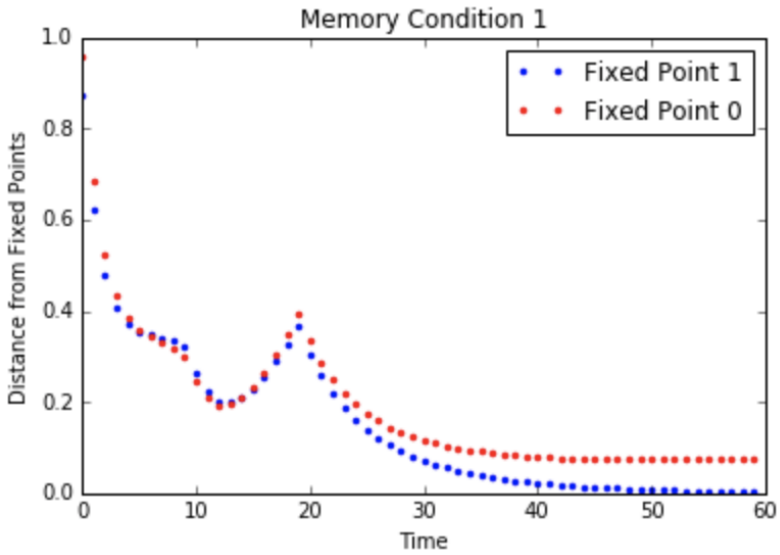
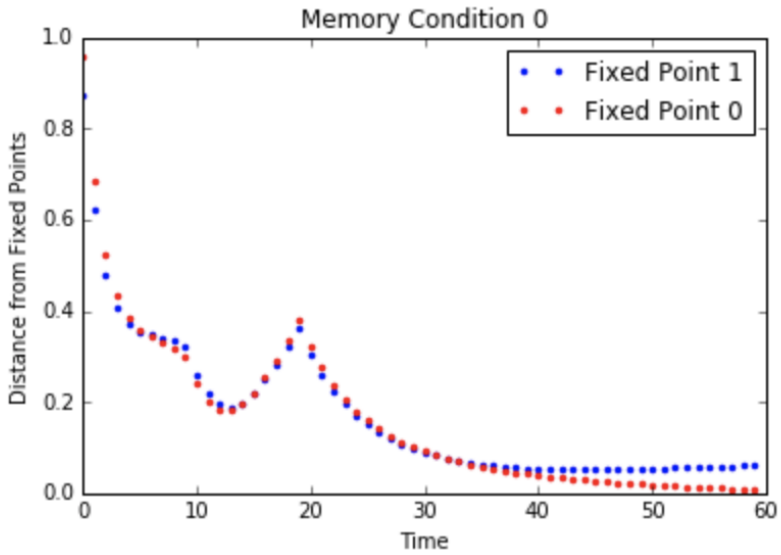
Memory Task

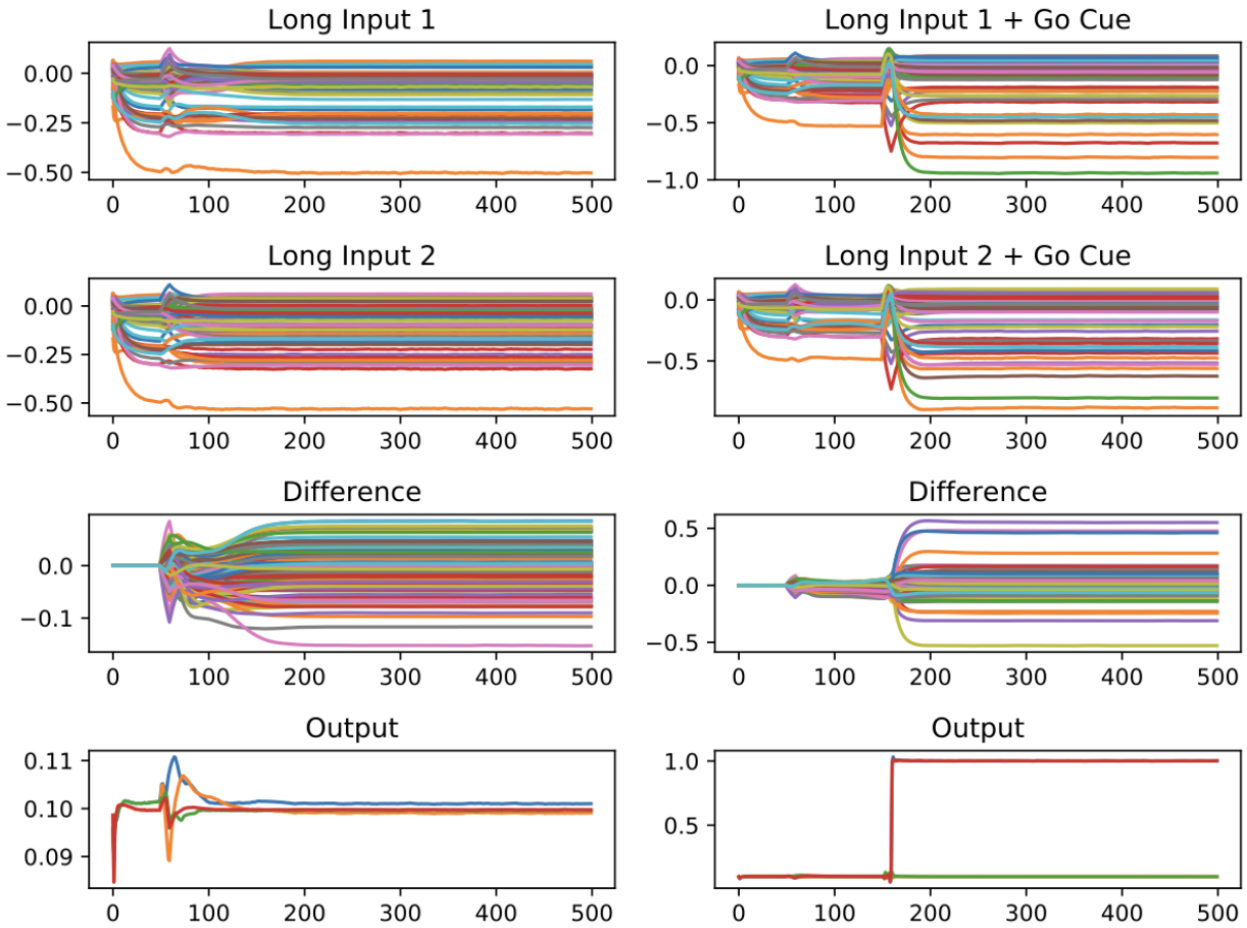

Out
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
50
100
150
200
250
300
State
1
0
−1
−2
−3
−4
−5
0
50
100
150
200
250
300
Input
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0
50
100
150
200
250
300

Memory Task

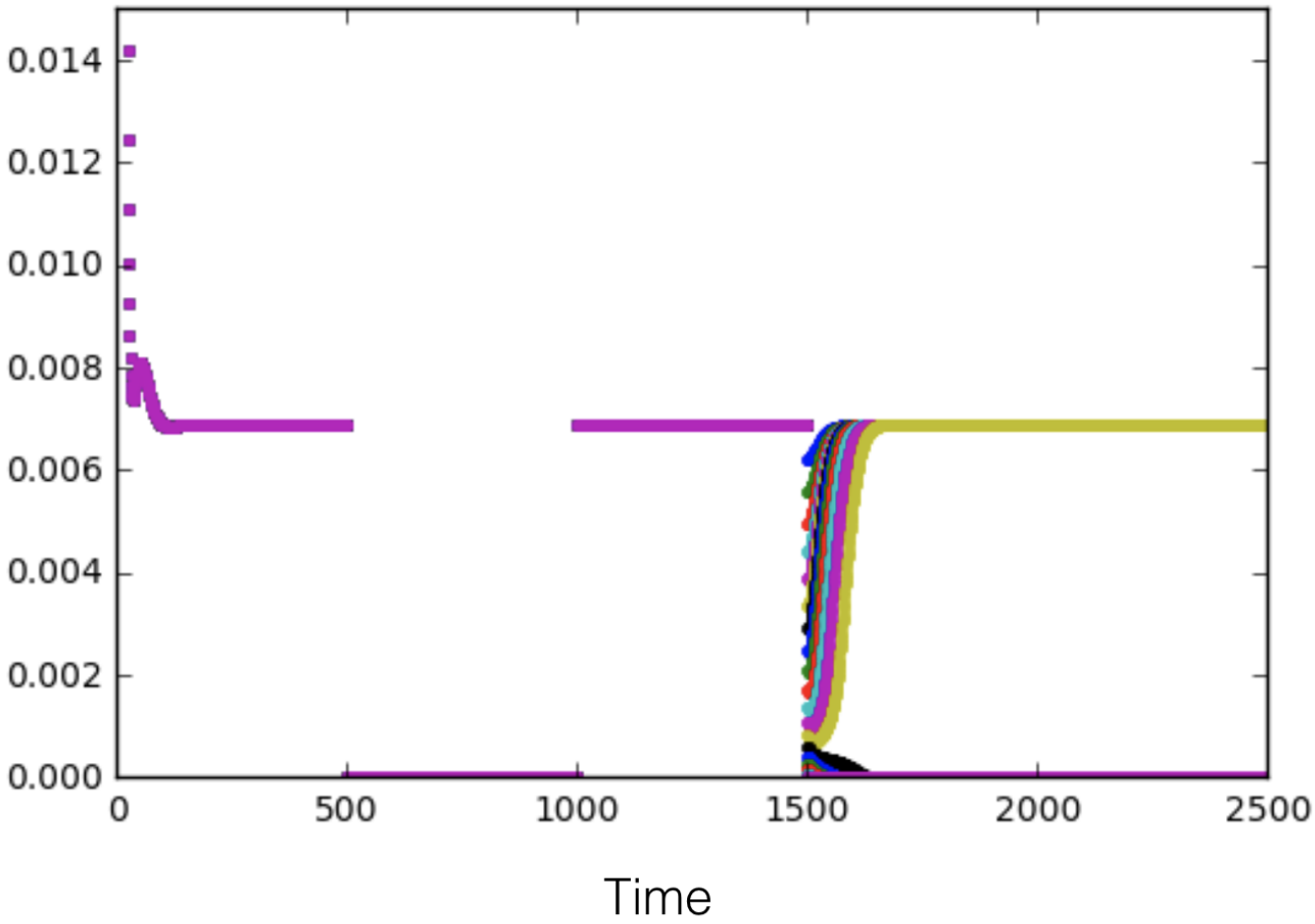

Memory Condition 1
Fixed Point 1
Fixed Point 0
Distance from Fixed Points
Time
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60


Memory Condition 0
Fixed Point 1
Fixed Point 0
Distance from Fixed Points
Time
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60

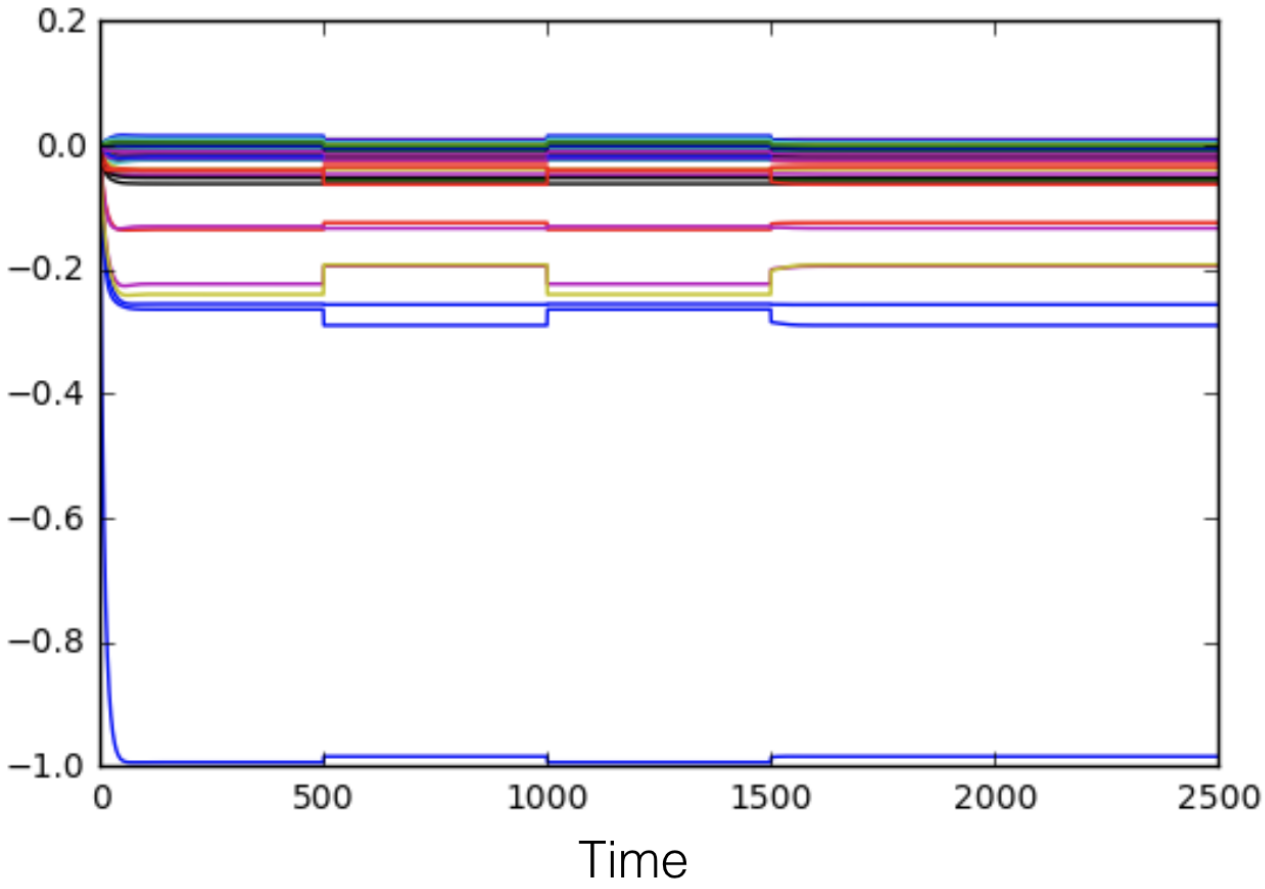
Memory Task


Long Input 1
Long Input 1 + Go Cue
Long Input 2
Long Input 2 + Go Cue
Difference
Difference
Output
Output

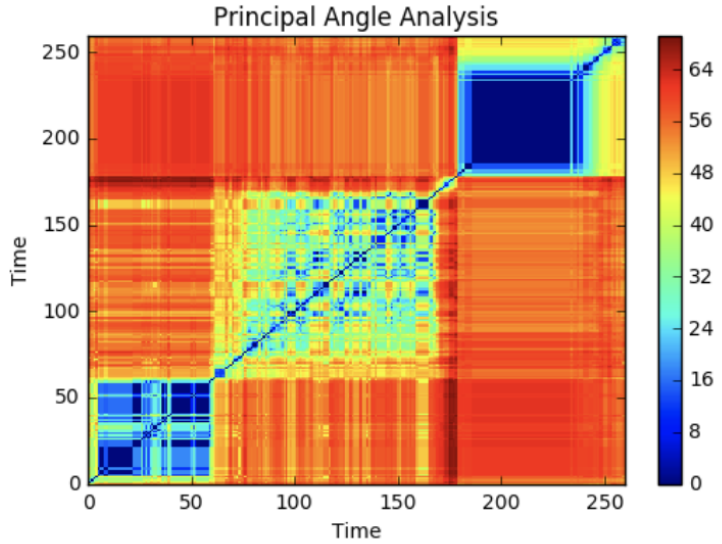
Memory Task

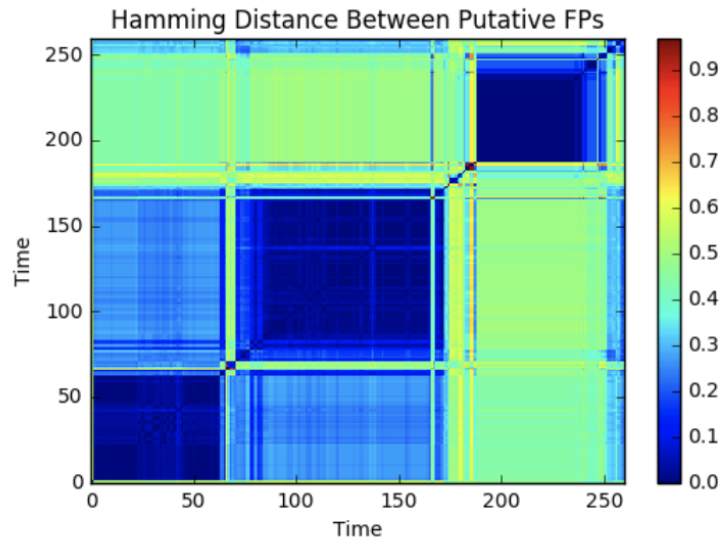

0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0
500
1000
1500
2000
2500
Time

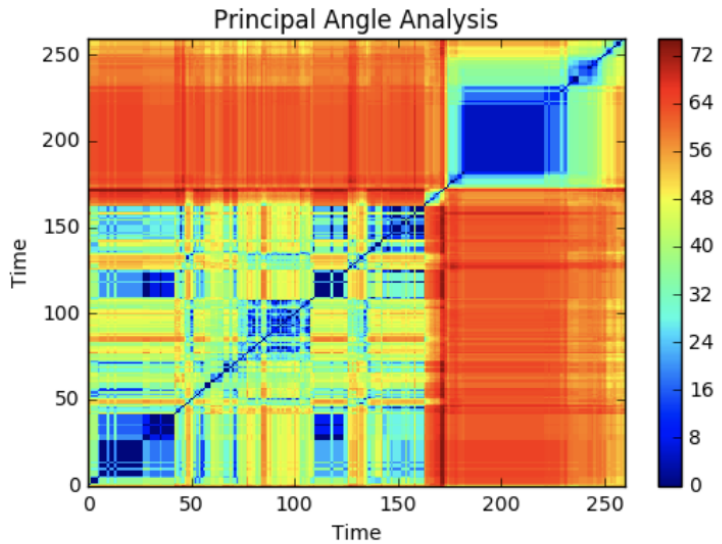
Memory Task

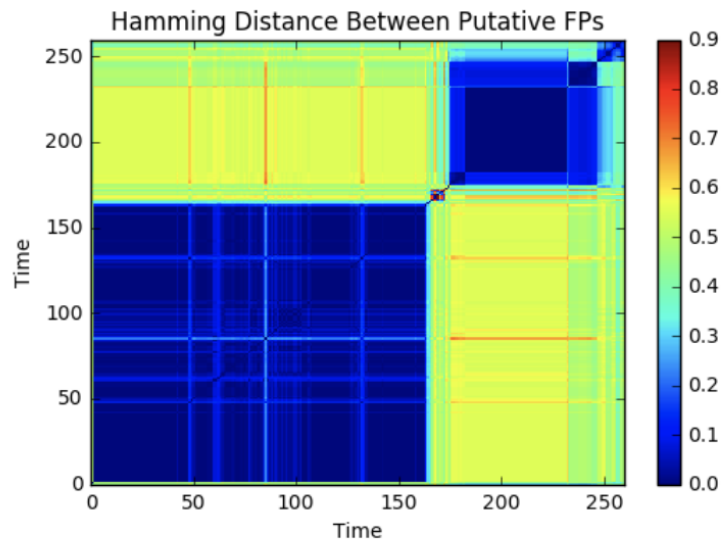


Memory Task

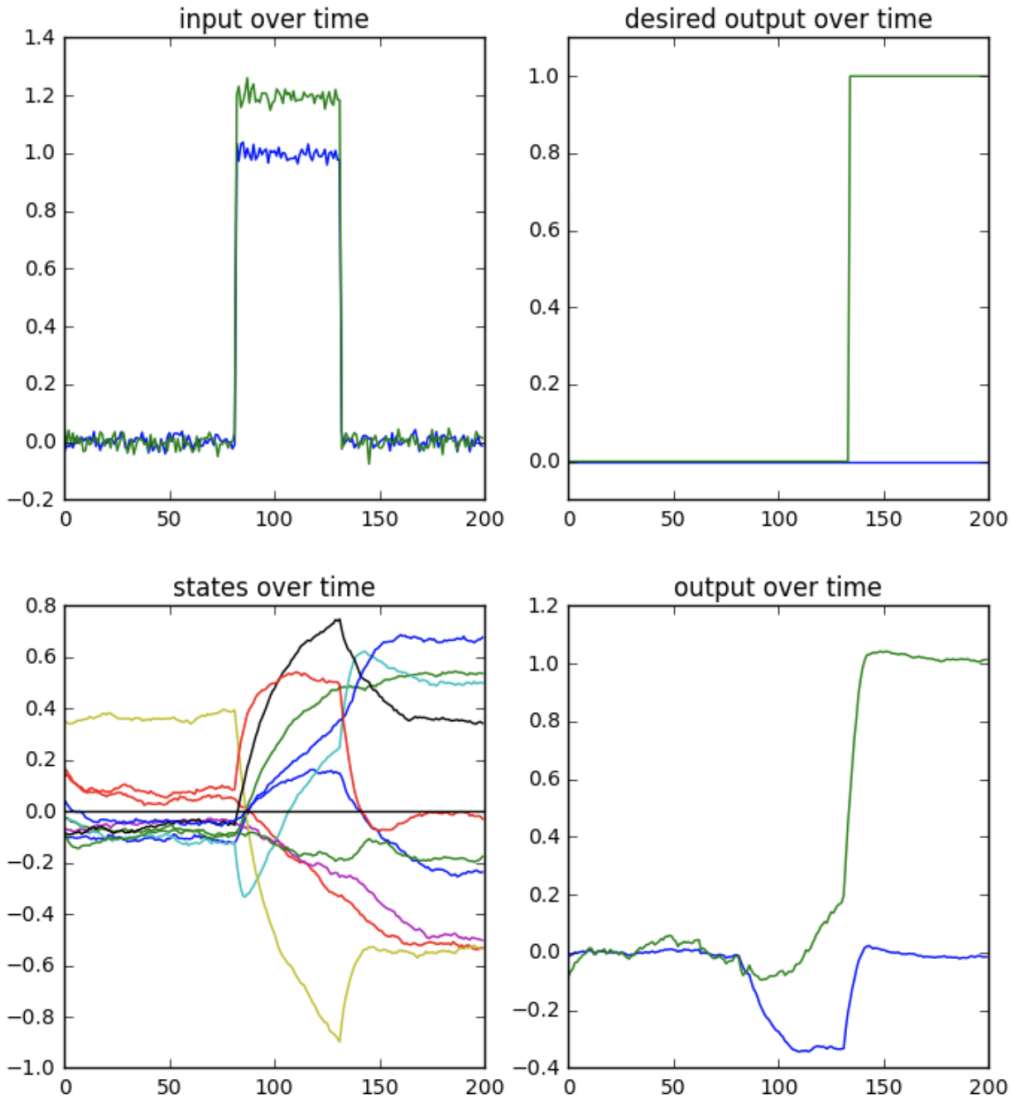

Principal Angle Analysis
Time
Time
0
50
100
150
200
250
64
56
48
40
32
24
16
8
0


Hamming Distance Between Putative FPs
Time
Time
0
50
100
150
200
250
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0

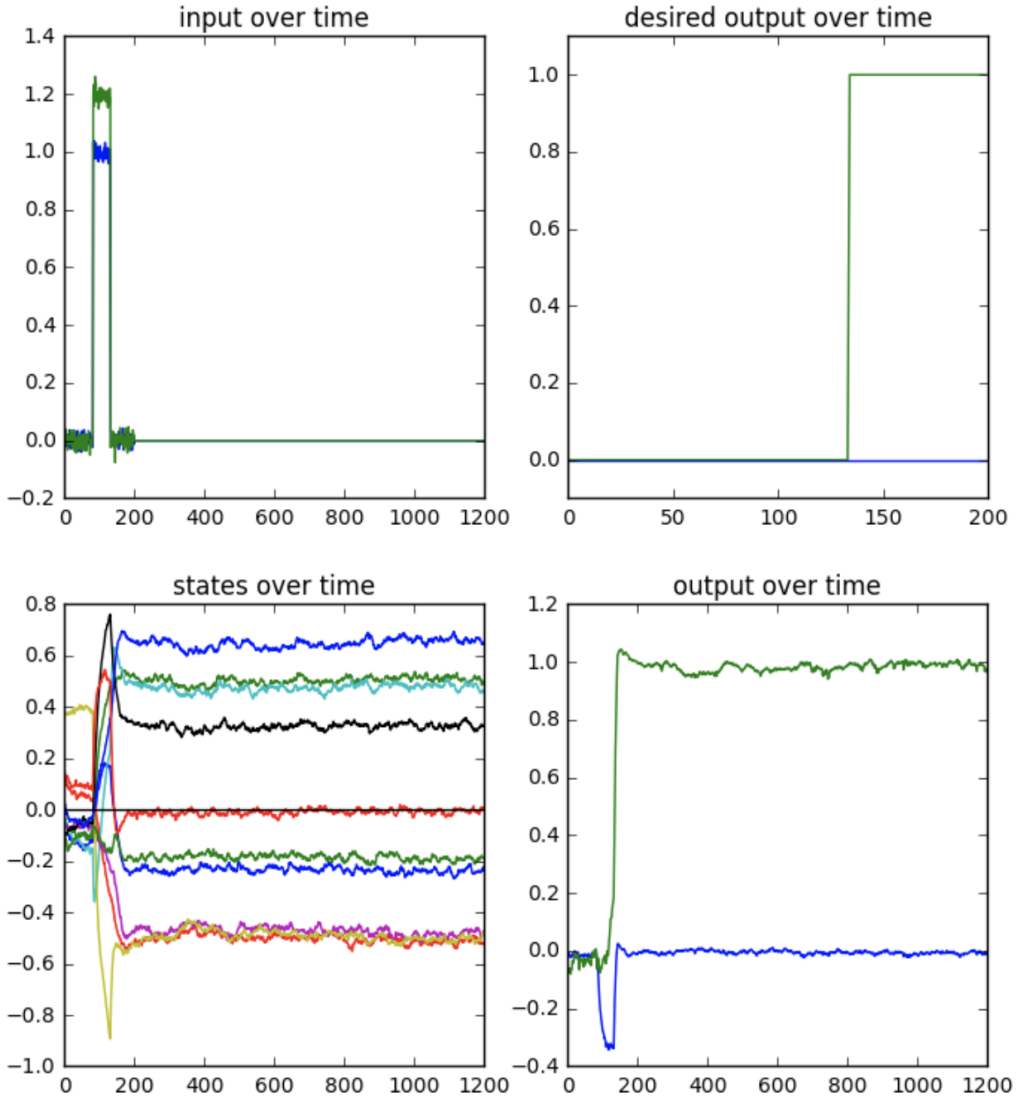

Principal Angle Analysis
Time
Time
0
50
100
150
200
250
72
64
56
48
40
32
24
16
8
0


Hamming Distance Between Putative FPs
Time
Time
0
50
100
150
200
250
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0

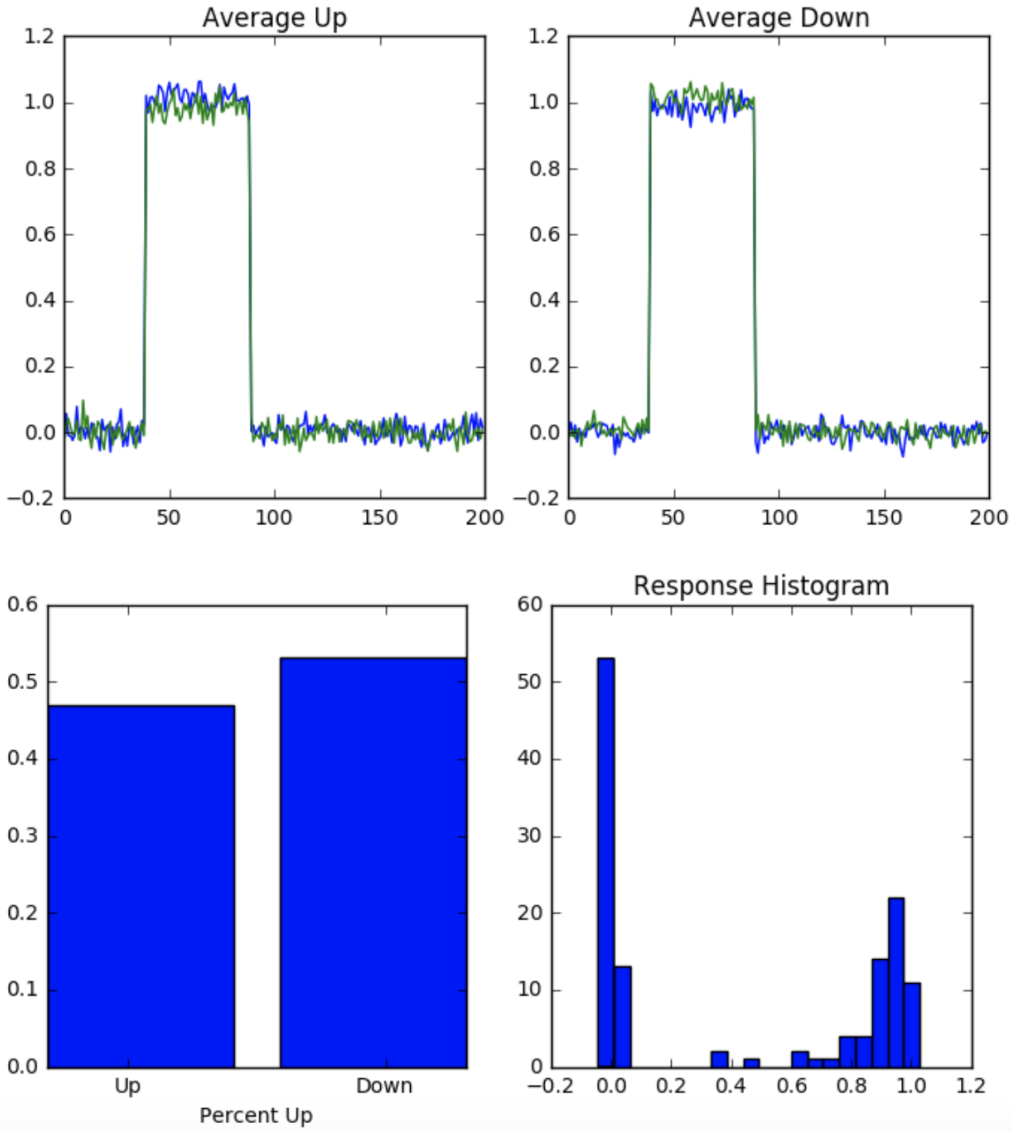
RDM Task


input over time
desired output over time
states over time
output over time

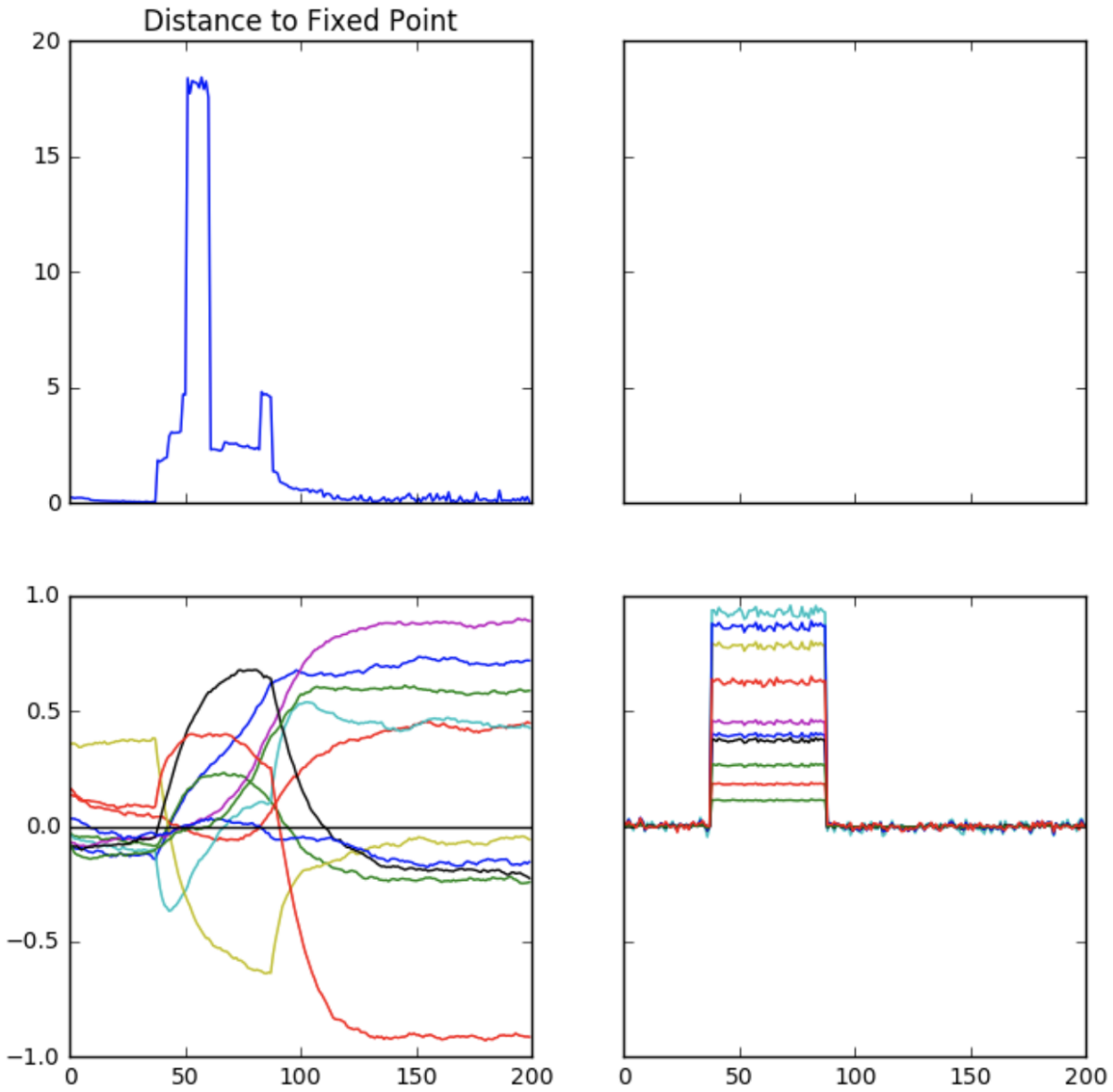
Long Time Scales



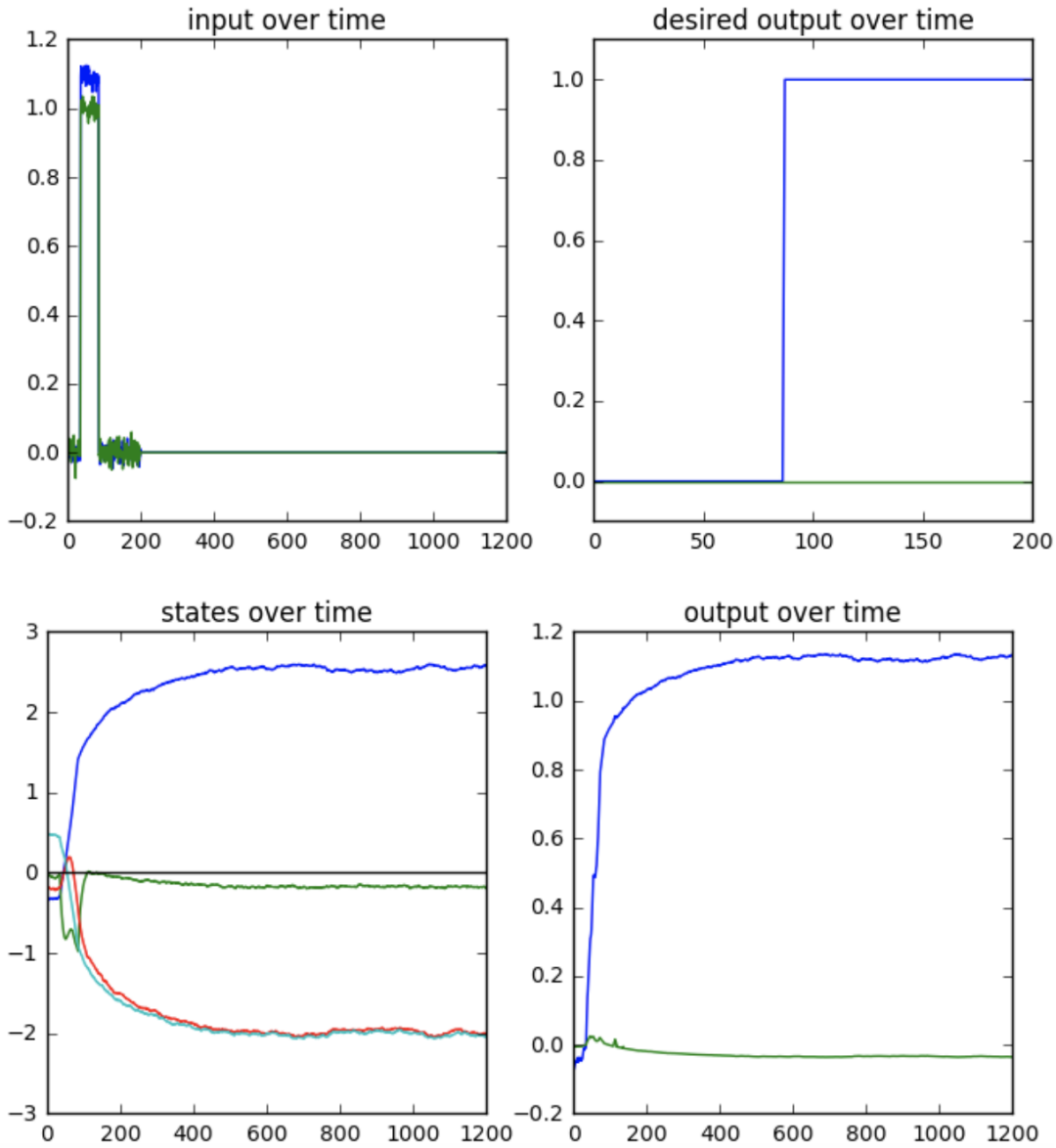
Random Inputs



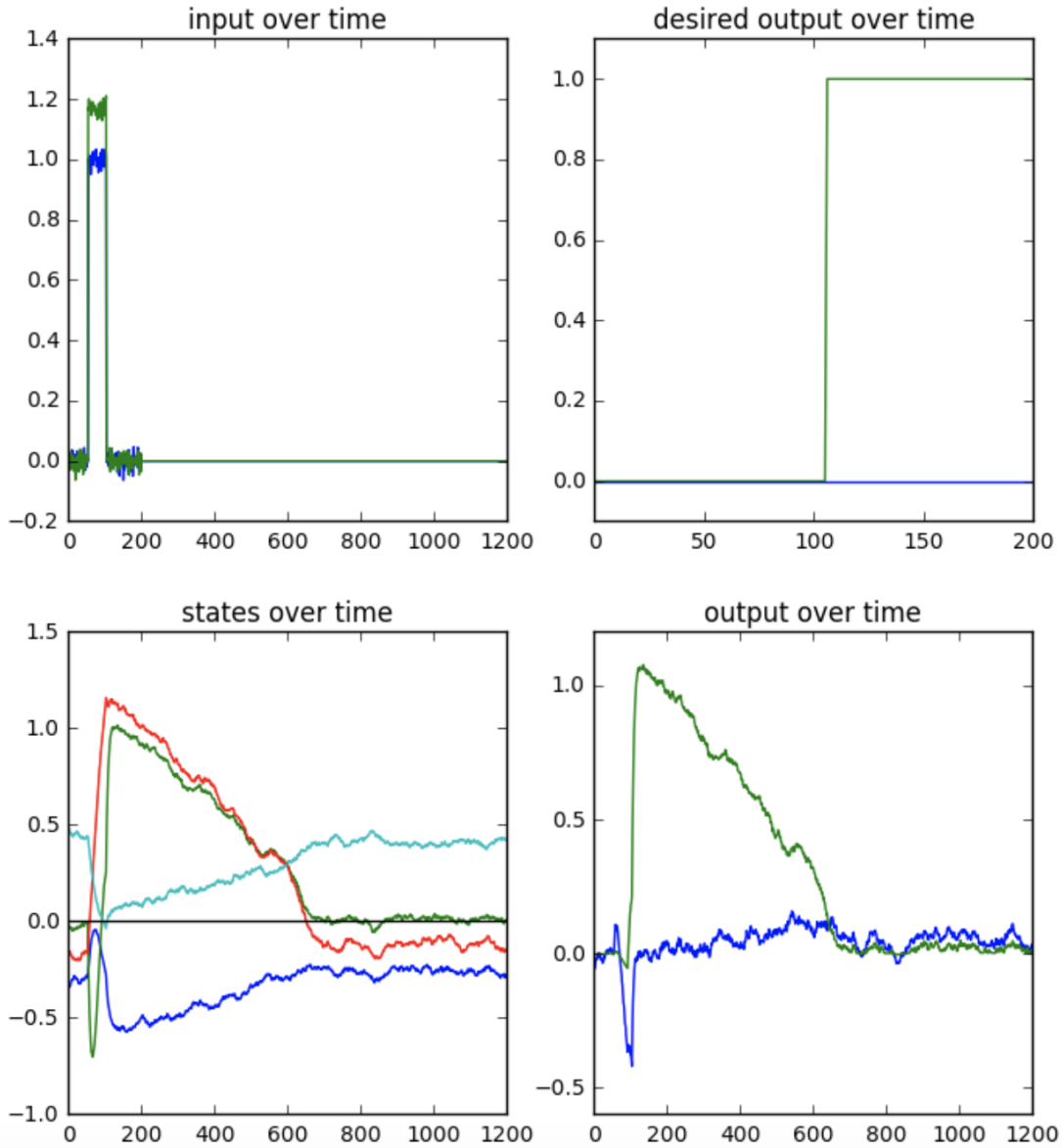
Distance to Fixed Point



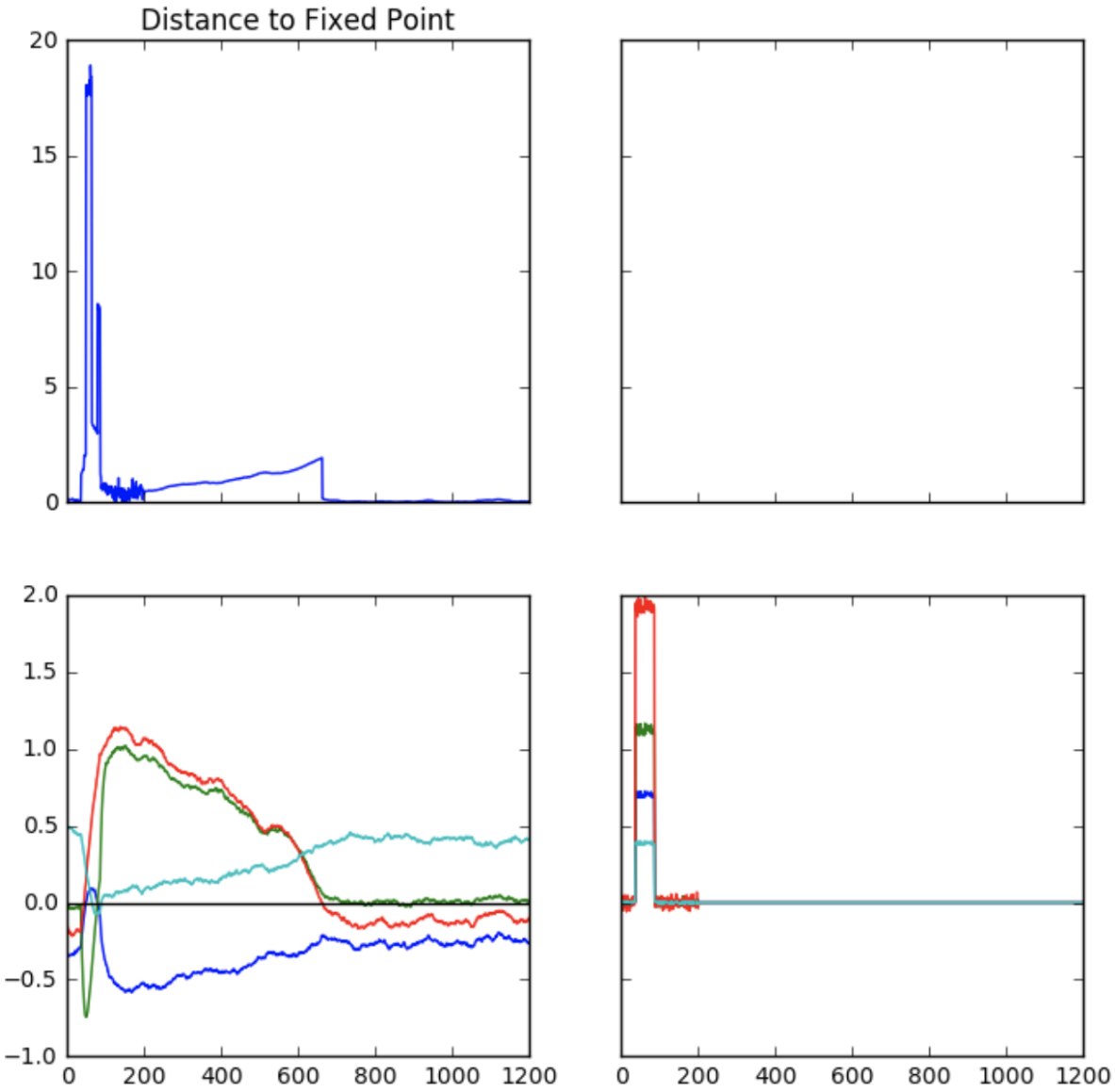
4 Neuron Model



4 Neuron Model



4 Neuron Model



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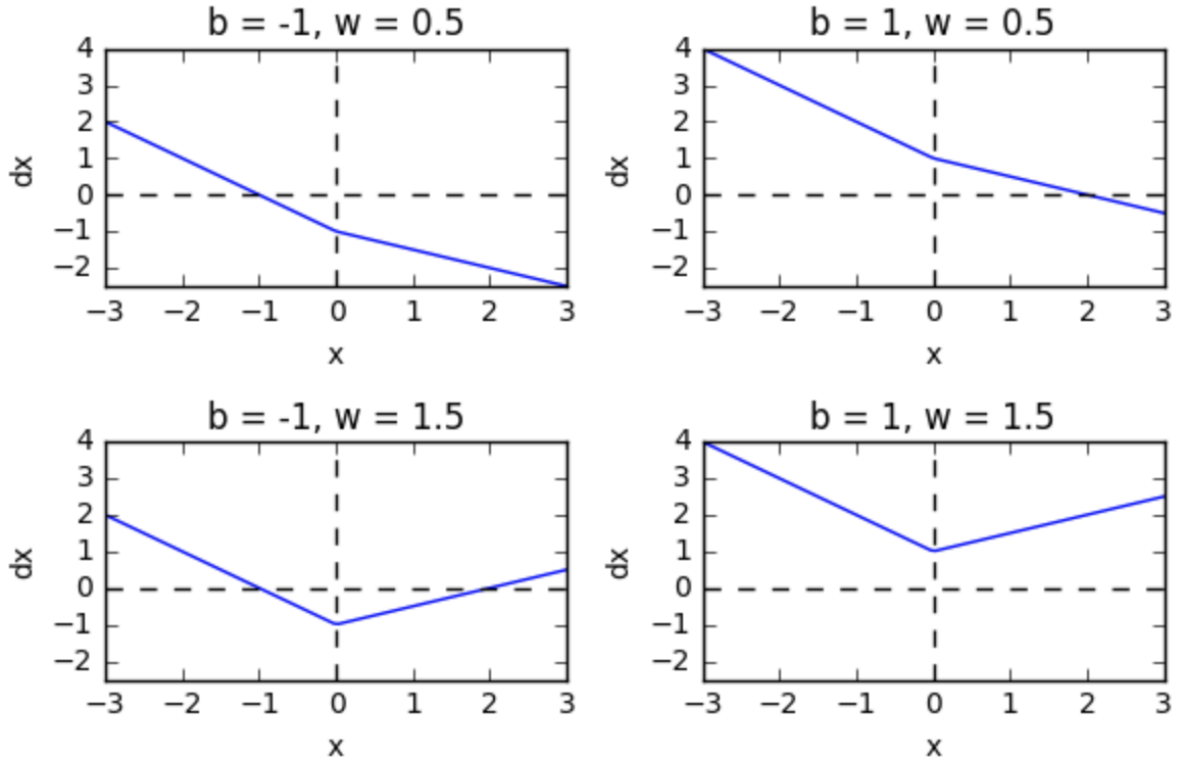
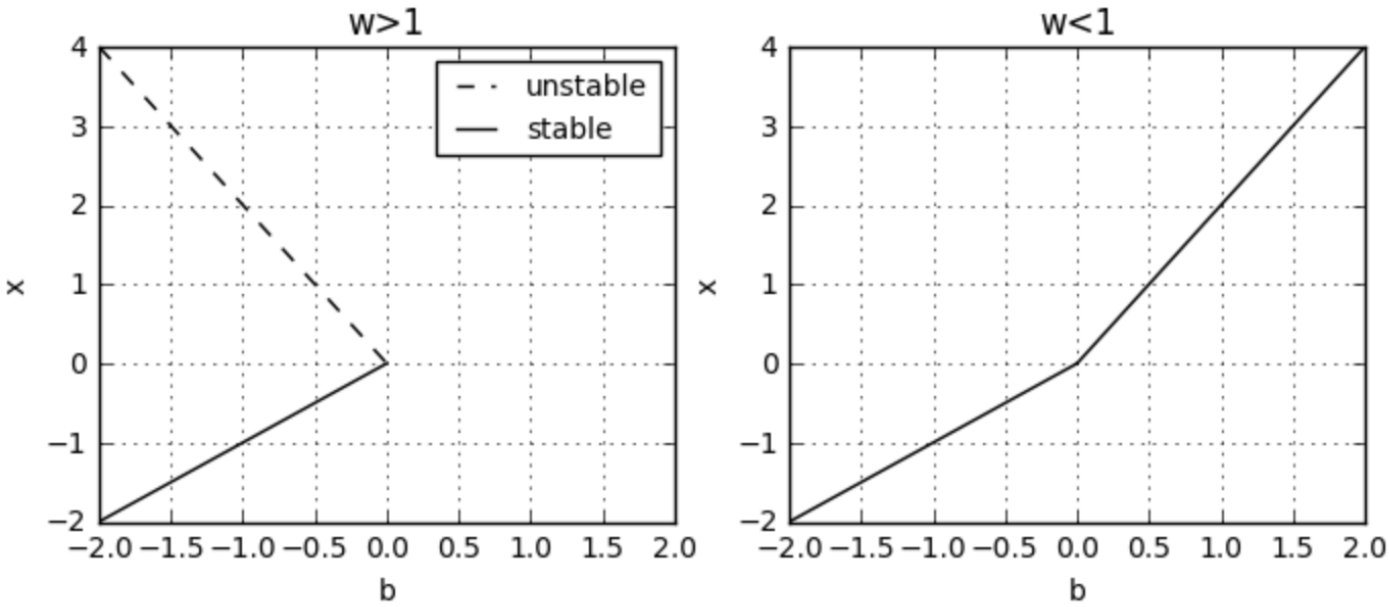
1-D ReLU Networks

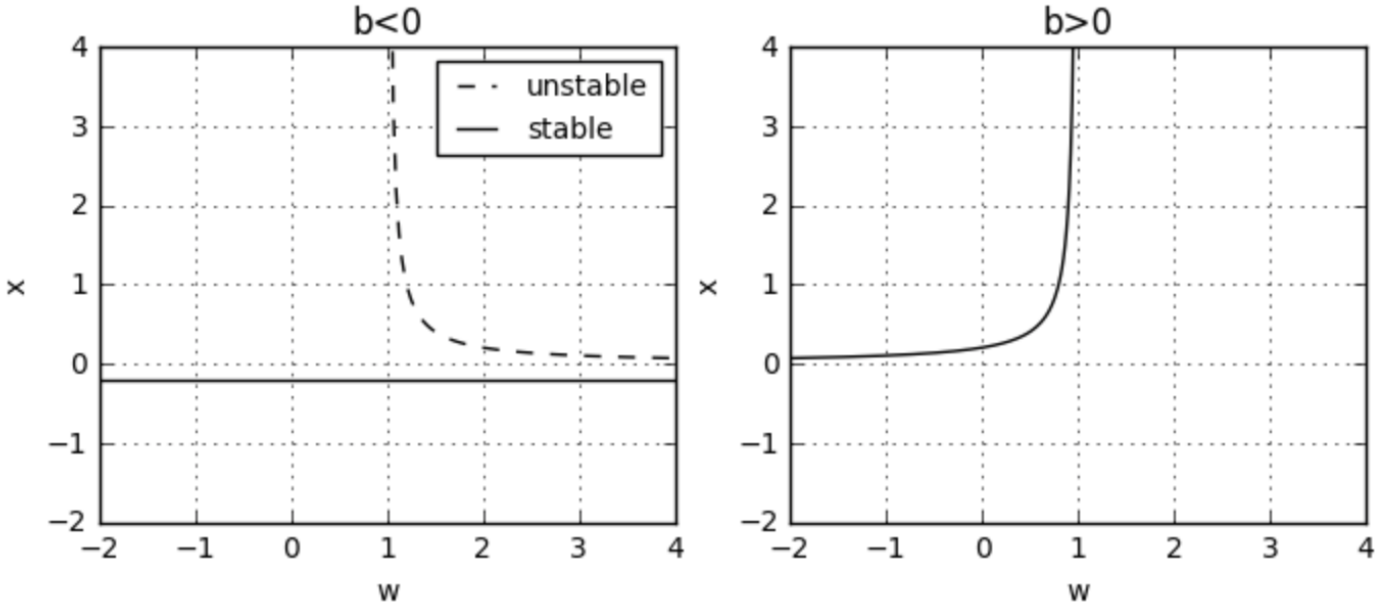
$$\dot{x} = -x + wf(x) + b$$
$$f(x) = max(x, 0)$$



1-D ReLU Networks

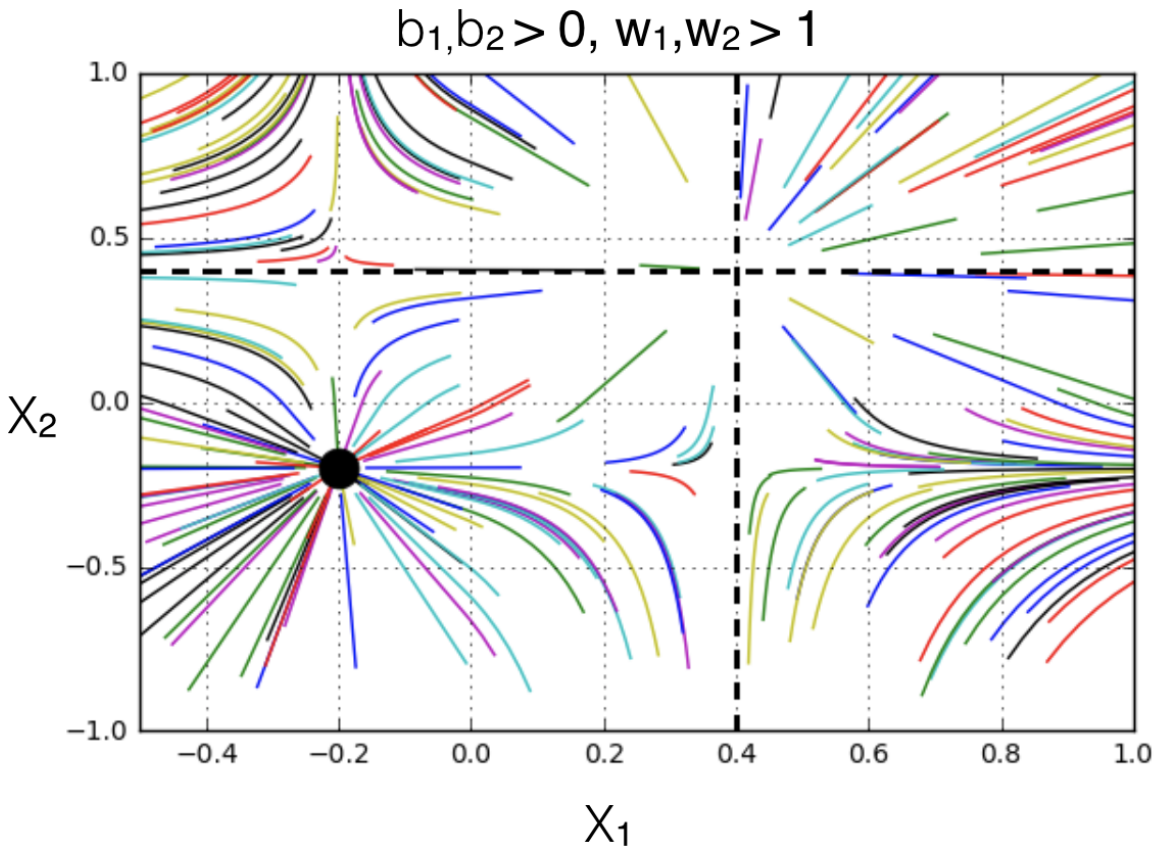


1-D ReLU Networks



2-D ReLU Networks

$$\dot{x}_i = -x_i + \sum_{j=1}^{n} w_{i,j} f(x_i) + b_i$$



2-D ReLU Networks